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A NUMERICAL CODE FOR THE STUDY OF THE REACTOR SPATIAL DYNAMICS IN TWO GROUPS

by

E. VINCENTI, R. MONTEROSSO

(Euratom) and

A. AGAZZI (C. Gavazzi S.p.A. - Milan)

1964



Joint Nuclear Research Center Ispra Establishment - Italy

Scientific Data Processing Center - CETIS Reactor Physics Department

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CONTENTS

Pa	g.e
	5 V.

	Preface	7
I.	The power excursion reactor and its simplified geometrical configuration	9
II.	The physical equations and their transformation into a system of linear equations	11
III.	The finite difference equations	15
IV.	The parts of the code and their function	20
v.	Initial conditions	21
VI.	Dynamic calculation	25
VII.	Comparison between the conventional method of the kinetic equations and the direct solution of the diffusion equations	28
VIII.	Some examples of calculation	32
	References	51

Br	Ħ	radial buckling
С	Ξ	delayed neutrons' precursor density
с	=	core specific heat $\frac{cal}{cm^3 \circ C}$
$^{D}\mathbf{f}$	=	fast diffusion coefficient of core
D_{f}^{r}	=	fast diffusion coefficient of reflector
D_{f}^{k}	=	fast diffusion coefficient of the K _{th} region
D _t	H	thermal diffusion coefficient of core
$\mathtt{D}_{\mathtt{t}}^{\mathtt{r}}$	÷	thermal diffusion coefficient of reflector
\mathbf{D}_{t}^{k}	=	thermal diffusion coefficient of the κ_{th} region
F	=	heat released by one fission (7.66 x 10^{-12} cal)
ĸ	=	infinite multiplication factor
^K ef:	e=	effective multiplication factor
L	=	diffusion length
10	=	life time of a thermal neutron in an infinite medium
1	=	life time of a thermal neutron in a finite reactor
n	Ξ	neutron density
R^k	=	K _{th} region
Т	=	temperature (Kelvin)
То	Ξ	temperature of the cold reactor
V	=	thermal neutron velocity
w	=	fast neutron velocity
α	Ŧ	$\frac{1}{T}$ = reciprocal of the reactor period
β	Ŧ	total fraction of delayed fission neutrons
Δt	H	time step
$\Delta \mathbf{z}_{k}^{k}$	=	mesh increment in the K _{th} region

 $\lambda = \text{precursor decay constant (weighted average)}$ $\nu = \text{fission neutrons per fission}$ $\Sigma_a^c, \Sigma_a^r = \text{absorption cross section of core, reflector}$ $\Sigma_p = \text{absorption cross section of the rod equivalent poison}$ $\phi = \text{thermal flux}$ $\Psi = \text{fast flux}$ $\tau = \text{Fermi age of core}$ $\tau_r = \text{Fermi age of reflector}$

- 5 -

PREFACE

This report describes a numerical code for the study of the spatio-temporal dynamics of a reactor. It is written for the reactor TESI in particular, which operates in conditions of prompt criticality. This example was chosen because the classical method of the kinetic eq. is based on the assumption that the reactor is very near criticality. The numerical method, based on the direct solution of the time-dependent diffusion equation, in condition of prompt criticality, should give more accurate results than the classical one.

Although a very special type of reactor is studied here, this work is meant to be a preliminary work for a more general code for power reactors.

The code described in this report is only for one space dimension.

The case of two space dimensions is being studied and will be the subject of another report.

We are indebted to Mr. Foggi and Mr. Ricchena (T.C.R.) for many informations on the core of the reactor TESI and on the point dynamics and for their valuable contribution during the discussions. We are also grateful to Mrs. Tamagnini (CETIS) for the calculation made with the Pineto Code and to Mr. Green and Mr. Caligiuri (CETIS) for the calculation on the analogue computer.

§ I. The power excursion reactor and its simplified geometrical configuration

This report contains the description of a numerical code written for the IBM 7090, and to be employed for the study of the spatio-temporal dynamics of the reactor TESI.

This reactor is meant for studying the destruction of a fuel element of any other reactor caused by a flash of high neutron flux. It operates in the following way: from being critical at a very low power, it is made prompt critical. The flux rises very rapidly, and, as the reactor is not cooled during the transient, the temperature in the core rises accordingly. The core has a large negative temperature coefficient and therefore when a certain value of the flux is reached, the reactor becomes undercritical and after a pulse of very short duration the flux decreases.

It is provided with a cooling system, which can bring the core to its initial temperature in two or three hours and prepares the condition for a new pulse. This cooling, however, has no effect during the transients of a few seconds which will be studied here. The temperature rises as the integral of the flux and tends to its final maximum value.

The core of reactor TESI is a vertical cylinder of 145 cm height made of a homogeneous mixture of fuel and graphite. There is a radial reflector and also an upper and lower reflector 60 cm thick. The control rods are of two types. The first type is placed at the interface between core and radial reflector. When they are completely introduced they separate the core from the reflector. The other type of rods are immersed from the top into the core. We will consider them as moving all together in the axial direction with the same depth of insertion. We shall consider substituting these rods by an equivalent poison which is homogeneously distributed in the rodded part of the reactor.

Since we dispose of a programme with one spatial dimension only i.e. the z vertical dimension, we shall consider, instead of the finite cylindrical core, a horizontal infinite slab with a thickness equal to the height of the core and with upper and lower reflectors. In the cylinder there is a horizontal neutron leakage, which does not take place in the infinite slab; to compensate this the absorption cross section is increased by the quantity B_p^2 D.

Along the z axis, see Fig. 1, we consider a mesh-points $\{z_j\}$ (i=1,...L); in the region R 1 (lower reflector) with a mesh increment Δz_1 ; in the region R 2 (core), mesh increment Δz_2 ; in the region R 3 (upper reflector) mesh increment Δz_3 . On each interface between core and reflector is a point of the lattice z_M , z_N , each region must contain at least one internal point.

The code can operate with a maximum number of 100 points. Fig.1.



§ II <u>The physical equations and their transformation</u> into a system of linear equations

One of the purposes of this code is to give a description of the flux distribution in the core and reflectors and of the deformation of this distribution during the transient, due to the movement of the control rods, to the temperature reaction and to the presence of reflectors. Although in reactor dynamics it is usual to adopt the one group approximation, <u>two energy groups</u>, fast and thermal, will be considered here. With one group there is no thermal source in the reflector, whereas with two groups there is a source of thermal neutrons also in the reflector due to the slowing down of the fast neutrons; this gives a much better spatial distribution of fluxes.

There is only <u>one group of delayed neutrons</u> with a decay constant λ equal to the properly weighed average for the six actual groups. This approximation is good enough because the reactor operates as prompt critical and the delayed neutrons have an effect only on the tail of the transient.

The <u>heat balance equation</u> contains only the term of the heat accumulation and that of heat production, which must be equal because, as said before, no heat is removed during the transient.

The following are the two groups' diffusion equations, the equation of the balance of precursors of the delayed neutrons and the equation of the heat balance.

$$(II-1) \quad D_{\mathbf{f}} \frac{\partial^2 \Psi}{\partial z^2} - (\frac{D_{\mathbf{f}}}{\tau} + D_{\mathbf{f}} B_{\mathbf{f}}^2) \Psi + k(1-\beta) \Sigma_{\mathbf{a}} \phi + \lambda C = \frac{1}{w} \frac{\partial \Psi}{\partial t}$$

$$(II-2) \quad D_{t} \frac{\partial^{2} \phi}{\partial z^{2}} - (\Sigma_{a}^{C} + D_{t} B_{r}^{2} + \Sigma_{p}) \phi + \frac{D_{f}}{\tau} \Psi = \frac{1}{v} \frac{\partial \phi}{\partial t}$$

$$(II-3) \quad \frac{\partial C}{\partial t} = k \sum_{a} \beta \phi - \lambda C$$

(II-4)
$$\frac{\partial T}{\partial t} = \frac{F}{c} \frac{k}{\nu} \sum_{a} \phi$$
 (see List of Symbols)

The equations (II-1) and (II-2) are written for points in the core; in the reflectors they have the following form:

$$(II-5) \quad D_{f}^{r} \frac{\partial^{2} \Psi}{\partial z^{2}} - (\frac{D_{f}^{r}}{\tau_{r}} + D_{f}^{r} B_{r}^{2})\Psi = \frac{1}{w} \frac{\partial \Psi}{\partial t}$$

$$(II-6) \quad D_{t}^{r} \frac{\partial^{2} \phi}{\partial z^{2}} - (\Sigma_{a}^{r} \Sigma_{p} + D_{t}^{r} B_{r}^{2}) \phi + \frac{D_{f}^{r}}{\tau_{r}} \Psi = \frac{1}{v} \frac{\partial \phi}{\partial t}$$

The term B^2D which appears in the diffusion equations increases the cross-sections of a quantity equivalent to the radial leakage which is not considered in this slab geometry. In equations (II-2) and (II-6) the term Σ_p indicates the control rod equivalent poison, which is present only at those points of the lattice (Fig. 1 p. 4) which are in the rodded region. Equations (II-3) and (II-4) of the precursors of delayed neutrons and of heat balance respectively are to be considered only in the core region; the reflectors are thermically separated from the core and are always at 20 °C. The fluxes Ψ and ϕ are supposed to be zero at the outer boundary of the reflectors (Points $z_1 = 0$, $z_L = H$)

$$\Psi (0,t) = \Psi (H,t) = 0$$
(II-7)
$$(H = height of reactor)$$

$$\phi (0,t) = \phi (H,t) = 0$$

The system of the two diffusion equations is quasi-linear because the cross section, the neutron velocities and the quantities τ , k, c, which appear in their coefficients, are temperature-dependent and therefore they are indirect functions of the flux ϕ .

It is through these variable coefficients that the temperature feed-back takes place.

The cross sections and the neutron velocities are calculated according to

$$\Sigma_{a}^{C}(z,t) = \Sigma_{a0}^{C} \sqrt{\frac{T_{o}}{T(z,t)}} ; \Sigma_{p}(z,t) = \Sigma_{p0} \sqrt{\frac{T_{o}}{T(z,t)}}$$
$$w(z,t) = w_{o} \sqrt{\frac{T(z,t)}{T_{o}}} ; v(z,t) = v_{o} \sqrt{\frac{T(z,t)}{T_{o}}}$$

where T_o is the uniform temperature of the cold reactor at the initial state, and Σ_{ao}^{c} , Σ_{po} , w_{o} , v_{o} are values corresponding to temperature T_o.

The quantities k, τ , c are obtained in this code by linear interpolation on tables given as input data.

We consider in a small time interval Δt the system of the two quasi-linear parabolic equations (II-1), (II-2) or (II-5), (II-6) as linear; in other words we suppose that

- 13 -

the coefficients remain constant during this interval. This is possible only when they change very slowly with time and for small Δt .

At every time step Δt , the coefficients are given new values, which are determined according to the temperature reaction and the position of the control rods.

§ III - The finite difference equations

The two diffusion equations can be written in the following form:

(III-1)
$$D_{f}^{k} \frac{\partial^{2}}{\partial z^{2}} \Psi - A^{k} \Psi + B^{k} \phi + C^{k} = \frac{1}{w^{k}} \frac{\partial \Psi}{\partial t}$$

(III-2)
$$D_t^k \frac{\partial^2}{\partial z^2} \phi - E^k \phi + F^k \Psi = \frac{1}{v^k} \frac{\partial \phi}{\partial t}$$

where k(k = 1, 2, 3) is the index of the region; lower reflector, core and upper reflector respectively. The meaning of A^k, B^k, C^k, E^k, F^k is evident.

The height of the reactor has been divided into mesh-points $\{z_i\}$, (i = 1,2,...,L) with $z_1 = 0$ and $z_L = H = height of the reactor.$

On each interface between core and reflector is a point of the lattice, and each region contains at least one internal point; $\Delta_i = z_{i+1} - z_i = \Delta z^k$ is constant in each region \mathbb{R}^k , and can vary from region to region.

Let us consider the ith point of the lattice.

Fig. - 2



The coefficients D_f , A, B, C, $\frac{1}{w}$ and D_t , E, F, $\frac{1}{v}$ depend on the temperature, and on the position of the control rods; therefore they vary from point to point of the lattice.

$$z_{i-\frac{1}{2}} < z < z_i$$
 and $z_i < z < z_{i+\frac{1}{2}}$.

.

In these intervals the coefficients are not supposed to vary.

For the sake of brevity we will carry out the calculation only for eq.(III-1); for eq.(III-2) we will give the final expressions.

$$\int_{z_{i-\frac{1}{2}}}^{z_{i}} \frac{1}{w} \frac{\partial \Psi}{\partial t} dz = \left[D_{f} \frac{\partial \Psi}{\partial z} \right]_{z_{i}}^{z_{i}} - \left[D_{f} \frac{\partial \Psi}{\partial z} \right]_{z_{i-\frac{1}{2}}}^{z_{i-\frac{1}{2}}} \int_{z_{i-\frac{1}{2}}}^{z_{i}} (A\Psi - B\phi - C) dz$$
(III-3)

$$\int_{z_{i}}^{z_{i}+\frac{1}{\Psi}} \frac{1}{\psi} \frac{\partial \Psi}{\partial t} dz = \left[D_{f} \frac{\partial \Psi}{\partial z} \right]_{z_{i}+\frac{1}{\Psi}} - \left[D_{f} \frac{\partial \Psi}{\partial z} \right]_{z_{i}+\frac{1}{\Psi}} - \int_{z_{i}}^{z_{i}+\frac{1}{\Psi}} (A\Psi - B\phi - C) dz$$
(III-4)

where

$$\begin{bmatrix} f(z) \end{bmatrix}_{z_{i}}$$
 and $\begin{bmatrix} f(z) \end{bmatrix}_{z_{i}}$ +

are respectively the left and right limit of f(z) in $z = z_i$.

As fluxes and neutron currents are continuous throughout the reactor it is:

$$\left[D_{f} \frac{\partial \Psi}{\partial z} \right]_{z_{i}} = \left[D_{f} \frac{\partial \Psi}{\partial z} \right]_{z_{i}}$$

Adding (III-3) and (III-4) we obtain:

$$\int_{z_{1-\frac{1}{2}}}^{z_{1+\frac{1}{2}}} \frac{1}{w} \frac{\partial \Psi}{\partial t} dz = \left[D_{f} \frac{\partial \Psi}{\partial z} \right]_{z_{1+\frac{1}{2}}} - \left[D_{f} \frac{\partial \Psi}{\partial z} \right]_{z_{1-\frac{1}{2}}} - \int_{z_{1-\frac{1}{2}}}^{z_{1+\frac{1}{2}}} (A\Psi - B\phi - C) dz$$

$$(III-5)$$

The derivatives $\frac{\partial \Psi}{\partial z}$ are calculated with the central differences:

$$\begin{bmatrix} \frac{\partial \Psi}{\partial z} \end{bmatrix}_{z_{i-\frac{1}{2}}} = \frac{\Psi_{i-\frac{1}{2}-1}^{n}}{\Delta_{i-1}}; \begin{bmatrix} \frac{\partial \Psi}{\partial z} \end{bmatrix}_{z_{i+\frac{1}{2}}} = \frac{\Psi_{i+1}^{n} - \Psi_{i}^{n}}{\Delta_{i}}$$

The derivatives with respect to the time are approximated according to:

$$\begin{bmatrix} \frac{\partial \Psi}{\partial t} \end{bmatrix}_{z_{i}} = \frac{\Psi(z_{i}, t_{n}) - \Psi(z_{i}, t_{n-1})}{t_{n} - t_{n-1}} = \frac{\Psi_{i}^{n} - \Psi_{i}^{n-1}}{\Delta t}$$

The integrals are approximated according to the formula:

$$\int_{z_{i-\frac{1}{2}}}^{z_{i+\frac{1}{2}}} f(z) dz = f_{i}^{-} \cdot \frac{\Delta_{i-1}}{2} + f_{i}^{+} \frac{\Delta_{i}}{2}$$

equation (III-5) then becomes:

$$O = \left[\frac{1}{w} \frac{w - w - 1}{\Delta t}\right]_{z_{i-}} \cdot \frac{\Delta_{i-1}}{2} + \left[\frac{1}{w} \frac{w^n - w^{n-1}}{\Delta t}\right]_{z_{i+}} \cdot \frac{\Delta_{i}}{2} - D_{i+\frac{1}{2}} \cdot \frac{w^n - w^n}{\Delta_{i}} + \frac{\omega_{i+1} - w^$$

$$+ D_{\mathbf{i}-\frac{1}{2}} \frac{\Psi_{\mathbf{i}-1}^{\mathbf{n}}\Psi_{\mathbf{i}-1}^{\mathbf{n}}}{\Delta_{\mathbf{i}-1}} + \left[A\Psi_{\mathbf{n}}^{\mathbf{n}} - B\phi_{\mathbf{n}}^{\mathbf{n}} - C\right]_{\mathbf{z}_{\mathbf{i}}} \cdot \frac{\Delta_{\mathbf{i}-1}}{2} + \left[A\cdot\Psi_{\mathbf{n}-B}^{\mathbf{n}}\phi_{\mathbf{n}-C}\right]_{\mathbf{z}_{\mathbf{i}}} \cdot \frac{\Delta_{\mathbf{i}}}{2}$$

(III-6)

If \underline{z}_i is an internal point of region \underline{R}_k (K = 1,2,3; -reflector; core, reflector) then $\begin{bmatrix} D_f \end{bmatrix}_{i-\frac{1}{2}} = \begin{bmatrix} D_f \end{bmatrix}_{i+\frac{1}{2}} = D_f^k$ and $\Delta_{i-1} = \Delta_i = \Delta z^k$. The functions A, B, C, ware continuous in z_i and equation (III-6) becomes:

$$-\frac{D_{f}^{k}}{\Delta z^{k}}\Psi_{i-1}^{n} + \left[\frac{2D_{f}^{k}}{\Delta z^{k}} + \left(\frac{1}{w^{k} \cdot \Delta t} + A_{i}^{k}\right) \Delta z^{k}\right]\Psi_{i}^{n} - \frac{D_{f}^{k}}{\Delta z^{k}}\Psi_{i+1}^{n} - \Delta z^{k} \cdot B_{i}^{k} \cdot \phi_{i}^{n} = \Delta z^{k} \left(C_{i}^{k} + \frac{\Psi_{i}^{n-1}}{w^{k} \cdot \Delta t}\right)$$
(III-7)

If $\underline{z_i}$ is a point on the interface between $\underline{R_{k-1}}$ and $\underline{R_k}$ then eq.(III-6) becomes:

$$-\frac{D_{f}^{k-1}}{\Delta z^{k-1}} \Psi_{i-1}^{n} + \left[\frac{D_{f}^{k-1}}{\Delta z^{k-1}} + \frac{D_{f}^{k}}{\Delta z^{k}} + \left(\frac{1}{W_{i}^{k-1} \cdot \Delta t} + A_{i}^{k-1}\right) \frac{\Delta z^{k-1}}{2} + \left(\frac{1}{W_{i}^{k} \cdot \Delta t} + A_{i}^{k}\right) \frac{\Delta z^{k}}{2}\right] \Psi_{i}^{n} - \frac{D_{f}^{k}}{\Delta z^{k}} \Psi_{i+1}^{n} - \left(\frac{B_{i}^{k-1} \cdot \Delta z^{k-1}}{2} + \frac{B_{i}^{k} \cdot \Delta z^{k}}{2}\right) \phi_{i}^{n} = \left[C_{i}^{k-1} + \frac{\Psi_{i}^{n-1}}{W_{i}^{k-1} \cdot \Delta t}\right] \frac{\Delta z^{k-1}}{2} + \left[C_{i}^{k} + \frac{\Psi_{i}^{n-1}}{W_{i}^{k} \cdot \Delta t}\right] \frac{\Delta z^{k}}{2} \quad (III-8)$$

A similar result can be obtained with the integration of equation (III-2).

The two diffusion equations of the fast and thermal groups are then transformed into the system of 2 X (L-2) linear equations:

$$\begin{array}{c} -\mathbf{r}_{i1} \cdot \Psi_{i-1} + \mathbf{p}_{i1}\Psi_{i} - \mathbf{t}_{i1} \cdot \Psi_{i+1} - \mathbf{s}_{i}\phi_{i} = q_{i1} \\ (\text{III-9}) \\ -\mathbf{m}_{i1} \cdot \Psi_{i} - \mathbf{r}_{i2}\phi_{i-1} + \mathbf{p}_{i2} \cdot \phi_{i} - \mathbf{t}_{i2}\phi_{i+1} = \mathbf{q}_{i2} \\ \left[\mathbf{i} = 2, \dots, (L-1) \right] \end{array}$$

We have supposed that $\phi_{l} = \phi_{L} = \Psi_{l} = \Psi_{L} = 0$. The coefficients depend on the temperature, on the position of the control rods and on the distribution ϕ_{i}^{n-1} and Ψ_{i}^{n-1} calculated at the preceding time interval.

A subroutine calculates the coefficients for every point of the lattice at every time step.

The solution of this system can be obtained with an iterative method using alternatively Ψ as source of ϕ and ϕ as source of Ψ , or with a direct method. Both methods were thoroughly described in another report (EUR - 5% e"Comparison between the solution by an iterative and a direct method" by Monterosso and Vincenti).

The distribution Ψ_i and ϕ_i are calculated at every time step by another subroutine.

The coefficients (which must be calculated at every time step according to the temperature reaction and the position of the control rods) are presumed to be constant during the interval Δ t which must therefore be sufficiently small. The choice of Δ t is based on a compromise between the precision and the time of calculation.

§ IV The parts of the code and their function

This code consists of two parts:

a) Initial condition:

this section calculates for the cold reactor:

- the uniformly distributed poison Σ_p for which the reactor is overcritical with a required stable period \cdot

- a critical uniformly distributed poison $\Sigma_{po}^{},$ or the critical depth of insertion of the control rods -

- the corresponding distribution $\Psi(z)$, $\phi(z)$, C(z)at the steady state for a required power (this power must be sufficiently small for the reactor to be considered as remaining cold)

b) Dynamic calculation:

this section simulates the movement of the rods and calculates the evolution of flux and temperature in space and time during the power excursion.

§ V - Initial conditions

- Calculation of the uniformly distributed poison Σ_p corresponding to a desired stable period T

When the reactor is not critical, after an initial transient, the flux increases exponentially according to $\phi(t) = \phi_0 e^{at}$, $(a = \frac{1}{T})$, until the temperature reaction takes place.

A poison Σ_p , as a first approximation, is introduced in the diffusion equations; after a certain number of time steps, the code calculates $\alpha = \frac{1}{T}$, and corrects the value of Σ_p in the sense of reducing the error $(\alpha - \overline{\alpha})$, where $\overline{\alpha} = \frac{1}{T}$. The calculation is repeated until $(\alpha - \overline{\alpha}) < \varepsilon$, where ε is an arbitrarily small quantity given as input data.

To avoid the temperature reaction, which perturbs the stable period, these calculations start from an initial arbitrary flux distribution at a very low power ($\phi \approx 10^{10} \frac{n}{cm^2 sec}$); and the reactor is considered as remaining cold during the time of this calculation.

To calculate the first approximation of $\boldsymbol{\Sigma}_p$ we use the formula

$$\overline{a} = \frac{1}{\overline{T}} = \frac{\delta k_{eff}}{1} \qquad (V-1)$$

even though this is valid only near criticality (in our case the reactor is prompt critical).

It is:

$$1 = \frac{1}{(\Sigma_{a}^{C} + \Sigma_{p}) \cdot v} \cdot \frac{1}{(1+B^{2} L^{2})}$$
 (V-2)

$$k_{eff} = 1 - \frac{k_{\infty} \frac{\Sigma_{a}^{c}}{\Sigma_{a}^{c} + \Sigma_{p}}}{(1 + B^{2}L^{2}) (1 + B^{2}\tau)}$$
(V-3)

where

δ

and

$$L^{2} = \frac{D_{t}^{C}}{\Sigma_{a}^{C} + \Sigma_{p}}$$
(V-4)

substituting in (V-1) and rearranging:

$$\Sigma_{\rm p} = \frac{k_{\infty} \Sigma_{\rm a}^{\rm C}}{(1+{\rm B}^2 \tau)} - {\rm B}^2 D_{\rm t}^{\rm C} - \Sigma_{\rm a}^{\rm C} - \frac{\sigma}{\rm v}$$
(V-5)

- Calculation of the critical uniformly distributed poison Σ_{po}

With the same iterative method it is possible to determine Σ_{PO} taking $\bar{\boldsymbol{a}} = \frac{1}{\bar{\boldsymbol{\pi}}} = 0$, starting from:

$$\Sigma_{\rm po} = \frac{k_{\rm o} \Sigma_{\rm a}^{\rm C}}{(1+B^2\tau)} - B^2 D_{\rm t}^{\rm C} - \Sigma_{\rm a}^{\rm C}$$
(V-6)

It is interesting to remark that, for large values of \overline{a} , the $\Sigma_{\rm p}$, calculated by the code with the iterative method, is considerably different from the value obtained with formula (V-5); this is due to the fact that the deduction of the formulae (V-1) to (V-4) is based on simplifying assumptions valid only near criticality. For $\overline{a} = 0$, however, the

 Σ_{po} calculated with (V-6) is almost the same as the value calculated by the code with the iterative method. The small difference is due to the inexactitude of B² in (V-6). In fact TESI is a reflected reactor and B² is the buckling of an equivalent bare reactor, the dimensions of which are determined by a reflector saving calculation. By substituting in (V-6) the final value of Σ_p , it is possible to calculate a more exact value of the equivalent buckling B².

- Critical depth of insertion of the control rods

If the effect of the control rods is equivalent to a uniformly distributed poison Σ_{pB} , then it must be:

 $\Sigma_{p_B} + \Sigma_{p} > \Sigma_{p_O}$

When the control rods are inserted the reactor is undercritical; when they are completely withdrawn the flux evolves with the stable period \overline{T} before the temperature reaction takes place.

At a certain depth of insertion of the bank of control rods, the reactor is critical. To find it, the code utilizes an iterative method (regula falsi): at every successive position of the rods, a is calculated and the iterations stop when $|a| < \varepsilon$.

- The distribution of fluxes at steady state

After having obtained the critical Σ_{po} , or the critical position of the rods, the code repeats the calculation of the fluxes until the stable critical distribution $\Psi(z)$, $\phi(z)$, C(z) is obtained. These distributions are symmetrical or asymmetrical accordingly. The code can now reduce the fluxes to values corresponding to a desired power. This power, of course, must be sufficiently small to have a negligeable heat production.

The iterative calculations will be repeated a number of times. At every iteration the thermal flux $\phi(z)$ is multiplied by the factor

$$F = \frac{A \int_{core} dz}{\int_{core} \phi(z) dz}$$

where A is the average flux in the core corresponding to the desired power. At every iteration, the C(z) are calculated according to:

$$C(z) = \frac{k \cdot \beta \Sigma}{\lambda} \cdot \phi(z)$$

After a number of iterations the fast flux $\Psi(z)$ also reaches the value of steady state in equilibrium with its normalized source $\left[k(1-\beta)\sum_{a} \phi(z) + \lambda C(z)\right]$ (see p. 6 eq.(II-1)).

§ VI - Dynamic calculation

The programme reads an initial distribution $\Psi(z,t_0)$; $\phi(z,t_0)$; $c(z,t_0)$ and $T(z,t_0)$ at the time t_0 and the control rod position $xz(t_0)$. Starting from these values it calculates their evolution in space and time according to the diffusion equations and the movement of the control rods.

The calculation can start from a critical distribution obtained by "Initial Conditions" at the time $t_0 = 0$. It is also possible to reinitiate, from the last distribution at the time $t_0 = t$, an interrupted calculation.

<u>Control rods</u>. The movement of the control rods is simulated in a special subroutine. They can be withdrawn instantaneously, or with a finite velocity which can be changed during the extraction, or reinsertion, according to any given law of movement.

The coefficients p_{i2} of the thermal group equation are calculated with or without poison for all the points of the lattice. The subroutine decides which points are in the rodded or in the non rodded region and assigns to p_{i2} the poisoned or the non poisoned value.

The point z_i of the non rodded region, in the vicinity of the rod, will be partially poisoned and the total absorbtion cross-section is:

$$(\Sigma_{ao} + \delta \Sigma_{po}) \sqrt{\frac{To}{(T(z_{i,t}))}} \qquad 0 \le \delta \le 1$$

where the factor δ is obtained by interpolation according to the position of the rods xz(t) between the two points z_i and z_{i+1} , see Fig. 3.



In this way the total equivalent poison in the reactor is expressed as a continuous function of the position of the rod, independently of the subdivision of the space in mesh points.

Temperature reaction. Equation (II-4) of page 6 is transformed into the explicit system of L equations:

$$\frac{T_{i}^{n} - T_{i}^{n-1}}{\Delta t} = \frac{F}{c_{i}} \frac{k_{i}}{\nu} \Sigma_{ai} \phi_{i}^{n} \qquad (i = 1 \ 2 \ \dots L)$$

The T_i^n are the unknown values of the temperature at each point z_i of the lattice at the time t_n ; T_i^{n-1} are the known values calculated at the time $(t_n - \Delta t)$; Σ_{ai} , k_i , C_i are temperature dependent and therefore they have different values from point to point, their values were calculated according to the temperature distribution T_i^{n-1} at the time $(t_n - \Delta t)$; ϕ_i^n is the thermal flux corresponding to the lost calculation at time t_n .

The values of Σ_{ai} , k_i , C_i are therefore delayed with respect to the values of ϕ_i^n . The error due to this inconsistency can be reduced by diminishing Δt . Many tests were made using several Δt . The fluxes and temperatures calculated

converge to a final value for decreasing Δt . The experience has demonstrated that for $\Delta t \leq 10^{-3}$ sec no practical improvement can be obtained (see Report EUR-596 e).

With the temperatures T_i^n the new τ_i , c_i , k_i are calculated by interpolation between the tabulated values, and the new Σ_a , Σ_p , w, v are calculated with the formulae:

$$\Sigma_{ai} = \Sigma_{ao} \sqrt{\frac{To}{T_{i}^{n}}}; \quad \Sigma_{pi} = \Sigma_{po} \sqrt{\frac{To}{T_{i}^{n}}}; \quad w_{i} = w_{o} \sqrt{\frac{T_{i}^{n}}{To}};$$
$$v_{i} = v_{o} \sqrt{\frac{T_{i}^{n}}{To}}$$

These values corresponding to the time t_n are introduced in the coefficients of the system (III-9) to calculate the new distribution of Ψ_i^{n+1} , ϕ_i^{n+1} , C_i^{n+1} at the time $t_{n+1} = t_n + \Delta t$.

§ VII - Comparison between the conventional method of the kinetic equations and the direct solution of the diffusion equations

The conventional method for studying the dynamics of a reactor is based on the kinetics equations

(VII-1) $\frac{dn}{dt} = \frac{k_{eff}(1-\beta)-1}{1} + e^{-B^2\tau} \cdot \lambda \cdot C$

$$(VII-2) \qquad \frac{dC}{dt} = \frac{k_{\infty}\beta}{l_{\Omega}}n - \lambda C$$

The following equation gives the law of the temperature variation

(VII-3)
$$\frac{dT}{dt} = \frac{F}{C} \frac{k}{v} \Sigma_{ao} \cdot v_{o} \cdot n$$

These equations contain the space average values of n, c and T and do not consider the geometrical configuration of the reactor. From now on we will call this the point-method. The one used in our code will be called the spatialmethod.

The value of k_{eff} is determined by the control rods' position and also by the values of k_{∞} , τ , L^2 , which are known functions of the temperature. It is therefore possible to calculate the function $k_{eff}(T)$, which, introduced into equation (VII-1) gives the temperature reaction.

This set of equations, together with the function $k_{eff}(T)$

can be studied with the analogue computer, or with the digital code AIREC or PINETO.

However, this method is based on simplifying assumptions, which will be now explained:

a) The equations (VII-1) and (VII-2) are deduced from the one-group time-dependent diffusion equation. This deduction is possible only by supposing that the flux is separable relatively to the time and space variables, i.d. the flux

$$\phi(\mathbf{r},\mathbf{t}) = \mathbf{R}(\mathbf{r}) \cdot \mathbf{T}(\mathbf{t}) \quad (\text{VII}-4)$$

actually it should be:

$$\phi(\mathbf{r},t) = \sum_{n=1}^{\infty} \operatorname{Rn}(\mathbf{r}) \cdot \operatorname{Tn}(t) \qquad (\text{VII}-5)$$

we consider only the first mode.

This is true only at criticality and can be assumed for states very close to criticality. The reactor TESI, however, operates in conditions of prompt criticality.

b) The expressions of k_{pff} and 1 appearing in (VII-1)

$$k_{eff} = \frac{k_{\infty}e^{-B^2\tau}}{(1+B^2L^2)}$$
 $1 = \frac{1_0}{(1+B^2L^2)}$

contain B^2 (Buckling) or the first of the proper values of the solution (VII-5). As the reactor is reflected, B^2 can only be obtained considering an equivalent bare reactor, the dimensions of which are determined by a reflector saving calculation. As before this calculation has a meaning only at criticality or close to it. c) The temperature reaction given by the function $k_{eff}(T)$ can be calculated only using a value of T averaged throughout the core. The temperature however is different from point to point and its spatial distribution changes according to the production of heat. This deforms the shape of the flux distribution and influences the reactivity. This effect can not be taken into account using this method.

d) The k_{eff} depends on the position of the control rods xz and on the temperature T. These two variables in the function $k_{eff}(T,xz)$ are non separable. However the only possible way of determining k_{eff} with the analogue computer or with the Airek Code is to consider their effect separately.

These simplifying assumptions and their consequent effects on the precision of the results can be avoided by the method of the direct solution of the time-dependent-diffusion-equation considered in each point of the reactor core and reflectors.

It is interesting to compare the results of both methods for the same case. The <u>choice of a suitable example</u> for this comparison is however rather difficult because the two methods have a completely different approach to the problem.

In this example, to avoid the deformation of the flux distribution due to the movement of the control rods, these are withdrawn instantaneously (which corresponds to a step of reactivity in the point-method). This instantaneous extraction of the rod avoid furthermore the difficulties explained in d) ; in fact only the temperature reaction on reactivity takes place during the power excursion. The control rods at the interface between core and radial reflector are kept completely inserted. By this means it is possible to calculate exactly the radial buckling $B_r^2 = \left(\frac{2.405}{R}\right)^2$, where R = radius of core. The error of the buckling determination [b) p. 24] is limited to the axial buckling.

Fixed a value of \bar{a} , the Σ_p can be calculated with the iterative method of p.15. Then using the formulae (IV-1) to (IV-4) it is possible to calculate the corresponding δk_{eff} to be used in the kinetic equations.

§ VIII - Some examples of calculation

The Code Costanza can plot automatically the curves by a CALCOMP DATA-PLOTTER. See Figures 7 to 11.

a) Control rods extracted with constant velocity

Fig. 7 shows the curves of $\Psi(t)$ and $\phi(t)$ obtained with an instantaneous expulsion of the control rods, and by extracting the rods with constant velocity in 0.15 sec. The poison left in the core is, in this case, $\Sigma_p = 0.00007$ cm⁻¹. At the time the control rods are completely extracted, the temperature reaction has not yet reached its effect, therefore the curves $\Psi(t)$ and $\phi(t)$ are almost the same in both cases, they are shifted forwards for the ramp extraction. In Fig. 8 are the curves of the average temperature T(t).

Figs. 9 and 10 are the spatial distributions $\Psi(z)$ and $\phi(z)$, when the control rods are extracted in 0.15 sec, at the time 0.075 sec when the rods are still in the core, and at the time 0.241 sec when the maximum of $\overline{\phi}(t)$ is reached.

Fig. 11 shows the temperature distribution of 0.501 sec. Fig. 12 is a page of the listing of the IBM 7090 with the input data and the tables of k_{∞} , τ and specific heat. In Fig. 13 are the initial distributions and Figs. 14, 15, 16 are the prints corresponding to the times 0.1, 0.24, 0.501 sec.

(For the meaning of the symbols see the WRITE-UP of the Code Costanza; some remarks and descriptions of symbols have been written with a type-writer on the listing it-self).

If the rods are extracted with a low velocity, the oscillations take place. Fig. 17 reports the function $\bar{\phi}(t)$ corresponding to a rod velocity of 40 cm/sec. In Fig. 18 the non symmetrical flux and temperature distribution and the control rods' positions are plotted, corresponding to the peaks of Fig. 17 at the times t = 2.63 sec and t = 3.62 sec.

b) Rod movements for constant power

One of the interesting operational conditions of TESI consists in reaching in a short time a certain power level and in maintaining it as lons as possible, according to the temperature reaction and to the total built-in reactivity.

The desired power can be reached in the shortest time by an extremely rapid expulsion of a certain number of control rods. Once the required power has been reached, the reactor must again become critical and some rods are shot into the reactor again. If in the meantime the temperature has reached a considerable level not all the extracted rods are reinserted. Some of them remain out to compensate the temperature reaction. After this first phase, the temperature reaction increases continuously and this will be compensated by again extracting the control rod at a reduced velocity. The total expulsion and reinsertion of the first phase are obtained with pneumatic devices, with a maximum permissible acceleration of 100 g. The movement at reduced velocity of the second phase is obtained by mechanical engines. An additional subroutine of the code simulates the expulsion and reinsertion of the rods. The reduced number of rods can easily be simulated by using a convenient value of Σ_{pB} in the rodded region. This subroutine determines continuously the position of the control rods according to the power wanted and the temperature reaction. The movements are kept within the limits of the maximum mechanical speed possible. Fig. 19 contains the curves of the rod position as function of tiand of the average temperature $\overline{T}(t)$. Fig. 20 conme tains the n(t), average neutron density, as function of time.

















COSTANZA - CONTROL RODS EXTRACTED IN 0.15 SEC - SPR=0.00007

DZ 1	DZ2	DZ3	DRF	DCF	DRT	DCT
0.14500E 02	0.14500E 02	0.14500E 02	0.11000E 01	0.10660E 01	0.98600E 00	0.95500E CO
DT	SARZ	SACZ	WZ	٧Z	SPZ	τz
1.00000E-03	0.26350E-03	0.24770E-02	0.30800E 07	0.24860E 06	0.58338E-03	0.29300E 03
BU	В	DL	SPR	PREC	VNU	FC
0.34400E-03	0.67600E-02	0.76790E-01	0.70000E-04	1.00000E-04	0.24700E 01	0.76600E-11
ΧZ	TF	VKP	TRP	TCP	CSP	
0.	0.50000E 00	-0.55500E 00	0.	0.50200E-04	-0.57500E-02	
M = 5	N = 15	L = 19	INT = 6	KL = 20	KI = 0	K S = 0
	т	VK	Ť	AUR	TAUC	CS
	0.29300E 03 0.39300E 03 0.49300E 03 0.59300E 03 0.69300E 03 0.79300E 03 0.99300E 03 0.109300E 04 0.112930E 04 0.12930E 04 0.15930E 04 0.15930E 04 0.15930E 04 0.15930E 04 0.15930E 04 0.15930E 04 0.15930E 04 0.16930E 04 0.18930E 04 0.18930E 04 0.12930E 04	0.18315E 01 0.18255E 01 0.18195E 01 0.18145E 01 0.18055E 01 0.18055E 01 0.18055E 01 0.17995E 01 0.17995E 01 0.179933E 01 0.17990E 01 0.17990E 01 0.17890E 01 0.17880E 01 0.17855E 01 0.17855E 01 0.17855E 01 0.17855E 01 0.17855E 01 0.17855E 01 0.17855E 01 0.17855E 01	06666666666666666666666666666666666666		0.40200E 03 0.39700E 03 0.393060E 03 0.39360E 03 0.38850E 03 0.388500E 03 0.388500E 03 0.388500E 03 0.388500E 03 0.388180E 03 0.388180E 03 0.38900E 03 0.37900E 03 0.377800E 03 0.377800E 03 0.3774500E 03 0.3772500E 03 0.3772500E 03 0.377250E	0.25200E-00 0.35000E-00 0.44000E-00 0.55500E 00 0.60000E 00 0.66000E 00 0.66000E 00 0.66000E 00 0.66000E 00 0.75000E 00 0.75000E 00 0.75000E 00 0.76000E 00 0.778000E 00 0.78600E 00 0.79800E 00 0.79800E 00 0.79800E 00
Fig. I2	T = temperatur VK = K _e infini	es [°K] te multiplication	1 factor 1	AUR = Fermi-Ag AUC = Fermi-Ag CS = Specific	ge in the reflector: ge in the core heat of core	$\begin{bmatrix} cm^2 \\ cm^2 \end{bmatrix}$

RISOLUZIONE	CON	METODO	ITERATIVO	INT =	6

		CONDIZIONI	INIZIALI			
T0 = 0.	XZ =	0.				
Z	TR	тс	С	PH 1	PH2	N
0.145000EE022 0.295000EE022 0.435000EE022 0.435000EE022 0.55825000EE003 0.1016050EE003 0.11459500EE003 0.14595000EE003 0.1459500EE003 0.1459500EE003 0.1459500EE003 0.1459500EE003 0.14595000 0.14595000 0.14595000 0.1459500000000000000000000000000000000000	0.29300E 03 0.29300E 03	0.29300E 03 0.29300E 03	0.26334E 07 0.32161E 07 0.439979E 07 0.439979E 07 0.48577E 07 0.47409E 07 0.43997E 07 0.43997E 07 0.32161E 07 0.32161E 07 0.26334E 07	0.54178 0.54178 0.13901 0.30251 0.30251 0.130579 0.135746 11 0.156968 11 0.16968 10 0.16968 0.16968 0.1008 0.16968 0.1008 0.0008	0. 0. 15561E 10 0.32396E 10 0.504838E 10 0.659529E 10 0.966809E 10 0.11071E 11 0.12164E 11 0.12164E 10 0.80529E 10 0.80528E 10 0.659485E 10 0.659485E 10 0.52485E 10 0.5561E 10 0.5564E 10 0.5561E 10 0.5561E 10 0.5561E 10 0.112164E 10 0.112164E 10 0.12164E 10 0.12164E 10 0.12564E 10 0.5564E 10 0.5564E 10 0.5561E 10 0.5564E 10 0.5561E 0.5561E 0.556	0.655 93074300055 93074300055 93074300055 0.12062394200055 0.2202931200055 0.2202931200055 0.2202931200055 0.2202931200055 0.2202931200055 0.2202931200055 0.2202931200055 0.2202931200055 0.220315 0.2005 0.2555 0.25555 0.255555 0.255555 0.255555 0.255555 0.255555 0.255555 0.255555 0.255555 0.255555 0.255555 0.255555 0.255555 0.255555 0.255555 0.255555 0.255555 0.255555 0.255555 0.255555 0.0555555 0.0555555 0.0555555 0.0555555 0.0555555 0.0555555 0.0555555 0.0555555 0.05555555 0.0555555 0.0555555 0.0555555 0.0555555 0.0555555 0.0555555 0.055555555
VALORI MEDI		0.29300E 03	0.39937E 07	0.13723E 11	0.99999E 10	0.40225E 05

Fig. I3

The calculation is made with the iterative method, with a maximum of 6 inner iterations.

TO = 0.10000	ITER = 100						
XZ = 173.99998	VB = 0.1740	00E 04 REP =	0.114054E 03	DP = 0.379130#	14 PHI = 0.4	+27778E 10 VM =	0.248600E 06
Z	TR	TC	с	РН1	PH2	N	
0.14500E 02 0.29000E 02 0.43500E 02 0.58000E 02 0.72500E 02 0.72500E 02 0.10150E 03 0.11600E 03 0.13050E 03	0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03	0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03	0.27129E 07 0.33090E 07 0.39729E 07 0.45149E 07 0.48577E 07 0.486400E 07	0.20561E 11 0.52886E 11 0.11547E 12 0.24434E 12 0.24034E 12 0.50560E 12 0.57201E 12 0.50595E 12 0.55595E 12	0. 0.67748E 11 0.13968E 12 0.21450E 12 0.27497E 12 0.32873E 12 0.32875E 12 0.42897E 12 0.42897E 12 0.44855E 12 0.444259E 12 0.44212E 12	0. 0.27252E 06 0.56186E C6 0.86282E C6 0.11061E 07 0.15547E 07 0.15546E 07 0.18043E 07 0.16537E 07	
0.15950E 03 0.17400E 03 0.18850E 03 0.20300E 03 0.21750E 03 0.23200E 03 0.24650E 03 0.26100E 03	0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03	0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03	0.44823E 07 0.39319E 07 0.32667E 07 0.26722E 07	0.48565E 12 0.39266E 12 0.28848E 12 0.16864E 12 0.79733E 11 0.36505E 11 0.14188E 11 0.	0.35576E 12 0.27887E 12 0.21340E 12 0.16340E 12 0.11872E 12 0.73270E 11 0.34318E 11 0.34318E 11	0.14310E 07 0.11218E 07 0.85842E 06 0.65728E 06 0.47754E 06 0.29473E 06 0.13805E 06 0.	
VALORI MEDI		0.29300E 03	0.40837E 07	0.46025E 12	0.35137E 12	0.14134E 07	
Fig. I4	$TO = time$ $ITER = number of time steps$ $XZ = position of the control rods$ $VB = control rod velocity$ $REP = reciprocal of the reactor period = \frac{I}{T}$ $DP = \frac{d\vec{\Psi}}{dt}$		TR = ter $TC = ter$ $C = pro$ $PHI = far$ $PH2 = thr$ $N = ner$	mperature in ref mperature in cor ecursor density st flux ermal flux utron density	lector [•] K • c/cm ³ n • n/cm³		
	PHI = $\int_0^t \overline{\varphi}(t)$	dt		Under ea correspo	ch column are re nding average va	ported the lues.	
	VM = neutron	velocity of the	thermal group				

T0 = 0.24000	ITER = 240						
XZ = 260.99996	VB = 0.	REP =	0.274671E 01	DP = 0.675901E	19 PHI = 0.	562321E 17 VM =	= 0.499579E 06
Z	TR	TC	с	PH1	PH2	Ν	
0.145000E 022 0.295000E 022 0.435000E 022 0.43805000E 022 0.5885000E 023 0.11500E 033 0.1135500E 033 0.1159500E 033 0.1459500E 033 0.1459500E 033 0.1459500E 033 0.1459500E 033 0.12594500E 033 0.125946500E 033 0.2217500E 033 0.2246500E 033 0.2246500E 033	0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03	0.97205E 03 0.10748E 04 0.11711E 04 0.12440E 04 0.12485E 04 0.12885E 04 0.12885E 04 0.12845E 04 0.12845E 04 0.12440E 04 0.12748E 04 0.10748E 03	0.69883E 12 0.83910E 12 0.97482E 12 0.97482E 13 0.11453E 13 0.11453E 13 0.11453E 13 0.10800E 13 0.97482E 12 0.83903E 12 0.69883E 12	0. 0. 0. 0. 0. 0. 0. 0. 0. 0.	0. 37724E 18 0.77519E 19 0.11975E 19 0.20030E 19 0.20030E 19 0.23990E 19 0.229137E 19 0.229137E 19 0.229137E 19 0.229137E 19 0.229137E 19 0.229137E 19 0.229030E 19 0.229030E 19 0.23930E 19 0.23930E 19 0.23930E 19 0.23930E 19 0.23930E 19 0.237519E 19 0.237519E 18 0.37724E 18	0.151752E 13 0.3118705E 13 0.4430687E 13 0.4430687E 13 0.4422288E 13 0.4482088E 13 0.555886804E 13 0.55529268680E 13 0.55529268680E 13 0.55529268680E 13 0.5552926800 0.4288078E 13 0.4288778E 13 0.4431782E 13 0.4431782E 13 0.4431782E 13 0.4431782E 13 0.4431782E 13 0.4431782E 13 0.4431782E 13 0.4431782E 13 0.4431782E 13 0.45576800000000000000000000000000000000000	
VALORI MEDI		0.11832E 04	0.99446E 12	0.15622E 19	0.24641E 19	0.49909E 13	

Fig. 15

TO = 0.50100 XZ = 260.99996	ITER = 501 VB = 0.	RÊP = -	-0.330473E-00	DP = -0.384903E	15 PHI = 0.	137855E 18 VM =	0.623441E 06
Z	TR	rc	С	PH1	PH2	Ν	
0. 14500E 02 0.29000E 02 0.43500E 02 0.58000E 02 0.72500E 02 0.87000E 03 0.10150E 03 0.114500E 03 0.135950E 03 0.15950E 03 0.15950E 03 0.174050E 03 0.20300E 03 0.20300E 03 0.20300E 03 0.20300E 03 0.21750E 03 0.2175E 03 0.2175E 03 0.2175E 03 0.2175E 03 0.2175E 03 0.2175E 03 0.2175E 03 0.2175E	0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03 0.29300E 03	0.14884E 04 0.16630E 04 0.18230E 04 0.19439E 04 0.20180E 04 0.20180E 04 0.20180E 04 0.20180E 04 0.19439E 04 0.18230E 04 0.18630E 04 0.14884E 04	0.14859E 13 0.17649E 13 0.20249E 13 0.22245E 13 0.23476E 13 0.23476E 13 0.23476E 13 0.23476E 13 0.22249E 13 0.22249E 13 0.17641E 13 0.14859E 13	0. 0.25244EE 14 0.644772EE 14 0.140755EE 15 0.296322EE 15 0.4603125EE 15 0.4603125EE 15 0.4603125EE 15 0.72573EE 15 0.7251254EE 15 0.7251254EE 15 0.46033229EE 15 0.46033229EE 15 0.42968522EE 15 0.4295852EE 15 0.42968522EE 15 0.4295852EE 15 0.42958522EE 15 0.4295852EE 15 0.429585585585585585585585585585585585585585	0. 0.17073E 15 0.351997E 15 0.54797E 15 0.744578E 15 0.91378E 16 0.128485E 16 0.137845E 16 0.137845E 16 0.137845E 16 0.137845E 16 0.137845E 16 0.1378576E 15 0.744797E 15 0.547974E 15 0.351992E 15 0.3519745 15	0.68676E 09 0.14159E 10 0.22042E 10 0.17183E 10 0.17569E 10 0.18317E 10 0.221136E 10 0.21136E 10 0.21136E 10 0.21136E 10 0.21136E 10 0.18317E 10 0.18369E 10 0.18369E 10 0.171842E 10 0.220465E 10 0.48476E 09	
0.26100Ē 03 Valori medi	0.29300E 03	0.18427E 04	0.20597E 13	0. 0.62475E 15	0. 0.11645E 16	0. 0.18963E 10	

Fig. I6









References

AIREK:	"Generalized Reactor Kinetics Code";
	Atomics international, No.4980.
COHEN, E.R.:	"Some Topics in Reactor Kinetics";
	Geneva Conference 1958, Volume 11, P/629 USA.
FOX, L:	"Numerical Solution of Ordinary and Partial Differential Equations";
	Pergamon Press - 1962
KAPLAN, S.:	"Some New Methods of Flux Synthesis";
	N.S.&E. 13 - 1962.
P.D.Q.:	"An IBM-704 Code to Solve the Two-di- mensional Few-Group Neutron Diffusion Equations";
	WAPD-TM-70
RICHTMYER, R.D.:	"Difference Methods for Initial-Value Problems";
	Interscience Publishers, Inc., New York.
SCHECHTER, S.:	"Quasi-Tridiagonal Matrices and Type- insensitive Difference Equations";
	AEC Computing and Applied Mathematics Center - TID-4500, NYO-2542.
STAB:	"A Kinetic, Three-Dimensional, One- Group Digital Computer Program";
	AEEW-R.77.
VARGA, R.S.:	"Matrix Iterative Analysis"; Prentice-Hall, Inc.
WACHSPRESS, E.L.:	"Digital Computation of Space-Time Va- riation of Neutron Fluxes";
	KAPL - 2090.
WANDA:	"A One-dimensional Few-Group Diffusion
	Equation Code for the IBM-704".
R.MONTEROSSO and E.VINCENTI	"Finite Difference Method for Solving the Spatio-Temporal Diffusion Equation in the Two-Group Approximation"
	EUR 596.e

References

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C.BONA, C.TAMAGNINI

PINETO - A Fortran Code for Systems of Quasi-Linear Differential Equations, Particularly Suitable for Nuclear Reactor Dynamics.

To be published.

To disseminate knowledge is to disseminate prosperity - I mean general prosperity and not individual riches - and with prosperity disappears the greater part of the evil which is our heritage from darker times.

Alfred Nobel

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