Analysis and visualization of synchronicity in European business and growth cycles and the implications for ECB monetary policy

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Abstract

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1 Introduction

The synchronicity of cycles in real output is a subject that has attracted increased attention in the field of economics, largely because not only is the international synchronicity of turning points of business cycles a "stylized fact", but also because within a monetary union, given the absence of labor mobility and federal transfers, synchronicity of growth cycles is an important pre-requisite for optimal application of a single monetary policy. Given the fact that business cycles are largely the only cycle that economists recognize in national income data, it is perhaps natural to study the synchronicity of these cycles, and yet for monetary policy the dynamic of GDP growth at other frequencies is also important. Given that monetary policy usually operates at roughly a monthly level, the dynamic of GDP at even quarterly levels has implications for the implementation of monetary policy across different countries or jurisdictions. As the optimal currency area literature suggests that synchronization of business and growth cycles is an important consideration for adoption of a single currency, monetary policy is more easily formulated if these cycles in real GDP growth are similar between the member states that have qualified and subsequently adopted the euro.

In this paper the synchronicity of business and growth cycles is first assessed assessed, both statically and dynamically, and then the paper goes on to look at the potential problems that these findings might raise for a single monetary policy in the euro area in the future. In the quantitative part of this paper, a new approach is taken to assessing synchronicity in real GDP growth, that of using a variation on recurrence plots and recurrence quantification analysis (RQA). RQA has its origins in physics, but is now used in many disciplines such as climatology, physiology, biology, chemistry, acoustics and astronomy. This technique is particularly suited to analysis of nonlinear dynamical systems, and so is likely better suited to analysis of business and growth cycles in real GDP than are linear econometric-based time series methods.

2 RQA and recurrence plots: a crash course

RQA is now only 15 years old (see Zbilut and Webber Jr. (1992) and Webber and Zbilut (1994)) but the notion of recurrence has a much longer pedigree in mathematics (see Feller (1950)). Recurrence plots first originated from work done in mathematics and physics but now have a considerable following in a variety of fields[1]. There are now several excellent introductions available to RQA and recurrence plots, not least those by Marwan, Romano, Thiel, and Kurths (2007) and Webber Jr. and Zbilut (2005).

2.1 A simple example of auto-recurrence

To illustrate that basic ideas behind recurrence analysis, using the example from Webber Jr. and Zbilut (2005)[2] we consider the system of waves on the sea as measured from buoys. With a dataset of 226 hourly measurements of the wave heights over roughly 9.4 days, it is clear from figure 1A that tidal flows have some form of periodicity, although the movement clearly does not follow any simple deterministic function. To illustrate the basic premise behind recurrence plots and RQA, if dots are drawn to indicate whenever the wave height is at 0.9 feet, 25 points are obtained indicating each occasion that the wave height returns to 0.9 feet. By plotting these points in figure 1B for this series against itself, we obtain the "auto-recurrence" plot.

Of course, just looking at one particular point probably doesn't tell us a lot about the dynamics of the series, so assume that we are interested in how close to a particular wave height the wave dynamic is at any specific point in time - that is, a "local" recurrence plot. But recurrence plots can be generalized so as to visually display proximity to any particular wave height, not just one specific wave height, and these "global" recurrence plots are shown in figure 2 below, in plot C. One parameter which measures the proximity to any particular wave height is also varied so as to give a "spectrum" of how close another section of the time series is to the wave height in another section is. The parameter for detecting proximity ("radius") is increased in plot B to 0.9 feet, so the blue areas show all parts of the auto-recurrence that are within 0.9 feet of one another. Clearly the white areas of the plot indicate significant departures of over 0.9 feet from any other part of the series. In plot C, the radius parameter is varied to five different ranges: blue (0-0.2 feet), cyan (0.2-0.4 feet), green (0.4-0.6 feet), yellow (0.6-0.8 feet) and red (0.8 feet and over).

[1] Norbert Marwan's website catalogues all the articles published using recurrence plots and RQA, and is a veritable mine of information on this topic. See http://www.recurrence-plot.tk

[2] This extremely useful introduction to RQA is also available from the NSF website at www.nsf.gov/sbe/bcs/pac/mnbs/mnbs.jsp

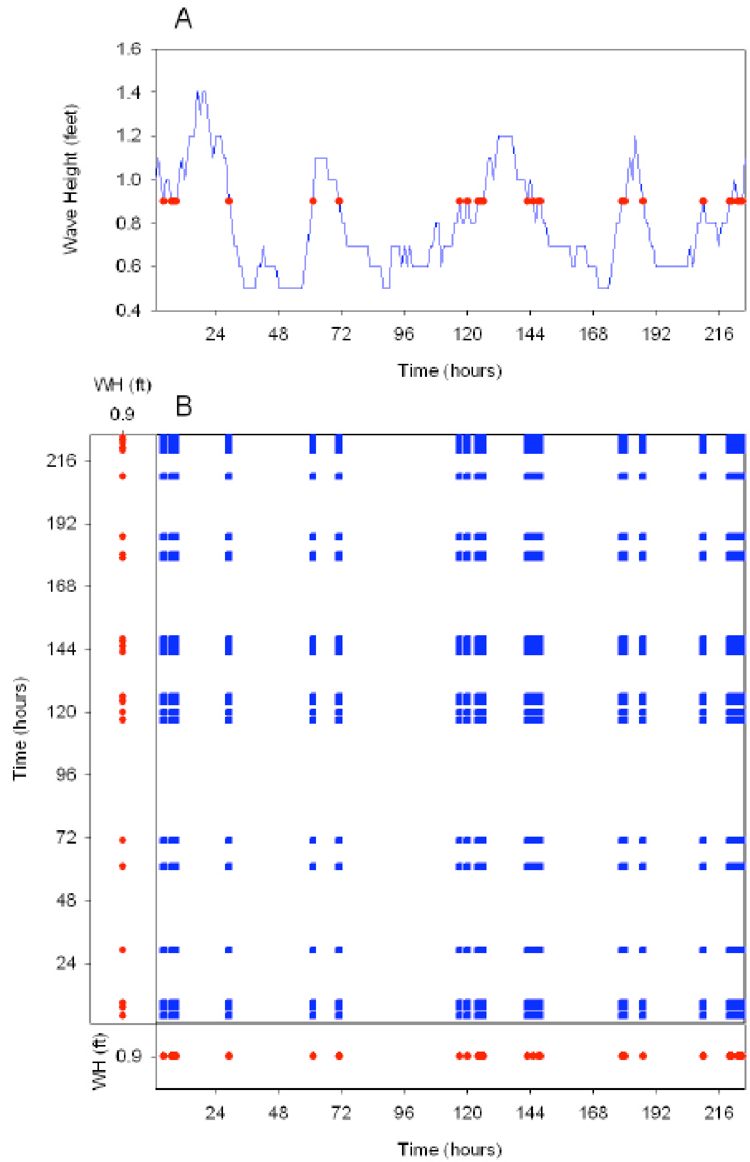

Figure 1: Wave height measurements recorded hourly from byoys located 33 nautical miles south of Islip, LI, NY, USA.

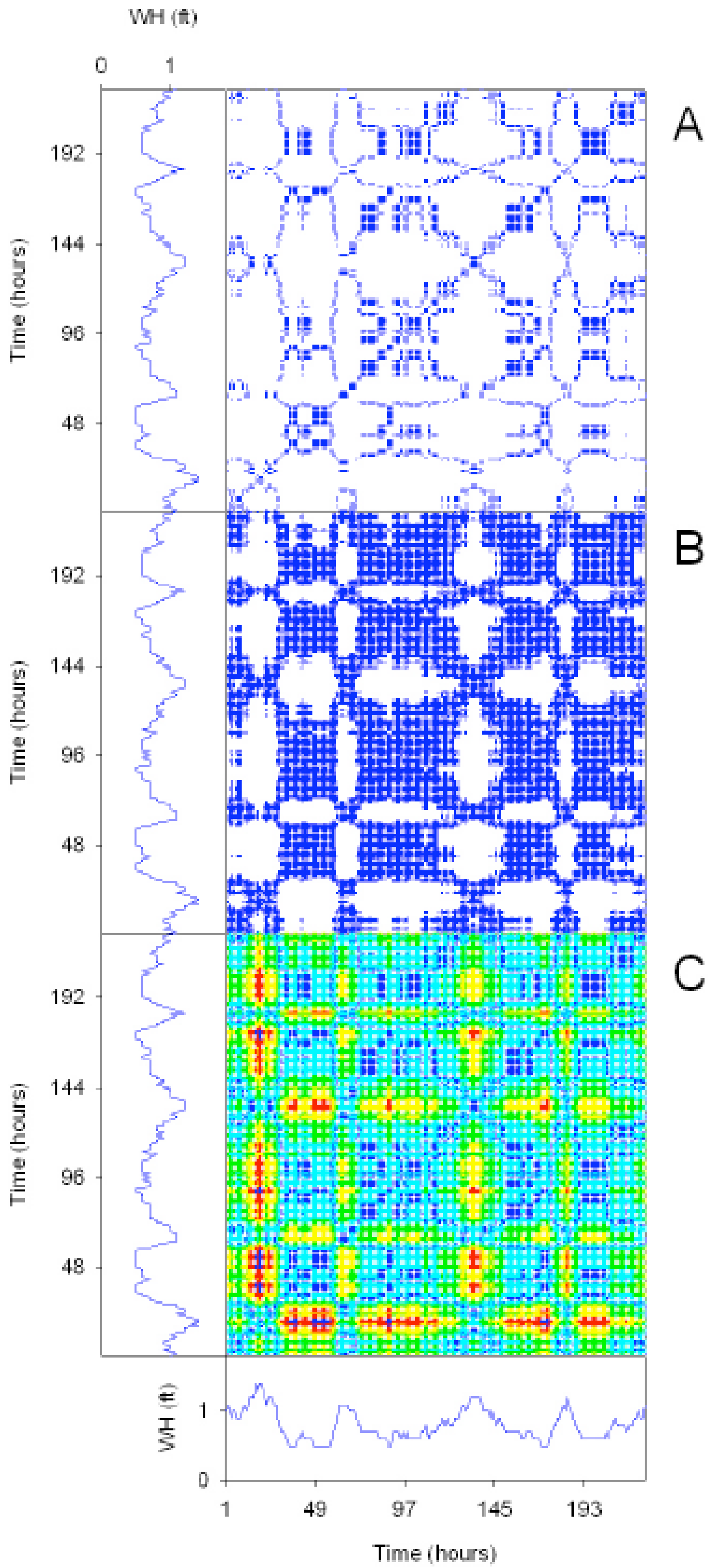

Figure 2: Recurrence plots of wave height data

2.2 A simple example of cross recurrence

Cross recurrence plots were a recently introduced into the physics literature by way of Marwan, Thiel, and Nowaczyk (2002) and Romano, Thiel, Kurths, Kiss, and Hudson (2005) and are a simple way of looking at recurrence between time series. From this, one can study the synchronization of non-linear time series. For example take two trigonometric series, one representing the movement of a pendulum, and the other the velocity of the pendulum. This can be represented by a sine and cosine function respectively as per equation 1 below:

$$x_t = \sin(2\pi t) \tag{1}$$

$$y_t = \cos(2\pi t) \tag{2}$$

which graphically looks as per figure 3. Clearly both x and y are not synchronous, but are phased by a quarter of a cycle.

When preparing the recurrence plot, as both series possess the same scale, a similar exercise can be used as per the auto-recurrence plot, except that we will be comparing the level of x_t against the level of y_t and then blackening the area of the recurrence plot when these levels are similar. Figure 4 shows the resulting lattice-type plot that shows when these two series are at similar levels. If the series were synchronous in time, then this would result in a dark diagonal line up the middle of the plot (- implying that at any point in time, the first series was close in value to the second series). In this instance it is clear that if lagged (by a quarter cycle, or what is known in the signal processing literature as a "half-sample delay"), the series would be synchronous.

To enhance these results and to find how the series are synchronized together, the recurrence plot can first be thresholded - in other words rather than using a specific criteria to assess whether a point is similar between the two series, to measure the distance between the points in one series and another series at any given moment in time. To best illustrate the thresholding, figure 5 shows red for points which are very similar between the series and blue for points that are very different between the two series. Clearly these periodic series are completely deterministic and yield a beautiful lattice pattern reminiscent of the patterns created by looking into a child's toy kaleidoscope from the mid twentieth century. A line of synchronicity can be estimated for the data and this is plotted using a thick black line. As can easily be seen, the line of synchronicity shows that a half sample delay in the first (sine) series will be consistent with synchronicity between the two series.

The start of the line of synchronicity is at the origin, but this is just a default starting point - the program used for this analysis could start at (50,0) instead, which would yield a

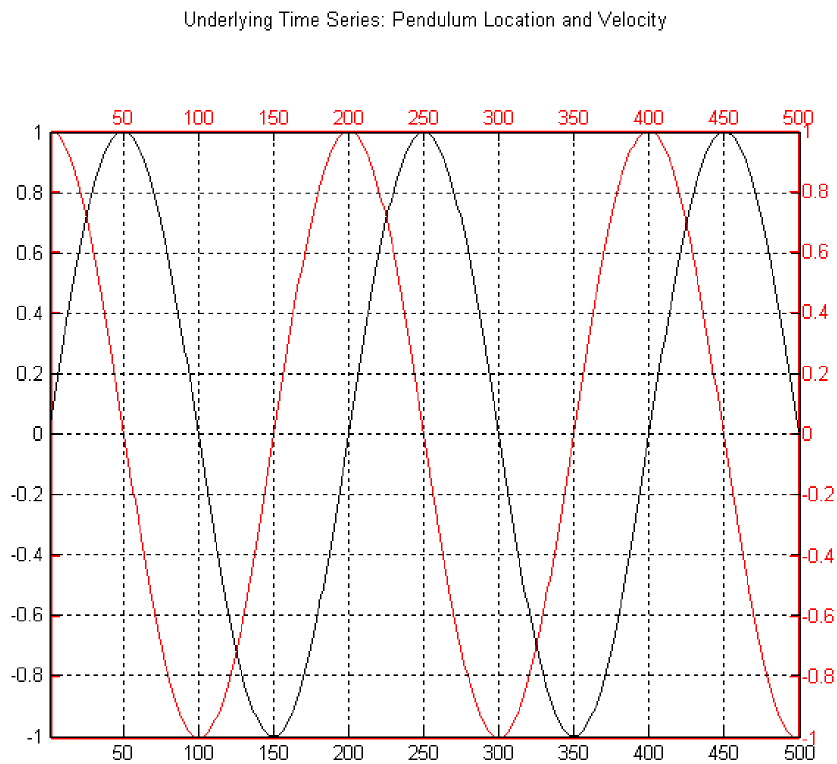

Figure 3: Pendulum location and velocity against time

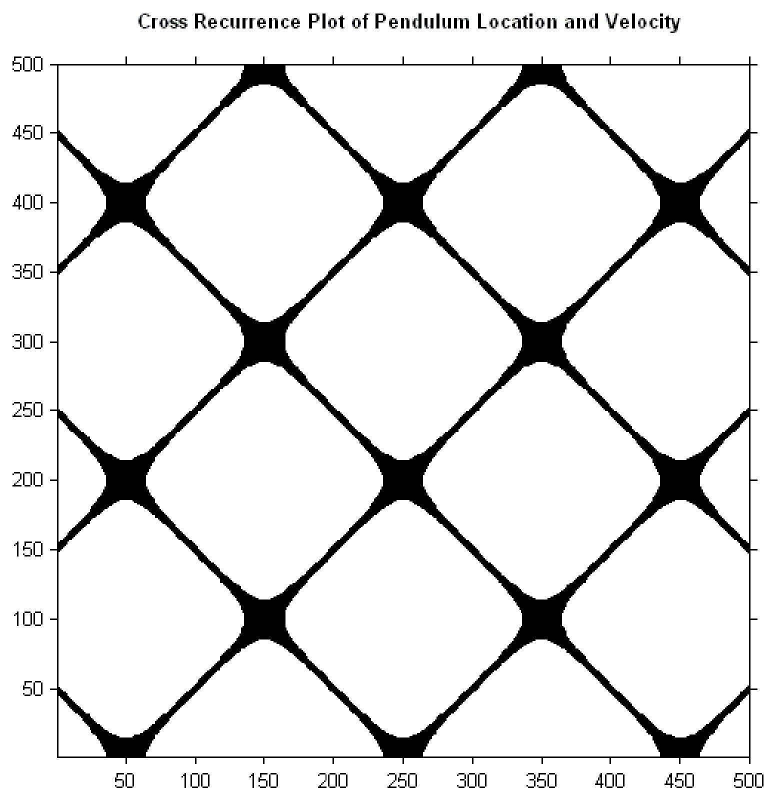

Figure 4: Cross recurrence plot of pendulum location and velocity

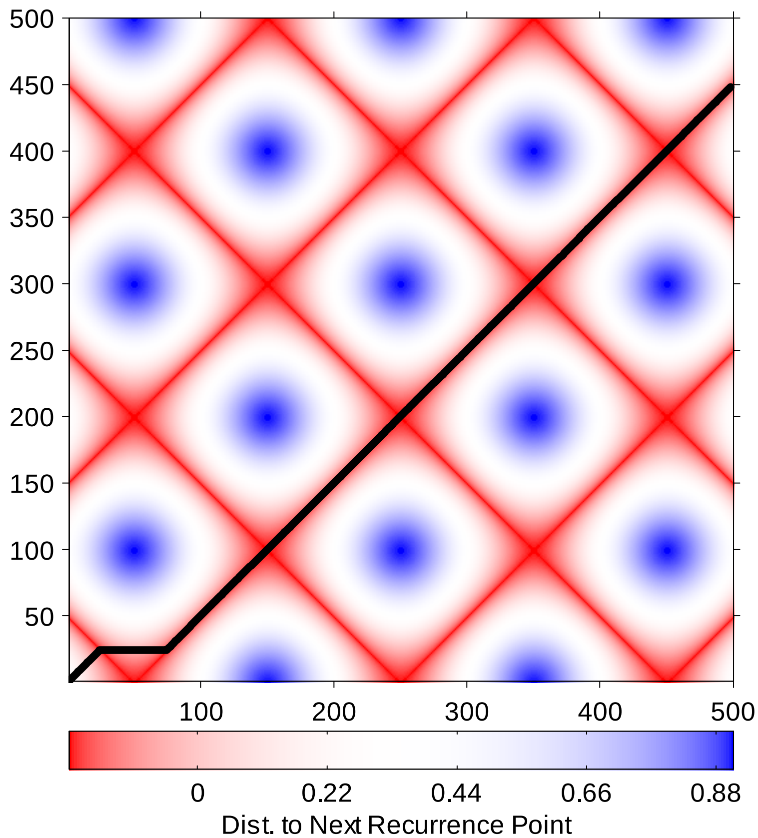

Figure 5: Thresholded recurrence plot for pendulum with line of synchronicity

straight line for the line of synchronicity. Obviously the results above are for deterministic systems, and when systems are non-linear and stochastic, these neat results will no longer appear. Nevertheless, in applications in the physical sciences and elsewhere, recurrence plots are able to reveal interesting features of dynamical systems, so obviously they likely have some potential in the social sciences as well.

2.3 Some math and CRQA measures

Pretty pictures are all well and good, but analysing these plots for what they contain is now a major part of recurrence analysis, in the form of RQA. RQA was initiated by Zbilut and Webber Jr. (1992) and has now been introduced into mainstream physics through the study of nonlinear dynamics. A good summary is available in Webber Jr. and Zbilut (2005). As our focus is on cross recurrence plots, following Marwan and Kurths (2002), the emphasis is on measures for cross RQA.

First, using Takens' embedding theorem (see Takens (1981)), the recurrence plot is a way of analysing the dynamics of phase space trajectories in deterministic systems. Takens' embedding theorem states that the dynamics can be approximated from a time series x_k sampled every t by using an embedding dimension m, and a time delay, τ, by a reconstruction of the phase-space trajectory $\overrightarrow{y}_t$, where:

$$\overrightarrow{y}_t = (x_t, x_{t+\tau}, ..., x_{t+(m-1)\tau}) \tag{3}$$

The choice of m and τ are based on methods for approximating these parameters, such as the method of false nearest neighbors and mutual information for m and τ respectively. When using cross recurrence plots, the choice of m and τ are assumed to be the same.

Second, following Marwan, Thiel, and Nowaczyk (2002) the cross recurrence plot is definied by:

$$\mathbf{CR}_{i,j} = \Theta(\varepsilon - \|y_i - z_j\|) \tag{4}$$

where $i, j = 1, ..., N$, y_i and z_i are two embedded series, ε is the predefined "threshold", $\|\|$.is the norm (for example a Euclidean norm) and Θ is the Heaviside function. This gives a cross recurrence matrix $\mathbf{CR}_{i,j}$ which contains either 0s (the white areas in the plots) or 1s.(the black areas in the plots). To get the contoured plots shown above, ε is varied to predetermined values.

Third, in an auto-recurrence plot, the main diagonal is always present, as every point in the series is identical to the same point in the series, so there will always be a diagonal line (1's down the main diagonal of the $\mathbf{R}_{i,j}$ matrix), once all points in the series are considered (- see figure 2 for example). In the cross recurrence plot if this line is present, the two series are identical, but this is obviously a special case. A line, if it appears in the cross-recurrence plot, implies similar dynamics, but these maybe offset from the main diagonal, implying phasing of the two cycles (- see figure 5 for example)..This line, if it can be identified, is termed the "line of synchronization" or LOS.

Fourth, complexity measures can be derived to characterize the cross-dynamics of a given series. For two series these will be characterized as diagonal lines (not necessarily on the main diagonal), which demonstrate similar dynamics maybe at different points in time. Following Marwan and Kurths (2002) the distributions of the diagonal line lengths can be written as $P_t(l)$ for each diagonal parallel to the main diagonal, where $t = 0$ denotes the main diagonal, $t > 0$ denotes diagonals above the main diagonal (a lead) and $t < 0$ denotes diagonals below the main diagonal (a lagged dynamic). Given this diagonal measure, the following measures can be extracted from the cross recurrence plot:

i) recurrence rate. This is defined as

$$RR(t) = \frac{1}{N-t} \sum_{l=1}^{N-t} l P_t(l) \tag{5}$$

which represents the probability of similar dynamics occurring with delay t.

ii) determinism. This is defined as

$$DET(t) = \frac{\sum_{l=l_{\min}}^{N-t} l P_t(l)}{\sum_{l=1}^{N-t} l P_t(l)} \tag{6}$$

which represents the proportion of long sequences of dynamics in all similar dynamics. A deterministic system will have a high DET while a stochastic system will have a low DET.

iii) average diagonal line length, $L(t)$, where:

$$L(t) = \frac{\sum_{l=l_{\min}}^{N-t} l P_t(l)}{\sum_{l=l_{\min}}^{N-t} P_t(l)} \tag{7}$$

This measure shows the average duration of these similarities in the two series.

iv) entropy measure. This refers to the Shannon entropy of the probability $p(l)$, and is defined as:

$$ENTR = -\sum_{l=l_{\min}}^{N} p(l) \ln p(l) \tag{8}$$

This is a measure of the complexity of the recurrence plot.

High values of $RR(t)$ indicate a high probability of occurence of the same state in both systems, and high values of $DET(t)$ and $L(t)$ indicate a long time span for this synchronization. This technique also allows distinguising positive correlation and negative correlation between the series.

Fifth, other measures which are the analogues of RQA auto-recurrence measures are possible, as follows:

v) ratio. This is defined as:

$$RATIO = \frac{DET}{RR} \tag{9}$$

This measure has been shown to uncover transitions in dynamics for physiological data.

vi) laminarity. This is a measure of tangential motion, and refers to the distributions of the vertical line lengths in the recurrence plot, which can be written as $P_t(v)$, analogous to the diagonal lines in the plot. Analogous to determinism, laminarity is defined as:

$$LAM = \frac{\sum_{v=v_{\min}}^{N-t} vP_t(v)}{\sum_{v=1}^{N-t} vP_t(v)} \tag{10}$$

where high values of LAM denote motions that are not opposite in direction of trajectory but are not similar in direction either.

vii) trapping time. This refers to the average length of these vertical measures and is analogous to $L(t)$ above. It is defined as:

$$TT = \frac{\sum_{v=v_{\min}}^{N-t} vP_t(v)}{\sum_{v=v_{\min}}^{N-t} P_t(v)} \tag{11}$$

3 Data and embedding

3.1 Data

To analyse synchronicity in economic growth, quarterly real GDP data was used,.transformed into log year over year changes. This is analogous to year-over-year percentage growth rates, and corresponds to probably the most widely used economic growth data for policy purposes. Quarter over quarter data was analysed, but little synchronicity was found in the data, possibly because of the erratic nature of the real GDP data series. Data used in this study was sourced from a variety of different national statistical offices, while the euro area aggregate was obtained from the ECB's euro area quarterly model database[3]. The transformed data runs from 1971 thru 2004Q2.

Figure 6 below shows the data for several groups of countries - the core EMU member states are shown first, then peripheral EMU member states in figure 7, and lastly non-EMU countries/member states are shown in figure 8

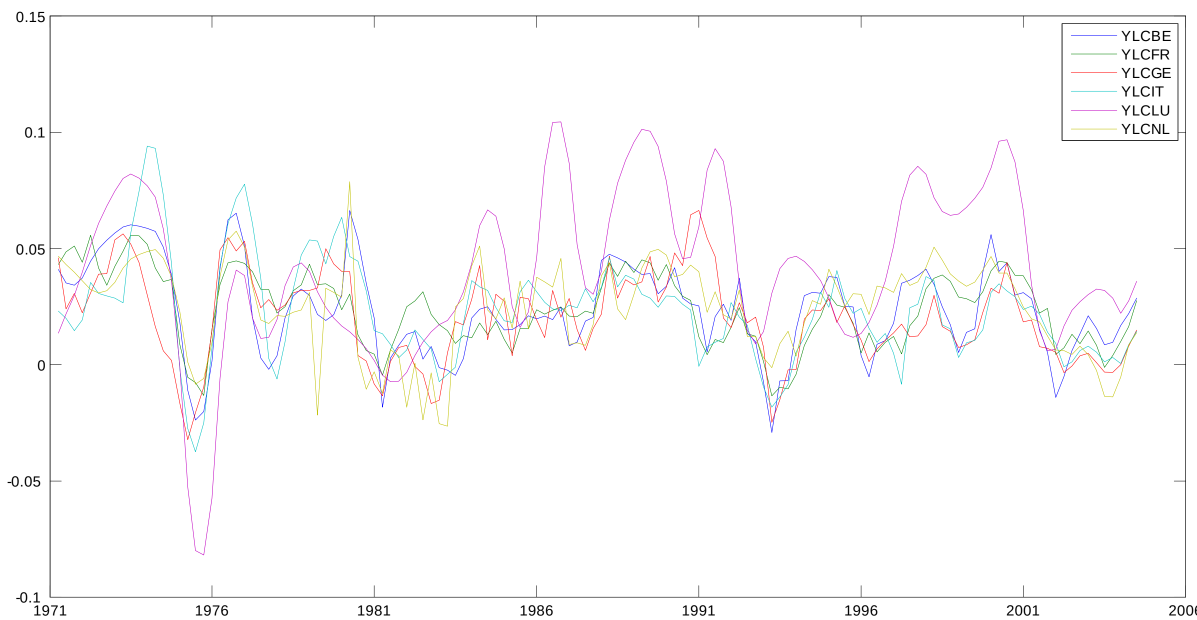

Figure 6: Economic growth rates for core EMU member states

[3]This data can be obtained from the Euro Area Business Cycle Network at www.eabcn.org

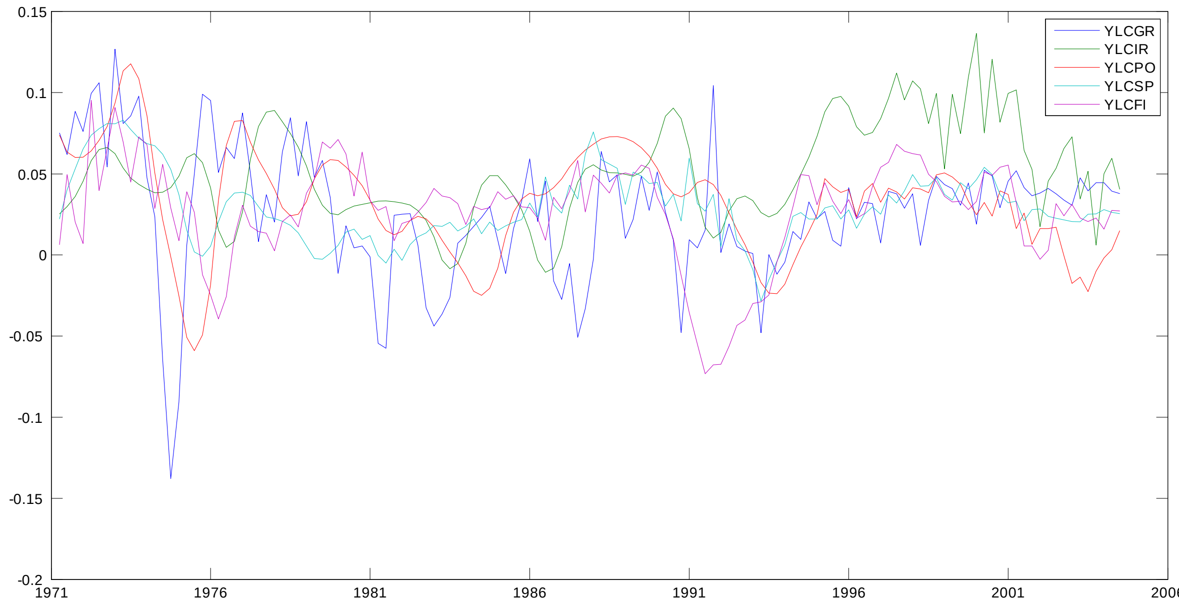

Figure 7: Economic growth rates for peripheral EMU member states

It should be noted that (apart from Luxembourg) the growth rates of the EMU core countries are very similar and since the inception of EMU appear to be dispersed even less. With the peripheral EMU members the dispersion is much greater, and reflects the core-periphery dichotomy. Clearly in economic terms the dynamics of economic growth in the EU countries is going to be affected according to the time period under consideration.- the pre-EMU exchange rate mechanisms ("snake", early EMS, "new" EMS) and the situation post-ERM crisis in 1993 through to full-blown EMU. Lastly countries outside the EMU and the EU appear to have some simmilarities in growth profiles, but significant departures from any discernable trend line.

3.2 Data embedding

Very little research has been done with recurrence plots and RQA with macroeconomic variables, but notable exceptions to this are Zbilut (2005) and Kyrtsou and Vorlow (2005). While Zbilut stresses that the notion of correct embedding is important when dealing with non-stationary data, Kyrtsou and Vorlow notes that recent research suggests that the choice of embedding parameter in experimental data may not be a crucial issue In terms of the

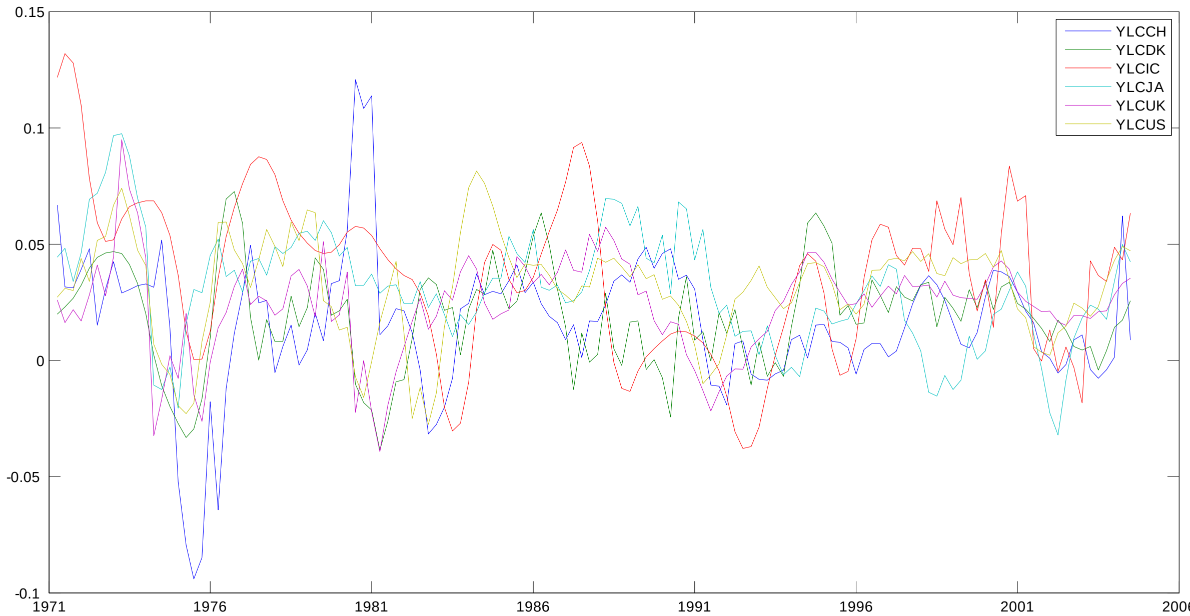

Figure 8: Economic growth rates for non-EMU countries

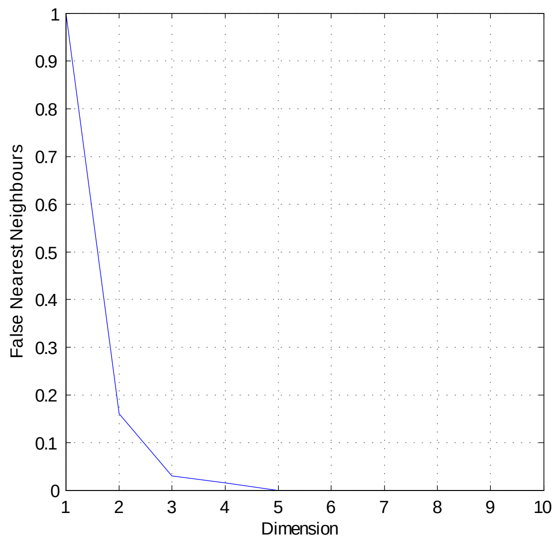

Figure 9: False nearest neighbor plot for euro area real GDP growth

two parameters that need to be set for RQA, m and τ, Kyrtsou and Vorlow set these two parameters to be 1 in all their recurrence plots, following some research done by **?**) that suggests that experimental data does not need to be embedded. In terms of setting values for the radius, ε, they follow Webber and Zbilut (1994) and set a threshold level of the lower 10% of the maximum distance between the embedded vectors. In this research, as Kyrtsou and Vorlow (2005) only analysed CPI, industrial production, interest rates and unemployment, we need to first explore traditional methods for determining embedding and time delay, just to confirm that the logic used by Kyrtsou and Vorlow (2005) is sound.

Here we use euro area real GDP growth rate and use the French real GDP growth rate for determining the optimal lag. First, in figure 9 the false nearest neighbors method is used to calculate the optimal embedding dimension, where the optimal embedding dimension whould be chosen where the amount of false nearest neighbors almost disappears (see **?**)). It can be clearly seen for euro area real GDP growth, the optimal embedding dimension would be either 2 or perhaps 3.

Now to determine τ, the time delay, either the mutual information criterion can be used, or maximal correlations for a given lag length. In this case it is mostly an academic exercise, as for implementation of monetary policy, time delays should not be a factor between member states in terms of their suitability to be subject to a single monetary

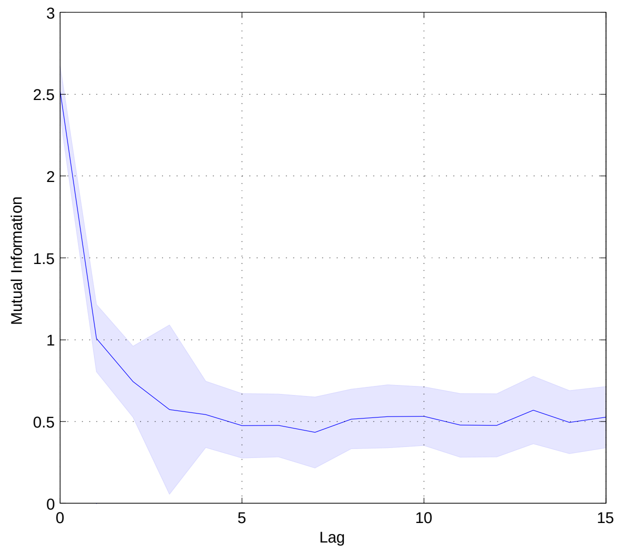

Figure 10: Mutual information plot for euro area real GDP growth vs French real GDP growth

policy. Nevertheless, to check on mutual information between euro area real GDP growth and that of France and Finland, mutual information plots are shown in figures 10 and with standard errors. As Zbilut (2005) makes clear, one way of choosing the lag is to find the first local minimum of the autocorrleation or mutual information plot. It can be clearly be seen that the minimum occurs at 7 lags for France, and for Finland the minimum occurs at 5 lags. In this research the advice of Zbilut is used: "Given that much financial data is discrete, a lag of one is usually sufficient".

Given the above analysis, we use an embedding parameter of either 1 or 2 and up to 2 lags depending on the data and the CRP results.

4 Cross recurrence plots and CRQA for the euro area

To facilitate both a limited and clear presentation of results, the countries are split into three groups and analysed separately, depending on their relationship with the euro area. The three exercises that are done here area as follows:

i) core EMU member states vs Germany.

ii) peripheral EMU member states vs the euro area aggregate.

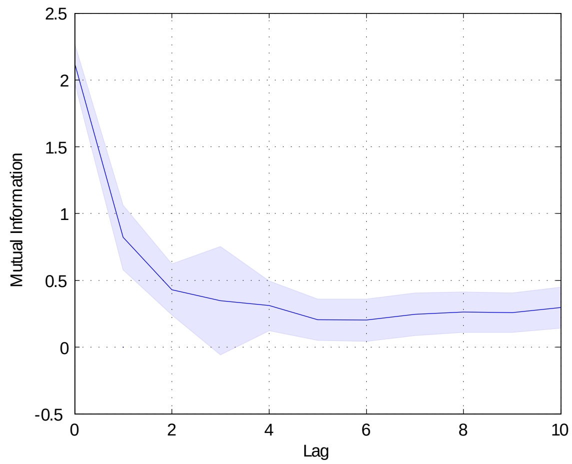

Figure 11: Mutual information plot of euro area real GDP growth vs Finnish real economic growth

iii) other European and other countries vs the euro area aggregate.

The logic here is to first look at the level of synchronicity within the core of the euro area, while avoiding the correlation bias that undoubtedly will arise from large member states having higher weights in any euro area aggregate. Second, clearly those in the periphery of the euro area have smaller weights, and anecdotal evidence suggests that they are less synchronous with the euro area as a whole (- for example Ireland, Greece and Finland). Lastly for those outside of the euro area, the objective is to see if synchronicity has increased or decreased, and how similar it is to other member states or countries.

4.1 Core euro area member states

Here we look at cross recurrence between core euro area member states and Germany. In figure 12 the original series are shown as well as the CRP for France vs Germany. The line of synchronicity clearly deviates from the main diagonal during the 1980s, returning in the early 1990s, then departing once again only to return towards the main diagonal in recent years. Notably in recent years French real GDP growth appears to slightly lag that of Germany in terms of cycles, and hence the line of synchronization has moved above the leading diagonal indicating that more recent French data point gives the same rate of growth as a German growth rate from some quarters before.

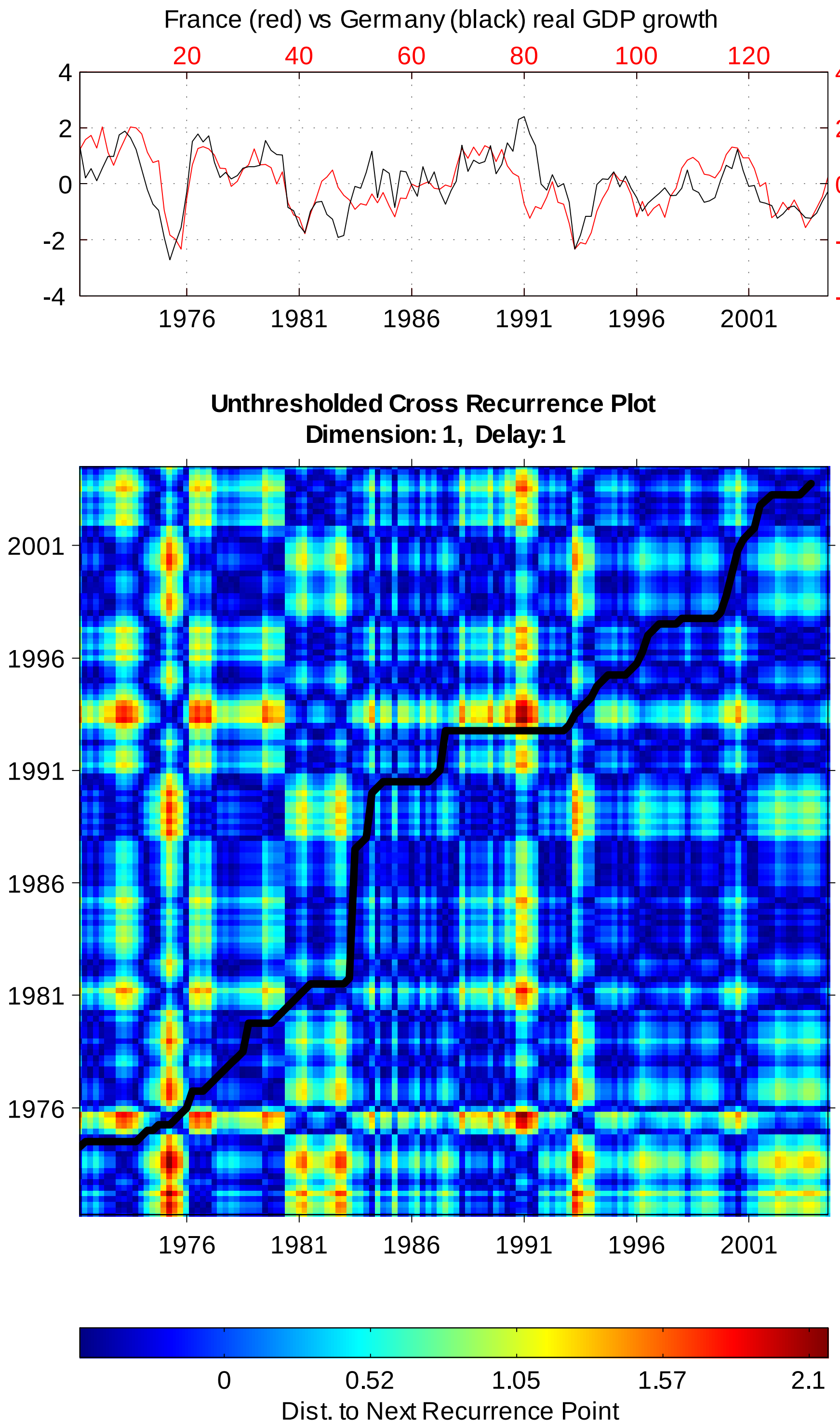

Figure 12: CRP for France vs German real GDP growth

The main feature to note in the CRPQA is that the recurrence rate appears to be increasing again which tends to suggest similar dynamic features appear in both time series. The laminarity rate also appears to have been erratic and falling over time, indicating less vertical structures in the recurrence plot and therefore higher recurrence in the dynamic structure of the series.

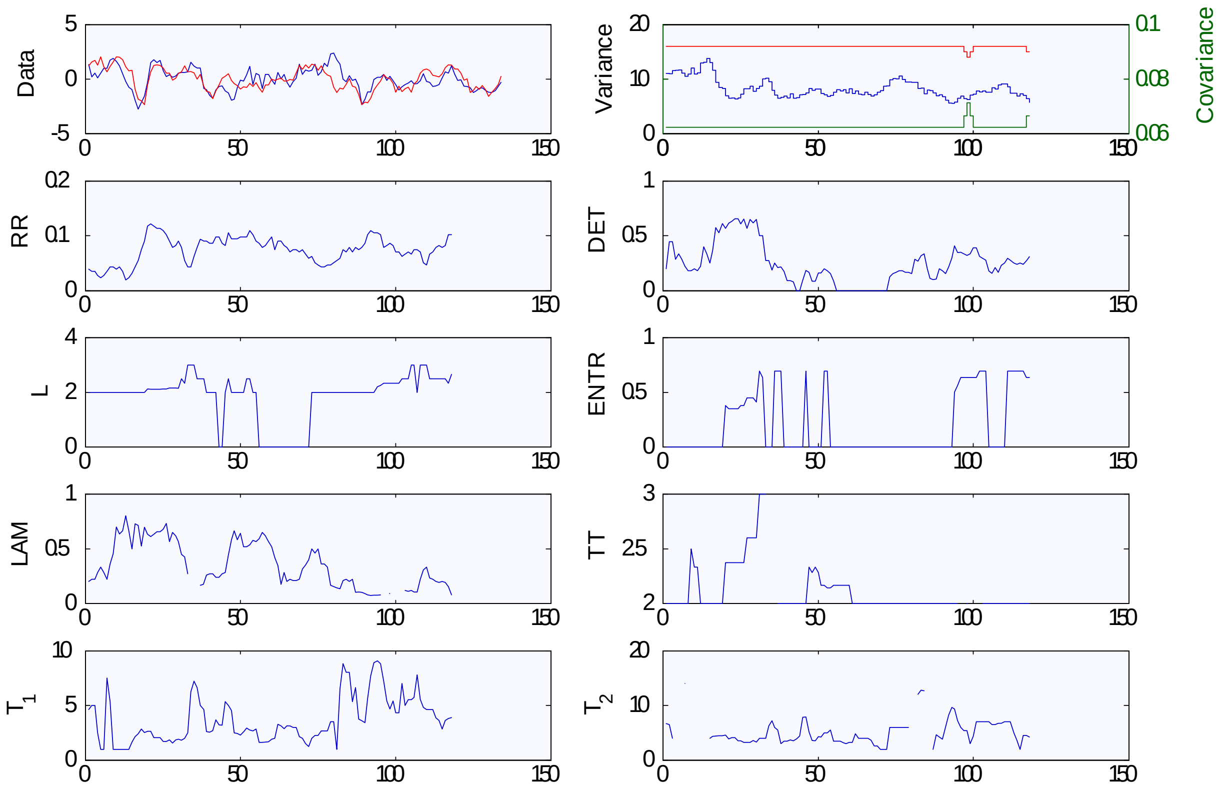

Figure 13: CRQA for French vs German real GDP growth.

The Belgian recurrence plot appears below in figure 14. As a line of synchronization could not be resolved for an embedding of dimension 1, this parameter was increased to 2 and a line of synchronization was subsequently located. Apart from the late 1980s and early 1990s, Belgium seems to be well synchronized with Germany, suggesting that growth patterns were similar..At the end of the series though growth rates appear to have diverged, with the result that the line of synchronization diverges from the leading diagonal.

The main features of the CRQA for Belgium in figure 15 are similar to that of France here, with an upturn in recurrence and laminarity varying through time, although here in

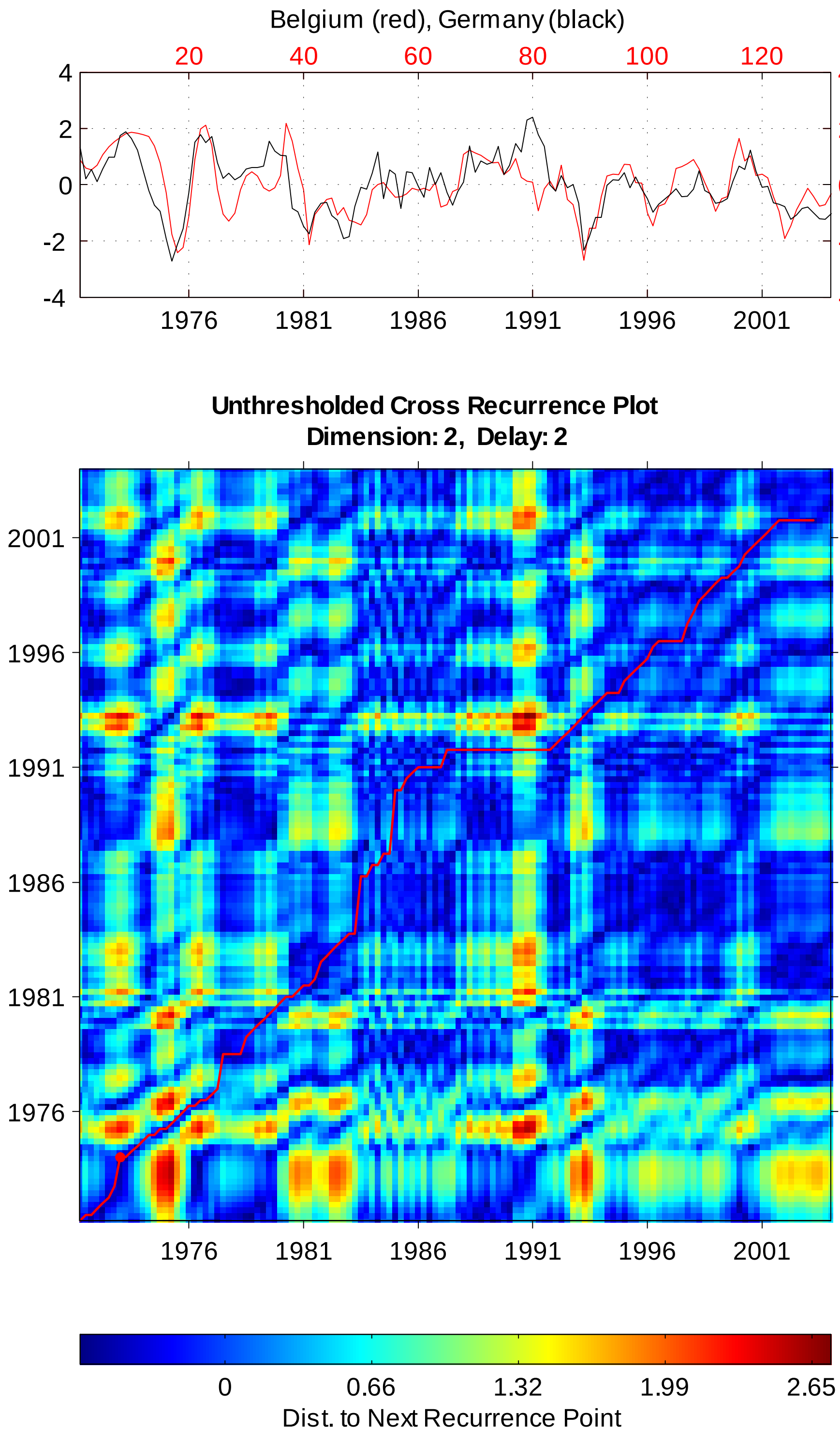

Figure 14: CRP for Belgian vs German real GDP growth

a less systematic way. As these CRPQA plots are similar for most of the euro area core member states, we omit them for other member states for this section of the paper.

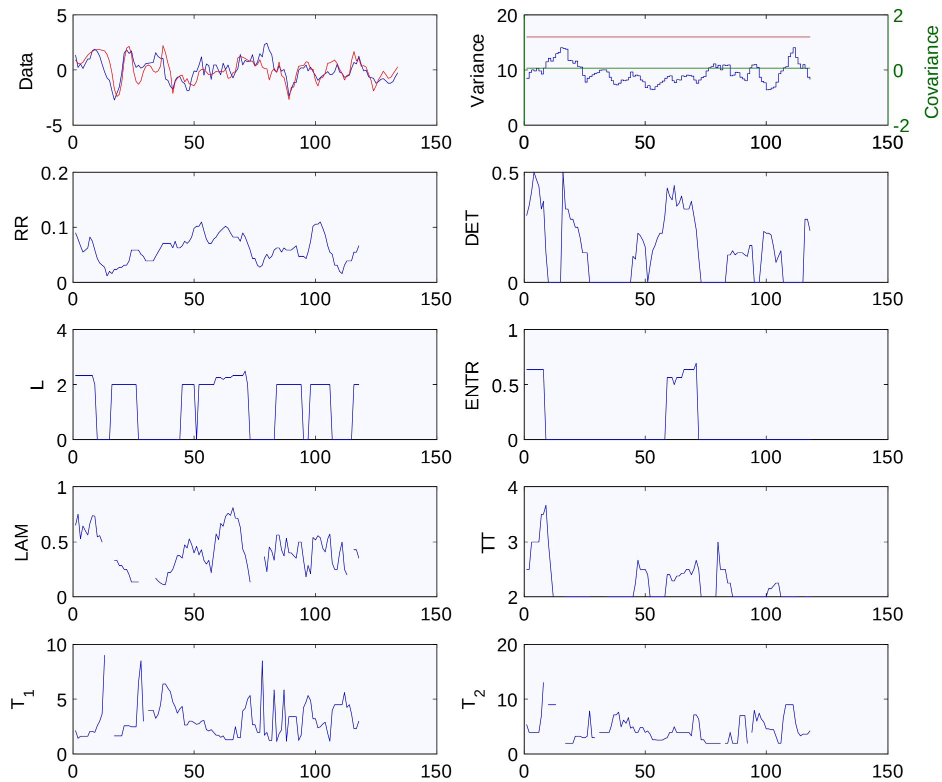

Figure 15: CRQA for Belgian vs German real GDP growth

Figure 16 shows the cross recurrence plot for Italian real GDP growth. Here the nature of the plot appears to have changed through time, with clearly greater synchronicity, especially since the inception of EMU in 1999. The incidence of diagonal lines has increased since the mid-1990s, which indicates that synchronicity has increased significantly compared with previous periods.

In the case of Luxembourg in figure 17 the line of synchronization strays further from the main diagonal as it can easily seen from the plots of the two series that Luxembourg's real GDP growth rate is less consistently in sync with that of Germany. Interestingly though, divergences in the line of synchronicity do not last for more than a few years, before synchronization is restored once again.

Figure 18 shows the recurrence plot for the Netherlands against Germany. Interestingly

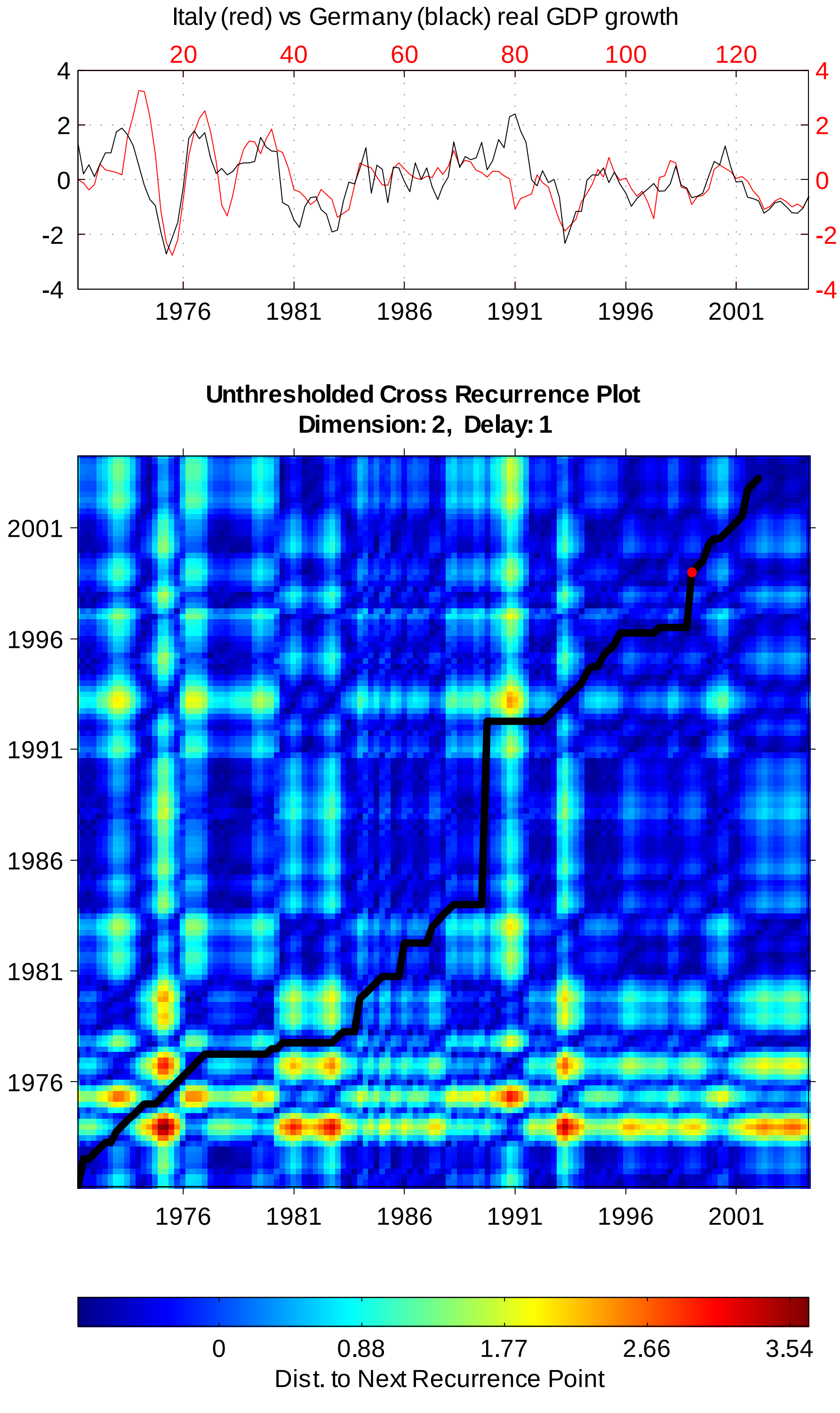

Figure 16: CRP for Italy vs German real GDP growth

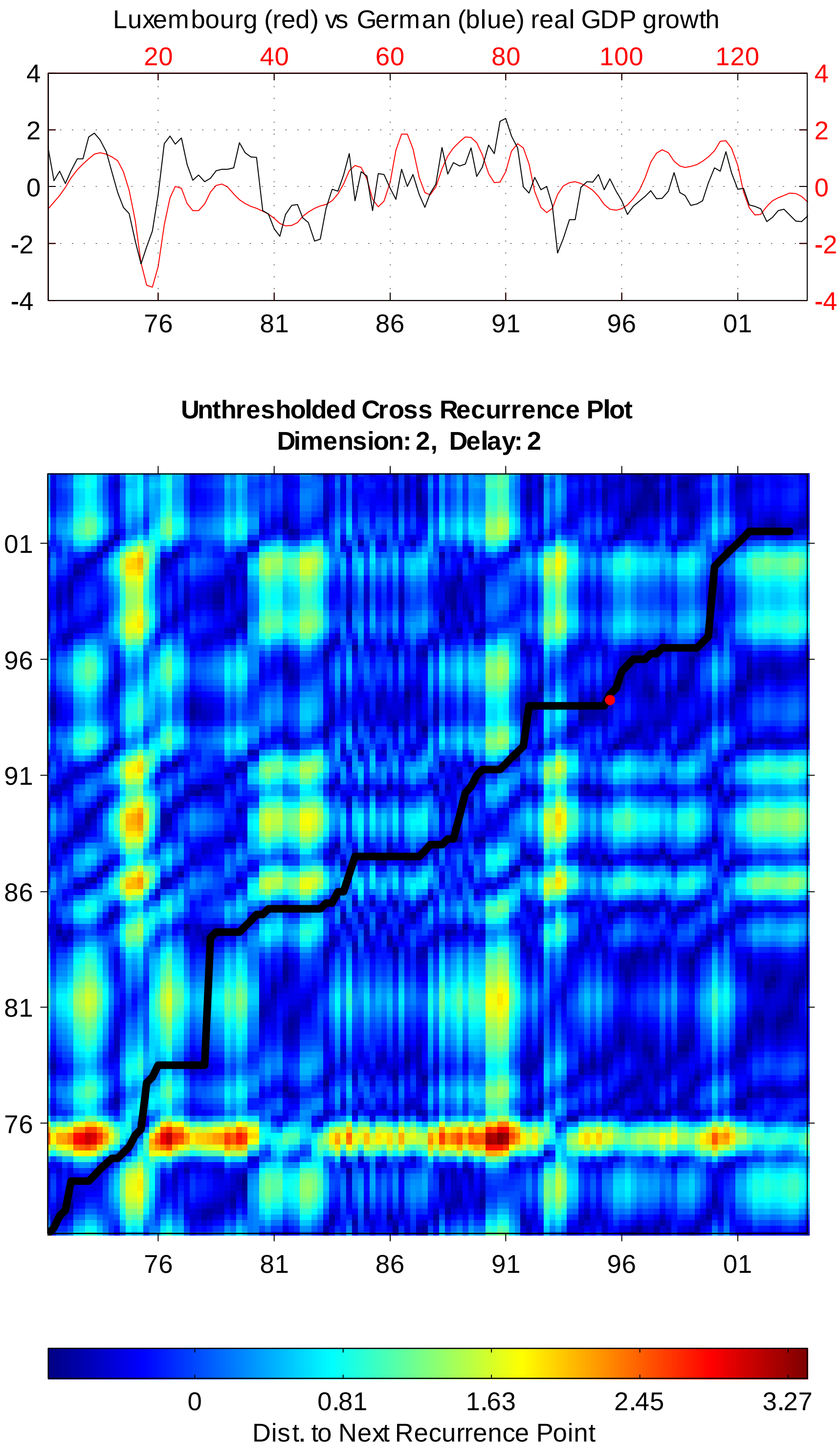

Figure 17: CRP for Luxembourg vs German real GDP growth

the divergences with German growth rates through the 1980s are much less marked than for other countries, but here growth rates clearly diverged significantly during the period 1995-2000, which is reflected in the line of synchronization becoming vertical and then horizontal through this period.

4.2 Peripheral euro area member states

In the case of peripheral euro area member states, cross recurrence plots are calculated against euro area real GDP growth. For Finland, in figure 19 the major recession in the early 1990s is clearly notable in terms of the red and yellow horizontal band across the plot. In the late 1970s and early 1980s Finland appears to have been leading the euro area in terms of growth patterns but y the late 1980s Finland starts to lag behind the euro area growth dynamic. Also the most recent data used in this study appeared to suggest that these was a divergence in growth rates after a period of increased synchronization in the early part of the introduction of ECB monetary policy. The structures in the cross recurrence plot appear to be much more vertically aligned, which tends to suggest a low degree of recurrence, which could cause a problem for Finland in the future. This is borne out by the CRQA plots which are shown in figure 20, where high values of DET and L suggest a long time span in which both systems visit the same regions.

In the case of Greek real GDP growth, shown in figure 21, first note that the scale on the top plot is now (-5, 5), indicating the much wider movements in real GDP that Greece has had relative to the rest of the euro area. There is little synchronicity in the late 1980s and then subsequently, since the inception of EMU, the line of synchronicity has become almost vertical, implying a large divergence in growth rates with the euro area.

In figure 22 Ireland also experiences large departures in its line of synchronicity from the leading diagonal. Here, the red points represent "pins" which attach the line of synchronicity to known points when growth rates were identical between the euro area and Ireland. In the recurrence plot, as the yellow and red areas attest, growth rates in Ireland have been quite dissimilar to those of the euro area, plus although there is some movement in similar directions, these dynamics appear to be much more extreme for Ireland. In figure 23 the CRQA analysis for the cross recurrence plot is shown, and it is interesting to note that recurrence rates reached a minimum in around 1996, and then has increased but not substantially Laminarity, however, appears to fluctuate between high values and low values, which reflects the large number of vertical structures in the cross recurrence plot.

Figure 24 shows the cross recurrence plot for Portugal, and here it is clear that the

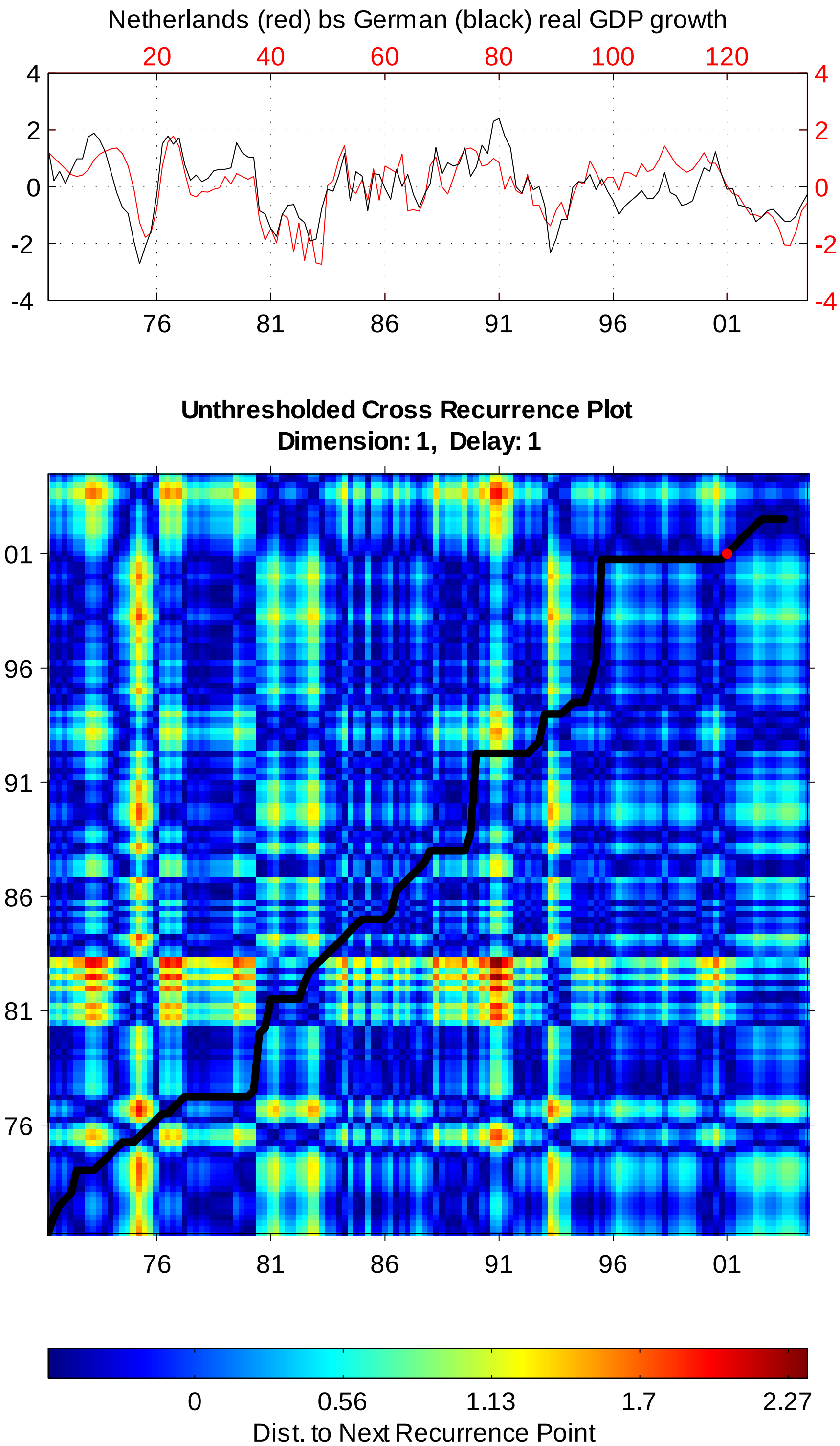

Figure 18: CRP for Netherlands vs German real GDP growth

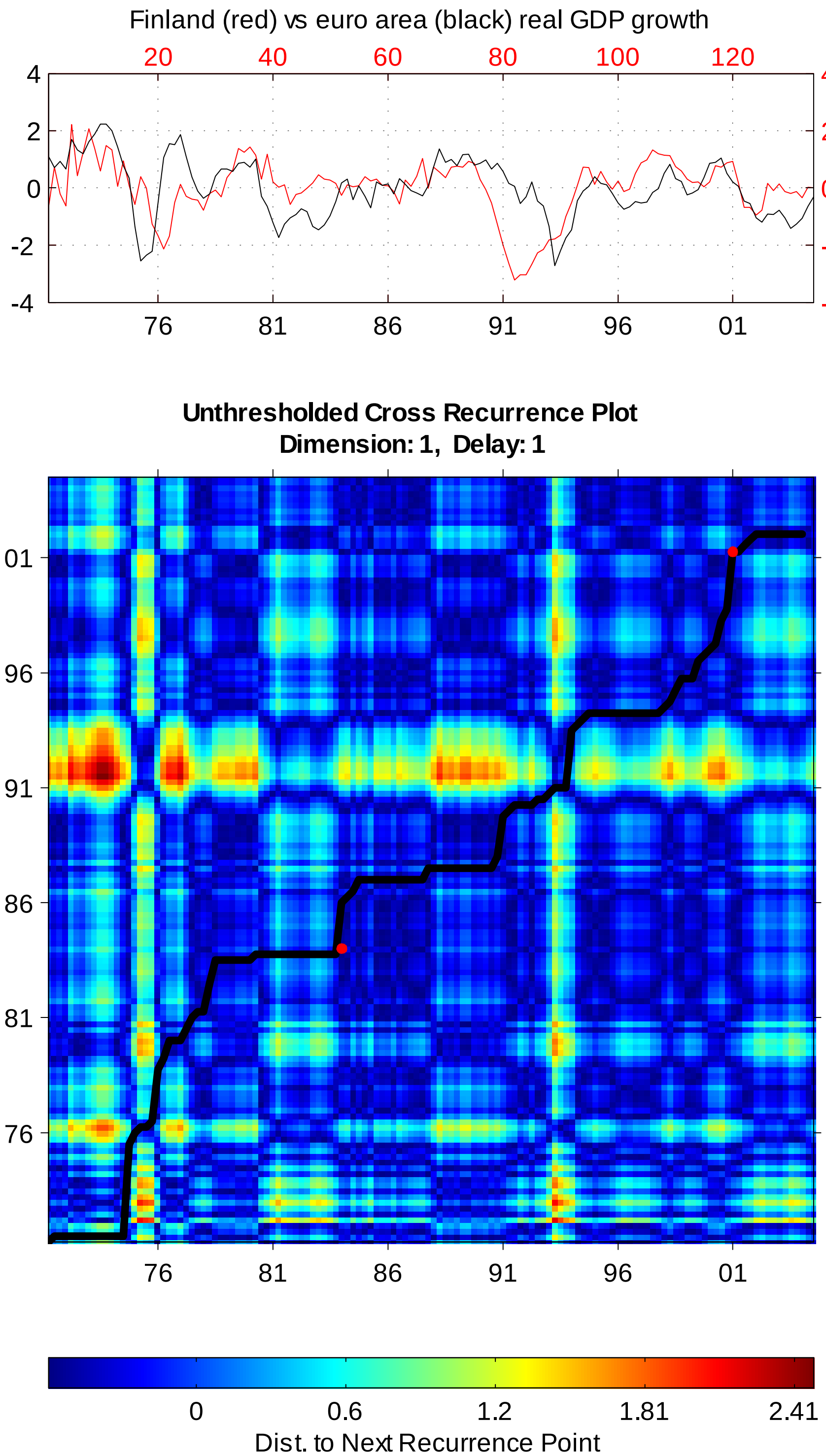

Figure 19: CRP for Finnish vs euro area real GDP growth

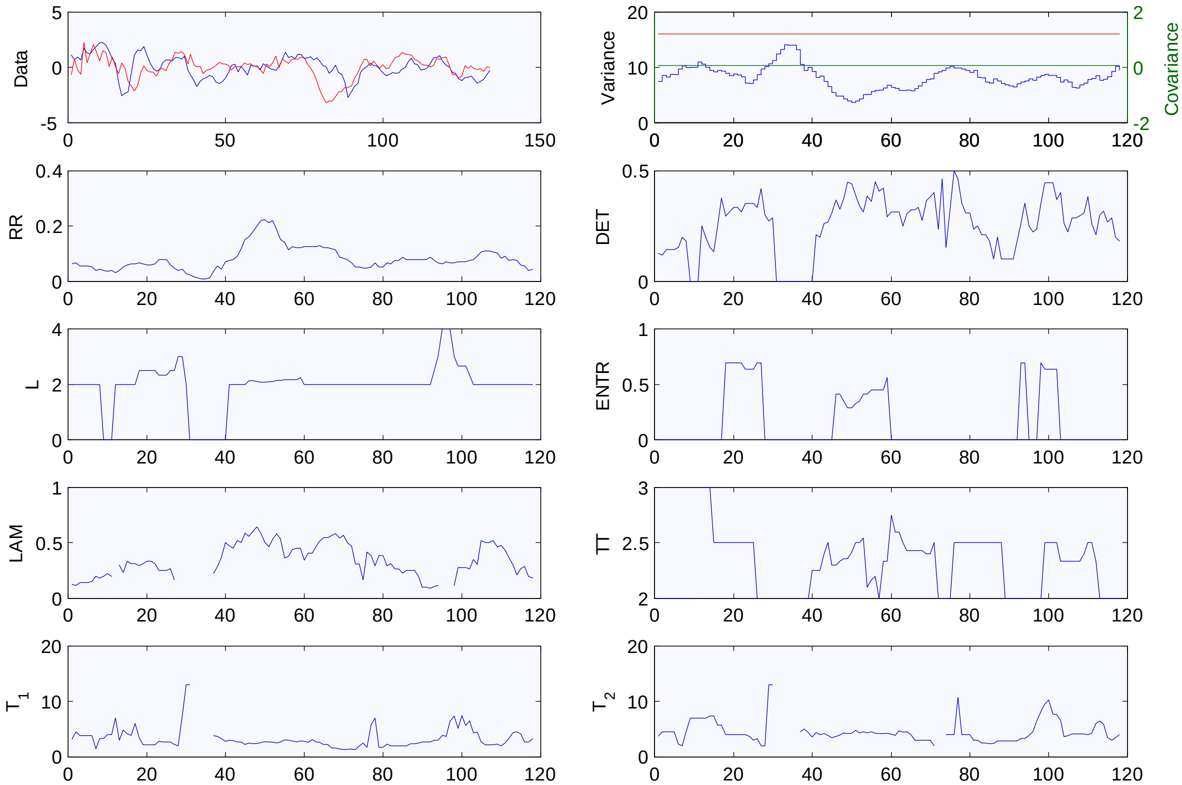

Figure 20: CRQA for Finnish vs euro area real GDP growth

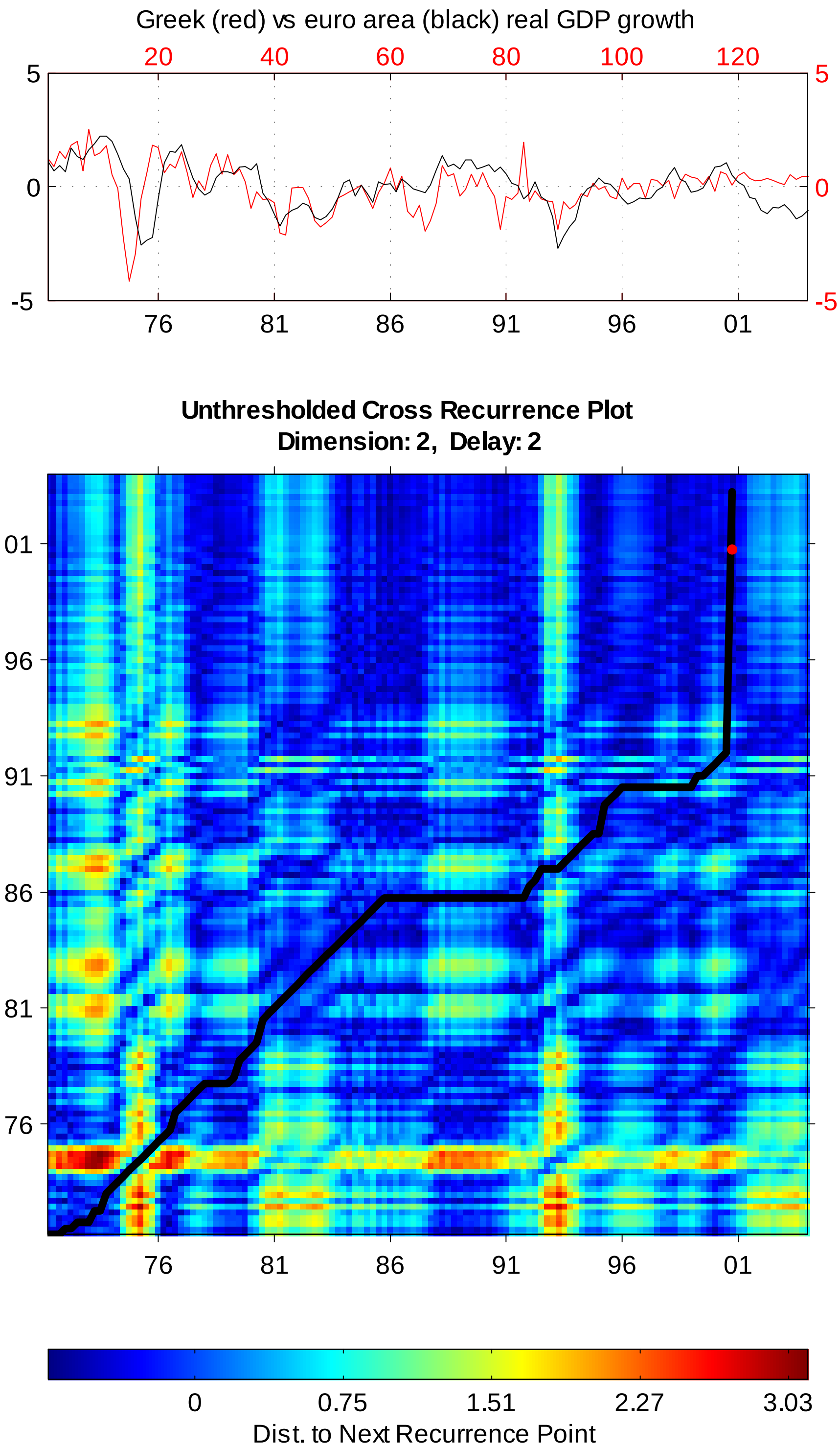

Figure 21: CRP for Greek vs euro area real GDP growth

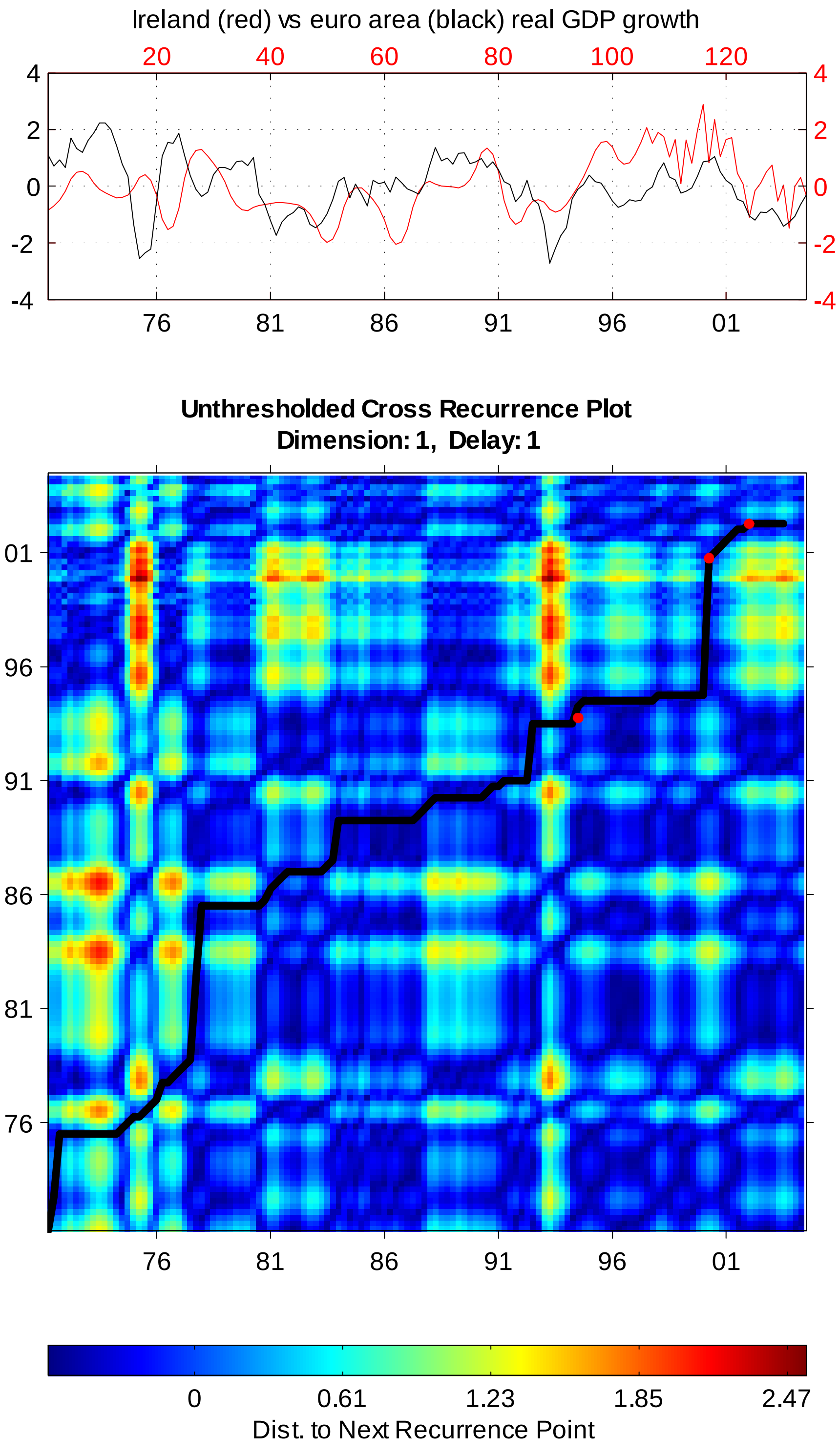

Figure 22: CRP for Irish vs euro area real GDP growth

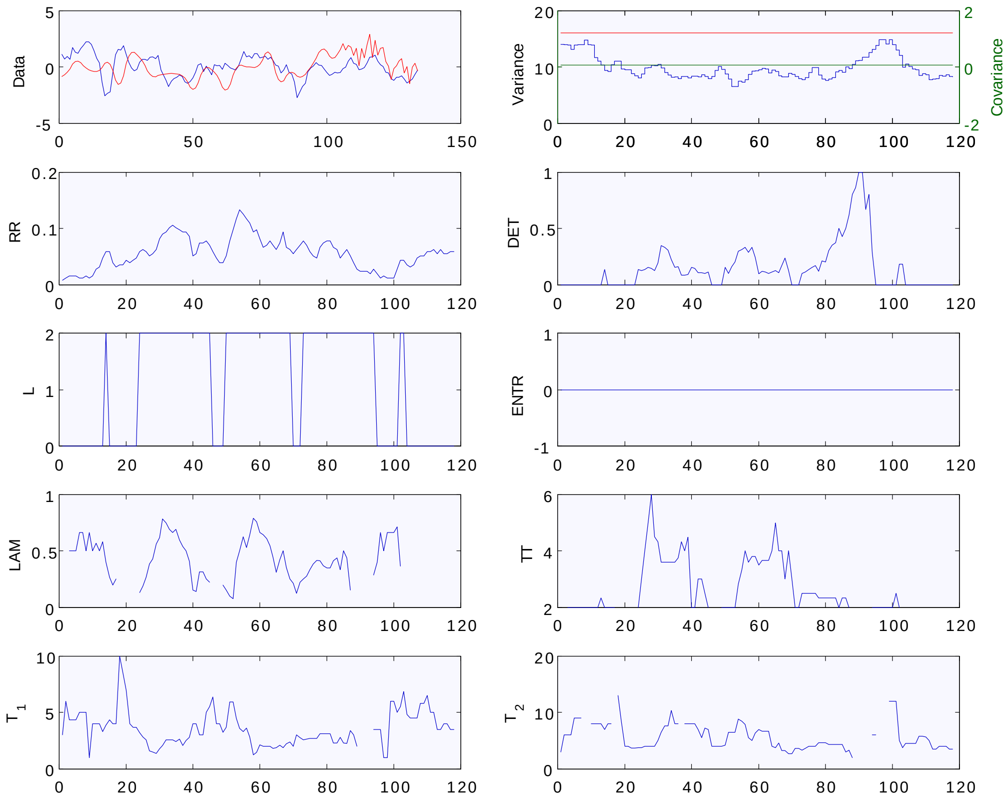

Figure 23: CRQA for Irish vs euro area real GDP growth

movement in real GDP for Portugal largely mirrors the movements in euro area real GDP, with the exception of the period from 1982 to 1985. The cross recurrence plot shows a mix of diagonal type line structures and box-like structures, indicating some laminarity, but also synchronous movements in Portuguese real GDP. Since the inception of EMU Portuguese real GDP seems to have moved mostly in step with euro area real GDP.

On the other hand, Spanish real GDP growth was very synchronous with euro area real GDP growth from 1986 through until 2001, but more recently growth rates appeared to have diverged, leading to the line of synchronization moving away from the leading diagonal in the plot (shown in figure 25).

4.3 Non euro area countries

For non euro area countries or member states, cross recurrence plots are constructed against euro area real GDP growth rates. The Swiss real GDP growth rate is shown in figure 26, and as expected mirrors the euro real GDP growth rate, giving diagonal structures throughout most parts of the plot, although there were clearly significant departures in the mid-70s and also in the early 80s - these departures are clearly indicated by the red and yellow horizontal bands showing no similar points anywhere in the euro area real GDP growth series. In recent years though there is clearly some divergence from the main diagonal, and particularly since late 2001.

Denmark's cross recurrence plot for real GDP growth against that of the euro area is shown in figure 27. Here, apart from the mid1980s, levels of Danish real GDP growth are quite similar to that of the euro area although from 1987 to roughly 1991 there were clearly substantial differences, and also from 1993 to 1996 there were also large divergences from euro area growth levels. Danish real GDP is fairly synchronous though with euro area real GDP, although the dynamics appear to be quite different and so the line of synchronization had to be "pinned" so as to not diverge significantly from the central diagonal.

The case of Iceland is also interesting, as figure 28 demonstrates. Here there is very little similarity in the dynamics of real GDP throughout the series, and this is reflected in the recurrence plot. The line of synchronization clearly does not follow the leading diagonal, and the shape of the line of synchronization suggests that Iceland's real GDP leads that of the euro area for the most part, probably because of the dip in real GDP in 1984 and the peak in real GDP in 1987/88. Lately the line of synchronization has returned to the leading diagonal in the plot, signifying a similar dynamic.

In the case of the UK, figure 29 reveals a line of synchronization that clearly diverges

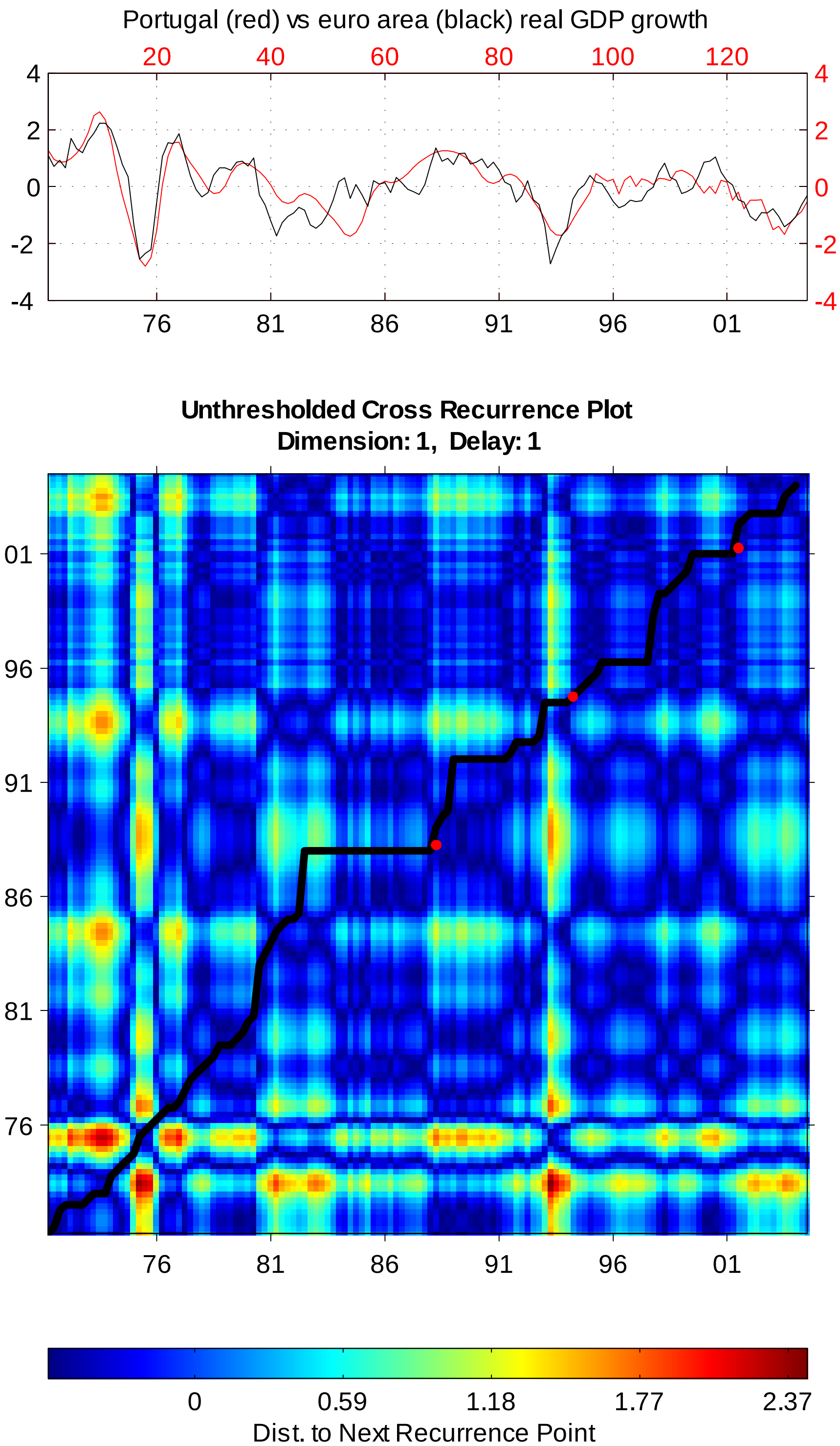

Figure 24: CRP for Portuguese vs euro area real GDP growth

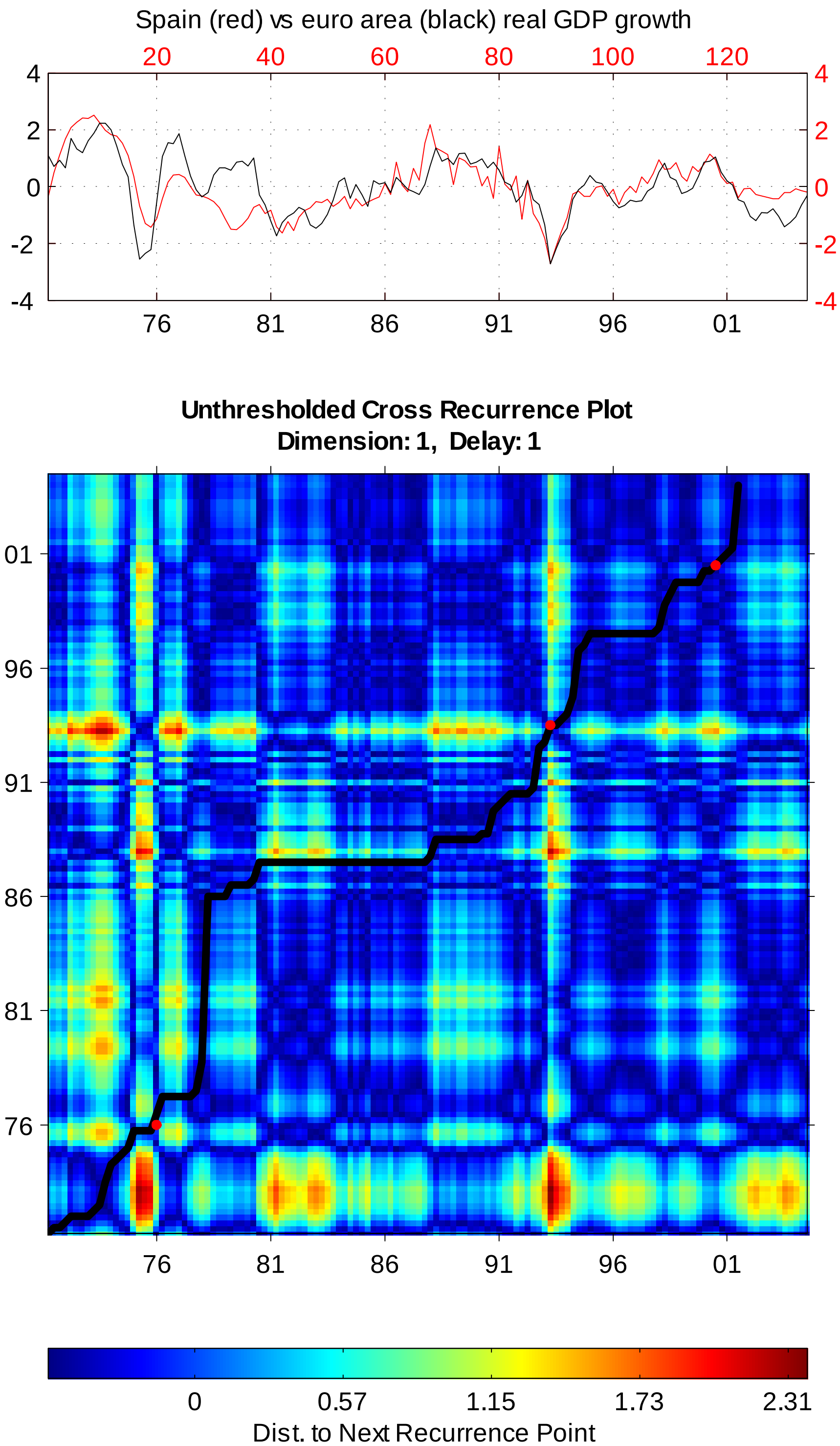

Figure 25: CRP for Spanish vs euro area real GDP growth

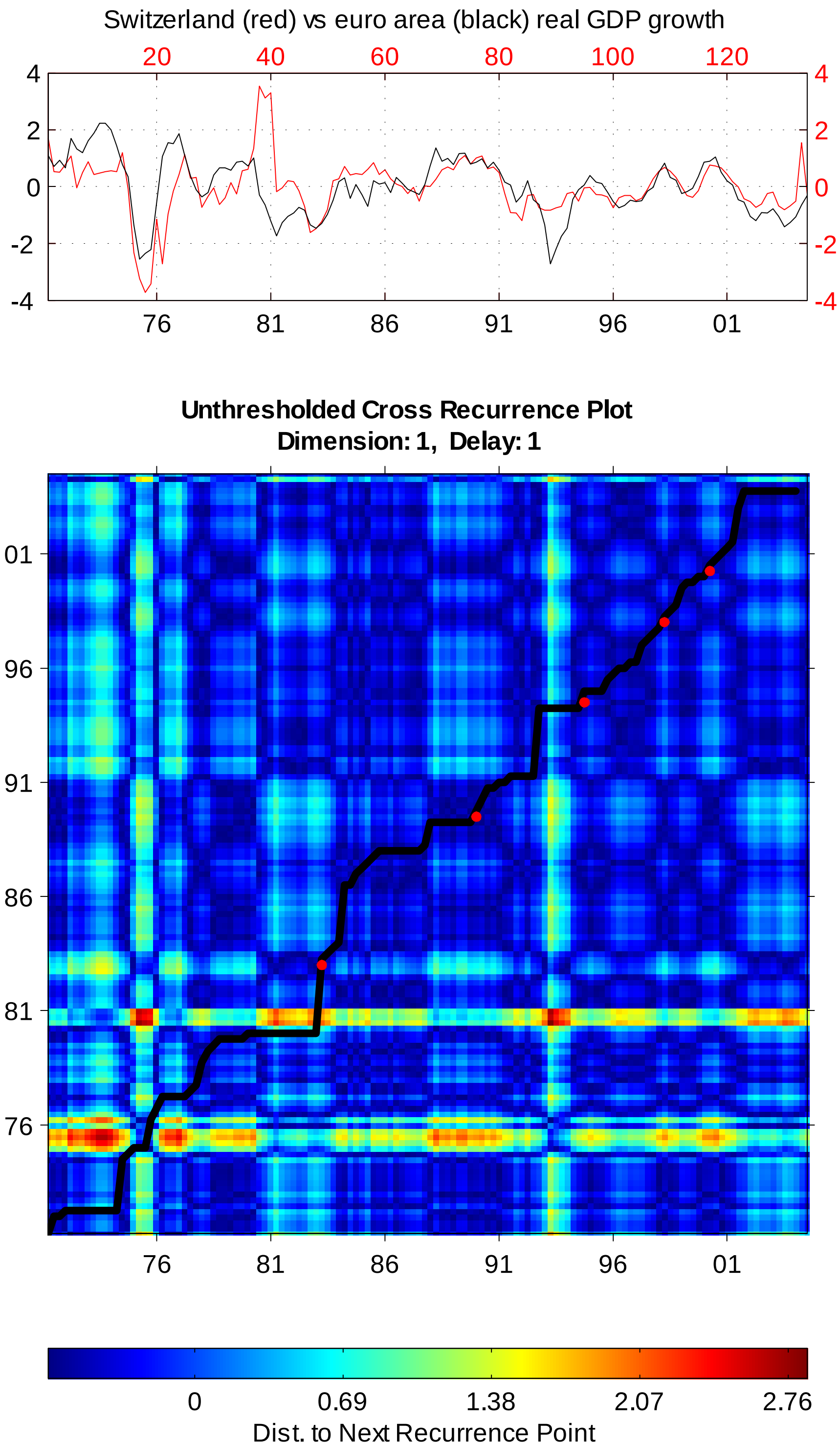

Figure 26: CRP for Swiss vs euro area real GDP growth

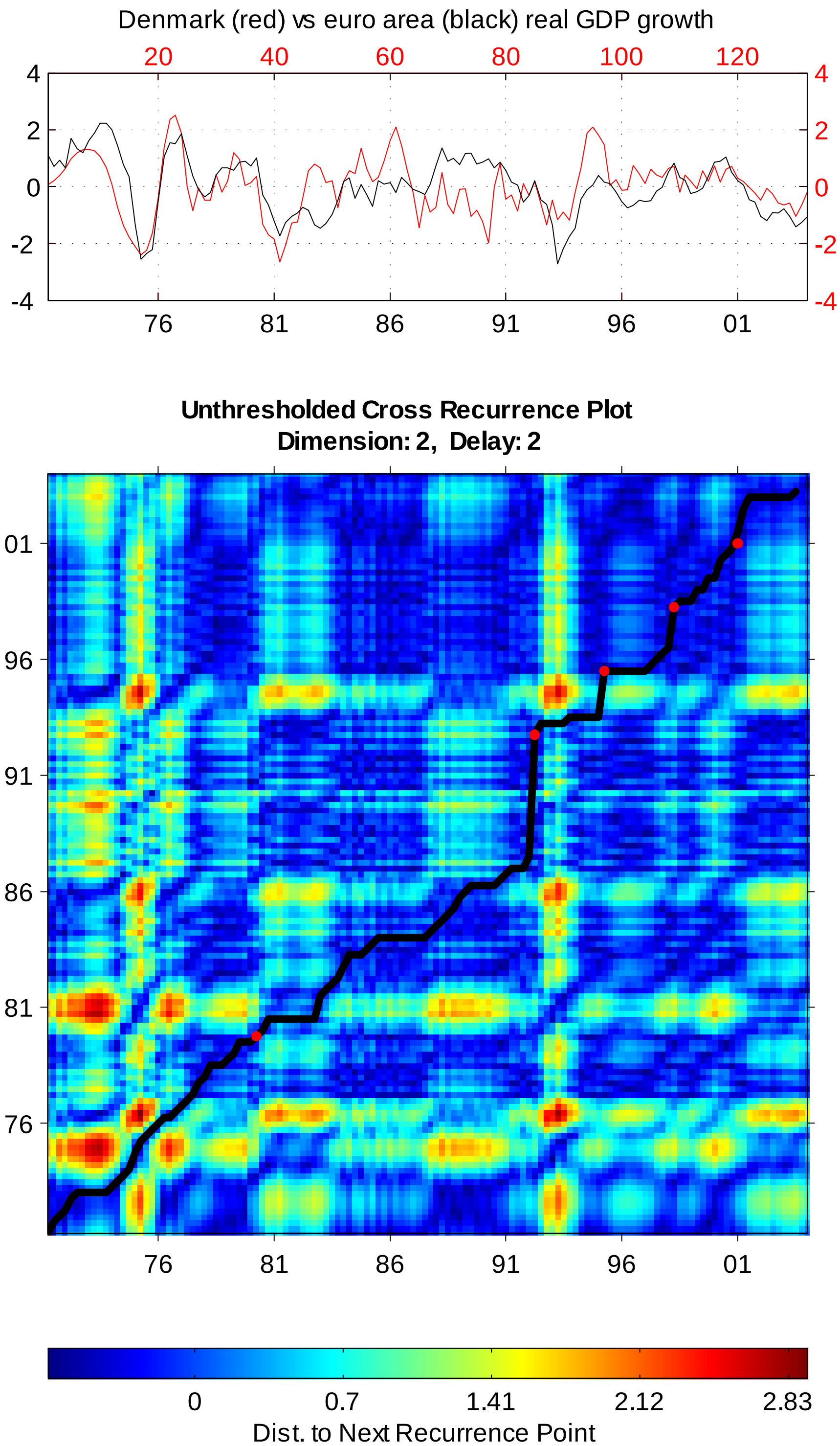

Figure 27: CRP for Danish vs euro area real GDP growth

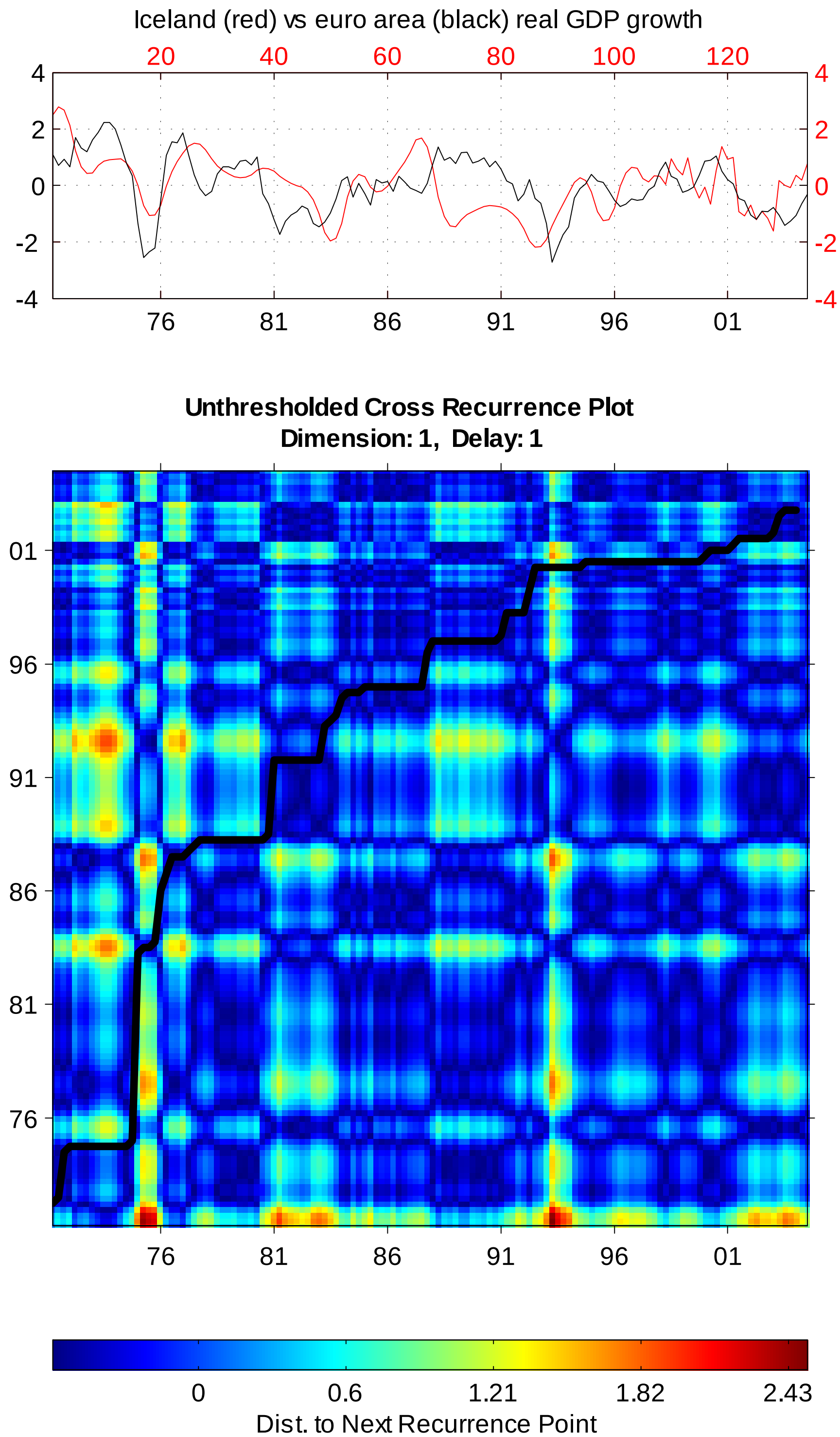

Figure 28: CRP for Iceland vs euro area real GDP growth

frequently from the central diagonal.and appears not to reflect the lagged relationship that has been found elsewhere between the UK and the euro area. Clearly there was a rough degree of synchronicity in the series up until around 1988, a lead relationship opened up for the UK beyond this date, although from around 2000 this relationship has also disappeared. Beyond that point the two growth paths appear to have had quite different dynamics.

In figure 30 the cross recurrence plot for US real GDP growth vs euro area real GDP growth is shown. In this case the usual business cycle "stylized facts" clearly come into play, with lagged synchronization of turning points clearly evident, and this is reflected in the line of synchronization, which shows a lead for the US from the late 1980s up until around 2002. In this case, looking at the CRQA measures in figure 31 shows an increase in the recurrence rate through time, suggesting an increase in similarity of dynamics between the two series, although the size of the recurrence measure is still not that high. The laminarity index appears to have peaked in 1996 and then has declined post 2001 as US and euro area real growth appeared to move in similar directions. Indeed the lagged effects between the series come through in the diagonal lines that appear in the plot and cause the determinism measure to be consistently positive beyond around 1993. This shows that similar dynamics between the US and Germany are at play in determining economic growth, and might suggest that a common factor such as globalization is impacting both economies.

5 Implications for ECB monetary policy

5.1 Some theoretical background

Any graduate student in international economics who has studied the real business cycle school in macroeconomics knows that business cycle turning points are roughly correlated across countries - and this is now considered a "stylized fact" in macroeconomics. The recognition of the synchronicity of turning points in business cycles across countries (noted by Backus and Kehoe (1992) and Backus, Kehoe, and Kydland (1995)) also has the implication that the economic growth dynamic between these turning points (usually the recessions or peaks of business cycles) can be radically different between countries. This implication has given rise to the notion and then study of growth cycles in the context of the dynamic of economic growth between these turning points. This line of inquiry has found that there are significant differences between countries (see Kontolemis (1997) for a recognition of growth cycles between these turning points and Zarnowitz and Ozyildirim

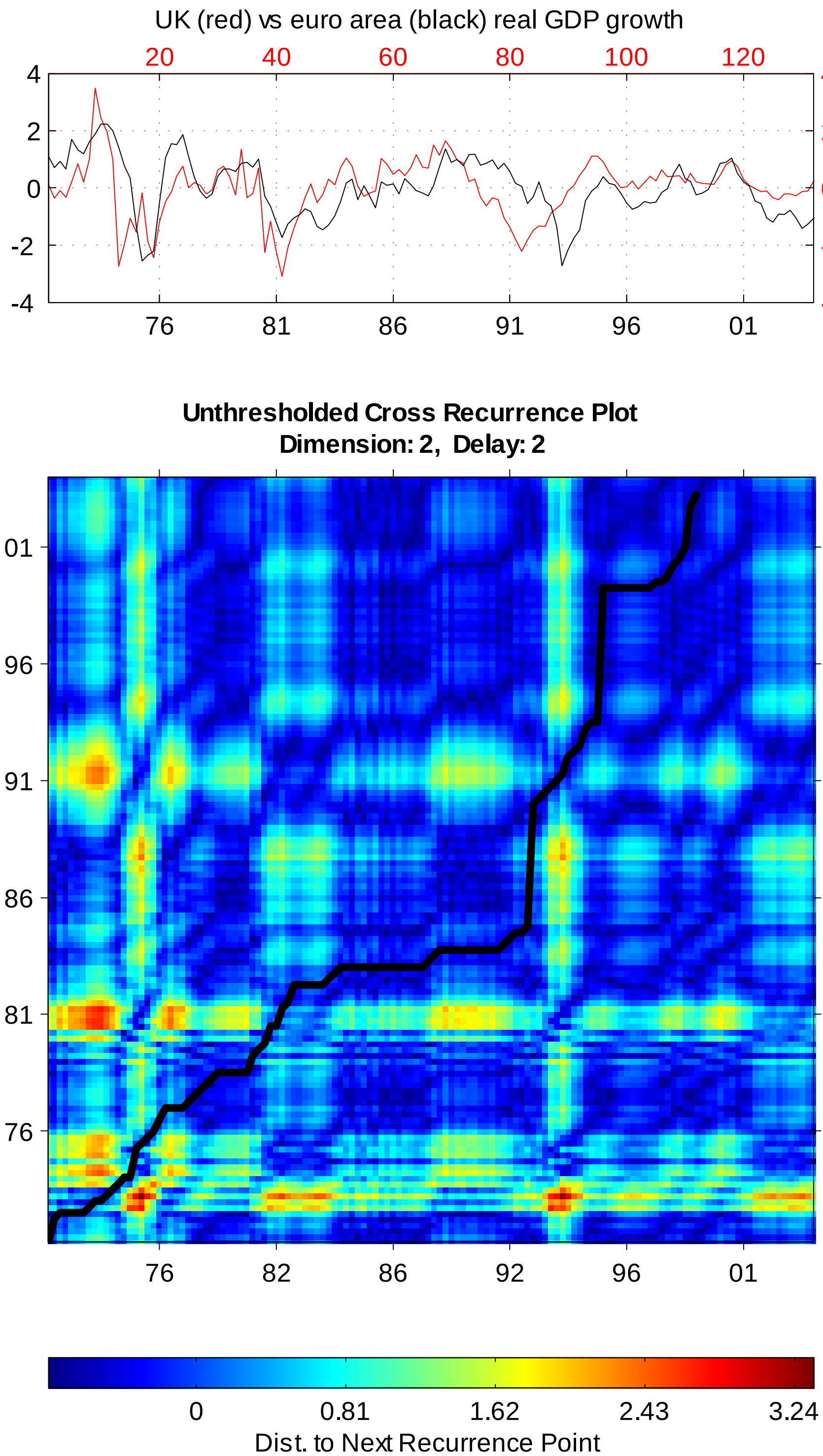

Figure 29: CRP for UK vs euro area real GDP growth

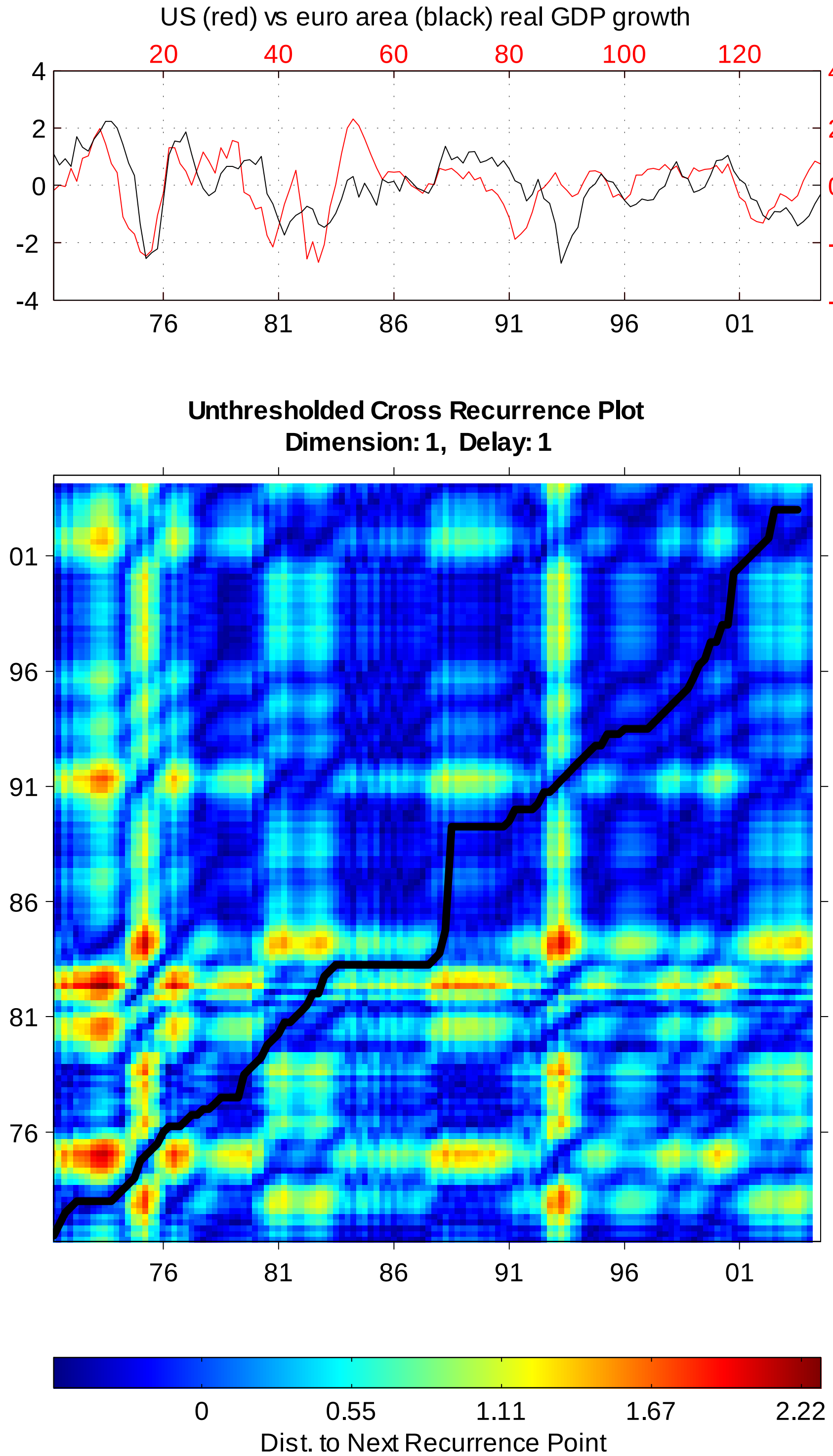

Figure 30: CRP for US vs euro area real GDP growth

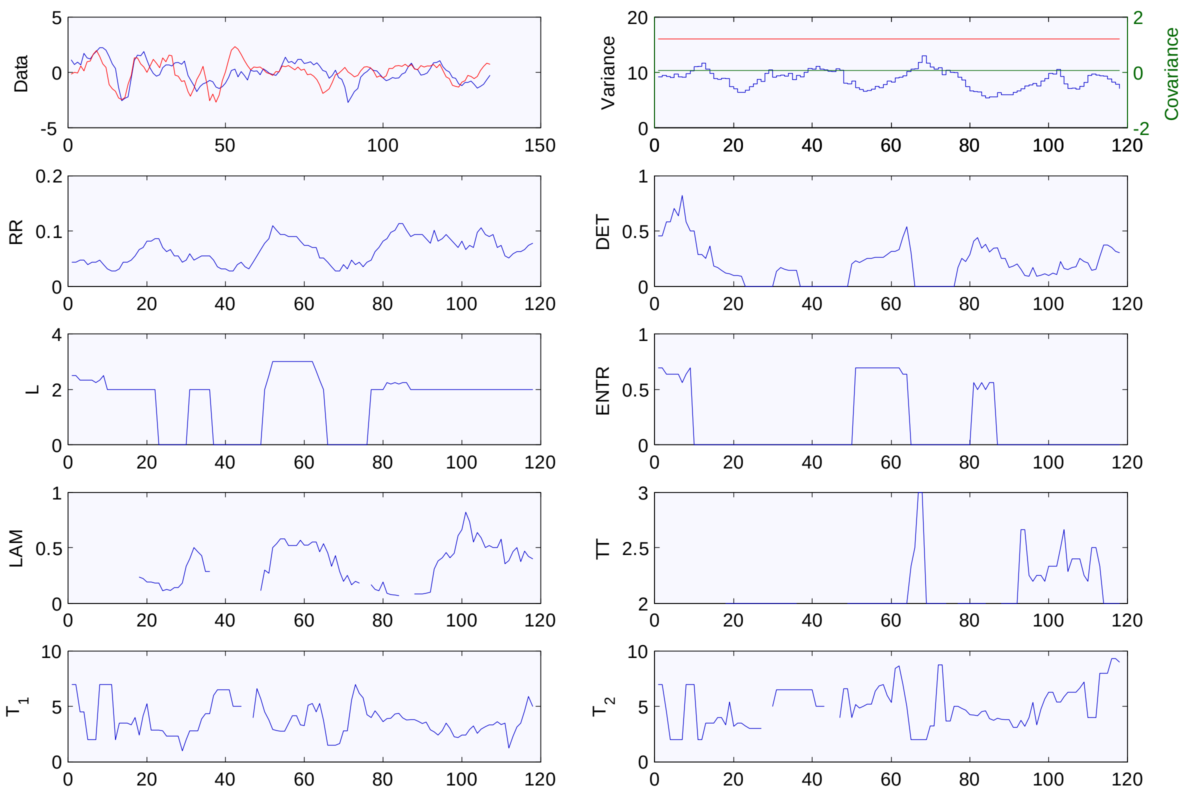

Figure 31: CRQA for US vs euro area real GDP growth

(2002) for measurement of these cycles in relation to the business cycle). From an empirical perspective there have been some efforts to empirically extract these cycles for measurement and comparison across countries using other statistical techniques such as wavelets (see Crivellini, Gallegati, Gallegati, and Palestrini (2004) and Crowley and Lee (2005)).

In the euro area context, there has been a recognition for some time that with firstly closer cooperation in monetary policy (under the ERM and in the run up to EMU) and then secondly the shift to the adoption of the euro within the EMU process, that synchronization of euro area growth rates would likely increase. But measuring this has been more problematic for a variety of reasons - notably the short data span available and the exceptional circumstances surrounding events in the early part of this decade (9/11, Iraq invasion, German structural problems etc). Despite these issues, there has been a variety of empirical research of different types done on this topic, with notable contributions by Artis and Zhang (1997) who first recognized the existence of a separately identifiable European business cycle, followed up by Artis and Zhang (1999) with further evidence on the same, and then mostly studies that have tried to measure whether the "European business cycle" has become stronger since the inception of EMU and the introduction of the euro and a single ECB monetary policy (see Altavilla (2004), Sensier, Artis, Osborn, and Birchenhall (2004), Valle e Azevedo (2002), De Haan, Inklaar, and Sleijpen (2002), and Süssmuth (2002)).

This is an important issue for the ECB, and for a myriad of reasons. First, optimal currency area (OCA) considerations (see Mundell (1961)) suggests that similar growth rates in member states will ease the problems associated with the differential impact of monetary policy on these countries. Second, not just growth rates matter, but also the dynamics of growth also matter - thus the idea that similar frequency growth cycles between countries in a monetary union will also ease the problems of implementing monetary policy across a collection of member states or countries. And lastly, the OCA theory also suggests that even without this increased synchronicity of business and growth cycles, increased mobility of factors of production can offset this by and so aid implementation of monetary policy as resources can flow from one country to another to offset the differential impact of monetary policy within a monetary union. Although labor and capital mobility have increased post-EMU inception, it is still acknowledged that language and cultural barriers impose greater barriers to mobility of factors of production than they do in many other monetary unions (such as the US or Canada). Fifth, another offset to lack of synchronization can be found in autonomy of fiscal policy, perhaps at a national or member state level, or at the supranational level. This has caused considerable concerns in the euro area in past years, as the Stability and Growth pact (SGP) appeared to severely limit member state fiscal policy so as to counterbalance ECB monetary policy and its differential impact on certain member states (- for example Germany). Clearly in the euro area there is some lattitude to use fiscal policy to offset the impact of an inappropriate monetary policy, but the scope can be limited, depending on current public debt levels and any existing structural budget deficit considerations.

So, given the above, it is clear that synchronicity of business cycle and growth cycles becomes important in the context of a sustained and "crisis-free" monetary union..

5.2 Do cross recurrence plots/analysis have any implications for ECB monetary policy?

Cross recurrence plots have now been fully developed and accepted in other disciplines such as physiology, mathematics, physics, chemistry, biology, medicine, neurology, pathology, psychology, astronomy, geology, matallurgy, forestry, botany, genomics, proteomics, semionics, linguistics and finance as a legitimate empirical approach to studying real world systems. Because economics is a discipline where data is extremely unreliable and prone to measurement error, there is perhaps a larger caveat that needs to be placed upon these results than might be given elsewhere, but nevertheless the recurrence plots and analysis of synchronization using this relatively new technique can, I believe, shed some new light on monetary policy in the euro area.

The first implication for ECB monetary policy is that varying degrees of synchronicity are clearly apparent across the euro area, with certain groupings apparent here:

i) the core of the euro area, Germany, France, Italy, Belgium and the Netherlands are clearly well synchronized, and this "coupling" of growth rates has clearly increased since the introduction of a single monetary policy. The exception here is Luxembourg, but perhaps this should not be a concern as Luxembourg is relatively small and has a high level of labor mobility (into and out of the country).

ii) the periphery of the euro area appears to fall into 3 groups:

a) those member states that clearly have managed (through whatever means[4]) to have high levels of synchronicity with the rest of the euro area throughout the

[4] Trade or appropriate use of fiscal policy both spring to mind.

whole period under consideration - notably Portugal.

b) those member states that have traditionally had relatively low synchronicity with the euro area but now have had an apparent increase in synchronicity, perhaps due to trade effects and an ex-post monetary policy impact on growth rates - for example Finland.

c) those member states that had established a high degree of synchronicity, but have divergent growth rates with that of the euro area - for example Spain; and

d) those member states where synchronicity has changed little, and may be a problem in the future - for example Greece and Ireland.

iii) those countries within europe who could potentially join the euro area, but are ineligible or who prefer to remain outside - for example Denmark and Switzerland

iv) those countries for which euro area membership would be inadvisable, given current levels of synchronization with euro area growth rates. The best examples here are Iceland and the UK.

Given the above groupings, the question then becomes whether ECB monetary policy could prove wholly inappropriate for some of the peripheral member states - with the member states concerned being identified as Spain, Greece and Ireland, and could precipitate a crisis if these member states decided to pull out of the euro area.

6 Conclusions

Cross recurrence plots and quantification offers a new and unique non-linear approach to studying time series and their interaction over time. Although originally used as a tool to analyze deterministic time series, they also have now been applied in more "experimental" environments where stochastic time series are considered the norm. One of the advantages of using cross recurrence plots for the analysis of complex systems is that they can reveal interesting features of the dynamics and assess the degree of synchronicity between any given time series.

In this paper cross recurrence plots were used to analyse real GDP growth between euro area member states and other European countries against some kind of benchmark such as German or euro area real GDP growth rates. Synchronicity was found to be high between the core euro area member states, with the exception of Luxembourg, suggesting that monetary policy is not having a differential impact on these countries and that they are increasingly suited to being part of a monetary union. Outside of this core, there is no single common characteristic that can be identified as giving rise to non-synchronous real GDP growth cycles. Although a convergence in synchronicity for Finland appears positive, other countries such as Spain, Greece and Ireland, have persistent problems with synchronicity in the euro area, either because of different levels of real growth, or because of cyclical differences with the euro area. This should clearly be a concern for the future for these countries, and might decrease the public perceptions of the desirability of being part of the euro area in these countries.

Clearly there is little if anything the ECB can do about this, but it does suggest that with even more member states likely to be taking steps to join the euro area in the future, other means of securing and maintaining the euro area as an optimal currency area should be explored with greater urgency.

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