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nuclear science and technology

APPLICATION OF THE METHODS OF STATISTICAL ANALYSIS OF PSEUDO-RANDOM SIGNALS TO THE INTERPRETATION OF FIELDS TRIALS ON THE ATMOSPHERIC TRANSPORT OF ACTIVE PRODUCTS AND TO ESTIMATE OF CONFIDENCE LIMITS

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TABLE OF CONTENTS

ABSTRACT

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An attempt is made at identifying the most important factors which introduce difficulties in the analysis of results from tests on pollutant dispersal : the unsteadiness of the phenomenon, the effect of external uncontrollable parameters, and the inherent complexity of the problem itself.

The basic models for prediction of dispersion of passive contaminants are discussed, and in particular a Lagrangian approach which seems to provide accurate results.

For the analysis of results many problems arise. First the need of computing for the results the statistical quantities which describe them: the mean, the variance and higher order moments are important.
It is shown that there is no easy solution if the duration and/or the number of independent "events" to be analyzed are too limited. The probability density function provides the most useful information, but is not easy to measure. A family of functions is recalled which predict reasonably well the trend of the pdf. Then the role of intermittency is shown in some detail. Its importance cannot be underestimated and its relationship to pdf and the effects on measurements are shown to be rather complex.

Finally, an example is made to show the effects of the variance of external factors.

1. INTRODUCTION

The purpose of the present research is to determine the limits of confidence which can be associated with concentration measurements in full scale tests, where most of the dominant elements governing the phenomenon are not under full control.

These may be an incomplete knowledge of the meteorological conditions, the uncertainties of the characteristics of the release source, and even accuracies of the measuring instruments.

Knowing that the problem of turbulent diffusion is one of the most complex in fluid dynamics, adding all the uncertainties associated with experiments in real atmospheric flows, makes the task very daunting. Really, one should start from the analysis of the solution of Navier-Stokes and diffusion equations for stochastic boundary conditions.

For the present research, the author hopes to arrive at some useful conclusions of practical interest, but so far as the work advances the problem becomes so wide and takes on so many facets that a sort of unitary view of the problem still seems difficult to achieve.

The present report is not intended to provide any conclusion, but just illustrates some of the subjects available in the literature and which have attracted the interest of the author as useful in the present context. Of course some discussion will be made and some conclusions drawn, but the author himself is not fully satisfied nor absolutely sure that they should not be revised in future research.

For the problem stated above, there are at least two possible ways of looking for a solution. One is to start from the available full scale results or wind tunnel measurements and to try to correlate them with appropriate rules. This approach which can be considered as a trial and error one is in fact not so bad, not being based on a crude attempt to fit a curve into the available data, but on what can be considered an educated guess based on accumulated knowledge of average behaviour of dispersion phenomena.

The second approach is based on the analysis of the fundamental equation governing the dispersion phenomena in turbulent flow fields. The hope here is to be able to understand and master the essential elements involved without running into the problem of too big mathematical difficulties in the solution. That the problem in general is not so easy has already been said and the sort of difficulties which may be encountered when trying to deal with the whole situation is clearly illustrated in the analysis of Farmer. Nevertheless, if one really wants to arrive at conclusions supported by an understanding of the phenomena involved, this is the only way to approach the problem.

Again there are two possibilities of looking at it; one which may be called microscopic and the other macroscopic. Without going into too much detail, one can establish the difference between the two by considering that in the first one the real mechanism of turbulent dispersion is analyzed and the final results obtained by the logical consequences of each step. In the second only the gross features are looked at, such as mean concentration, meandering, intermittency and so on, the finest details being hidden or covered by more or less local coefficient of diffusion of universal nature and acting at a scale much smaller than the one of interest.

The problem of the accuracy or better of the statistical relevance of the measured results is also important; especially if one has to take into account the averaging effect of the finite size of the instruments themselves and of the region of the flow over which they respond.

In the following it will be attempted to provide some, at least qualitative, answer to the above questions.

2. SOME EXPERIMENTAL OBSERVATIONS

This type of approach, based on the fitting of experimental data with empirical correlation is widely used and well documented in the literature. As mentioned before, such work is far from being as trivial as it seems from the above statement, because if consistent results are looked for, they should be based not only on sound experimental evidence, but also on a correct interpretation of the fundamental mechanism of turbulent dispersion. Experimental data can here be considered as originating either from full scale or wind tunnel simulated tests, although the main field of application of such an analysis is for the interpretation of full scale results. Accounts, descriptions and justification of such approaches are given by Csanady, Larsen, Cats and Haltslag, Nieuwstad, Tsukatani, to quote some examples.

In each case the problem is that of determining the dependence of air pollution frequency distribution on meteorological data. The starting point can probably be traced back to Larsen who suggested in 1969 that air pollution frequencies in complex source areas are approximately described by a log-normal distribution of the probability.

There are some good reasons to suggest such a conclusion, and in particular the fact that this can be considered as the limit to be achieved when dealing with large sequences of measured air pollution concentration. In fact, it probably can be considered as a limit situation which is reached when it is assumed that air pollution concentrations are independent and identically distributed.

As always the proof of any theory rests on the availability of a good set of experimental data : ideally this will require the measurement at a given site over a long period of the entire probability distribution function of the concentration.

Long sequences of observation are required because air pollution concentration depends on the random fluctuation of weather and also on the variability of emission from the source itself.

Also the actual concentration depends very much on the local topography near the source emission and near the monitoring point.

But probably the wind velocity and direction (and their variabilities) are the main meteorological factors determining local concentrations. It should also be stated that actual, local concentration in itself is not a useful parameter being a random variable, and so likely to be described only by a large set of information. Depending on the type of pollutant and on its possible effect, what is more useful is to be able to describe, or predict, the statistical properties of say maximum values, or of exceeding prescribed values for a specified time or to be within certain prescribed limits.

It is in this context that the availability of air pollution frequency distribution model is helpful because it allows the further estimation of the required quantities. It is also important here to note that there are a few different situations which may require different analysis. Without going into the details it can be expected that different approaches should be used, let us say, for the case of distributed sources as opposed to point sources, for the case of the data being the results of a number of individual short time releases opposed to continuous sources and so on.

More important (as will be discussed later on because it involves the applicability of the models themselves) is the difference between sources near the ground and elevated ones.

It seems that the applicability of the above described model is best suited to the case of distributed ground sources, which clearly best approach the limit condition consisting of dealing with a large number of independent realisations.

For the same reason it seems that such an analysis is least well suited for the prediction of a "one-off" release of pollutants of short duration.

Even in the best conditions there is strong evidence (see above references), that the log normal distribution does not always fit the data in the best possible way (it should be stated again that the problem here is not to determine the centerline trajectory of pollutant release nor its mean concentration. These definitely are important parameters, even more so because the mechanism of turbulent transport is the same whether one looks for the instantaneous values or the mean one, and after all the mean concentration is nothing else than the cumulative probability of the instantaneous realisation. The relations one is looking for are, as stated before, those giving the probabilities of having certain specified values for a specified duration).

Most of the results show some kind of variation with respect to the log normal model. Such discrepancies are not always large and it is always difficult to attribute them to a true behaviour, to the fact that the available data are insufficient for a correct statistical analysis, or that (as is often said) eventual background levels are difficult to separate from the rest. The fact that these differences are often small is not sufficient to say that they can be neglected. If this can be done is some cases, when the conditions are critical, either because of the nature of the pollutant or because of the site, further refinement is required.

There are furthermore some other reasons for the discrepancies, which may seem more relevant, such as the fact that the conditions of independence are not (or cannot be) reached in the actual situations. Another reason could be the dominant effect of large scale eddies in the atmosphere.

The fact that better approximations can be obtained by using different distributions for concentration frequencies, and in particular some with more free parameters than the log normal one, does not really solve the problem, which is to know which one has to be applied when an assessment has to be made about the limits of confidence to be associated with the result of a given test or of a given survey of site conditions.

3. MODELS FOR DISPERSIONS OF PASSIVE CONTAMINANTS IN TURBULENT FIELDS

Reviews of models for pollutant dispersal in turbulent flows abound. References describing the problem are in Hama, Csanady, Robins, Farmer, not to mention the fundamental book of Monin and Yaglom, and the extremely interesting even if very complex paper by Lin. In view of this there is no point in remaking in full such a review here except to mention a few points which are considered of primary concern.

For the sake of simplicity, all the following analyses and examples will be made considering a time dependent, unidimensional or plane flow field. Extension to multi dimensional problem and to sheared flows is discussed in the literature, (but it is not evident that the word easily can be used for such an extension).
From a practical standpoint contaminants can be classified as :

- Passive
  - no action on the flow field;
  - uniform density, no gravity forces;
  - no momentum excess or defect;
  - in practice simply a tagged or marked particle.
- Active
  - action on flow field;
  - density difference, gravity forces;
  - etc.
- Reactive
  - chemical reaction, change in volume, etc.

The following analysis will essentially deal with passive contaminants.

3.1 Notes on terminology

It was said that some definition of terms to be used to describe the turbulent transport contaminants is required. It is in particular important to make a clear difference between the meaning of terms such as "diffusion" and "dispersion".

DISPERSION is a general term and in most contexts it can be used to describe all the phenomena associated with turbulent transport.

DIFFUSION is a much more restrictive phenomenon and when associated with turbulence it should be used with care. It is important to note in this context that all molecular transport phenomena can be considered as "diffusive" from a macroscopic standpoint. This is true when the Knudsen number is large enough, which is the case for all practical problems of interest in our context. It was already said that for a turbulent flow the situation is more complicated because one is always dealing with scales which cannot be neglected in comparison to mean flow scales. Dispersion can be considered as diffusion only if it can be assumed to take place in a system which has no memory of its past history. This means that the future state of each particle in the system is only determined by the present status of the system, independently from the way taken to arrive at it.

As a consequence it may be stated that any variable in a system which satisfies the above conditions must have a correlation scale small enough in comparison to the time involved in the computations. In a discretized system the variable W_t must be uncorrelated for successive time steps, that is, its correlation is defined by Dirac delta function. Such systems are stochastic. Continuous ones, defined as MARKOVIAN systems, are characterized by a correlation function of the type :

$$R_0 = \exp\left(\frac{-t}{T}\right)$$

Diffusion can only take place in such systems, as opposed to dispersion. Because the notion of diffusion is encountered very often in turbulence problems (eddy diffusivity, gradient diffusion, and so on) such a definition is important and clearly is bound to set limits to the applicability of some theories. The existence of such limits is well understood in most cases, but the full implications of the limits are so stringent that sometimes they are neglected in practical applications. Nevertheless, this does not always lead to easily identifiable consequences.

It should be noted that the above correlation implies that variables have no microscale, but a macroscale T, and very often it is used to desribe the longitudinal correlation of velocity in homogeneous isotropic turbulence (Ref. 15,3).

The mean concentration at a given point and time C(x,t), form a source at x_0, with intensity $C(X_0)$ will be defined as :

$$\bar{C}(x,t) = \int_{-\infty}^{\infty} P(x,t,x_0)\ C(x_0)\ dx_0 \qquad (1)$$

where $P(x,t,x_0)$ is the probability of a particle originating from x_0, being at the time t at the point x.

This simply means that one is looking at all the possible random trajectories of the particles generated in x_0, for all the possible sources x_0 (Ref. 12).

It should be noted that the above equation goes backwards, i.e., from a fixed observation point to any source point. When required such an approach can be reversed if it is more practical. Second, it reduces the problem to that of determining particle trajectories so it is essentially a Lagrangian approach to the problem. The problem of determining the particle trajectories in turbulent flows can be done using the assumption that one is dealing with a stochastic system. The knowledge of the trajectories provide the necessary information on the shape of probability $P(x,t,x_0)$. Otherwise the equations for the transport of P can be written without considering each single trajectory : it will be briefly shown that this is equivalent to considering the problem from an Eulerian standpoint.

In all cases to solve the problem the statistics of the velocity field must be known in their entireness, which is a very demanding request.

Simple models (such as gradient models, or eddy diffusion model) only require knowledge of the "gross features" of turbulence, namely of typical macroscales, and variance of velocity. From this simple example it is evident why they are much more widely employed. But this is also a reason for a number of their major limitations.

In the above approach the actual path from x_0 to x is not important in itself, but is determined by the characteristic of the flow field. Not all the possible positions x can be obtained, from x_0, so for a given elapsed time there will be regions where the probability is identically equal to zero. The properties of the boundary of the region of interest are also important and a detailed analysis is given in reference 12. The details of this model will be discussed later on.

3.2 The gradient diffusion equations

On the other hand, by making the statistical average before the determination of the paths of the particles, one may define the concentration C as a continuous property which is, as such, transportable in an Eulerian framework.

Then one can write the transport equation (or conservation equation) for C as :

$$\frac{DC}{Dt} = \frac{\partial}{\partial x_j}\left(K\,\frac{\bar{C}}{\partial x_i}\right) \qquad (2)$$

where K is the molecular diffusivity.

With the above observations it could be stated that equation 2 is a "diffusion" equation. From the above considerations, and considering the definition 1, if the flow field is turbulent, the concentration C at a given point and time C(x,t), will also be a random variable, with average value $\bar{C}$.

Making the usual separation, one writes :

$$C = \bar{C} + c$$

(where for a time dependent flow $\bar{C}$ has the meaning of an ensemble average and not time average).

Then, limiting the case to a plane parallel flow for simplicity

$$\frac{\partial C}{\partial t} + U\,\frac{\partial C}{\partial x} = \frac{\partial}{\partial x_i}\left(K\,\frac{\partial C}{\partial x_j}\right) + \frac{\partial \overline{uc}}{\partial x} + \frac{\partial \overline{uv}}{\partial y} \qquad (3)$$

The three dimensional components u,v,w appear because of the three dimensional nature of turbulence which is essential (the contribution on w disappears because of lack of gradients in that direction).

The important terms here are ($\overline{uc}$) and ($\overline{uv}$). They describe the tranport of C generated by the turbulence motion. They are important first because they are a priori unknown, and second because their effect may be that equation 3 is no more a simple "diffusion" equation. This is because the scale over which their action is felt may be not negligible in comparison with the mean scale associated to the gradients of C (as is the case when only K is acting). In fact in most practical situations they are so large that the effect of K can be neglected and the corresponding term suppressed in the equation (see § 3.5).

The way of determining the ($\overline{u_i'c}$) terms is the whole problem of turbulence modelling, which remains largely empirical or phenomenological, because there is no "theory of turbulence" as such available. A good analysis of the problem can be found in reference 1.

The problem is that the correlation $\overline{uc}$, a "gross feature" of turbulence, is unknown, and for that matter very difficult to measure. Results are available, (references 7 and 14, for example), but are very limited. In any case it is impossible to write an exact equation for the evolution of $\overline{cu}$ which does not include further unknowns of higher order. Unless one goes back to a stochastic approach, but then one is changing one's model.

The simplest approach is to analyse their dimensions :

$$\overline{uc} = \left[C \frac{L}{T}\right] = \left[\frac{C}{L} \frac{L}{T} L\right] \qquad (4)$$

In the second expression it is easy to see that they can be expressed as the product of a term with dimension of concentration gradients and a second with dimension of "diffusivity". So in a first simplified approach one may write (Ref. 10) :

$$\overline{u_i c} = \eta_i \frac{\partial C}{\partial x_i} \qquad (5)$$

There is nothing wrong about that, if one assumes for η a variable value

$$\eta = \eta(x,t)$$

except that this does not solve the problem and one has to make assumptions for the dependence of η on the other known parameters. For boundary layer flows this can be of the type (Ref. 7) :

$$\eta = u^{*}(y+y_0)\cdot f(\frac{y}{\delta}), \qquad \text{with } f = \alpha \text{ near the surface} \qquad (6)$$

u^{*} the shear velocity, and y_0 the roughness thickness.

A couple of points are important in this context. First is the fact that the $(\overline{u_i c})$ terms are only dependent on absolute position in the flow field (and absolute time, for time dependent flows. The same, because of 5, should be applied to η so that

$$\eta = \eta(x,y,t)$$

in absolute terms. So one is not allowed, in principle, to use expressions for η which are based on relative distance from the source (or elapsed time since the release). Even if this is sometimes adopted it seems (Ref. 12) that there is no ground; to justify it. Second, the above expression is of "gradient diffusion", and excludes any possibility of any kind of "bulk transport". This question is a delicate one in terms of turbulence modelling, but contributions of this type have in some cases been suggested (Ref. 32), and may take on more importance if the existence and role of "coherent structures" is fully understood and explained.

Finally, it transforms equation 3 into a pure diffusion equation, so it is likely to violate some of the rules stated at the beginning of the chapter. This is likely to be true near the source, where the dimensions of the region of "contaminated flow" are smaller than the typical dimensions associated to turbulent motions. If one considers equation 6, then the assumption is valid everywhere only for the case of sources located at ground level, where turbulence scale tends to zero value (Ref. 7).

3.3 The statistical approach

A completely different approach to the problem is based on the study of the statistical properties of turbulent dispersion, for passive contaminants. This was first suggested by Taylor (Ref. 31), and then widely analyzed and extended. Quickly summarizing, it considers the relative probability of a particle to disperse from the source x_0 as a function of time, taking into account the fact that the velocity at each successive location is not independent, or that there is "correlation" of velocity fluctuation over a finite distance. Then the "markers" are assumed to follow perfectly the velocity, and as a consequence tend to spread. The above considerations are along the same lines as the remarks made at the beginning of this chapter. A very comprehensive analysis of the problem can be found in reference 10, where the important extension to the relative dispersion between two markers is also analyzed.

In simple terms, considering a homogeneous flow, without mean velocity (or with uniform mean velocity but then having a moving reference), one can say that the average position (computed as an ensemble average for all the particles) of the markers is by definition :

$$\bar{X} = \int_0^t u(t)dt = 0$$

where u(t) represent the fluctuating velocity component.

The first useful statistical quantity which can be computed is the variance of the marker position, which is equivalent to the spread, or average size of the cloud over which it is likely to have presence of markers.
This can be expressed as :

$$\overline{X^2} = \int_0^t\int_0^t u(t)u(t')dtdt' \qquad (7)$$

The variance of the concentration is the first useful statistical quantity which can be computed, the mean value being identically equal to zero.
Using

$$R_0(dt) = \frac{\overline{u(t)u(t')}}{\overline{u^2}} \qquad (8)$$

one obtains

$$\overline{X^2} = 2\bar{u}^2 \int_0^t (t-t') R(t') dt'. \qquad (9)$$

Knowing the shape of R(t), $\overline{X^2}$ can be computed. But it should be noted that, as opposed to the previous approach, in this case only the spread can be computed, and not the distribution of contaminants.

Recalling that the correlation $R_0(t)$ and the spectrum of fluctuations $S(f)$ are related by the Fourier transform, one can write for the symmetric function R :

$$R_0(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega)\cos\omega t \; d\omega \tag{10}$$

and make the substitutions to obtain, with $f = \frac{\omega}{2\pi}$:

$$\overline{X^2} = u^2 t \int_0^{\infty} S(f) \left(\frac{\sin(\pi f t)}{\pi f t}\right) df \tag{11}$$

It is evident that the term $F(f,t) = \left(\frac{\sin(\pi f t)}{\pi f t}\right)^2$ acts on the spectrum as a low pass filter, or bandwidth function of the elapsed time, t. This presentation is very much favored by the author because it illustrates very clearly the different scales of the turbulence during the successive stages of turbulent dispersion.

Equations 9 and 11 can be solved if an expression is available for the autocorrelation function. Using the already mentioned formula

$$R = e^{-t/T}$$

where T has the meaning of the turbulence macroscale, one obtains (Ref. 9) :

$$\overline{X^2} = \overline{u^2}\, 2T^2 \left(e^{-t/T} + \frac{t}{T} - 1\right) \tag{12}$$

It is easy to see that the above relation tends, for large t (in comparison to T), to the same results as the gradient diffusion equation under similar conditions, i.e.,

$$\overline{X^2} \approx \eta t \qquad \text{if} \qquad \eta \approx \overline{u^2}$$

The important difference is that relation 12 depicts correctly the first phases of the phenomenon, when because of high correlation the spread X^2 tends to be proportional to (t). An extensive comparison of the two approaches with experimental results is made in reference 7.

The filtering effect indicated by relation 11 is illustrated in figure 1. It appears clearly that the most active scales of turbulence, in terms of dispersion, increase with elapsed time. A possible consequence of this effect will be analyzed later on.

3.4 The random walk approach

While references to this approach are mentioned in the literature and used in some applications (Refs. 2, 33), probably the most complete analysis is that made by Durbin (Ref. 12). In the referred paper almost all the relevant aspects are dealt with and the brief description given here only represents a simplified description.

Values of C(x,t) as given in equation 1 can be obtained by generating random walk for particles from a fixed source or to a fixed observation point. The description here will be very limited and it will not deal with all the subtleties nor with all the mathematical requirements necessary to provide an exact proof.

In a random system particle trajectories may be computed in a discretized way at successive constant time increments Δt, using what is known as the "drunkard walk", that is using random number generators. This drunkard walk has been extended by Wiener to continuous system. If ΔW is a random walk having the value ΔW_t at time t, the value at $T = N\Delta t$ is given by :

$$W_T = \sum_{1}^{N} {}_n \, \Delta W_{n\Delta t} \qquad \text{discrete system} \qquad (13)$$

or

$$W_T = \int_0 dW_t \qquad \text{continuous system}$$

The ΔW_t must have a certain number of properties, namely that they are independent of one another and statistically identical.

In this case, for the central limit theorem the probability density distribution of W_T will tend to be Gaussian.

If Y(t) is the trajectory of a particle it can be computed in such a system as the solution of the equation :

$$dY(t) = a(Y)dW_t \tag{14}$$

subject to a certain number of conditions (Ref. 12).

If one is interested in evolution in terms of time then a normalization can be made by assuming :

$$\overline{dW_t} = 0 \qquad \text{and} \qquad \overline{(dW_t)^2} = dt.$$

Equation 4 can be solved numerically, for large enough samples of dW_t and time steps, to obtain the statistical properties of the dispersion.

From equation 14 it is evident that

$$\overline{Y}(t) = a(Y)\ \overline{dW_t} \equiv 0$$

while the variance, or typical cloud size is of the type (with a = const.) :

$$\overline{Y^2}(t) = a^2 t \tag{15}$$

so within these limits this represents (see previous sections) a pure diffusion phenomenon. The extension to deal with the finite correlation of velocity fluctuations is analyzed in reference 12.

Also in non homogeneous turbulence, the mean velocity (in a Lagrangian sense) may be different from zero. This may be accounted for by modifying equation 14 to become :

$$dY(t) = b(Y)dt + a(Y)dW \qquad (16)$$

The meaning of a and b is somewhat similar to the Lagrangian displacement velocity as defined by Batchelor and discussed in reference 29.

Then, if the dispersion takes place in a flow with mean velocity $U(y)$, the longitudinal displacement can be computed as :

$$dX = U(Y)dt.$$

This approach is less often used than the other, so examples of application are illustrated. One is given, by comparison, in Appendix 1, two others are briefly discussed here. The first one, over-simplified because of the small number of time steps and "markers" used, involves a source at ground level. The surface is considered as fully reflecting in the sense defined in reference 12. Figure 2 represents the concentration at various tiles in a homogeneous flow, that a a = const, $b = 0$.

In figure 3 are the results for a "homogeneous shear" flow, where the diffusivity is $\eta = K(y+y_0)$, so similar to relation 6, and so $a = \sqrt{2\eta}$ and $b = K$ (the reason for these values will be shown later).

So in both cases the time shown in the figure could be replaced by the travelled distance $x = U \cdot t$.

The density of points plotted is the "representation" of the local concentration, and the solid line shown represents the position of the cloud centroid.

The appeal of the method for such computations is evident, even with the limited number of points used.

A practical drawback is shown by contrast in figure 4 where the final position of the cloud (for similar condition as in previous cases) is shown and where the same diffusivity is used but the mean velocity is : $U = C_1 + C_2 y$ (a crude representation of a gradient layer).

The limited number of markers used is now so dispersed that it becomes difficult to evaluate a mean value of concentration at each position (x,y).

It should be stressed that this is not a defect of the approach; simply it indicates that a very large number of markers (and of computations) should be used to obtain a useful result. This may be in fact the practical limitation on the usefulness of this technique, and is in a sense a pity because this seems one of the best suited approach of dispersal of buoyant (or heavy) gas. It should be hoped that such limits may be overcome.

The second example of application of this method is provided by the work carried out at VKI and given in references 33, 49 and 50. The last one is probably the more comprehensive and takes into account the full theoretical development outlined above and as suggested by reference 12. Henceforth some results will be presented here to give an idea of the suitability and power of the method. They are an attempt at predicting the results of the experiments by Schlien & Corrsion, Ref. 51.

What is measured is the spread from a line source of ground level and at some height inside a boundary layer. Heat is used as a tracer, but the temperature differences are kept low so that this can be considered as dispersion of neutral containment.

Comparisons of the experimantal results with the theoretical ones are shown in figures 5 for the position of the centroid and in figures 6 for the concentration (temperature) profiles in dimensionless form. It can be seen that the agreement of the two is pretty good and further work suggests that even more difficult situations can be treated with equally satisfactory results. This seems to apply in particular to the three dimensional source of reference 8.

It should be stressed, however, that the satisfactory results obtained are also due to a useful implementation of the theory in a numerical code. This is essential if the already mentioned limitations due to the finite (and relatively small) number of particles used are to be overcome, and requires a careful definition of the control values over which the averages are taken and of their evolution with distance.

3.5 The effect of molecular diffusivity

In all the above derivations it is assumed that the particle retains its relative concentration $C(x_0)$. This means that all molecular diffusivity phenomena can be neglected. This may not always be the case, so it is important to have a rough indication of its importance.

All passive scalars have a molecular diffusivity K (widely different values) and this has to be compared to the turbulent phenomenon of dispersion (the use of two different terms is made purposely). Molecular diffusion is a small scale feature, so it can be expected to act only at a level comparable, at most, to the microscale of the turbulent flow field, λ, and over distances of the same order (Refs. 2,27).

Turbulent transport in contrast, at least for its dominant features, may be associated with the macroscale L of the flow. So the relative importance of the two phenomena is given by the ratio ℓ/L. As well as for the case of turbulent versus laminar viscosity a ratio can be found of the type :

$$\frac{\nu_t}{\nu_\ell} = \sqrt{Re}$$

for the diffusivity one obtains :

$$\frac{L}{\lambda} = \sqrt{Pe}$$

where the Peclet number is given as

$$Pe = \frac{u'L}{K}$$

u' being the variance of the velocity fluctuations.

A very deep analysis of the problem is given in Tennekes & Lumley, and the result indicates that molecular phenomena can be neglected if $(Pe)^{1/2} \gg 1$, which is the case in atmospheric flows.

4. STATISTICAL PROPERTIES OF THE CONCENTRATION

4.1 The probability density function

From what was said before it should be evident that the "instantaneous" concentration at given point in space and time, $C(x,y,z,t)$, should be a random variable, because it is in itself the result of a random series of events.

Hence one will require the tools to describe such a variable in a concise, but nevertheless comprehensive way. Up to now the only statistical values introduced were the mean values, the variance, and the average cloud spread.

A useful description of a stationary variable can be obtained by means of its power spectral distribution. This is very important in terms of turbulence modelling and provides a deep understanding of the phenomena involved.

However, in the context of pollutant dispersal a more useful description of C is in terms of its probability density distribution, p.d.f. In terms of our description of concentration the p.d.f. determines the probability of a trajectory passing infinitesimally close to a given point (Ref. 12). In the general case this p.d.f. is function of position in the fluid and time. Alternatively (Ref. 12) it can be said that if a large number of markers are released from a source then the p.d.f. gives the fraction of these, that at a fixed time are in a small volume around a given point. By making use of conditional averages it is possible to determine the p.d.f., and its transport equation from a description as given in § 3.4.

Alternatively they can be derived directly from the diffusion equation of § 3.3, as indicated in reference 21.

Considering a unidimensional flow for simplicity one obtains :

$$\frac{\partial P}{\partial t} + \frac{\partial}{\partial y}\left[\left(V - \frac{\partial \eta}{\partial y}\right) P\right] = \frac{\partial}{\partial y}\left(\eta \frac{\partial P}{\partial y}\right). \tag{17}$$

where P indicates the p.d. function.

The similarity with the gradient diffusion equation for C is evident. There is nevertheless one extra term included in the convection term : instead of $V \frac{dP}{\partial y}$ one has to to include the effect of the diffusion itself as $\frac{\partial \eta}{\partial y}$. The result is that in a non linear problem the shape of the p.d.f. is modified as a function of the non homogeneity of the diffusivity (as opposed to what happens in most linear mechanical systems in random vibration analysis).

The term $\left(V - \frac{\partial \eta}{\partial y}\right)$ plays in the Eulerian framework the role of convection velocity. It can also be shown that it is related to the parameters a and b of section 3.4 by the very simple relations $b = V$ and $a^2 = 2\eta$. Again reference can be made to the formulation quoted in reference 29, for the meaning of this velocity.

When the mean convection is equal to zero (as it should be in the case of "homogeneous shear" flow discussed in the example), then $b = a^2$. This is the way in which the constants were determined in the quoted example.

4.2 Some practical problems related to p.d.f.

The p.d.f. is expressed by considering the probability of C(x,t) being smaller than a fixed value θ :

$$P(x,t) = P\left\{C(x,t) < \theta\right\}.$$

The range of P is from 0 to 1. Furthermore, C is a bounded variable, $0 < C < 1$, and so is θ.

The probability density is the likelihood of $C(x,t)$ being in the range $\theta-\delta\theta < C < \theta+\delta\theta$, so it can be obtained as the derivative of P with respect to θ as :

$$p(\theta) = \frac{\partial P}{\partial \theta} \quad \text{and} \quad \int_{-\infty}^{\infty} p(\theta)d\theta = 1; \quad \overline{C} = \int_{\infty}^{\infty} p(\theta)d\theta$$

For a random variable $p(\theta)$ has, typically, a Gaussian distribution. In the case of concentration it must also satisfy the condition that $p(\theta) \equiv 0$ for $\theta < 0$ and $\theta > 1$.

The importance of the p.d.f. in analyzing the hazard associated with the value of the concententration at a given point and time is now fully recognized. It is evident that in most cases the "mean value" (whatever its definition) is almost irrelevant. A flame can be ignited at a given point even if $\overline{C}$ is outside the limits of flowability, if it happens that at a certain moment its "instantaneous value" is within the above limits <u>and</u> at the same time there is a source of ignition. In the same way toxic levels may be exceeded if one happens to be at a given point when the "instantaneous concentration" is above a fixed value at a certain time or for a number of times (accumulated effect), no matter what is the mean value at that point. Practical mathematical formulations of this problem have been made by different authors; an extensive example is made in reference 27 and there is no need to repeat it here. For the case of the flame ignition a very good example is the experiments performed and reported in reference 27 by Birch. Fig. 7 qualitatively illustrates a typical result (it should be added that for this particular case Birch was able to explain the results by making use of the p.d.f. of concentration, but again this will not be discussed here as it is extensively dealt with in the quoted reference.

One point which needs to be added is that the likelihood of an "unwanted situation" does not depend only on the p.d.f. of concentration, but also on the probability that there is some triggering phenomenon sensitive to it (in other words it will be immaterial to have a toxic concentration level if there is no "observer" there likely to be affected). This requires the introduction of the joint probability of two events : for example that there is a given value of concentration, and simultaneously that the conditions at that point are such that it may result in damaging effects. The second condition is also likely to be, in most practical situations, a random variable, described by its own probability.

So the final hazard will depend not only on the p.d.f. of C, but also on the p.d.f. of the (let us say) "situation". Hence the likeliness of the final hazard could be expressed in terms of the joint p.d.f. :

$$P_j = P_j(c,s)$$

Unless what we called for simplicity concentration (C) and situation (S) are statistically independent variables there is no direct way of determining P_j from the individual p.d.f. of the two.

There is enough good evidence, experimental and theoretical to justify the assumption of a Gaussian shape for the p.d.f. in homogeneous turbulence and the fully developed turbulence in other flows (Refs. 34,38). This will solve (or at least simplify) the problem if it were not for the fact that the exceptions are associated with the situation of most practical interest. Two of them are particularly important in this context. The first, which is valid also for stationary flow, is that the edges of the turbulent flows with gradients (of velocity and/or concentration) are not sharply defined, but characterized by the so called phenomena of intermittency (Ref. 40).

The second exception happens when the shear flow of interest develops in an environment which is not stationary, or affected by a random component. To treat this will require to solve the relevant equations (at least those of the gradient diffusion type) for stochastic boundary condition. Such an attempt is made in reference 1, but it also shows that it is almost impossible to arrive at a general type of solution. Hence some form of phenomenological approach is the best alternative, at least from a practical standpoint. One of the most often quoted examples of such a phenomenon is the so called "meandering" of plumes,or puffs,of pollutant.

However, it is our opinion that in most situations (including situations where buoyancy plays a predominant role), the distinction between meandering and intermittency is a very thin one. Except when the scale (or time) of such phenomenon is large in comparison with the typical plume dimensions (and because this dimension increases with time elapsed, the scale of the external perturbation should be larger and larger as the distance from the source increases), there is no obvious way to make a clear cut distinction between the effects of large flow eddies (a stationary feature of the flow) and the effect of so called "external random perturbation".

Even if the curve in figure 1 may be of some help in trying to decide where to place the limit, one should admit that the distinction will never be a clear cut one.

Finally there is the important consideration that the p.d.f. is a statistical quantity and as such obtained by averaging techniques (be it as time averages or ensemble averages,depending on the problem,makes no difference), and cannot be depicted from the result of a single event. However there are situations where from a limited number of tests one must try to assess not only what are the confidence limits of the result, but also to try to determine how they can be representative of a more general behaviour. Considering the previous remarks on possible use of the p.d.f. for the determination of hazard this point seems to be of outmost interest.

Due to their importance the above mentioned problems will be discussed in a separate chapter.

5. EFFECTS OF MEASUREMENT INSTRUMENTS

5.1 The sensor effects

A problem always associated with all evaluations of experimental results and also with their comparison to theory, is that the measured value may be different from the ideal one.

The point raised here is not that of the errors which are (or could be) associated with any measurement, but rather a matter of definition.

In a theoretical approach an "infinitesimally small value around a given point", means that the dimensions considered tend to zero and, as an example, the value of the concentration at one point at a given time really means at x and t, and not at $(x \pm \Delta x)$ and $(t \pm \Delta t)$, no matter how Δx and Δt are small.

In practice any instrument is limited in its possibility to give a value of a quantity in the sense defined before. The spatial dimensions(be it a length, a surface, or a volume) over which is making the measurement (and the time duration) can be small, but they are almost inevitably finite.

This will be completely irrelevant if the quantity to be measured is uniform and steady, but not if one is in presence of space and time gradients. It is almost trivial to say that the extent of the problem will be dependent on the relative dimensions of the probe and of the scales associated with the variation of the quantity to be measured.

The result is that the probe will be making a sort of averaging of the spatial and time variations over its dimensions.

There are different ways to approach the problem, but before discussing them it will be useful to mention a few other possible effects.

The first one which comes to mind is the possible intrusive effect of the probe on the quantity to be measured. If the probe, or for that matter its support or any other element, is so large as to modify the flow field, then one will be measuring not only an averaged value, but an average of values which are modified by the presence of the probe itself. This consideration is so obvious that it will not be discussed here. Furthermore, it is so dependent on the interaction that it will be impossible to treat in general terms.

A second effect, which is related to the above, and which is of particular importance in the field of concentration measurements is the one related to the fact that in some cases probes are not just "sensing" the flow properties but require samples to be taken for analysis. The sampling may be continuous or in the form of finite burst. If one is interested, as in the present case, in time dependent measurement, then only the first case has to be considered.

Here, with the possible exception of truly isokinetic probes, it is difficult to evaluate the value of the flow domain which influences the sensor response; the path to the "sink" produced by the probe is different depending of the flow conditions. Also, it is difficult to evaluate the effect of such mode of action on fluctuations with time of the quantity to be measured.

Then one must also consider a point which is difficult to introduce in any simple treatment : namely the possibility of the probe having an inbuilt delay time. By this we mean that, apart from the averaging effect mentioned, and to be discussed later on, the sensor may react only after a fixed delay. This may be a consequence of its construction as in the case of the probes described in reference 35, or by any other cause. In principle such behaviour should be known, and accounted for in the probe specification, but the question becomes complex if the response delay is related to the value of the quantity to be measured or to any other quantity determining the field of interest (such as velocity or pressure, in the case of measurement of concentration, for example).

The last point is related to the threshold of sensitivity of the probe itself. That is,measurements may be impossible or unreliable when the value to be measured is below a certain small value. This is not so much a problem in itself, if this value is small, but raises the problem of the effect of the background level, or of the level of reference. By this we mean the fact that the background level of, say, concentration, is so small as to be undetected; but if the concentration suddenly jumps above the threshold level, then an output is present, which does not represent the actual increase in concentration, being interpreted as referenced to a zero value. It is true that in many cases this question (Ref. 28) is unimportant, but it should not be neglected for instance in the case of toxic gases, as opposed to the case of explosion hazard.

Coming back now to the determination of sensor response it is important to make a distinction between conditions in which the measurements are carried out.

The essential difference to be considered here, and this point has to be stressed, no matter how many times it has already been discussed, is between stationary and non stationary behaviour.

Stationarity is intended in the true sense of the term, and is not to be confused with steady : in a stationary phenomenon all the variables may be time dependent (turbulence being a typical example), but their statistical value is constant. This applies to the so called expected or statistical value of average, variance, spectral distribution and so on. Obviously there is no practical situation which corresponds to the absolute definition of stationarity, so a phenomenon should be considered as stationary if it is so over a time much larger than the time required to make all the necessary measurements.

The concept of stationarity in time could be translated to its equivalent in space, and in this case one speaks of homogeneity. The same considerations made before,for time dependence,can then be translated in terms of space dependence. However, while there is a large number of flow conditions of practical interest which can be considered as stationary, the number of practical situations in turbulent flows where the condition of homogeneity could be applied is much more limited.

In the domain of pollutant dispersion the classical examples of stationary and non stationary situations are the dispersion of a continuous plume, as opposed to that of a puff from an instantaneous source.

It is a matter of fact that in most cases of interest one has to consider that the problem to be analyzed is time dependent. This applies in particular to the heavy gas dispersal test (and to most accident-release situations).

The extent to which, in practice, the dispersion of a plume from a chimney can be considered as stationary may be the object of some discussion. In fact the source itself is constant, but the environment into which it disperses cannot always be considered as such. An example of such an effect is the change in meteorological conditions, such as wind velocity and/or direction. The limit here is again fixed by the rate at which such change takes place with respect to the observation time. To make a trivial example, if the environment is the same, the plume development under the action of a steady north wind will be the same as for the same plume under the action of a southern one, if the axes are aligned with the wind direction. However, the results will not be the same if during the observation time the wind direction changes and the axes are kept aligned to its mean value. This raises (or can be interpreted in terms of) the question of absolute versus relative diffusion discussed elsewhere. However, the author feels that this should not be extended too much to cover for instance the meandering problem, which taken in its strict definition should probably be considered as an inherent feature of turbulence.

On the other hand, one may ask the question of the possibility of making time dependent phenomena look like a stationary one. This question is possibly less "naive" than it seems, because of the way in which statistical properties are defined for the turbulence. In fact, if one is allowed to repeat the same time dependent experiment a large number of times, under identical external conditions, then averages could be made, at given (x,t),from the results of the ensemble of experiments. This is precisely the way in which "averages", which are in fact "ensemble averages", are defined for turbulent quantities. Time averages used in substitution are only an easier way to deal with the problem and justified by the existence of an ergodicity theorem.

So, in theory at least, there is no essential difference between the stationary and time dependent phenomena. The problem is that it is almost impossible to reproduce the experiment under identical conditions,and second that the number of tests required should be so large to make it virtually impractical.

The conclusion is then probably that one must consider all the problems of interest here as time dependent, and consider all instrumentation problems in this context.

If this is done then one should be able to have experimental information on the "statistical properties" over a limited number of events in a given boundary condition and extrapolate them to different ones.

The fact that we consider all the phenomena as time dependent is important in defining the response of the sensor because it makes impractical the use of the spectral representation and of the associated Fourier transform and transfer function in a straightforward manner. This is not to say that this is impossible or unjustified, because recourse could be made to so called "instantaneous spectra", or to a description of the time/frequency/amplitude domain (a technique widely used in speed analysis for instance, and more recently in the research in coherent structure in turbulence), but that the approach could be too complex to be justified (Ref. 39).

One advantage of these descriptions is that in some cases they are reversible (if the transfer function of the instrument is fully known) so that under some conditions it is possible to obtain the real value from the measured one. However, this applies in a simple way only for situations dependent on a single variable and their interest will decrease in any case when applied to the four dimensional problems of interest here.

A very interesting analysis of this problem is to be found in reference 36 which investigates the response of hot wire anemometers in turbulent flow field.

On the other hand, an often accepted relationship between the real and measured values is the one given in reference 27 as

$$Am = \frac{\int_D w(x',t')A(x',t')dV(x')dt'}{\int_D w(x',t)dV(x')\ dt'} \tag{18}$$

where the integration is over the four dimension D, defined by the volume dV around x and the time t. The size of the region D depends on the sensors. The function w(x,t) indicates the weight that the sensor gives to the local value of A, and is also function of the sensor. The integral at the denominator simply helps in normalizing the results.

The averaging effect of the above relationship on the quantity to be measured is self evident, as it is a fact that there is no way of reconstructing the true A(x,t) from the measured one. However, it seems also that it is based on a sort of superposition of effects, that is the result of the sum of local values of A and w over the value of the transducer, and gives no immediate indication of the sensor size on the result.

The result is then a fluctuating variable A_m (dampened in time and space), to which statistical considerations could be applied for the computation of mean value, variance, etc. A couple of points are also discussed in reference 28, in relation to this, to which we will come later.

However, for further appreciation of the effects of the sensor on the measured value, it is useful to tentatively try to evaluate directly its effect on the variance of A. This will be done simply for a one dimensional case.

If A(x,t) is a fluctuating quantity acting on a sensor of dimensions ℓ then, assuming $w \equiv 1$, one obtains :

$$A_m(x,t) = \frac{1}{\ell} \int_0^{\ell} A(x,t)dx \qquad (19)$$

For the variance of A_m, one must take into account the concept of simultaneity of fluctuation over the length ℓ, so, indicating with a the fluctuating component of A :

$$\overline{a_m^2} = \frac{1}{\ell^2} \int_0^\ell \int_0^\ell A(x',t)dx'A(x'',t)dx'' = a_m^2(x,t)$$

or if

$$\Delta x = x'-x''.$$

$$\overline{a_m^2} = 2\,\frac{1}{\ell^2} \int_0^\ell (\ell-\Delta x)\;\overline{A(x,t)A(x+\Delta x,t)}\,d\Delta x \qquad (20)$$

The product $\overline{A(x)A(x+\Delta x)}$, averaged, is the spatial autocorrelation of the function $A = R(\Delta x)$.

Obviously to go further one requires an expression for the autocorrelation (x). The choice of an expression of the type

$$R(x) = f(\frac{x}{L}).$$

has the drawback that it is based on the macroscale L of the fluctuations, while one is more interested here in the effect of small scale perturbations. In comparison to ℓ, (in general) L could be expected to be much larger.

To take a very simplified approach one may consider as reference the Taylor microscale of turbulent fluctuation λ (in a very crude way one may express the relationship between λ and L as $\ell \approx L\,Re^{-1/2}$). Then the autocorrelation near the origin can be expressed, in a power series development, as

$$R(\Delta x) = a^2 \left(1- \frac{\Delta x^2}{\lambda^2}\right) \qquad (21)$$

valid for $\Delta x \ll \ell$.

The integral in 20 can then be solved for two conditions, namely $\ell \ll \lambda$, using relation 21 and $\ell \gg \lambda$ taking $R(\Delta x) = a^2$ over the length of the integration.

This results in

$$\overline{a_m^2} = \overline{a^2} \left| 1 - \frac{\ell^2}{\sigma\lambda^2} \right| \qquad \ell \ll \lambda \qquad (22)$$

$$\overline{a_m^2} = \overline{a^2 L} \left| \frac{\lambda}{\ell} - \frac{\lambda^2}{\ell^2} \right| \qquad \ell \gg \lambda \qquad (23)$$

The two results are plotted in figure 8, with the dotted line representing the approximate expected behaviour over the full range of ℓ/λ values.

In laboratory conditions it is not unusual, or at least not impossible that the situation is close to case one, while in most practical application the second situation may be more likely.

In this context λ can be assumed as a typical dimension over which the quantity to be measured is extremely well correlated, that is the dimension of fluid pockets inside which it can be assumed to be practically constant.

It is easy to see that for the ratios ℓ/λ of the order of 10 the effect is already very large (however, it should be noted that the Taylor microscale ℓ is larger than the Kolmogoroff microscale often given as a reference value).

It should be noted, however, that to obtain the results one had to consider the likelihood of the simultaneity of the events on the length of the probe: this had led to equation 18 which <u>is</u> averaged with respect to time so while a_m^2 is function of t, $\overline{a_m^2}$ (in a steady state situation) is not.

To illustrate this one can make a single example : consider that a pocket of fluid is convected with velocity U over the probe for a time Δt. Inside this pocket (of dimension $U \cdot \Delta t \cdot \ell$), the containment (quantity to be measured) may be contained, with concentration 1, over 25% in a single lump, or, also with concentration 1 in a large number of smaller lumps also accounting for 25% of the total. Also the position of these lumps may be different within $U \cdot \Delta t \cdot \ell$. The real values of concentration are always either 0, or 1. The measured values will change as function of the above quoted parameters. The average measured will always be 0.25. In contrast, the measured values of the variance (evaluated over Δt) will be different, depending on the relative position of the lumps, as indicated schematically in figure 9.

Over the time Δt (and assuming a perfect time response for the probe) in cases (1) the measured variance is 0. In case (2) the measured variance should coincide with the real one. In case (3) it is smaller than the true one while in 4 it tends to 0 again. The above is a very sketchy illustration but indicates how the distribution of high concentration pockets in time and space may combine to an unrealistic measured value. The situation extended to the four dimensional case will look even more complex, but substantially not different : only in the case where the equal concentration lumps are larger than the four dimensional region sensed by the probe will there be an agreement in the results : the only situation which satisfies this condition in the figures is the case 2.

5.2 Analysis of results

It should be noted that the above determined effect on the measured value of the variance is not distributed uniformly over all the fluctuation components which contribute to it. As should already be apparent the smaller fluctuations are the most affected. In other words the high frequency range of the spectrum will be attenuated mostly (low pass filtering action of the sensor). If a wave number could be defined as the inverse of space dimensions

$$K = \frac{2\pi}{\lambda} \tag{24}$$

then it could be expected that for wave number below K as defined above the effect will be small, while for K above that the attenuation will be increasing with it. This is schematically shown in figure 10. The situation is in fact more complex because one has to take into account the three dimensional nature of K (which is a vector in turbulent flow fields) and the effect of time. (Ref. 36)

Nevertheless, it could be said that, to generalize, the high frequency (both in space and time) components will be dampened or smoothed, and even more important the high level short duration spikes reduced in amplitude and deformed. Now toward the edges of a cloud of contaminants (to return to the problem of interest here), these spikes are a dominant feature of the problem (because of the low value of the intermittency factor here), the elements which contribute to the typical twin peak shape of the probability density distribution. So one may expect that in this region there is a large likelihood of having a substantial difference between the true and the measured statistical quantities describing the concentration behaviour.

If the measured p.d.f. is not correct then all the statistical values associated with the concentration at a given point are likely to be affected, including the mean value. This is meant to say that the mean value is not reduced to the value of concentration averaged over the probe value (instead of the local one), but also that it can be lower than it.

However, the quantities more affected are those mostly associated with the fine scale structure of the flow, and the first which come to mind are the peak values or the extrema of of the p.d.f. So the ratio of external value to variance (also underestimated) may be substantially smaller when computed from measurements, than expected from theory. The same consideration applies to skewness and flatness (third and fourth moment of variance).

It goes without saying that the above conclusions are of importance when estimating the likelihood of having, for example, the concentration falling into the limits of flammability, or when trying to estimate the instantaneous or cumulative toxicity levels.

Two last points need to be mentioned for their effect on the measured one. The first is the non linearity of the response of a probe. The fact that when measuring a fluctuating quantity, the statistical values computed from the output do not coincide with the real one, should be considered as trivial, because if the calibration curve is known then "linearizing" the results before using them fully solves the problem and should always be applied. The second is much more critical and is the fact that the values of the weighed function w in equation 18 (and following) may be a function of the local value of the quantity to be measured. This is an almost intractable problem, but not unusual; to quote as examples two widely used instruments, it plays a certain role in the response of hot wire anemometers and to some extent in laser doppler velocimeter (the bias effect may be considered as a form of this phenomenon).

Few attempts have been made to analyze in full the problem in the general form. It should be stressed, however, that whenever possible a check should be made of its possible existence on the particular probe to be used.

The final point to be made, and this is not an original observation, may be to consider the fact that because there is nothing like a self-sufficient theory of turbulence, all the turbulence models are based, or validated, on the use of experimental data. These data being always to some extent affected by measurement errors as mentioned above, one may raise the question of how much the models themselves are biased by this effect. On the basis of the treatment of specific theoretical flow (like isotropic turbulence) and considering the accuracy which can be achieved in laboratory measurements, the above points should not be considered as the biggest worry at the moment in this domain. On the contrary it is true to say that new experimental techniques have put into perspective flow properties (like the existence of coherent structures) which were previously neglected in all theoretical approaches.

It should be stated, however, that attempts have been made to rewrite the equation for containment dispersal,based on measured values. This will usually lead to more complex formulations which will not be discussed here. A good account of this is given in reference 27.

In the above considerations, recourse had to be made to time averages, an approach which may be unacceptable in the analysis of time dependent phenomena. In an unsteady situation the evolution of A(x,t) may be like in figure 9 with the measured counterpart shown as $A_m(x,t)$. The smoothing takes place both because of volume and time averaging in the probe, so the mean and all the other statistical parameters are likely to be different. But the problem always remains of how to determine the statistical values associated with such a curve. In other words to determine what is, say,the evolution of the mean value of A(x,t) with time, if such a quantity can be defined. The point is raised because most of the simpler models predict the evolution in time of such mean values : for instance the evolution in time at a given point in space of the concentration associated to the passage of a puff release.

It goes without saying that the problem will be non existent in a "laminar flow" situation. It only arises because of the effect of randomness associated with the turbulent motion.

It is also evident that such a problem will not arise, or at least could be easily solved if one had available a large number of samples, such as those in figure 11, coming from individual but statistically independent tests. In such a case, recourse could always be made to ensemble averages, which, as said before, are the proper way to deal with such a problem. Also, a solution will be possible if results are available from non identical but sufficiently similar tests, but presented in the form of relative dispersion. The aspect of relative dispersion is dealt with in detail in reference 27, and, to summarize, simply consists of presenting the data from each test referred to the centroids of the puffs computed at equivalent times.

Neither of the above proposals is easy to put into practice, at least for full scale tests, because of the large number of the required experiments to be performed.

The above considerations are made assuming that one is following the development of a puff from an instantaneous point source, and which is convected away by a mean flow velocity over distances large in comparison to its typical diameter (which can be conveniently defined as the value of σ as a function of time).

For the case of releases in no wind conditions and measurements at relatively small distances, the figure may be interpreted as typical of the arrival at the sensor of the cloud front. This is a close representation of a number of full scale tests. All the considerations made before still apply, but another approach is now possible, even if the limits of its applicability have to be carefully considered for each case. This will be to make averages of results obtained along different radii at the same distance from the source. Hence a "mean" value can be obtained from the results, which can be considered as statistically independent if the distance between the measurement points is of the order of the macroscale and thus acceptable for most purposes. Strictly speaking this assumes an axisymmetrical cloud development in a homogeneous environment. In case of high winds acting on the cloud the same approach could still be made, if the average is made with respect to the drifting centroid, there again making use of the description in terms of relative dispersion. The main objection is that now the condition of homogeneity is not (or is not likely) to be satisfied. This is expected to be due not so much to non homogeneity in the atmospheric turbulence (and to a lesser extent not to the non homogeneity of the turbulence generated by the relative motion at the cloud interfaces) but to eventual effect of the relative motion in the rolling up at the cloud fronts, which are exposed to different wind velocities (in terms of magnitude and direction). This in our opinion seems to be the condition limiting the applicability of such averaging in the case of dispersal of heavy gas clouds.

So, as a general approach, it seems interesting to analyse the possibility of determining time dependent averages for a general situation. Typical examples of the sample measurement at different location (or elapsed times) are presented in figure 13 taken from reference 46. They illustrate the wide variety of behaviour which may be encountered and the difficulty of the associated analysis. The general rules for obtaining time averages is briefly resumed in the following approach.

Suppose that it is accepted to use time averages. Then the problem of the averaging time arises, because only finite times can be used. The average is defined

$$\bar{U} = \frac{1}{T} \int_{t_0}^{t_0+T} U(t)\, dt \tag{25}$$

Let $\langle U \rangle$ be the true mean value, equal to $\bar{U}$ for $T \to \infty$.

Then, because U(t) = <U> + u(t) :

$$\overline{U} - \langle U\rangle = \frac{1}{T}\int_0^T \left(U(t)-U\right)dt = \frac{1}{T}\int^T u(t)dt \qquad (26)$$

To estimate the error one must compute, say, the variance δ of the measured value with respecto to the true value of the quantity <U> :

$$\overline{\delta^2} = \left(\overline{U}-\langle U\rangle\right)^2 = \frac{1}{T^2}\int\int_0^T u(t)\cdot u(t')dt\,dt'$$

$$= \frac{1}{T^2}\,\overline{u^2}\iint_0^T \rho(t-t')\,dtdt' \qquad (27)$$

where ρ = autocorrelation. Posing $\tau = t-t'$:

$$\delta^2 = \frac{2\overline{u^2}}{T^2}\int_0^T (T-\tau)\rho(\tau)d\tau$$

$$= \frac{2\overline{u^2}}{T}\int_0^T \left(1 - \frac{\tau}{T}\right)\rho(\tau)\,d\tau \qquad (28)$$

For times T larger than , $\frac{T}{\tau} \gg 1$, one obtains :

$$\left(U_T - \langle U\rangle\right)^2 = 2\overline{u^2}\,\frac{\tau^*}{T} \qquad (29)$$

where τ^* is the time integral scale or macroscale. The above analysis essentially follows the approach given by reference 47 and as carried out in the book of Lumley and Panofsky (Ref. 48).

The requirement that a time average should converge for increasing T to a mean value, and that this value is always the same is called <u>ergodicity</u>.

A variable is ergodic if averages, from all possible quantities formed from it, converge.

This requires that the correlation goes to zero for large t, and also the variable becomes statistically independent on itself. As a constatation this trend is generally observed to be true in the case of turbulent flow fields.

τ is the time over which u(t) is correlated with itself, but also a measure of the time over which u(t) is dependent on itself. For t larger than τ, u(t) becomes statistically independent of itself so that τ is a measure of the time interval over which u(t) "remembers" its past history. Thus it can be deduced that different sample of the signal of duration larger than τ, can be considered as statistically independent events, and so averages made in this manner can be considered as equivalent to ensemble averages.

Considering equation 29, and following reference 48, an error ϵ can be associated to the time T used to compute the average. Defining ϵ as

$$\epsilon = \frac{\text{variance}}{\text{mean value}} \tag{30}$$

then one obtains

$$T \simeq 2 \frac{u^2}{U^2} \tau \frac{1}{\epsilon^2} \qquad (31)$$

which indicates the sensitivity of the integration time to the accepted value for the error. Since ϵ is small this already indicates the necessity of the existence of a large gap in the characteristic time scale for the evolution of the mean and of the variable part of the phenomenon. This is no problem if one is faced with a stationary situation, or if, as already suggested, independent samples are available to make ensemble averages.

If the required time T is small in comparison with the duration of phenomena,then this analysis can be extended to time dependent situation. This may be the case in the flat part, or "plateau" of the signal, but surely not for the front where even a qualitative estimate indicates that the rate of change of the mean is high. And, as a matter of fact it is probably along these interfaces that the most interesting phenomena take place, because they are the regions mostly affected by the intermittency, and thus by the dual-peak shape of the probability density distribution (and thus more likely to show instantaneous values largely different from the mean ones).

The analysis is carried further on in reference 48 for the case of instationary problems and will not be rediscussed here in detail,except to recall some useful formulae and conclusions. The problem is that of finding the mean value of a quantity variable with time,to which a second one with shorter time scale is superimposed. As stated in the quoted reference this situation may be unrealistic if the two variables are taken as statistically independent, if nothing else because some intermodulation of the two variables can be expected. (For instance, to quote from preliminary evidence from full scale tests on dispersion, it seems that the level of fluctuations decreases when the front of a heavy gas cloud is passing over the sensor, due probably to the high level of stability of such a situation). Nevertheless if one takes averages over a finite time, then the average approaches the value of the slowly varying component and its variance reaches a minimum, which is approximately given by reference 48.

$$\sigma^2 = \left(\overline{U_T} - \langle U \rangle\right)^2 \approx \frac{2\overline{u^2}}{T}\,\tau + U''^2\,\frac{T^4}{24} \tag{32}$$

where U" indicates the second derivative of the slow components, and represents the contribution of its instationarity.

The optimum time of integration can then be found by looking at the minimum of the variance and is given by the relationship

$$T_0 = \left(\frac{\overline{u^2}}{\overline{U''^2}}\,\tau 288\right) \tag{33}$$

In reference 48 it is shown that $\overline{U''^2}$ is the fourth moment of the spectrum U(t), so that the above relationship can be found to be proportional to 4/5 power of the ratio of the highest frequencies of the slow components, to the lowest frequency of the rapid one. So only if there is a "gap" between the two spectra which contains little or no energy there is the possibility of making meaningful averages. This seems to be the case for some meteorological phenomena, such as the wind velocity, but it cannot always be expected to take place in laboratory experiments. Indeed the results of reference 46 already quoted seems to indicate that there are situations where this is not the case.

It should be noted that making averages on a single event using integration times,as indicated above,is equivalent to using a running mean. This is equivalent to a filter characterized by a weighing function (Ref. 48) :

$$h(t) = \begin{cases} \frac{1}{T} & \text{for } -\frac{T}{2} < t < \frac{T}{2} \\ 0 & \text{otherwise} \end{cases} \qquad (34)$$

and a transfer function

$$H^2(\omega) = \frac{\sin^2 \omega T/2}{(\omega T/2)^2} \qquad (35)$$

So the above discussion is equivalent to saying that one needs a gap in the spectrum situated at around $f_0 = 1/2T$ or $\omega = \pi/T$, which is the point of cut off of the above filter. If such is the case and the low frequency component is sufficiently low with respect to f_0, it is passed through with small changes, whereas the highly fluctuating part is almost completely attenuated. Thus the mean is recovered and by further analysis the properties of the fluctuating part can also be treated, if required.

The problem is then dealt with in further detail in reference 48, to which the reader is referred for a more comprehensive vision of the problem. In other words it can be said that the problem of determining the mean value is a functional one, in the sense that the time dependent average can be defined as the function $\langle A(t) \rangle$ which satisfies the condition

$$\delta^2 = \frac{1}{T} \int_0^T \left[A(t) - \langle A(t) \rangle \right]^2 dt = \text{minimum} \qquad (36)$$

where T is the total observation time. One has obviously excluded the trivial solution $\langle A(t) \rangle \equiv A(t)$.

Even if the above value of $\langle A(t) \rangle$ can be determined (which is not evident), the determination of all the other statistical values remains problematic, because they are to be computed over the time τ introduced before. For these to be realistic, the condition stated before should hold, which means that they should be constant over such time intervals, while being dependent on the time at which observation started. This applies to the variance as well as to all the other properties, such as the p.d.f., and the condition is likely to become more difficult to be met, as the computed quality contains more information on the signal to be analyzed.

In view of the difficulty of the problem,it can be considered as a great advantage,if the information out of a single test can be analyzed on the basis of previously accumulated experience on the main features of its properties, obtained either from laboratory experiments or from theoretical evaluations.

This is closer to the situation where the problem is not to obtain "definite" values from one experiment, but instead, that of trying to assess the "typical correspondence" of the data from one (or a limited number of tests) made under not fully controlled external influences, in comparison to the expected general behaviour.

It can be heavily simplified if, as stated, independent samples were available and it seems that there is a tendency towards it. This is probably the more likely possibility to solve the problem associated with the full scale test results, as discussed in reference 56.

An an illustration some examples of application of the running mean average are shown in figures 14 to 15. It can be seen that even with two superimposed simple signals of wide difference in frequency the solution is not easy, due to the attenuation of the low frequency components. Of course, once the fundamental shape is recovered, (and is particular its frequency if it has one), some improvement with respect to amplitude response can be obtained, by using the relationship of the previous page. This means reconstructing a signal of the shape predetermined and modifying the amplitude until the given minimum in variance is reached. Then a more accurate evaluation can be modified and the reader is referred again to reference 48 for a discussion on optimum filters.

6. INTERMITTENCY AND p.d.f.

To illustrate this point recourse will be made to very simple graphical representations. It should be stressed immediately that they are not numerical simulations of turbulence (or not even attempts to do that) but idealized representations of possible situations, designed as a help to the description. As an example (Fig. 10) an attempt is made to indicate how difficult it may be in some cases to make a distinction between meandering and intermittency (remember that they are drawings and not a simulation of turbulence).

Intermittency is a typical phenomenon associated with turbulent flows where large gradients are concentrated in small regions of typically undisturbed flow. It is used to indicate the fact that the edges of such regions are not clearly defined in space and time (as is the case for laminar flow) but largely erratic and full of "indentations" (or hills and valleys). This is the case also for "stationary" flows, and is the case no matter whether the gradient considered is in terms of velocity, concentration, temperature. The very nature of this problem is not yet fully understood. Intermittency (as defined later) has even been considered a transportable quantity (Ref. 42) and a tremendous amount of new research has been triggered by the realization that turbulence contains, as a natural and inherent property, so called coherent structures. Leaving aside all these considerations, outside the scope of the report, the one important point to be recalled is that the local gradient and instantaneous values may be (and are in general) much larger than those associated with the mean flow. This is important from at least two aspects : first that the shape of the p.d.f. at a given point in space and time depends on the local level of intermittency; second that it may suggest that the possibility of bulk transport as opposed to gradient transport cannot be ruled out.

6.1 Definition of intermittency

If the edge of a shear flow looks like that of figure 11 then one may assume that there will be a number of points, P, in the region of interest where the quantity to be measured is present or not. Alternatively at a fixed point in space there will be moments when the quantity is present or not. Let us assume for the moment that the value of the quantity at that point is irrelevant : it is either there or not. It will then be possible to define an "indicator function" $\phi(x,t)$ which is either one or zero depending on the presence or not of the quantity of interest (be it concentration, temperature difference, turbulence, vorticity or whatever is relevant).

For the situation in figure 16 the function $\phi(x,\bar{y},\bar{t})$, at a given y will be as shown.

Alternatively an indicator $\phi(\bar{x},\bar{y},t)$ can be defined for a fixed point in space as function of time (and of the position of the point).

Then by making averages one may define the intermittency function γ as :

$$\gamma = \langle\phi\rangle \tag{37}$$

The type of average is not indicated because it will depend on the type of problem analyzed. Depending on it, it could (or should) be a time average or an ensemble average (the last one being always correct).

For a stationary problem, of the kind shown, one has :

$$\gamma = \gamma(x,y)$$

A typical example of γ for the concentration at some distance from a source at ground level is shown in figure 17 (Ref. 8).

As a general trend γ will tend to 1 near the "center" of the region of interest and to zero near the edges. It also has sometimes been suggested that the shape of γ (or of part of it) may be described by a Gaussian-like distribution function (Ref. 36).

The importance of the intermittency is evident if one tries to evaluate the real likelihood of concentration at a given point, as opposed to average values. The first step is to consider the "weighted" values, that is the average made only over the time (or region) when $\phi = 1$, thus excluding all the regions where the quantity measured is known to be identically equal to zero (Ref. 7). To do this one must recall some properties of γ, derived from its definition. For instance,

$$\gamma \;=\; \gamma^{n} \;=\; \gamma. \tag{38}$$

and

$$(\gamma-1)^{2} = (1-\gamma)^{2} = (1-\gamma). \tag{39}$$

Then if C_M is the mean average of (say) concentration, its "weighted" value is

$$C_W = \frac{C_M}{\gamma} \qquad \text{and} \qquad C_N - C_M = C_M \left(\frac{1-\gamma}{\gamma}\right) \tag{40}$$

If c(t) is the fluctuation around C_M, with variance $\overline{C^2}$, then if C defines the instantaneous value, the weighted functions are :

$$c_W(t) = C - C_W = C - \frac{C_M}{\gamma} = C_M \frac{(\gamma-1)}{\gamma} + c(t) \tag{41}$$

It results, taking the averages only for the time when C is present :

$$\overline{\overline{C_w^2}} = \frac{1}{\gamma}\,\overline{C_w(t)^2} = \frac{\overline{\overline{C^2}}}{\gamma} + C_M^2\,\frac{(1-\gamma)}{\gamma} \tag{42}$$

showing that local variances of the concentration fluctuation may indeed be much larger than the average one.

To the intermittency, or better, to its indicator function, can also be associated p.d.f. This is very simple due to the fact the ϕ is either 1 or 0. It results :

$$p(\phi) = a\ \delta(0) + b\ \delta(1) \tag{43}$$

δ indicating the Dirac function, and

$$\int_0^\infty \delta(t)\ dt = 1. \tag{44}$$

$p(\phi)$ must satisfy the conditions

$$\int_\infty^\infty p(\phi)a\ = 1 = \int_\infty^\infty \left(a\delta(0)+b\delta(1)\right)\ d\phi = a + b = 1 \tag{45}$$

The intermittency can then be derived as

$$<\phi> = \int_\infty^\infty \phi p(\phi)d\phi\ = b \tag{46}$$

and

$$\gamma = \frac{b}{a+b}\ \cdot\ = b = <\phi>. \tag{47}$$

The above derivations do not include time directly so it can be applied to time or ensemble averages as appropriate.

For other aspects of the importance of intermittency reference 57 indicates some conditions where the influence in toxicity or flamability is taken into account.

6.2 The combination of intermittency and concentration

For the "duration a" by definition the concentration is equal to zero, while for the "duration b", it is present and with statistical values typically as derived in the above weighted averages. So it is tempting to try to combine the two effects and derive a joint probability function of ϕ and C. There is no definite solution to this problem, so it can only be approached in phenomenological terms.

To visualize the approach,it could be helpful to schematize the likely shapes of the incoming form of a cloudy contaminant in the crude way shown in figure 18.

The dark regions indicate a presence of contaminants or $\phi = 1$. Then one can make different assumptions. The situation as presented is an unsteady one, so all the averages should be made as ensemble averages over a large number of events.

The first and simplest assumption is to consider that in all the dark areas the concentration is equal to 1. In this case if one makes the averages, the p.d.f. of the concentration will be identical to the p.d.f. of the intermittency. The mean value of C will equal the local value of intermittency γ, and so for the variance. Intermittency and correlation are perfectly correlated, and the joint p.d.f., $p(C,\phi)$ will reduce to two peaks as shown in figure 19a. Such a situation is too simplified to be realistic.

As a second attempt one may assume that the concentration in the dark spot is still uniform but that it has one of the possible values in the range 0 to 1. Then every time $\phi = 1$ the value of C (for an event) will be in the range 0 to 1, at random. The result now, over a large number of events will be more like in figure 19b. From the definition the p.d.f. of each variable can be derived from the point p.d.f. as :

$$\int_0^\infty p(C,\phi)d \;=\; p(C) \quad \text{and} \quad \int_0^\infty p(C,\phi)dC = p(\phi) \qquad (48)$$

In particular

$$\int_0^\infty p(C,1)dC = b = \gamma \qquad (49)$$

and

$$\int_0^\infty Cp(C,1)dC = C \quad \text{(at the point under consideration).} \qquad (50)$$

This is the mean value of C at that point and it is easy to see that it is strictly dependent on the local value of intermittency.

In changing the observation point from the edge of the cloud towards its center, the relative value of b with respect to a increases and so does the "apparent" or averaged value ot the main concentration. This is schematically shown in figure 19. However, from the same figure is apparent that the possibility (as opposed to the probability) of having dangerous concentration levels remains the same everywhere. This may be considered as a first step towards the interpretation of the mentioned Birch experimental results. It should also be noted that with the above assumption the value of $\overline{C}$ tends to 0.5 at maximum.

While better than the first one, this approach is still too simplified to be of true practical interest. The likelihood of concentration values cannot be assumed as equiprobable, but is expected to decrease near the extreme values. In other words, what is needed is a better knowledge of the actual p.d.f. of C as a function of the position in the clouds, independently of the effect of intermittency. However, this is a difficult question to answer. Concentration is a terribly difficult quantity to measure in its finer details, in spite of the recent improvements in measurement techniques already mentioned. It could be expected that the application of techniques based on digital image analysis of the visualization of a marker position and intensity will greatly improve this. Recent advances in this domain seem to indicate that this is not too far (Refs. 43,44) but the difficulty and complexity of this type of measurement (and the immense amount of data to analyze) should not be underestimated.

An evolution from a normal distribution at the center towards a highly skewed one at the edges could be expected. If an interpretation is made in terms of the joint p.d.f.,trends as shown in figure 20 could be assumed.

As a rule data about the p.d.f. of concentration are difficult to obtain in the available literature, especially if compared to same data for velocity and also temperature. The reason probably is to be found in the fact that detailed, and especially fine scale, measurements of concentration are far more difficult than for the other quantities,even in ideal laboratory conditions. Unhappily a direct comparison with, say, p.d.f. of velocity can be expected to be unrealistic, and results can be expected to be different because of the less "bounded" nature of velocity fluctuations. Also the vectorial, as opposed to scalar, nature of the velocity field should not be neglected in dealing with a fully three dimensional flow field,such as produced by turbulence. Comparison with data from temperature measurements may be more acceptable even if there is some evidence tending to indicate a slightly different behaviour of the two scalar quantities, i.e., of heat and mass transport.

It has been suggested (Ref. 55) that a proper normalization can be derived in some types of flows (typical of which are thin shear layers) such that all the interesting parameters have a value of zero at one edge of the flow and of unity at the other. Then the p.d.f will have non zero values only between the above specified limits. This will certainly make comparisons easier for all the different quantities, but will not change the essential difference in behaviour which has been mentioned.

To quote from reference 52 the knowledge of p.d.f. is largely lacking and yet is of crucial importance to the phenomenology of scalar qualities in turbulent flows, be it active or passive. As a result of the fact that scalars are bounded between 0 and unity the resulting fluctuations can involve highly skewed p.d.f. For example with a mean concentration C, it is always possible to find an intensity of fluctuations C'^2 such that, with bounds at 0 and 1, it will make the probability density distribution highly asymmetric. The importance of this in connection with reacting, and even toxic, flow is easy to understand. Furthermore for flow involving interfaces, scalar quantities frequently have zero or low values in the undisturbed region and a range of values within the turbulent region. Thus any analysis which disregards intermittency may not be accurate outside the fully turbulent flow region. If the non linear effect mentioned elsewhere (Ref. 57) is to be taken into account, as seen the case with toxic releases, then the problem becomes even more complex.

Typical results for the probability density function of concentration and temperature can be found in the papers of Robins et al. (Ref. 7); Birch et al. (Ref. 54); and Libby et al. (Refs. 52,53). Results obtained by using visualizations and digital image analysis, have also recently been made available by the work of Schon. The last referred paper is of particular interest because it deals not only with the p.d.f. (of temperature in this case), but also with the statistical description of the behaviour of the interface between the turbulent "contaminated" region of the field and the uncontaminated non turbulent one which surrounds it. The fact that it refers to a structure of a wake may make the direct applicability to the situations of interest here a little weak, if one considers that in the wake there is an important exchange of momentum, which is not always the case for pollutant dispersal. However, the interest of the results is such that it is considered useful to mention them here, if nothing else as a reminder of some general properties of turbulent interfaces derived from statistical aspects.

In the context of the simple presentation made at the beginning of the discussion, these results, presented in reference 53, are of particular interest. In fact they refer to the statistical properties, and in particular to the p.d.f. of the intermittency itself, or to be more precise of the turbulent/non turbulent interface. Some of them are reproduced in the figures 21 to 23 as the p.d.f. of the duration time of the turbulent part of the interface with respect to overall time of observation at the edge of a wake. Again it could be said that their direct applicability to the analysis of dispersion may be weak because of the difference in the driving forces involved (the interface is one of momentum deficit and not simply of difference of concentration, so other mechanisms are present to generate the turbulent fluctuations), nevertheless their illustrative purpose is self evident. In fact the intermittency can be considered as a "precursor" of the concentration level. As indicated schematically before in the illustrative examples, if one is outside the contaminated region the value of the concentration is identically equal to zero by definition, while inside it can take (at least it can be assumed to take) any value between 0 and 1. If as in the case of one of the possibilities considered this value is always 1, there in one limit case the concentration statistics will be identical to those of the intermittency. If it is "any value" then the long time average statistics will be those of the intermittency, weighted by those of the concentration in a fully turbulent region. This is a second limiting case. It is in this sense that the role of the statistics of intermittency could be seen as that of "precursor" of that of concentration ,while the p.d.f. is computed over continuous long enough averages, i.e., if intermittency is not considered apart by making for example conditional measurements. It should probably be noted here that while the use of standard probes leads to this, the use of visualization and optical analysis with only the plume seeded may fall in the other condition, and this may result in differences for p.d.f. and variance. This will also require that any p.d.f. function which is introduced to describe the concentration levels must take into account the adequate degrees of freedom of the phenomena which lead to it. This will be relatively simple in the two limit situations mentioned; the actual one can be expected to be a more complex and delicate mixture of phenomena and interaction, which can be interpreted as an increase in the degrees of freedom.

The analysis of Libby's data on turbulent interface statistics seems to indicate that a lognormal distribution is well suited to describe the properties of the p.d. function except at the lower tail, as illustrated in figure 23. The use of lognormal distribution for the description of the concentration p.d.f. has also been suggested. Typical of this approach is the relation derived by Csanady in reference 2 and referred to by Chatwin in reference 27, as a formula for the measured concentration probabilities. The actual derivation of the formula is based on the modelling of the physical phenomena and is suggested by experimental evidence. However, Birch and others as quoted in reference 27 proposed for similar conditions a truncated Gaussian distribution which can also be seen to fit the data. The important point to note is that it seems apparent that in both cases parameters to be determined from experiments should be fitted into the formulae to match the experimental results. While the meaning of these parameters can be shown to have experimental significance, one may still consider the possibility that they are a different way to introduce the required variability of p.d.f. in terms of skewness and kurtosis, which seems necessary in the analysis of such a phenomenon along the way already indicated. This should probably include the variability of the intermittency, the variability of the concentration in the contaminated flow region, as function of distance of the centerline (and thus of the actual value of the mean concentration and of the gradients) and and the eventual interaction of all these phenomena.

Along these lines it will be interesting to see if other families of probability distribution function cannot be used. Suitable candidates seem to be the χ^2 function and the truncated β function, because they already explicitly contain a certain number of "degrees of freedom" or variable parameters and the resulting shape is a function of these degrees of freedom. In particular the use of the latter is suggested by Rhodes in reference 55.

Before analyzing the function it is useful to come back to some experimental results obtained in laboratory conditions, and to make a few observations about their behaviour.

The data reported here are from Robins (Ref. 7), Birch (Ref. 54) and Libby (Ref. 52). While the first refers to dispersion in a surface layer from a ground source and the last comes from the analysis of dispersion inside a cylinder from a distributed wall source, they may be considered as representative of a similar phenomenon, while the second refers to a methane jet. Furthermore in this last case p.d.f. are not measured but reconstructed using the measured moments. The results are strikingly similar, and their use for comparison purposes may seem fully justified. The Robins data are presented in figure 24 as originally presented in the quoted reference. After some manipulation the results of figure 25 can be obtained, based on the values of the other parameter as quoted in the paper. Here the presentation is in the form of p.d.f. of the absolute concentration referred to (or normalized with respect to) the mean value of the concentration at ground level. Admittedly the number of manipulationscarried out from the original data to obtain this presentation (and the consequent necessary re-scaling of both the horizontal and vertical axis) is such that a good accuracy cannot be claimed for the figures as presented. However, it is felt that this may be a more self-explanatory presentation, as far as one is concerned in the evaluation of the statistical properties with distance from the centerline. One thing is very apparent : the change in shape from an almost normal distribution to a very skewed one as the distance from the ground increases (that is the mean value of concentrations decreases to 0)

The experiments of Libby et al. (Ref. 52) show similar trends, with the skewness of the p.d.f. increasing as the distance from the surface increases. Of interest is a comparison with the results for the velocity, plotted in the same figure 26, which indicates that the trend is much more Gaussian. This seems to indicate a considerable difference between the behaviour of vectorial and scalar quantities. The third set of results, from reference 54 and shown in figure 27, refers to a methane jet in steady surrounding fluid, so they may be considered to belong to a different family of flows, the effect of momentum difference being more important. Also they are apparently not direct measurements, but a reconstruction made on the basis of the values of the different moments. Their characteristic is to show for small values of the mean concentration a bivariate p.d.f., that is a peak at two different values, which was not evident in the other results referred here, even if such a trend is often considered to be the correct one from a theoretical standpoint. It is difficult to determine general rules for the structure of p.d.f. from the presented results, if nothing else because of the difference in flow situations. Probably the different responses of the different probes used for the tests should also be taken into account to some extent. However, some considerations about the evolution of the p.d.f. on a qualitative basis can be made. For instance the tail is always longer in the direction of high concentrations when the mean value decreases. This seems to be more the case for the values of reference 7, although the possibility of some error in the manipulation made here to obtain that presentation cannot be excluded. It seems also that the values of these high concentration tails seems to vary, as a first approximation faster than the local value of the variance but slower than the mean values. The aspect of the p.d.f. seems to indicate that small pockets of high concentration can be found for short times even when the mean values are low, confirming the results on flammability quoted in reference 27 The size of these pockets is difficult to evaluate, considering the presence of intermittency, which should smooth the results. It will be interesting to have for comparison the results obtained by optical techniques, such as those measured by Schon which due to the seeding technique used should be more close to what is called conditional sampling methods.

Coming back now to the problem of finding a general description of the p.d.f., there is a relatively wide choice of analytical functions with sufficient free parameters to be able to find something suitable to describe highly skewed probability density functions, and the variation of the skewness.

Amongst these, two seem to be particularly attractive, namely the χ^2 function and the incomplete β function. The latter was suggested as appropriate in reference 55, by Rhodes.

The analytical expression for the above relation, when applied to the description of p.d.f. is given in reference 56, for the χ^2 function as :

$$p(\chi_1^2\nu) = \left(2^{\nu/2}\,\Gamma\left(\frac{\nu}{2}\right)\right)^{-1}\left((\chi^2)^{\nu/2-1}\,e^{-\chi^2/2}\right) \qquad \begin{array}{l} 0 \leqslant \chi^2 \leqslant \infty \\ \nu = \text{integer} \end{array} \qquad (51)$$

where

$$\Gamma\left(\frac{\nu}{2}\right) = \begin{cases} \left(\frac{\nu}{2} - 1\right)! & \nu = \text{even} \\ \left(\frac{\nu}{2} - 1\right)\left(\frac{\nu}{2} - 2\right)\cdots\frac{1}{2}\cdot\sqrt{\pi} & \nu = \text{odd} \end{cases} \qquad (52)$$

The meaning of the χ^2 distribution is that of a p.d.f. of a $\chi^2 = \sum_1^\nu x_i^2$, where the x_i are ν independent and identically distributed random variables with zero mean and unit variance. The variable ν denotes the degrees of freedom of the system. For large values of ν the above distribution is almost Gaussian, while for ν small the skewness increases. The value of the skewness coefficient is $\sqrt{8}$ for $\nu = 1$

Examples of the shapes of the χ^2 p.d.f. for different values of ν are plotted in figure 28. It is easy to see that their shape can be adapted to approximately the shape of the measured p.d.f. on the condition that the values of ν are adapted to the observed values of the concentration parameters. In its standard formulation, as indicated in the equation above the mean values and the variance are related by a unique dependence on ν (respectively they are ν and 2ν), which may make the manipulation difficult if not impossible. Another important limitation is the fact that this function is not normally bounded by an upper limit at 1. However, this could probably be taken into account by considering the fact that its tail tends to zero rather rapidly and χ^2 needs to be a multiple of the concentration.

The p.d.f. can also be plotted as a function of χ, or to be more accurate of $|\sqrt{\chi^2}|$ which preserves the unique sideness of the function, as shown in figure 29. These seem to indicate that with some manipulations the agreement may be even better However, all the objections stated before remain.

From this standpoint a much better proposition is to use the analytical form of the incomplete β function, which is given in the same reference [56] as :

$$P(\chi,a,b) = \frac{1}{B(a,b)} x^{a-1} (1-x)^{b-1} \qquad 0 \leqslant x \leqslant 1 \qquad (53)$$

where

$$B(a,b) = \int_0^1 x^{a-1} (1-x)^{b-1} dx \qquad (54)$$

which is naturally bounded by the values 0 and 1 as required for the p.d.f. of any scalar quantity.

The beta function and the chi-square function are somewhat related because if X_1^2, and X_2^2 are two functions with chi-square distribution and freedoms ν_1 and ν_2 then the β function represents the p.d.f. of

$\frac{X_1^2}{X_1^2+X_2^2}$, and $a = \frac{\nu_1}{2}$, $b = \frac{\nu_2}{2}$. Apart from this a and b may take any value which is suitable for obtaining the required degree of skewness on the p.d.f. profile. Some example of the shapes which can be obtained from the β function are illustrated in figure 30, for different ratios of the two coefficient values. It should be noted that the effect is symmetric and it is sufficient to vary one. The problem of associating them to the properties of the concentration field is illustrated by Rhodes in reference 55.

Here they become the results of a combination of mean and variance values.

The formulas proposed are

$$a = \bar{C} \left[\bar{C}(1-\bar{C}) \;/\; \sigma^2-1\right] \tag{55}$$

$$b = (1-\bar{C})\left[\bar{C}(1-\bar{C}) \;/\; \sigma^2-1\right] \tag{56}$$

(note that the first formula is different from the one quoted, which is believed to contain a misprint).

where C has the meaning of the local normalized concentration and σ is the variance normalized with respect to the same mean values, as used for C. In the quoted reference, there is no derivation of these formulae, which are referred to in a different paper. A number of examples of application are given and in particular is indicated how well they fit with the values measured by Libby in reference 52. This is illustrated in figure 31 taken from the above reference.

Other computed values for results of the same reference are shown in figure 32.

Unhappily it was difficult to adapt the results of Robins (Ref. 7) so as to compute the values of a and b as indicated above. However, the shapes of the curves of figure 30 seems to indicate the possibility of obtaining correct results.

It is important to note that the χ^2 function and the β function are respectively a one parameter and a two parameter function. Once these parameters are fixed their shape is determined. This means that all the moment values are fixed. Concentrating on the β function means that once a and b are fixed, based on the values of mean and variance, all the higher order moments are also determined. The conclusion is that, for instance, the values of skewness and kurtosis (third and fourth order moment) are unique functions of the mean value and variance at a given point. The conclusion will be even more drastic for the χ^2 function which depends on only one parameter. This result would imply that there is a certain form of self similarity in the development of the p.d.f., and as a consequence in the physical phenomena they are intended to represent. In other words, higher order moments should be universal functions when plotted as function of $\bar{C}$ and σ. This seems to be the case if the good agreement obtained is considered,as for the case of reference 52. Furthermore, this seems to be confirmed, at least for jet flows, from the measurements of reference 54, which indicates that the distribution across the jet of skewness and kurtosis follows an evolution which is independent of the distance from the nozzle (a result which seems to be confirmed by other results on temperature for heated jets). Considering that both the mean value and variance of concentration can be plotted as self similar profiles across the jet, the condition stated before seems to be satisfied at least for this particular situation. Further analysis is obviously required to generalize the conclusion to all types of pollutant dispersal. If this is the case then a considerable simplification will be obtained in data analysis.

When considering analysis of measured quantities one should also take into account, as discussed before, the size of the measurement volume which has an averaging effect which may destroy or distort the finer details. To this the shape of the tails of the p.d.f. curves are likely to be more sensitive, with non negligible effect of the cumulative values of probability to be evaluated.

7. EFFECT OF EXTERNAL RANDOM CONDITION

To the previously mentioned effects,which contribute to make what is a complex problem almost intractable, one has to add one more contribution. This is the randomness, and the consequent lack of detailed knowledge of the meteorological conditions associated with a particular test. The main factor considered here is the likely change in direction and intensity of the external wind.

If one considers a puff released from a point source in a homogeneous turbulence with uniform mean velocity, then it will travel along a line growing in dimension approximately as $\sqrt{t}$.

In real cases this will be seldom the case, because a random component of large scale (in comparison to the puff size) may be expected to be present in both x and y components of velocity.

If the effect of such a random component in the transverse direction is superimposed,the possible paths of the puff will be more likely to be as shown in figure 33. That is, at a given distance from the source the cloud centroid is not fully determined but depends on the randomness of v. In a certain sense this can be considered equivalent to the meandering of a plume. The real problem is much more complex : as already mentioned there will be an interaction between the meandering and the spread of the plume. The solution to the problem can only be obtained in full if one is prepared to solve the relevant equations (as discussed in section 4) under stochastic boundary and initial conditions. Such an attempt in made in reference 1 but it seems difficult, for the moment, to hope to be able to solve most of the practical problems in this way.

As shown by Chatwin, and with reference to the figure 34, if one does not consider the absolute position in space of the cloud, then the problem is largely simplified.

The concentration properties for the cloud will always be, as a first approximation, the same. So for an observer located at the same position with respect to the centroid of the puff the external effect will be less important (negligible in the limiting case of the example). The difference and the full implications of taking an absolute or relative frame of reference for the description of the problem are analyzed in detail in reference 27.

The analysis will not be repeated here : it could be summarized by stating that in an absolute frame of reference one may analyze the problem making use of the joint probability functions in a similar way as that used before. This time it will be the probability of having at the same time the centroid in a given position and the probabilities of the concentration distribution in the puff itself.

So for random conditions in the y direction, the likely possibilities will look as shown in figure 35. This is a situation very close to that analyzed by Riethmuller in reference 41, for a continuous plume and which will be rediscussed later on. For the condition used, it can be expected that a Gaussian probability of the centroid position will result.

The only point which is important to make, but for which there is no easy solution, is that of the interaction of the random external events with the spread itself. The likely result to be expected is that very close to the source the large scale fluctuations will make the whole puff move erratically, while at larger distances it is its internal distribution of concentration which is affected.

An example of possible interaction is illustrated in figure 35 where the same plot as before is presented but for a non-homogeneous turbulence, of the type corresponding to $\eta \cong ky$, so one can take as a first approximation for the speed of the cloud :

$$dr = \sqrt{\overline{u^2}}\, dt \qquad \text{and} \qquad \overline{u^2} \approx ky. \qquad (57)$$

The velocity field is uniform along x, and has a vertical random component. The figure shows the results for two releases. It can be seen that not only the final cloud position is random but also that its typical dimension is changed, depending on the trajectory followed. It could be expected that, as illustrated, the results are a gross exageration of the real problem, but as stated this is not an attempt to evaluate them accurately.

The extent to which they happen in any real situation and how they can be taken into account could only be the result of more detailed work on the subject.

The idea was only to suggest that such a possibility could not be excluded completely.

To conclude we would like to come back, in a simplified form, to the very illustrative example discussed in reference 41. The reason is because we suspect that a term is lacking in one of the derivations.

Let us assume that a gradient dispersion model is used, leading to a gaussian distribution of the mean value of concentration in the plume. A uniform wind is assumed in the longitudinal direction, while the transversal component is taken to fluctuate randomly at very low frequency, to give at a distance x a plume centerline displacement Δy described by the probability distribution curve shown in figure 35. What we want to check is what is the probability of the concentration for an observer at point (x,y) as a function only of the large scale wind variability. Then

$$\overline{C} = f(x,y,\Delta y), \qquad \text{so} \qquad p(C,x,y) = p\left[C(\Delta y),x,y\right] . \qquad (58)$$

Note that the dimensions of p are those of [1/dimension of variable] so that the above gives the values of C in terms of occurrence of Δy. If one wants to express the results in terms of concentration as variable

$$p(C,x,y) = p\left(C(\Delta y),x,y\right) \frac{dC}{d\Delta y} \qquad (59)$$

which leads to a different presentation from that in reference 41. Typically, figure 37 shows how the mean concentration distribution is modified when averaged over very long times compared to the v fluctuations. Figure 38 shows for the same condition the p.d.f. of C which can be expected at different distances from the mean centerline. This clearly indicates how hazard limits can be misleading if based on mean values.

Nevertheless, this is only part of the full problem, the variability of the instantaneous value of C inside the plume in a relative frame of reference should now be taken into account and combined with the other. This is again a problem of determination of the joint probability function of the concentration in the plume and of the presence of the plume itself.

Conceptually it could be dealt with using an approach similar to that followed for the effect of the intermittency function.

8. CONCLUSION

An attempt has been made at estimating the effects of different factors on the evaluation of the statistical significance of data measured in a test,in the context of the statistical properties of the phenomena under discussion.

To approach the problem, the basic phenomena and equation of contaminants in a turbulent field are presented and briefly discussed. In particular, the so called random-walk model is compared with the more classical statistical theory of dispersion and the gradient diffusion type models. The interest of the above approach is that it will enable to predict detailed properties of the dispersion, and furthermore seems to be particularly apt to be extended to include gravity effects. It can also be shown that it may approach very well the solution of the equation for the transport of the probability density function of the instantaneous concentration. The analysis was not extended to cover all the problems of relevance in the establishment of correct higner level models of turbulent transport, which should be dealt with in another context.

The importance of the aforementioned probability density function is that it is probably the best tool available to really assess the hazard and confidence limits. Unhappily, it is not an easy function to deal with and this is further complicated by the relatively low amount of experimental data available. Furthermore, the concentration is not an easy parameter to measure in experiments, if its finer details are looked for. It was indicated that there is a great hope in this field, thanks to the development of new techniques, in particular the use of digital analysis of flow visualization.

So most of the discussion was limited to a qualitative analysis of the problems likely to be encountered and of their possible effects.

After a short discussion of the problems associated with the measurement techniques, again mostly in qualitative terms, an attempt has been made at analyzing the effects of random (or uncontrolled) external events on the results obtained.

Attention was drawn on the fact that the turbulent dispersion being in itself a random phenomenon, mainly dictated by the nature of flow field in which it takes place, it is very difficult to decide where to place the separation line between the randomness due to the nature of the problem and that related to external events. The problems of plume intermittency and meander were taken as examples, and an indication is given of a possible way to deal with them, but again because of the complexity of the problem and the lack of sufficient data, the discussion was essentially of qualitative nature. The figures used to illustrate the problem are not, and should not be interpreted as results of a numerical solution of the problem, and are not a numerical simulation of turbulence. They are simply a (and in most cases an exagerated) way to illustrate the problems discussed and have no quantitative significance.

Finally, an example from already published reports is given to illustrate the effects of a large external perturbation.

To conclude we would like to make a few remarks; first that the review made is necessarily incomplete and is likely to leave out a number of important phenomena. The second may be in the form of suggestions for future work : it seems that both in the theoretical and in the experimental domain there is large scope for more research. One will be on how it is possible to better integrate the experimental and theoretical approaches, taking into account the new possible development on both fields : new results from full-field measurements as opposed to the punctual one, (more common now) and from theory a better understanding of the nature and properties of coherent structures in turbulence.

It is possible that such an approach may lead to a better and possibly simpler way to deal with the very important properties of the probability density function. This could be a new field of activity for theoreticians. As an experimentalist, the author will be very interested in the possibility of having more tests carried out, especially full scale tests (for which a number of results are becoming available now) and also model tests on the available wind tunnel simulating the atmospheric boundary layer structure. And above all, to be able to answer the question on how much the model test results really can describe all the features of the full scale reality.

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APPENDIX 1

The purpose of the present section is to try to compare the results from the different dispersion models and to see under which conditions to describe the same phenomenon.

With reference to the previously made considerations let us consider the simple problem of the determination of the uni-dimensional spread of a pollutant from a source in a homogeneous turbulent field with mean velocity equal to zero. As usual the pollutant is considered as a passive one and can be identified as a "marked" particle,in no way different from the others except for the fact that it leaves a trace of its trajectory. Hence it will follow the random path generated by the random velocity field,retaining over a large observation time a center of gravity co-inciding with the release point. Statistical information can be gained by observing the trajectories of a large number of such particles, but because the previous considerations apply to each one of them, nothing will be gained from the analysis of mean values. The first useful results are obtained by computing as a function of time the variance of the particle trajectories.

With the usual notation if x is the particle position and v(t) the velocity, in a Lagrangian frame :

$$dx = A(x)dW_t \qquad \text{(A-1)}$$

where dW can be taken as a random walk in velocity.

For simulation purposes the above equation could be discretized to read, with A = const :

$$x(n+1) = x(n) + v\,(nT) \times T \qquad \text{(A-2)}$$

where T is the discretization step in time.

The marked particle trajectories can then be computed using for v a series of values obtained from a random number generator both to provide a sufficient number of particles at each step and for the successive ones. The generator used here will provide, as far as possible, equi-probable numbers, i.e., it has a flat probability density distribution. The results for the first step are shown as an example in figure 5. It should be noted that the problem is uni-dimensional, so all the points should fall on a single line, but for clarity they have been randomly expanded in the y-direction. "SigmaV" and "SigmaP" respectively represent the variance of the velocity and of the final position of the markers, defined as :

$\langle y \times y \rangle$ and $\langle v \times v \rangle$

where the brackets indicate averages taken over all the realizations. Also plotted is the p.d.f. of the markers position, which is approximately flat (good approximation considering the small number of points used at the first step).

Obviously, after the first step the position of each marker coincides with the end of the corresponding velocity vector and also the two variances are bound to coincide.

The next steps are computed in the same way : for each marker a new random velocity value is generated and the result added to the previous one. The end position of the marker is computed and plotted. Then the values of SigmaV (which should be unchanged) and of SigmaP are evaluated. An example of the results is presented in figures A1,A2. The change in shape of the markers' p.d.f. should be noted, because a clear trend is already apparent.

With this approach both the spread and the concentration distribution are estimated. It is important to note that even with a relatively limited number of points and steps the distribution tends to a Gaussian shape and the spread is linear. These are typical of a diffusive phenomenon, and in fact reflect the nature of the evolution in the system itself and not some mathematical properties of the equation used to describe it. This point is discussed in reference 12 in a wider context.

It is also important to note that because of the definition of equation 1 the resulting shape can be taken to be either the contaminant concentration or the p.d.f. of the tracer particles, which all come from the same point and have an initial concentration value of 1.

Because at each step the velocity value is obtained from a random number generator it could be expected that the velocity correlation is a Dirac delta function with :

$$
\begin{aligned}
R\ (dt) &= \left[v(t_1) \times v(t_2) \right] \\
&= 1 \text{ if } dt = 0 \\
&= 0 \text{ otherwise.}
\end{aligned}
$$

In a continuous system equation A-2 may be considered to be the discretized form of equation A-1, made for the sole purpose of being able to solve it numerically. In such a situation the result is that discretization errors inevitably creep in, in the form of higher order terms. They usually take the form of dissipative or diffusivity terms, but the discussion of their effects on the solution and on its stability are outside the scope of the present analysis.

A different approach can now be considered based on the following assumption, namely that : "The discretized form is assumed to be the correct representation of a physical continuous (analog) problem, and contains all the relevant information to describe it completely".

If this is accepted then one is allowed to look back at the original problem and determine the effective "environment" in which it takes place. It was assumed, for instance, that the velocity field was fully random and no correlation existed either between successive values of velocity or successive events. With the form given for the correlation, the power spectrum is that of white noise, that is flat over all frequency range. This cannot be true any more if one accepts the new descriptions.

The results can now be compared with the classical theory of statistical turbulent dispersion as first proposed by Taylor, but taking into account the restriction imposed on the physical system.

Two approaches are possible, based on the correlation or on the spectrum of the change in velocity.

Without going into the details it may be recalled that under the same conditions as for equation A-1, the spread of the contaminant concentration can be expressed as :

$$\overline{X^2} = \int_0^t\int_0^t u(t)u(t')dtdt'.$$

The variance of the concentration is the first useful statistical quantity which can be computed, the mean value being identical equal to zero.

Using

$$R_0(dt) = \frac{\overline{u(t)u(t')}}{\overline{u^2}}$$

one obtains :

$$\overline{X^2} = 2\overline{u}^2 \int_0^t (t-t')\ R(t')\ dt'. \tag{A-3}$$

Knowing the shape of R(t), $\overline{X^2}$ can be computed. But it should be noted that, as opposed to the previous approach, in this case only the spread can be computed, and not the distribution of contaminants.

Recalling that the correlation $R_0(t)$ and the spectrum of fluctuations S(f) are related by the Fourier transform, one can write for the symmetric function R_0 :

$$R_0(t) = \frac{1}{2\pi} \int_0^\infty S(\omega)\cos\omega t$$

and make the substitutions to obtain :

$$\overline{X^2} = \overline{u^2}t \int_0^\infty S(f) \left(\frac{\sin(\pi ft)}{\pi ft}\right)^2 df. \tag{A-4}$$

It is evident that the term $F(f,t) = \left(\frac{\sin(\pi ft)}{\pi ft}\right)^2$ acts on the spectrum as a low pass filter, of bandwidth function of the elapsed time t. This presentation is very much favoured by the author because it illustrates very clearly the different scales of the turbulence during the successive stages of turbulent dispersion.

Equation A-3 or equation A-4 can now be applied for different conditions expected to represent the phenomenon analyzed.

Case 1 - Under the conditions of equation 1, one has for R_0 a Dirac function, and the time step T for the random walk. Hence, it could be accepted that

$$\int_{\infty}^{\infty} R_0(t)dt = T \qquad \text{or} \qquad \int_{0}^{\infty} R_0(t)dt = \frac{T}{2} \tag{A-5}$$

indicating a correlation length of the order of $\frac{T}{2}$.

$$\overline{X^2} = \overline{2u^2}\,\frac{Tt}{2} = \overline{u^2}Tt \tag{A-6}$$

that is a linear function of t identical to what was obtained previously if T = 1 and $\overline{u^2} = 1$.

A comparison can be made with the gradient diffusion equation, which also assumes (by definition) an uncorrelated flow field in comparison to the computation time scale. This can be equation 5, which leads to :

$$\overline{x^2} = 2Kt, \qquad \text{that is} \qquad K = \frac{\overline{u^2}T}{2}, \tag{A-7}$$

which is the usual form of the diffusivity coefficient in homogeneous flows.

Because equation A-2 was solved for $a^2 = \overline{u^2}$ and $T = 1$ then from A-7

$$K = \frac{a^2}{2} = \frac{1}{2}$$

and

$$\overline{x^2} = \overline{u^2}\,t \tag{A-8}$$

again identical, as expected because it is a diffusion equation, to that obtained from the random walk approach. It should be noted that in this case one should be able to obtain also the concentration profiles.

<u>Case 2</u> - The same system can be considered as a continuous Markovian one for which A-2 is only an approximation (see Durbin). That is, one may consider to be in the presence of a more realistic simulation of homogeneous turbulence. In such cases the correlation is represented by an equation of the type : $R = e^{-t/\tau}$.

To keep things similar one may take $\tau = \frac{T}{2}$, equivalent macroscale of the turbulence.

Hence, the spread can be obtained by solving, analytically, equation A-3 or numerically, equation A-4, the spectrum being given by :

$$S(f) = \frac{2}{T}\,\frac{1}{\left[1+\left(\frac{2\pi f}{T}\right)^2\right]} \tag{A-9}$$

The analytical solution is obtained as :

$$\overline{x^2} = 2\overline{u^2}\,\frac{T}{4}\left(e^{-2t/T} + \frac{2t}{T} - 1\right) \qquad (A-10)$$

which only asymptotically tends to the solutions obtained before as, for example, equation A-8. This reflects the non diffusive nature of the system at small times, the correlation of the velocity fluctuation covering non negligible durations in comparison to those of observation.

Case 3 - Equation A-2 can be assumed to represent accurately, in discretization form, the continuous nature of the system. Under this assumption, the theory of information imposes that the original system contains no frequency larger than $\frac{1}{2T}$, that is, half of the sampling frequency $\frac{1}{T}$. Hence, the spectrum will be flat, but band limited to $f = \frac{1}{2T}$, and normalized to be

$$\int_{-\infty}^{\infty} S(f)df = 1. \qquad (A-11)$$

The corresponding correlation function is obtained, because of symmetry, as

$$R_0(t) = \frac{1}{2\pi}\int_0^{\infty} S(f)\cos(\omega t)d\omega = \frac{2}{2\omega}\int_0^{1/2T} T\cos(\omega t)d\omega_0 \qquad (A-12)$$

$$R_0(t) = \sin \frac{\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}} \tag{A-13}$$

with a macroscale equal to :

$$\int_0^\infty R_0(t)dt = \frac{\pi}{2}\,\frac{T}{\pi} = \frac{T}{2} \tag{A-14}$$

consistent with the previous one.

The shapes of the correlation curves are shown in figure A.3.

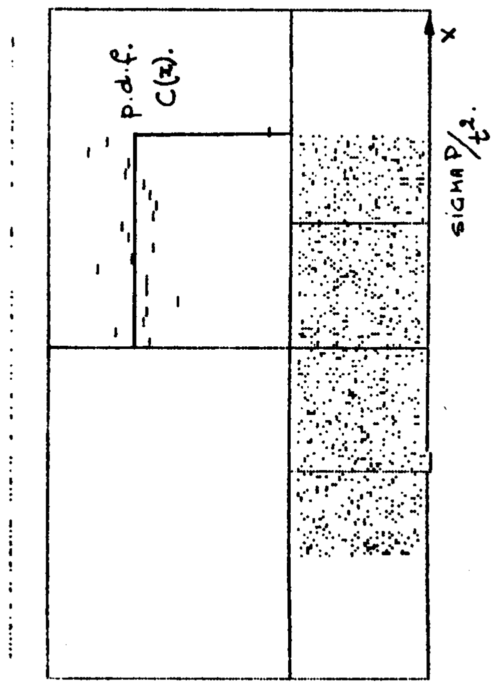
There are still more doubts about the compatibility of a system with a flat band limited spectrum and a uniform probability density distribution of the variable, but this question is left to be analyzed. The spread in this case is numerically computed from equation A-8.

The evolution of spread, or variance, SigmaP, of contaminant for all the cases are plotted as function of time in figure A.4.

Not surprisingly the results of case 3 are quite close to those for random walk.

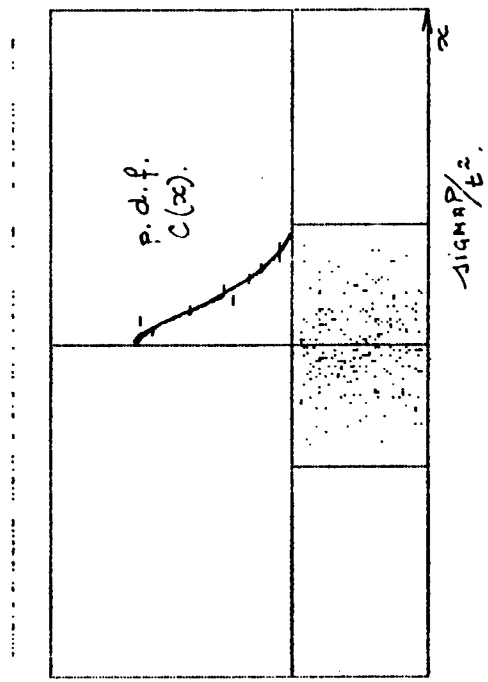
There are differences in the results, but the question is not to say which one is correct. Each one may be considered as correct for the system it is representing.

A more adequate question to ask is probably which system best represents the reality.



SIGMAV= 47 SIGMAP= 47 POINTS= 1000 TIME= 1
t^2

Fig. A.1 - Random walk dispersion



SIGMAV= 46 SIGMAP= 15 POINTS= 250 TIME= 10
t^2.

Fig. A.2 - Random walk dispersion

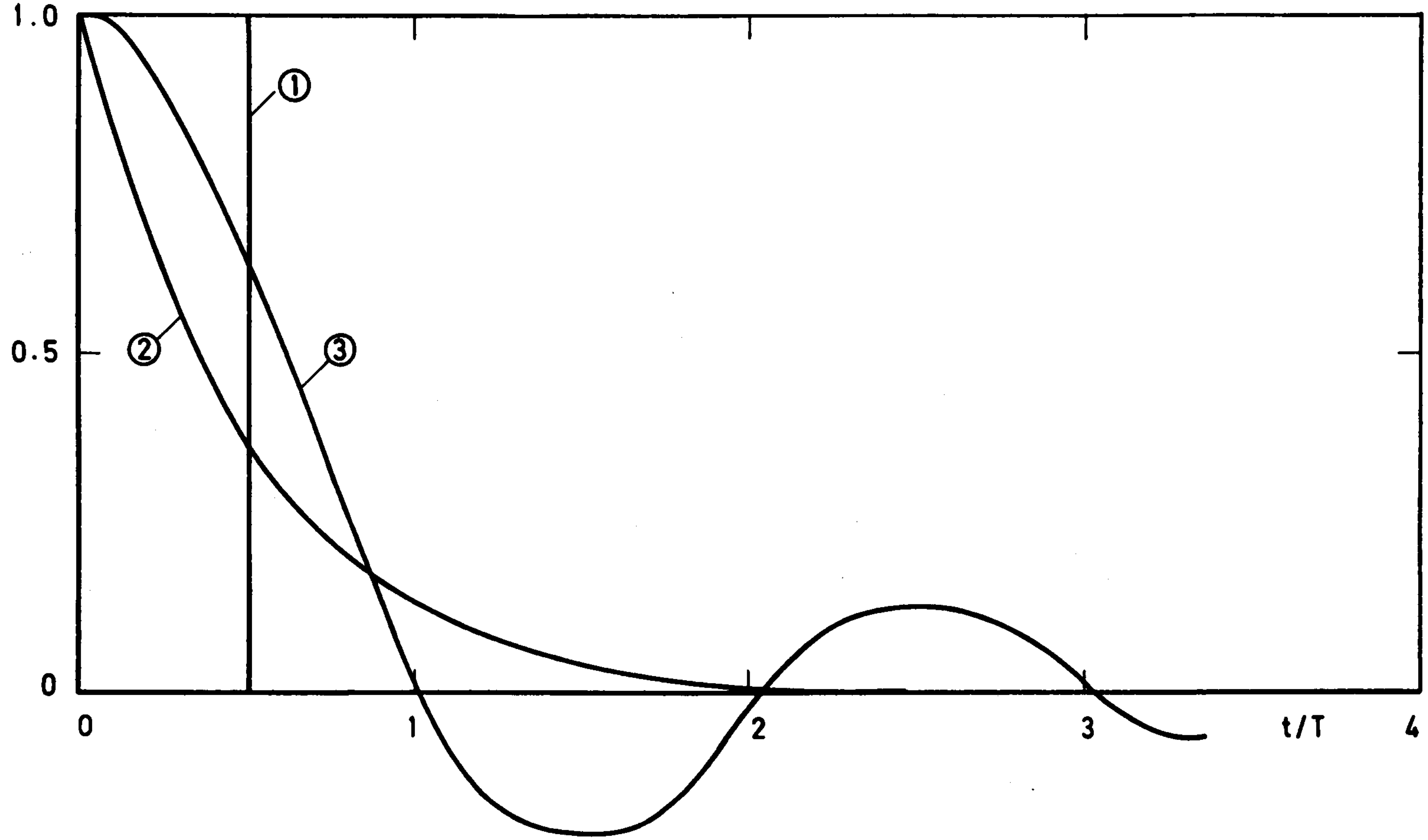


FIG. A 3 - CORRELATION FUNCTIONS

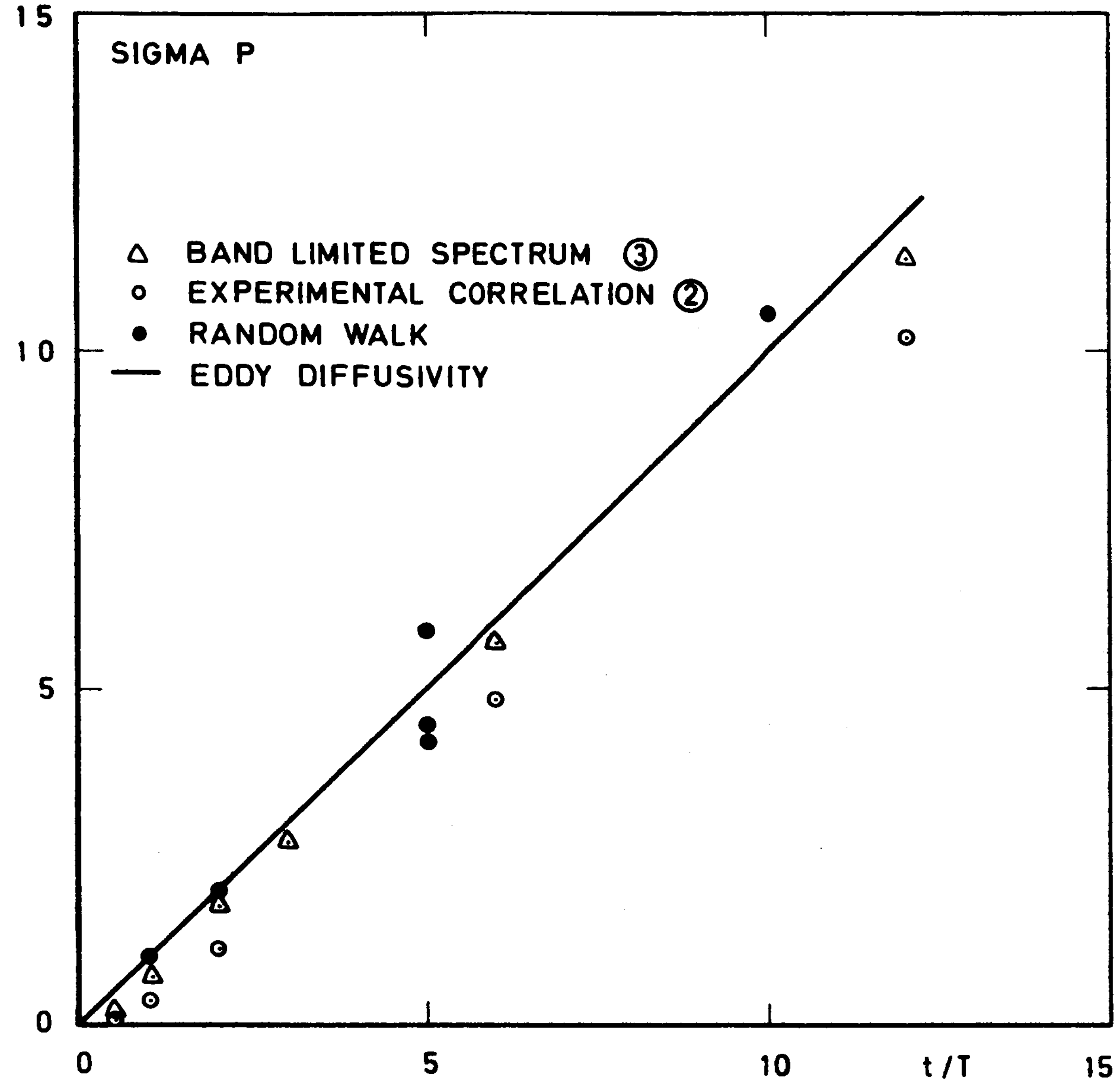


FIG. A 4 - EVOLUTION OF SIGMA P VS. TIME.

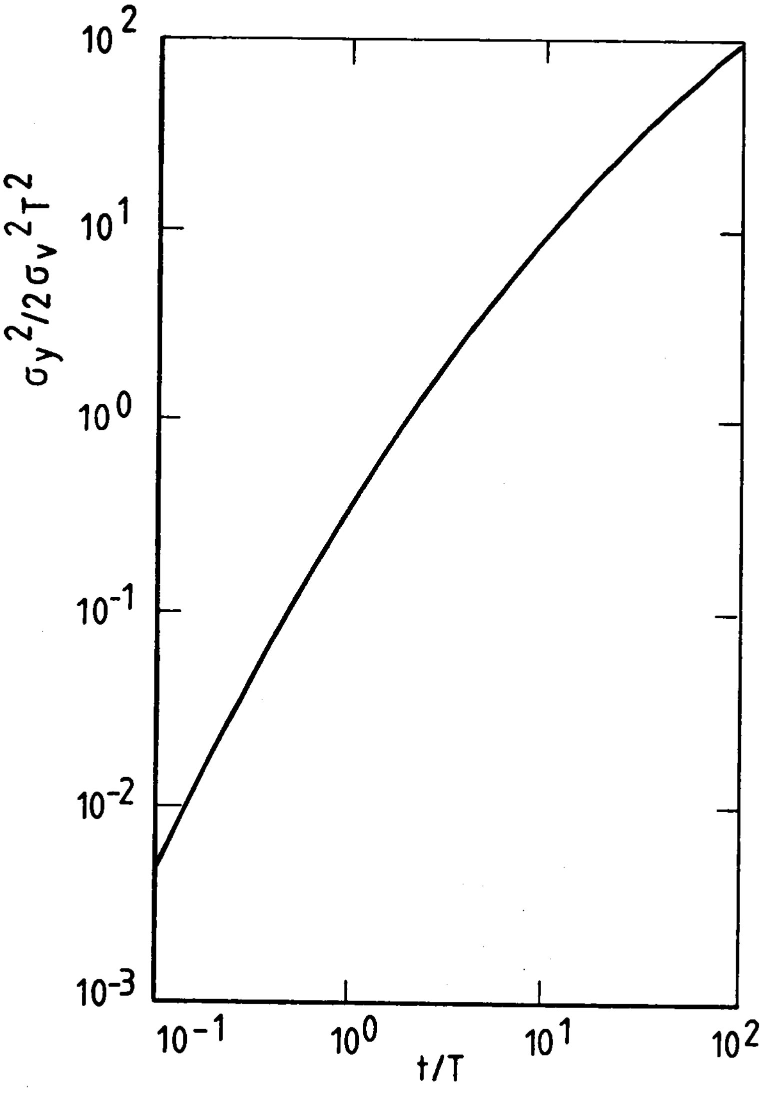
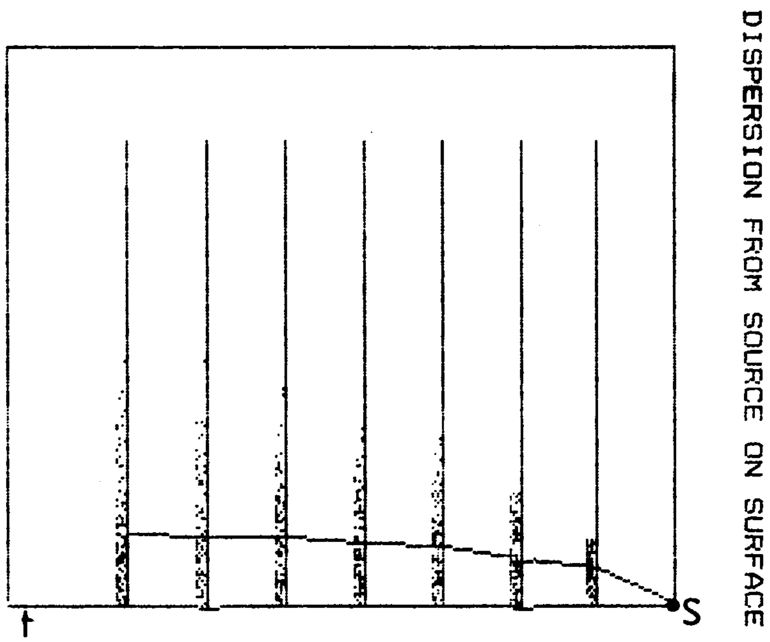


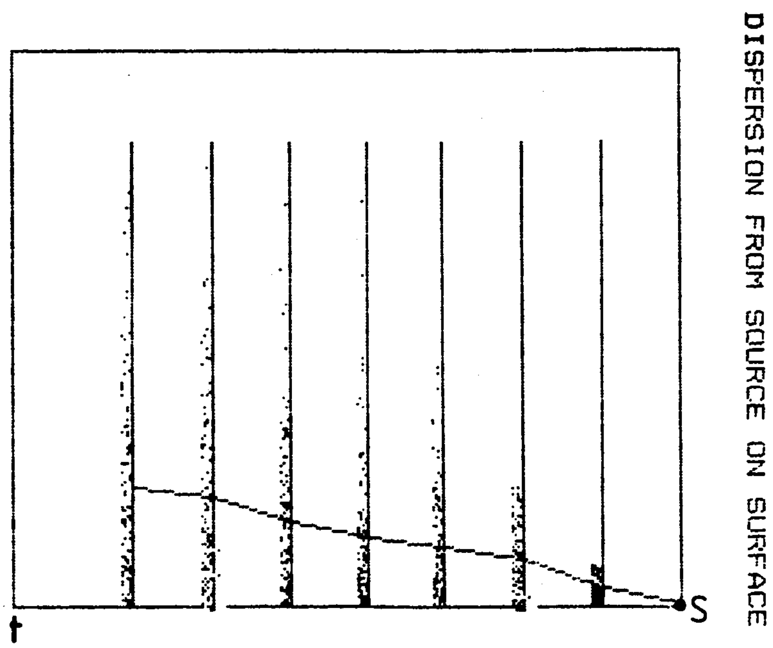
FIG. 1 - SOLUTION TO TAYLOR'S STATISTICAL DIFFUSION EQUATION FOR EXPONENTIAL CORRELATION COEFFICIENT, (ref.9).

$R(\tau) = EXP(-\tau/T)$



- HOMOGENEOUS

FIG. 2



- LINEAR DIFFUSIVITY

FIG. 3 -DISPERSION FROM A GROUND LEVEL SOURCE.

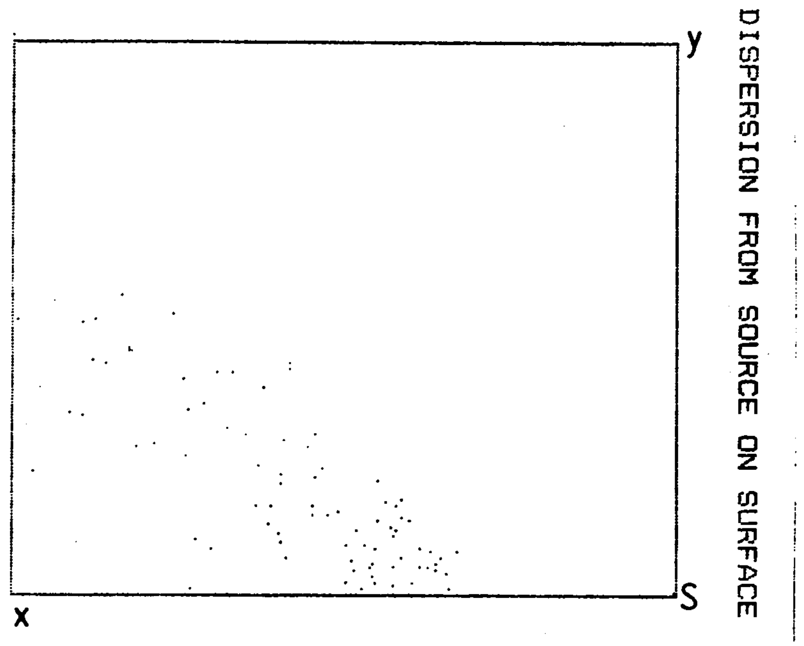


FIG. 4 -DISPERSION FROM AN INSTANTANEOUS POINT SOURCE IN SHEAR FLOW. CLOUD AT t = 7

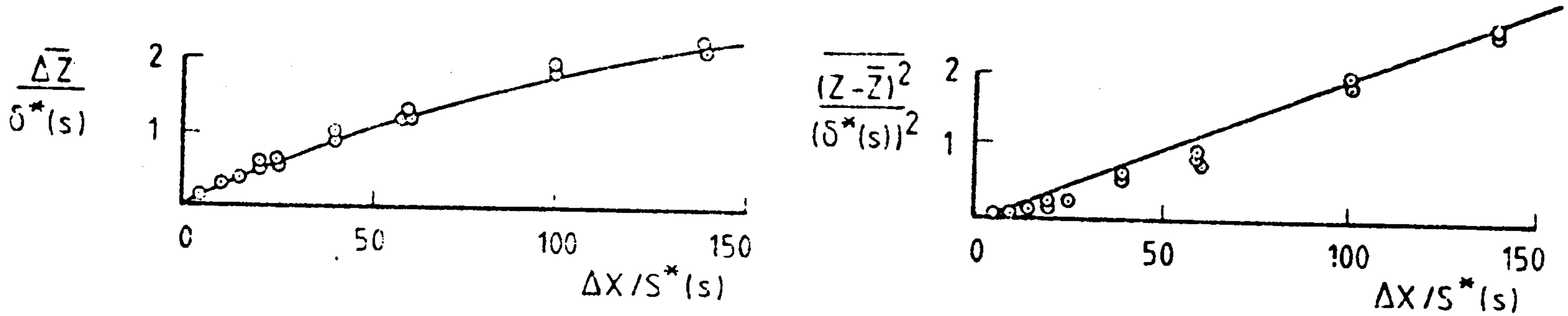


FIG. 5 - EVOLUTION OF PLUME CENTROID AND DISPERSION WITH DOWNSTREAM DISTANCE FOR A GROUND SOURCE. (From ref. 50)

——— PREDICTION

⊙ EXPERIMENT

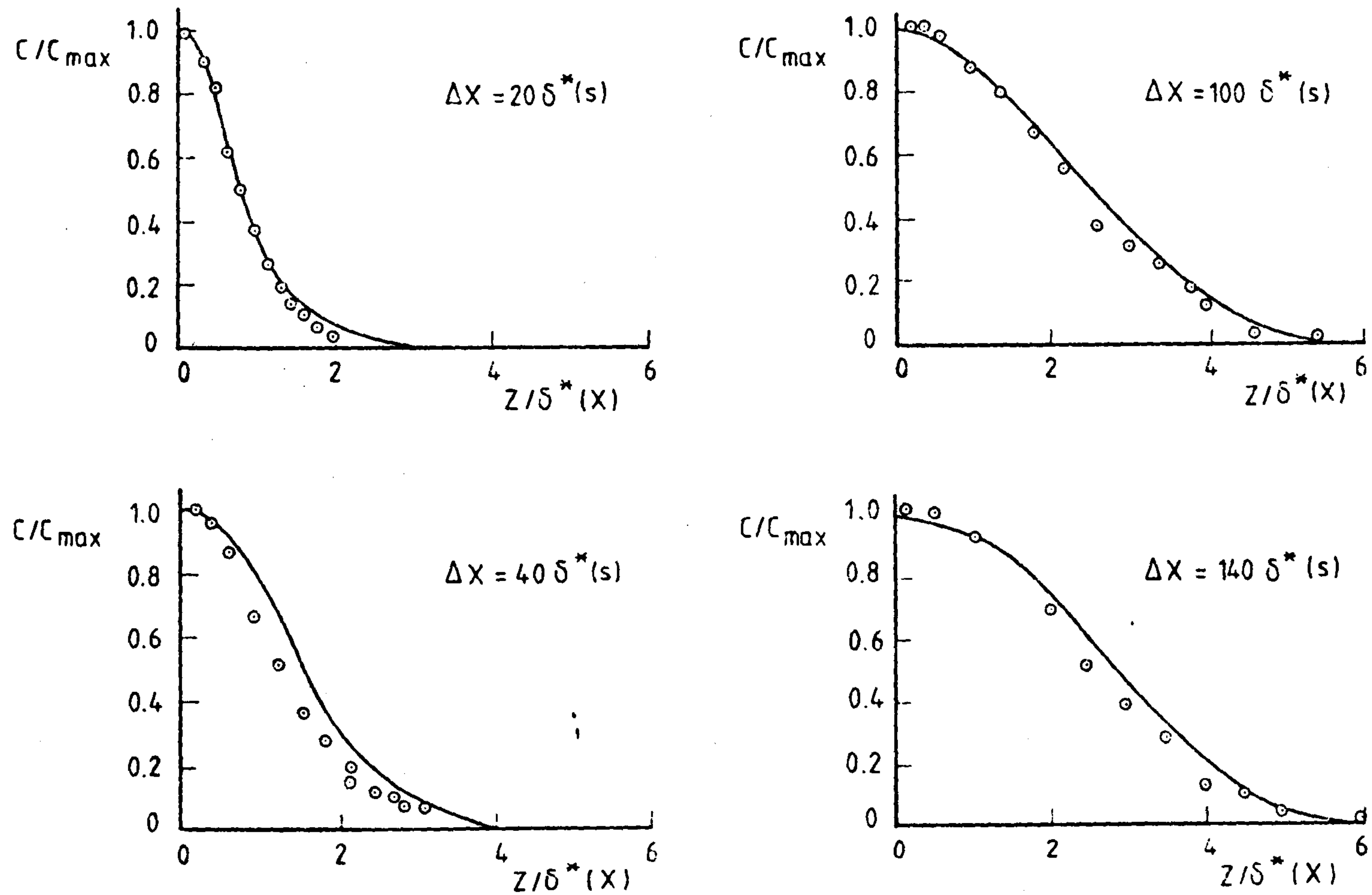


FIG. 6 - ADIMENSIONALIZED CONCENTRATION PROFILES FOR A GROUND SOURCE
—— PREDICTION
⊙ EXPERIMENT
(From ref. 50)

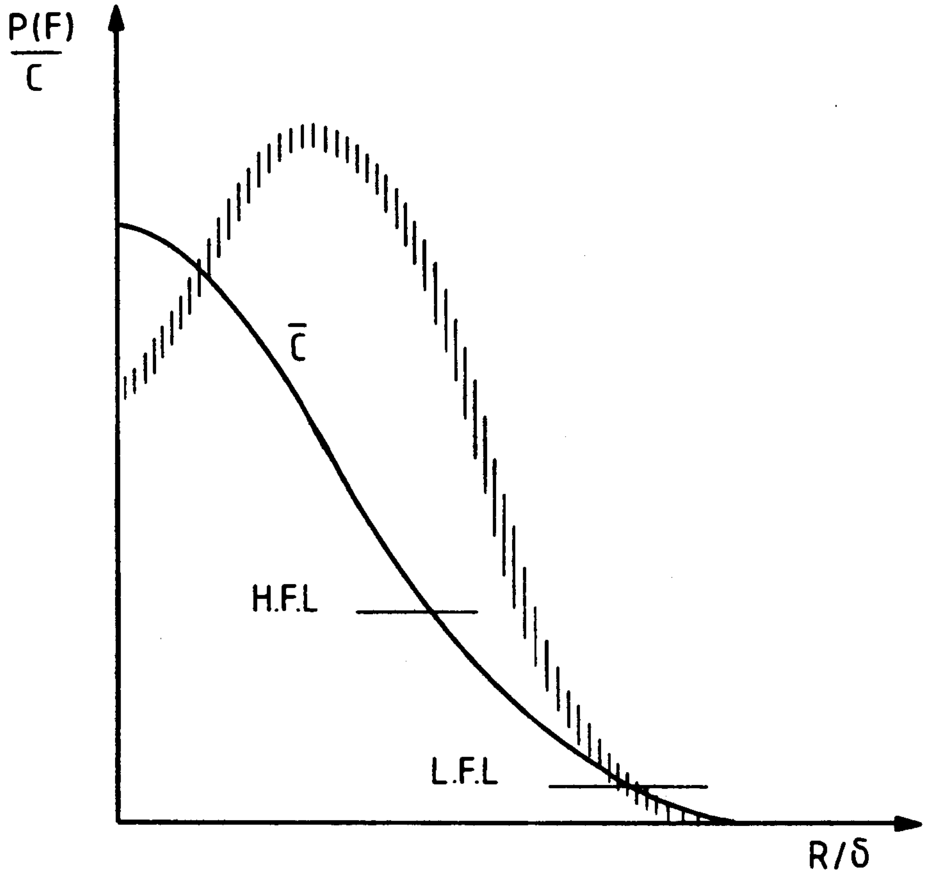


FIG. 7 - FLAME IGNITION PROBABILITY (III) AND MEAN CONCENTRATION (-) IN A ROUND JET. (Ref. 27)

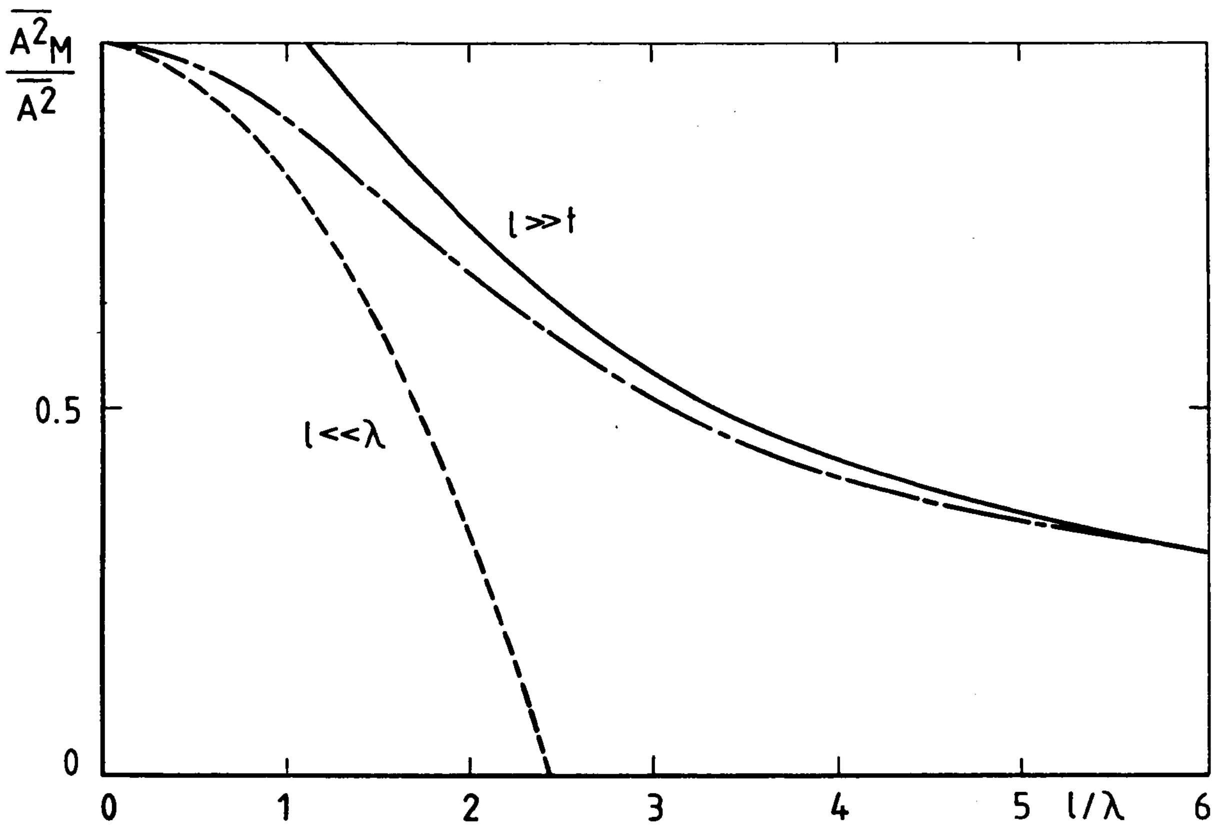


FIG. 8 - EFFECT OF PROBE DIMENSION.

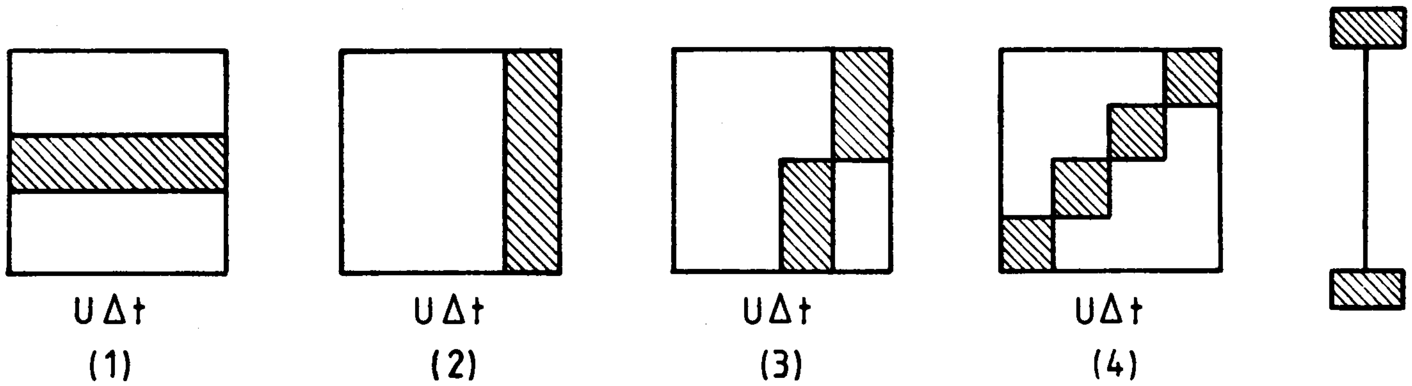


FIG. 9 -EFFECTS OF "CONCENTRATION DISTRIBUTION" IN THE REFERENCE VOLUME ON PROBE OUTPUT

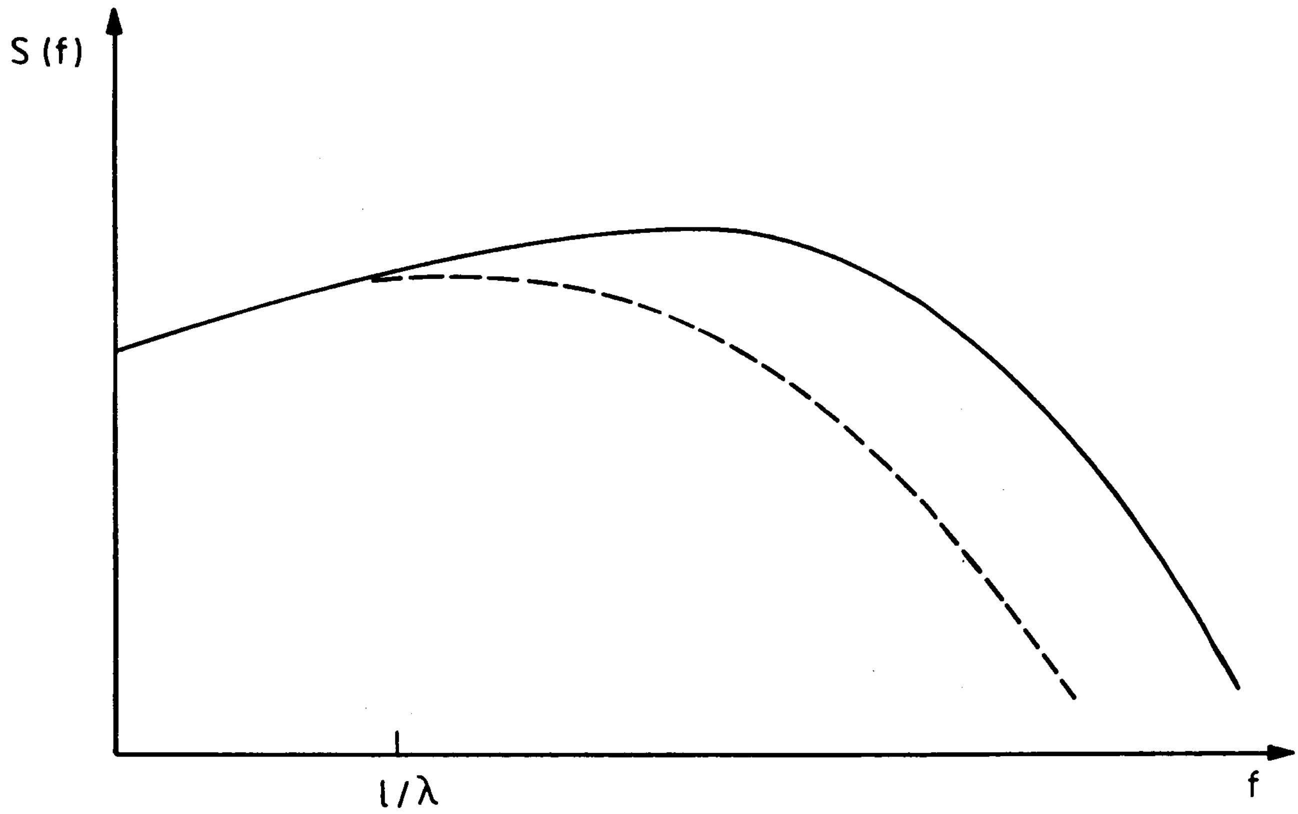


FIG. 10 - INDICATIVE EFFECT ON SPECTRAL RESPONSE.

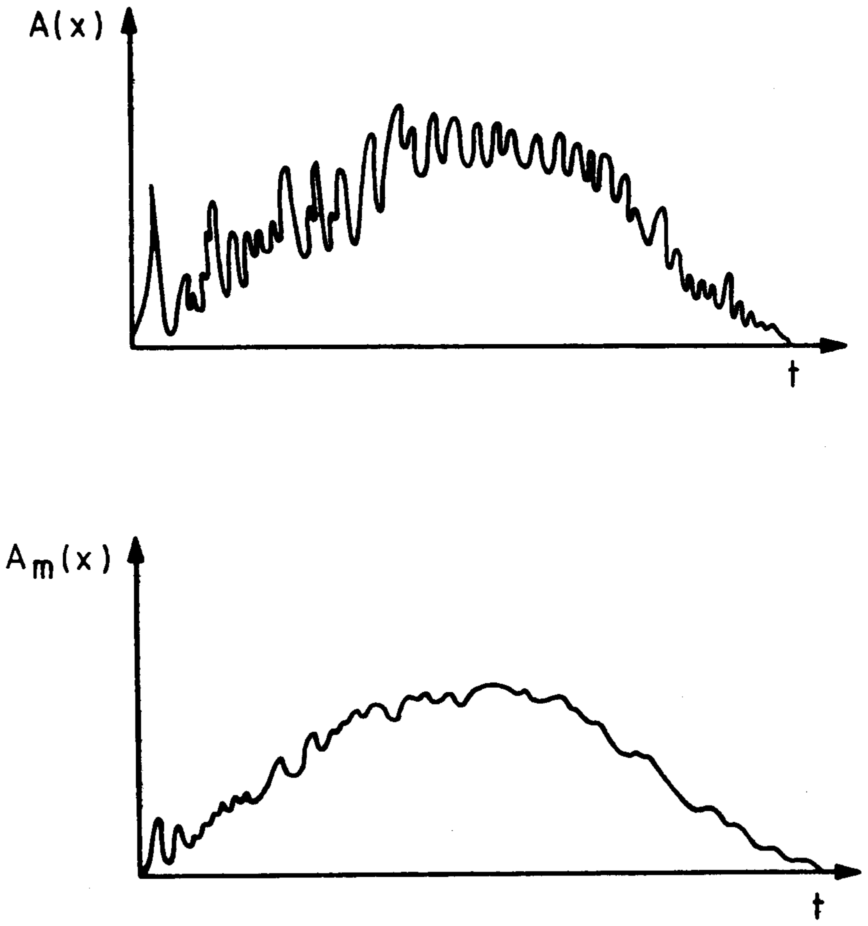
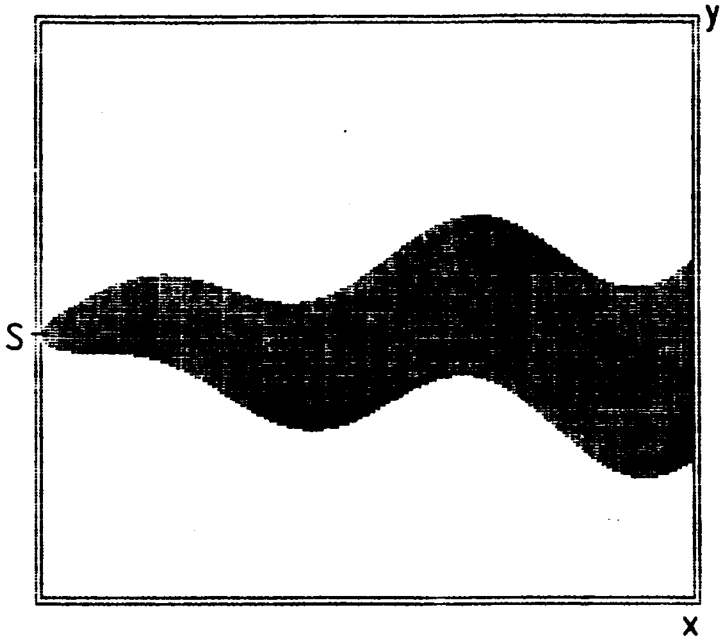
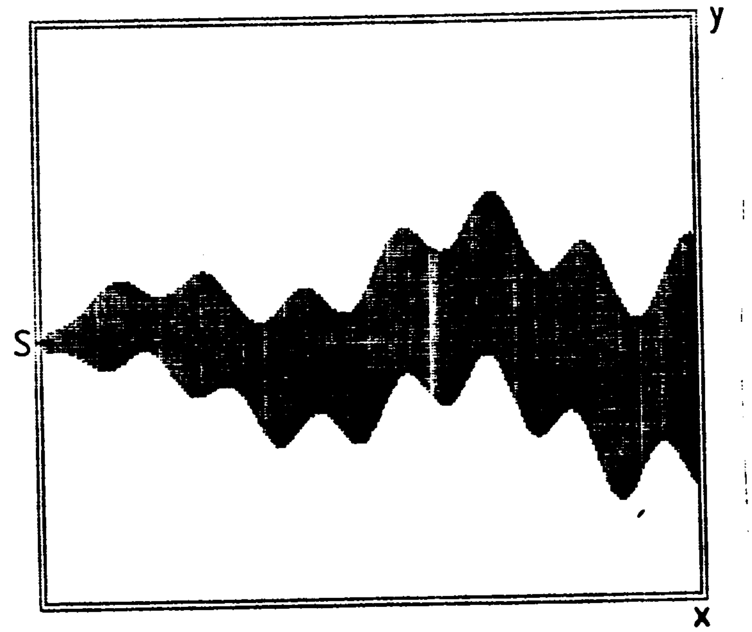


FIG. 11 -REAL VERSUS MEASURED RESULTS, SCHEMATIC EXAMPLE.



-MEANDERING OF A PLUME



-MEANDERING AND INTERMITTENCY

FIG. 12 - ILLUSTRATION OF MEANDERING AND INTERMITTENCY PLUS MEANDERING FOR A PLUME.

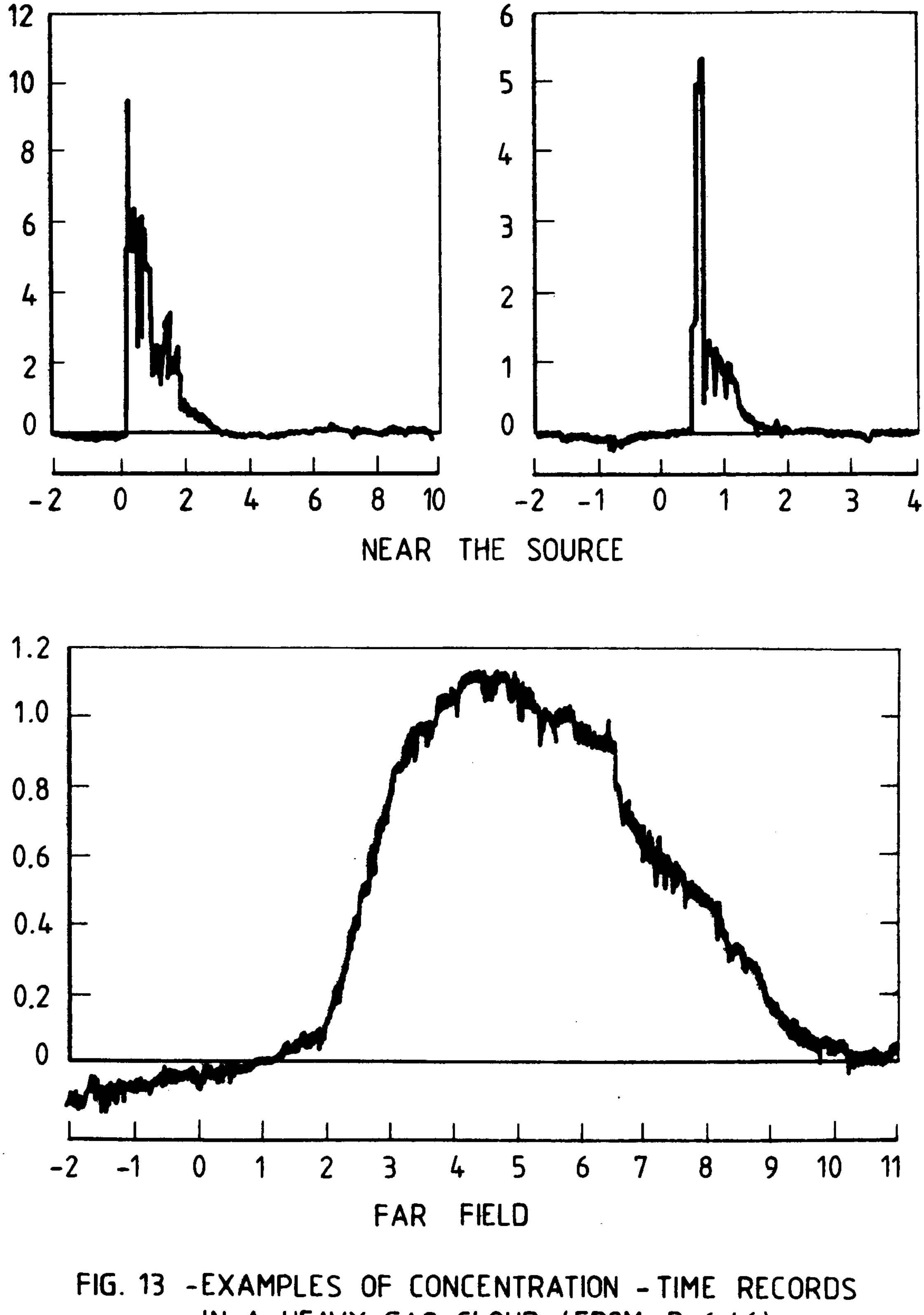


FIG. 13 -EXAMPLES OF CONCENTRATION - TIME RECORDS
IN A HEAVY GAS CLOUD. (FROM Ref. 46)
ORDINATE - CONCENTRATION, VOLUME PER CENT
ABSCISSA - TIME, SECS × 10^{-2}

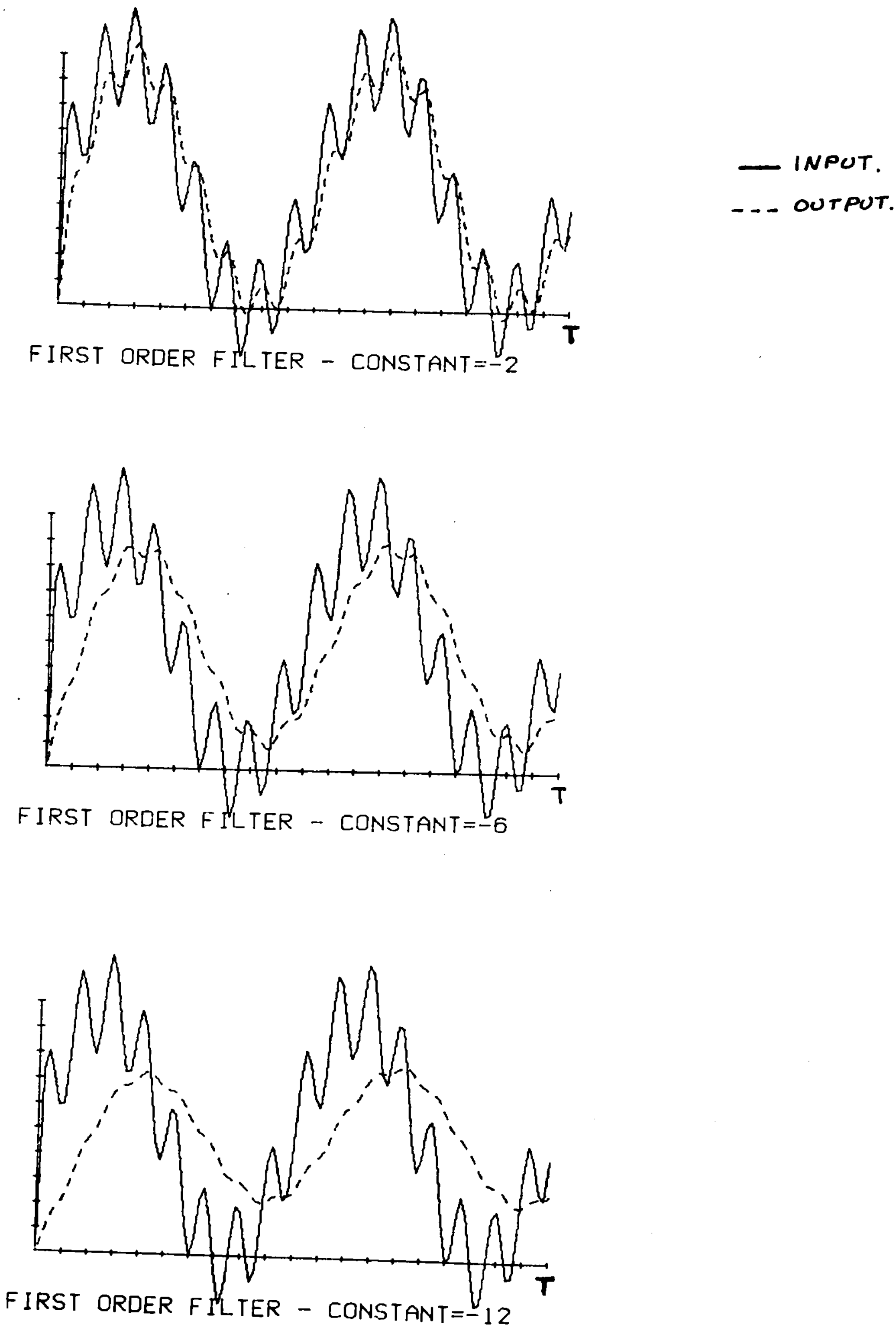
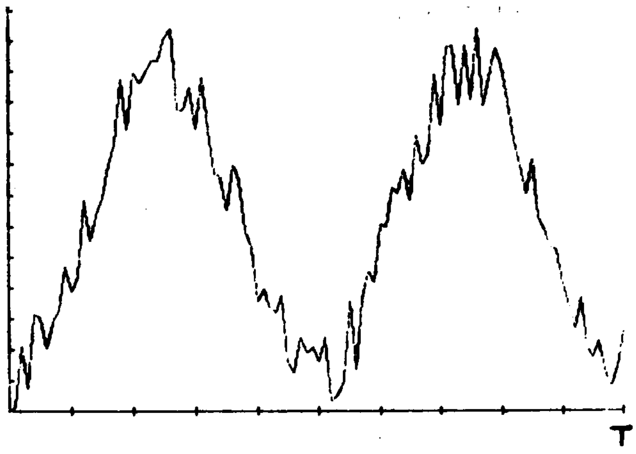
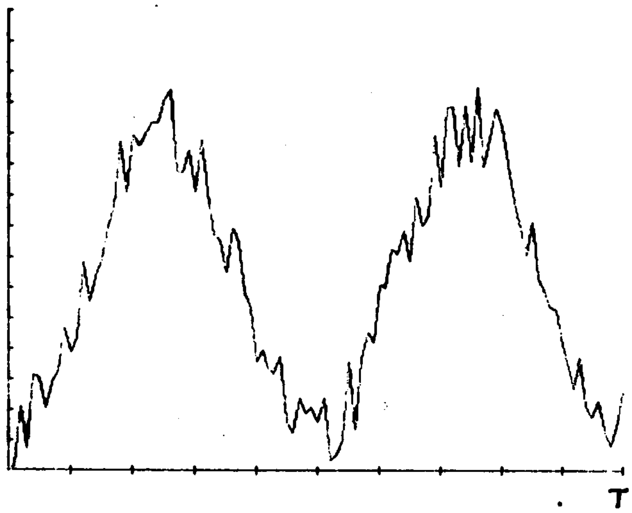


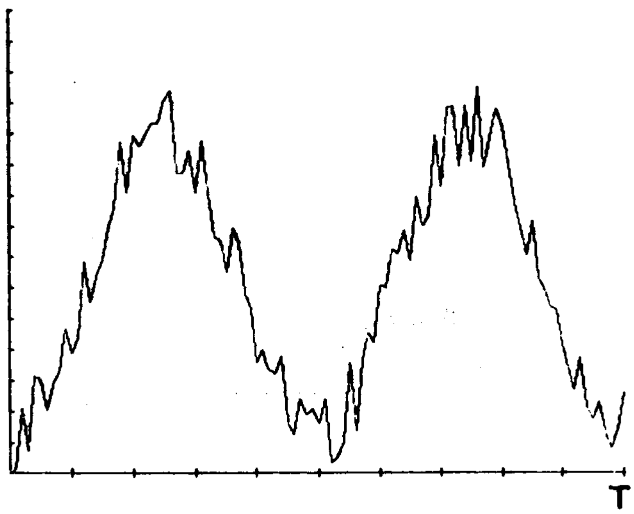
Fig.14 - EFFECT OF FILTER CONSTANT ON RECOVERY OF "MEAN" VALUE.



FILTERED ON 3 POINTS.



FILTERED ON 10 POINTS.



FILTERED ON 18 POINTS.

Fig. 15 - USE OF "RUNNING MEAN" TO RECOVERY OF "MEAN" VALUE.

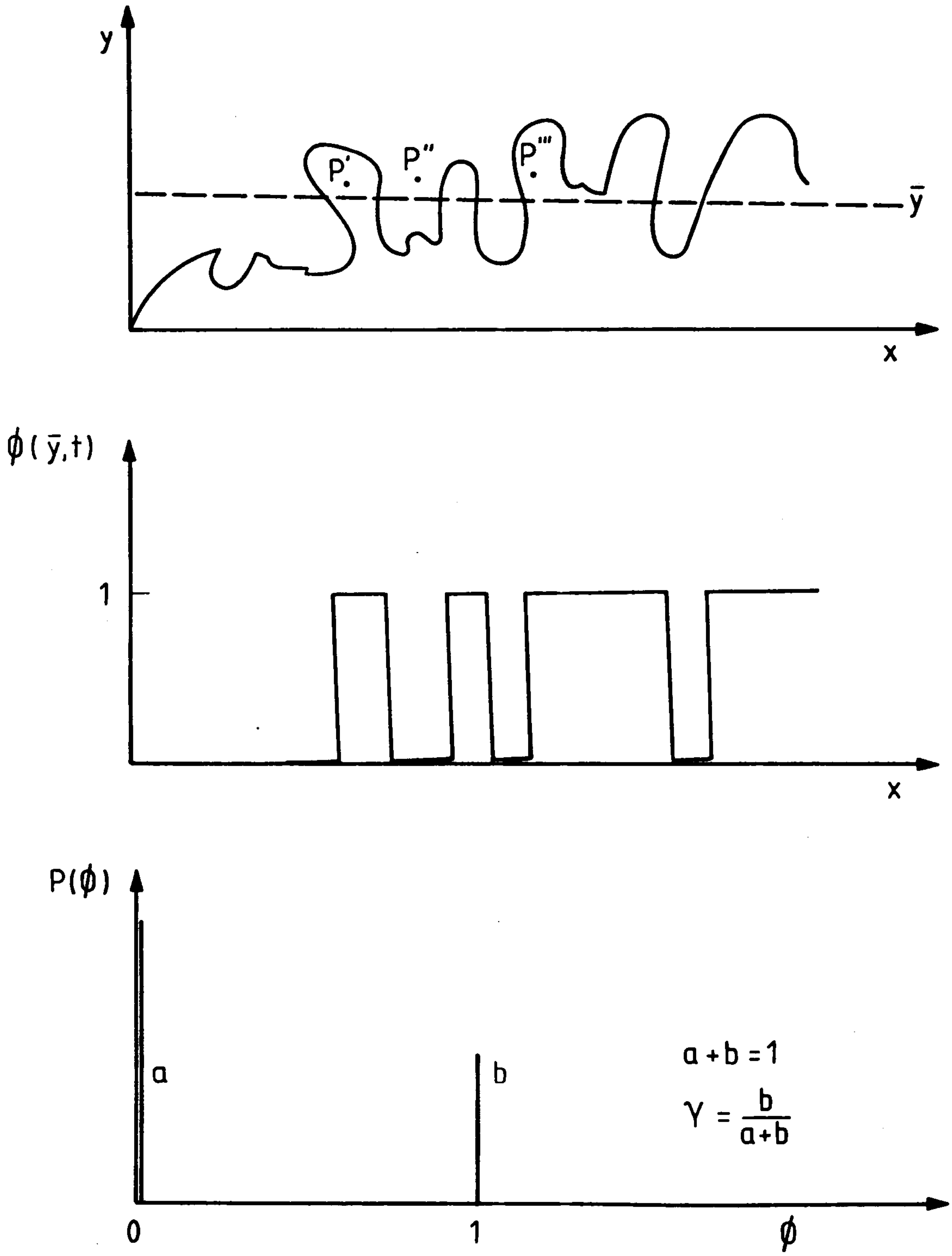


FIG. 16 - INTERMITTENCY AND INDICATOR FUNCTION

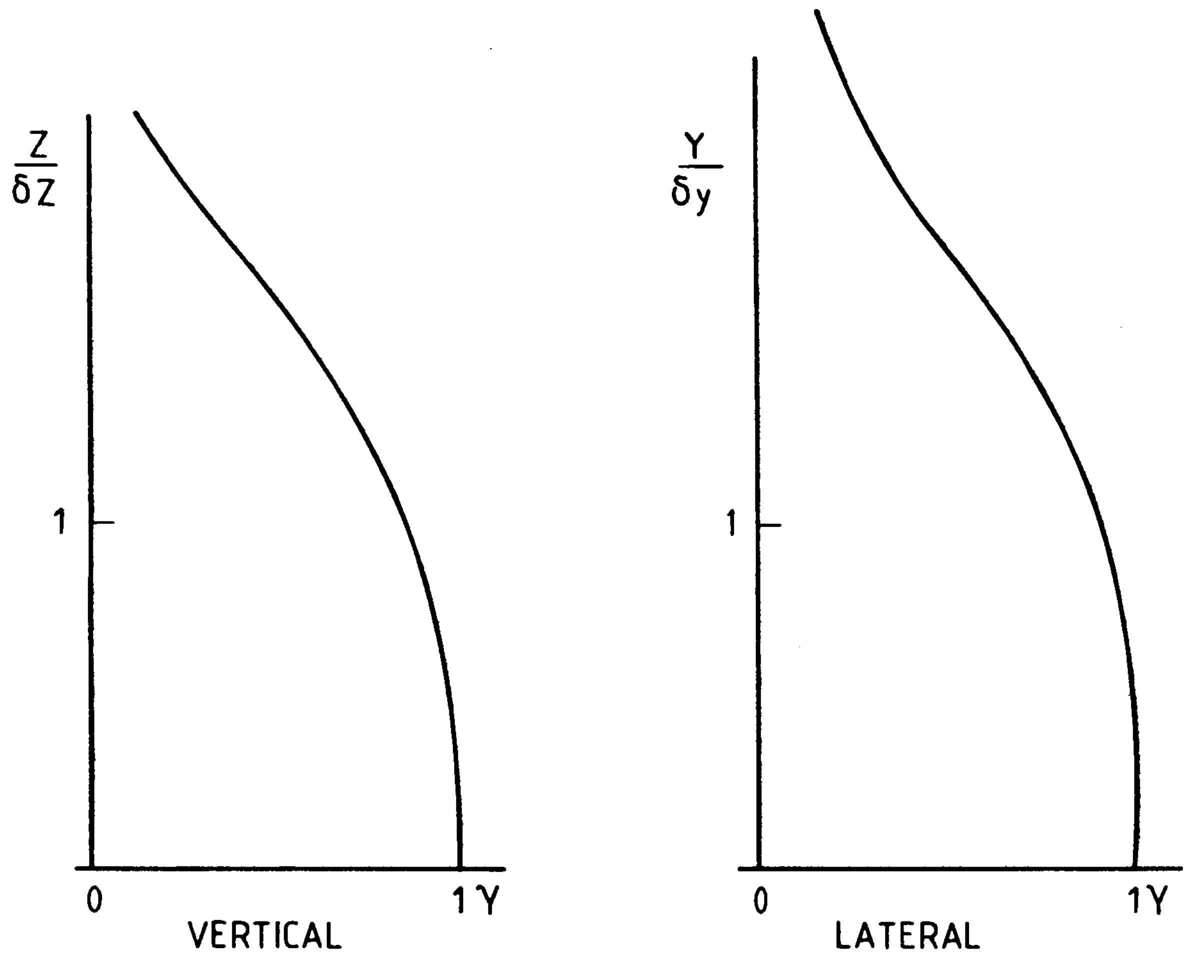


FIG. 17 - TYPICAL SHAPES OF INTERMITTENCY FOR 3-D PLUME AT SURFACE.
(ADAPTED FROM REF. 8)

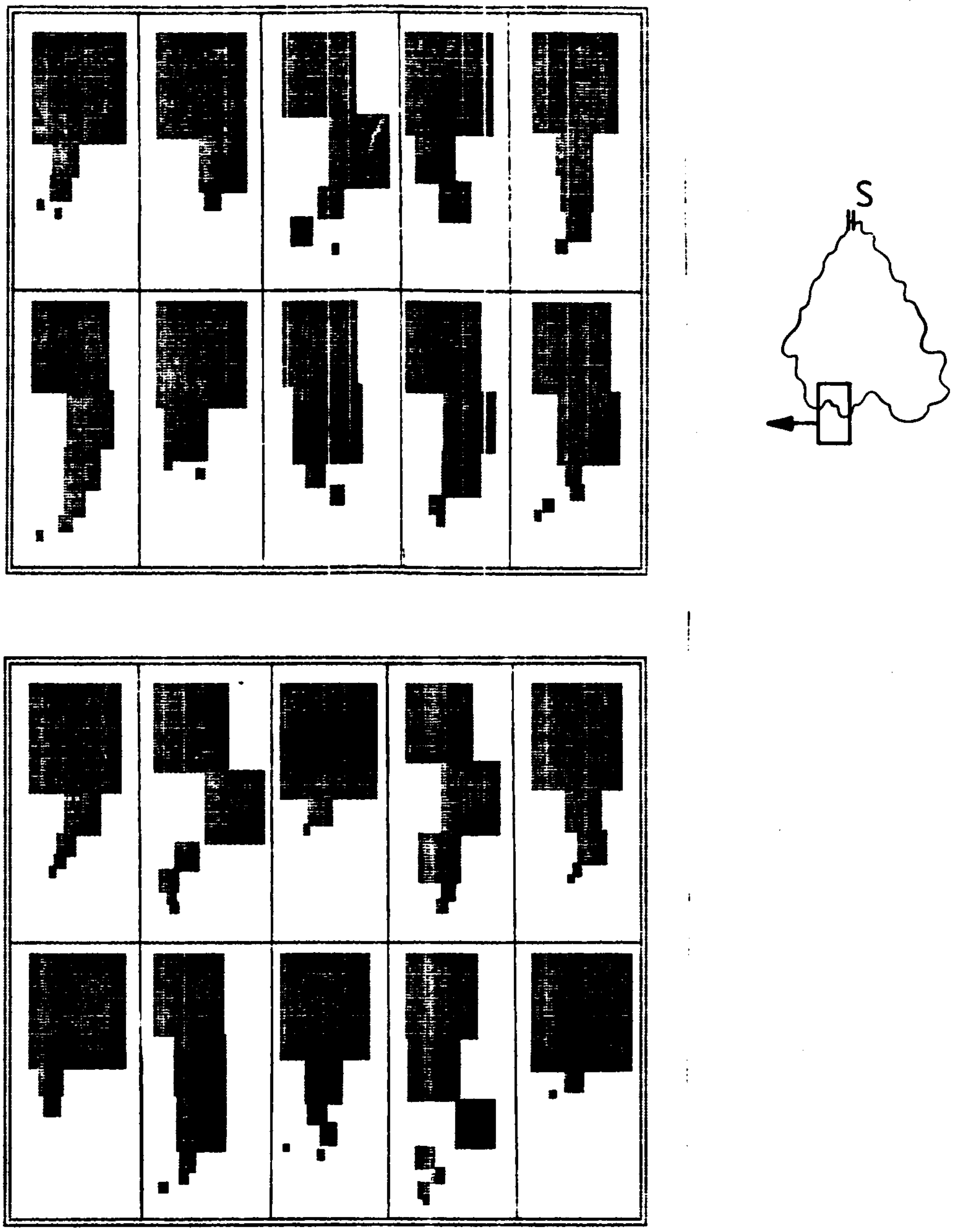


FIG. 18 - SCHEMATIC ILLUSTRATION OF THE INTERFACE OF CLOUDS OF CONTAMINANT. 20 INDEPENDENT EVENTS.

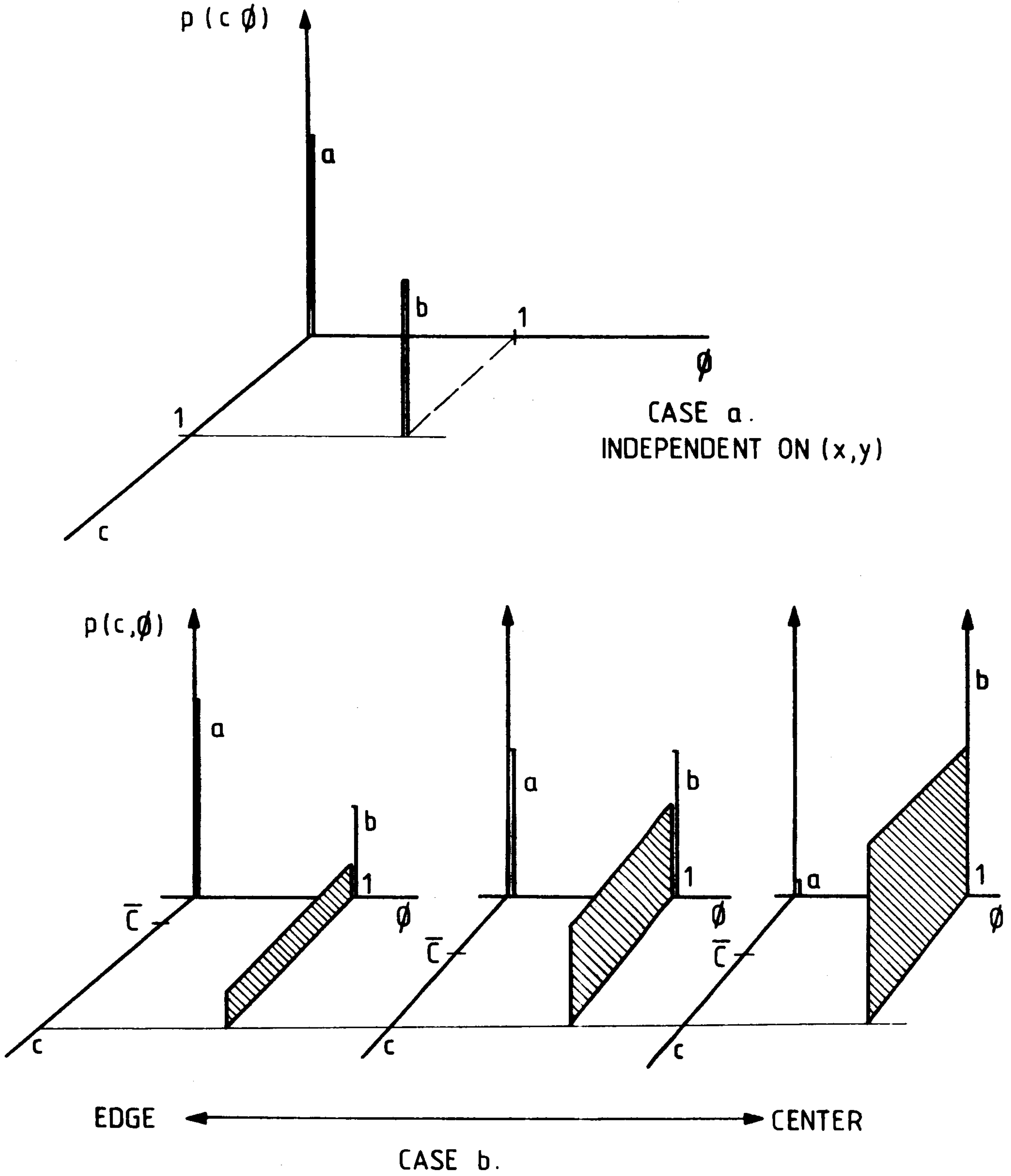


FIG. 19 - THE JOINT PROBABILITY OF C AND ∅

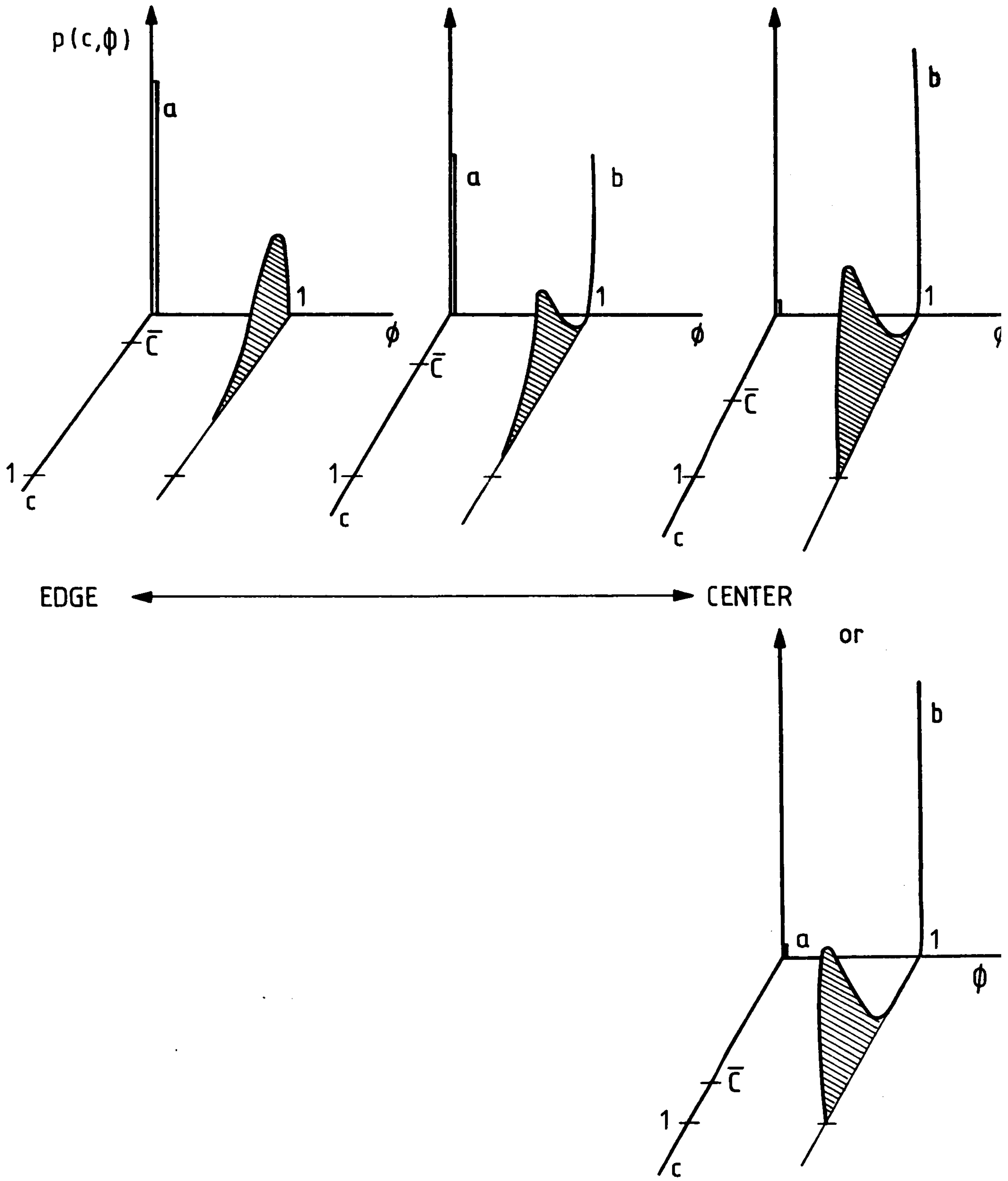


FIG. 20 - QUALITATIVE SHAPES FOR THE JOINT P.D. FUNCTION.

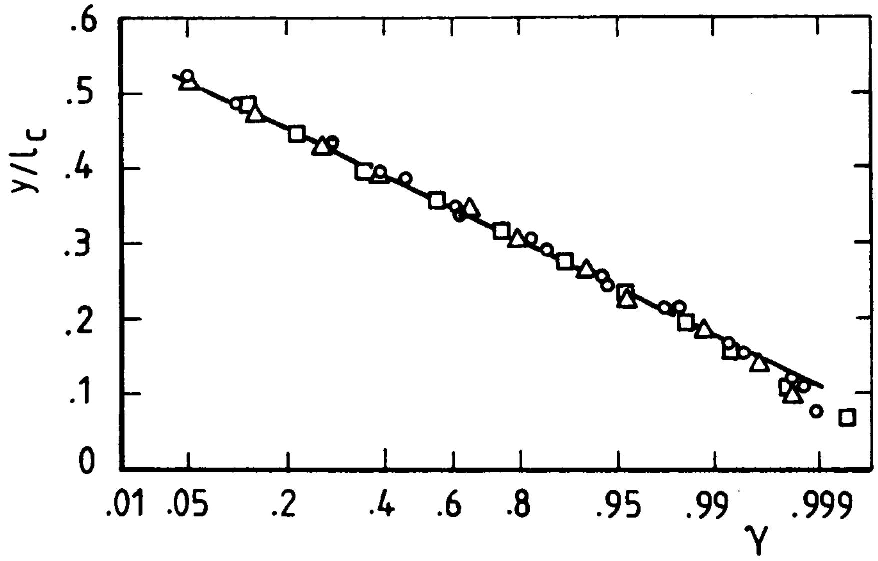


FIG. 21- INTERMITTENCY FACTOR, γ, ON GAUSSIAN PLOT. (From ref. 52)

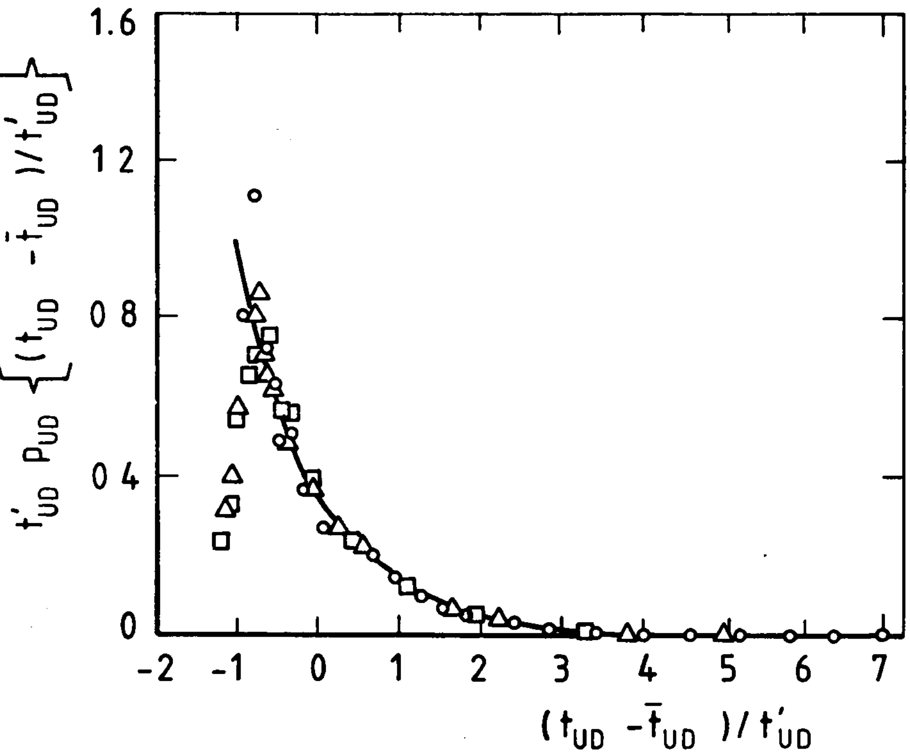


FIG.22- NORMALIZED PROBABILITY DENSITY FUNCTION OF THE TURBULENT LENGTHS AT X/d = 400 (From ref. 52)

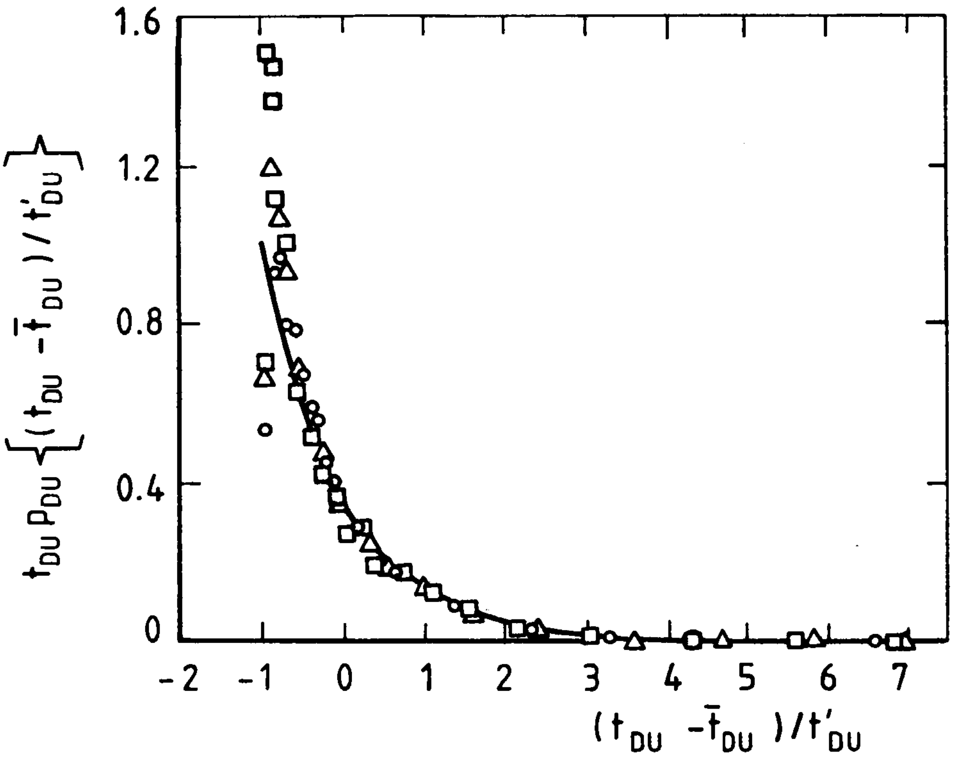


FIG.23 - NORMALIZED PROBABILITY DENSITY FUNCTION OF THE NON-TURBULENT LENGTHS AT X/d = 400 (From ref.52)

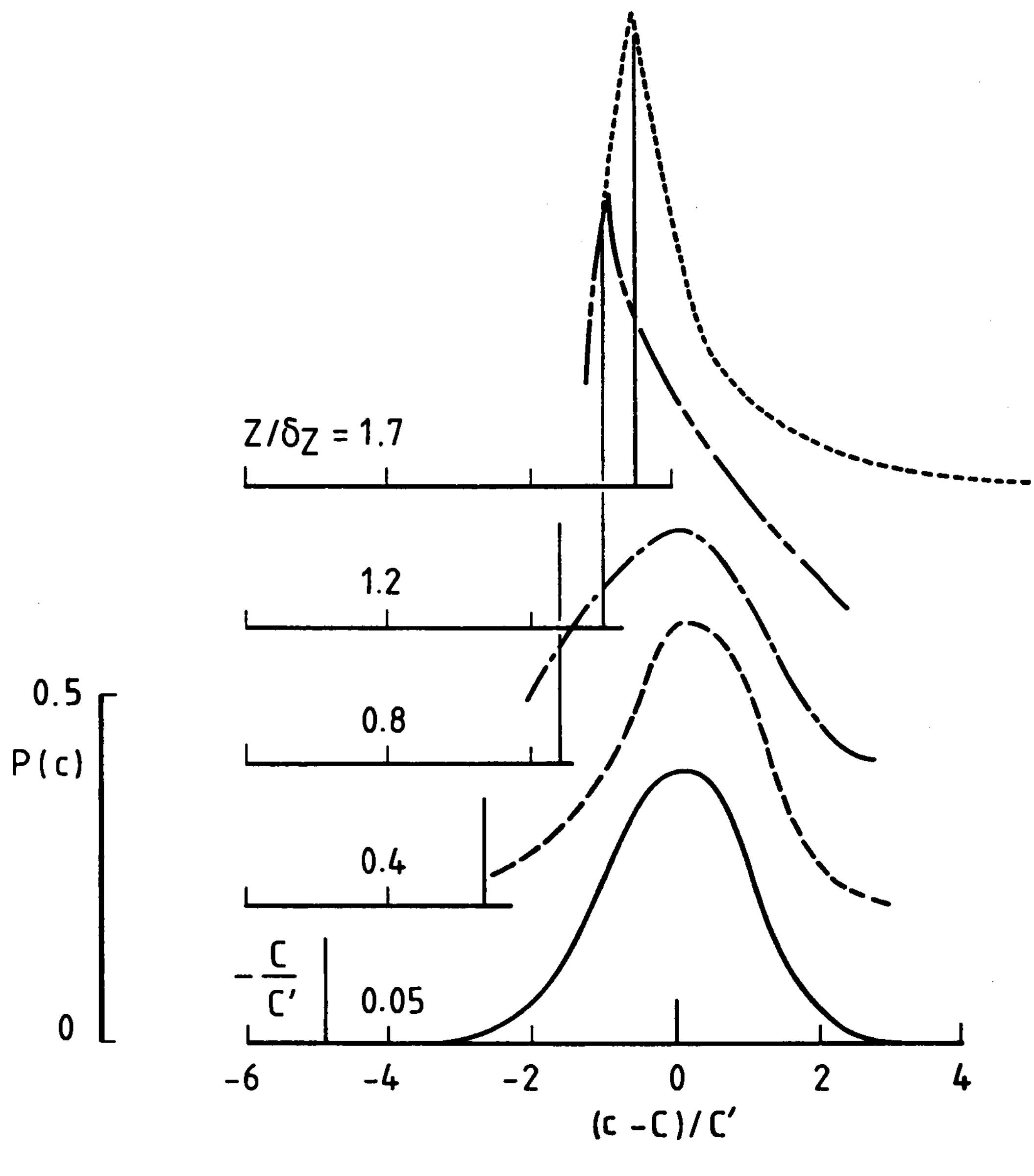
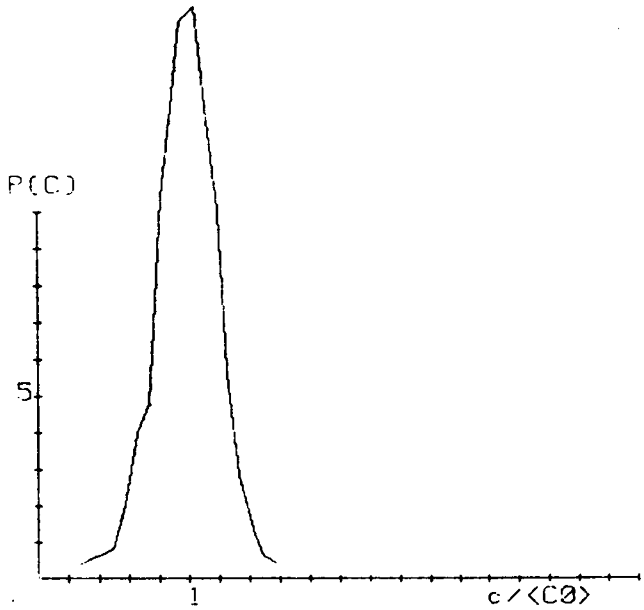
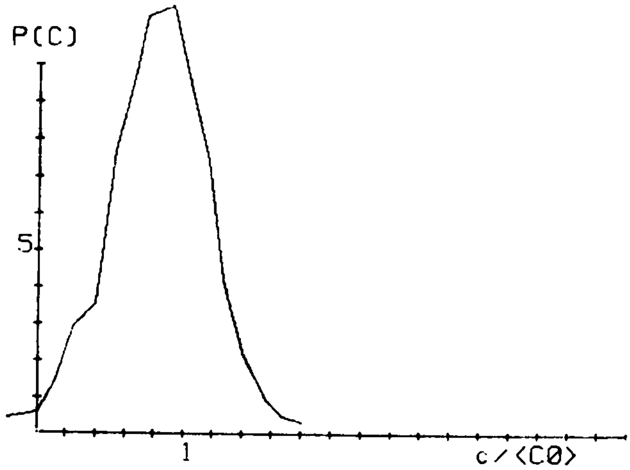


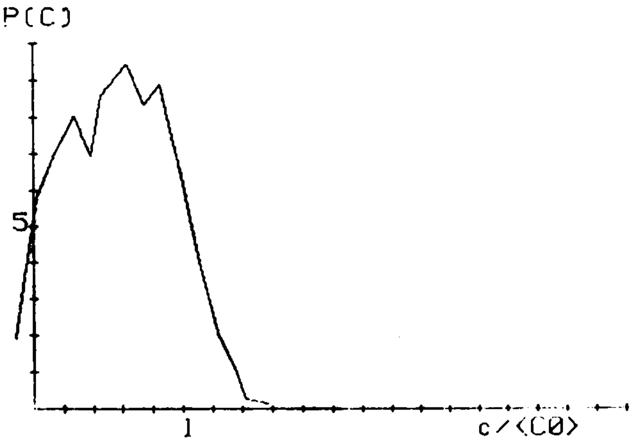
FIG. 24 - CONCENTRATION PROBABILITY DISTRIBUTION FUNCTIONS DOWNWIND OF GROUND LEVEL POINT SOURCE. (ref. 7)



P.D.F. OF CONCENTRATION AT Z/D=0.056

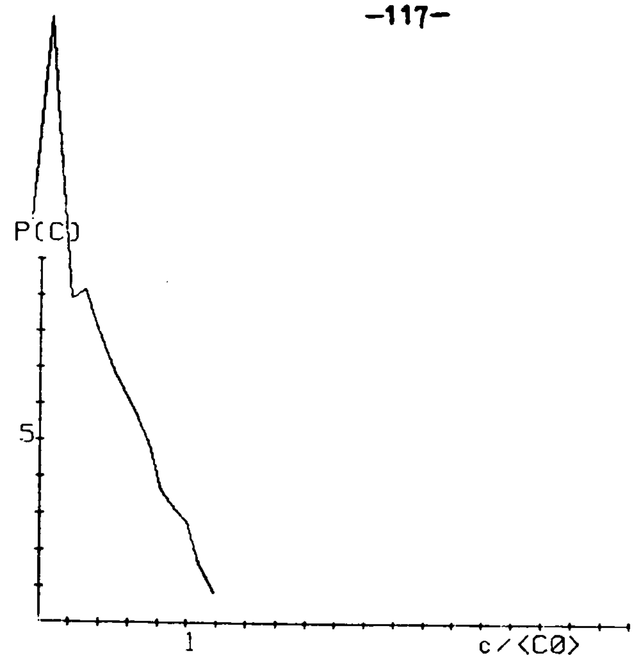


P.D.F. OF CONCENTRATION AT Z/D=0.4.

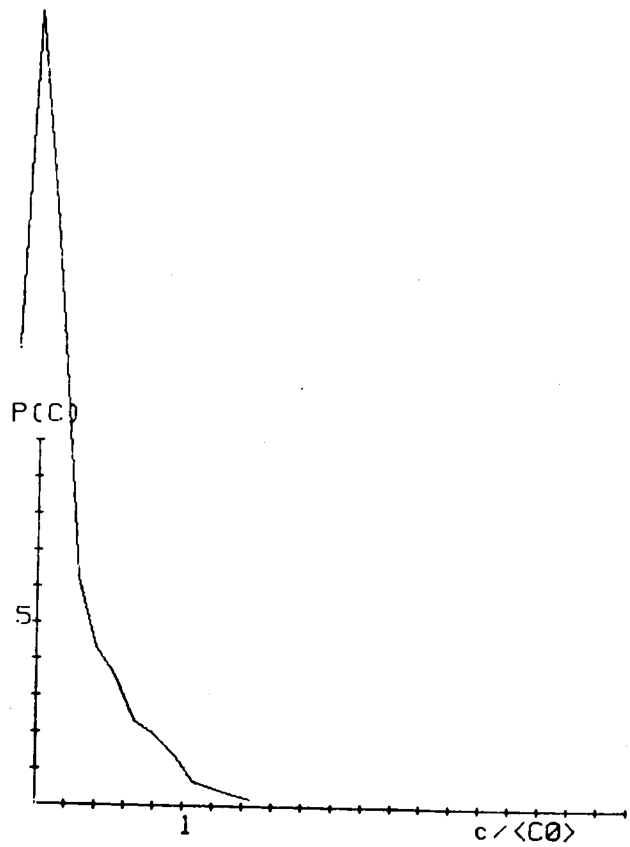


P.D.F. OF CONCENTRATION AT Z/D=0.8

Fig. 25 a . P.D.F. of CONCENTRATION - DATA REPLOTTED FROM REF.7



P.D.F. OF CONCENTRATION AT Z/D=1.2



P.D.F. OF CONCENTRATION AT Z/D=1.7 .

Fig. 25 b . P.D.F. of CONCENTRATION - DATA REPLOTTED FROM REF.7

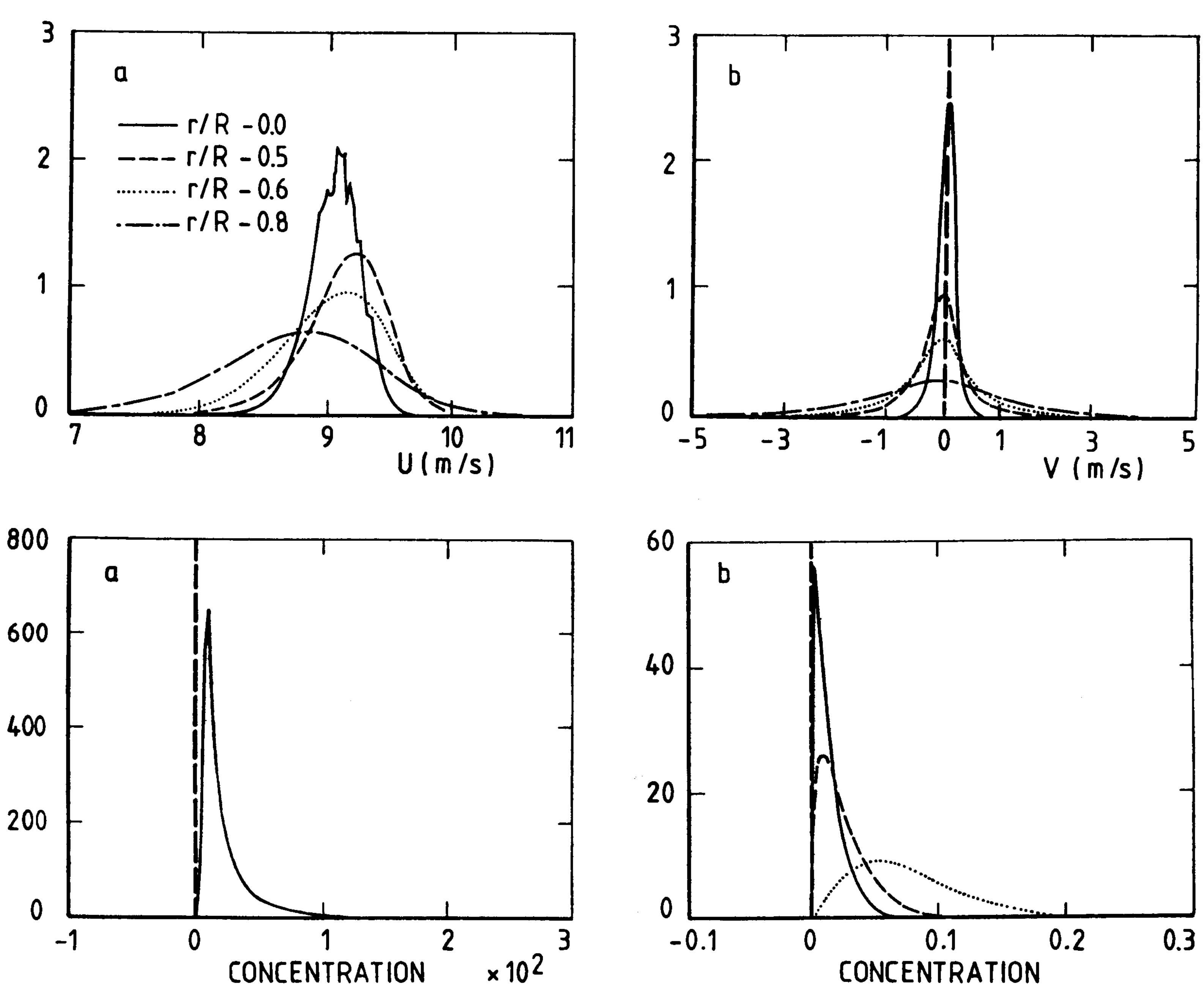


FIG. 26 - P.D.F. OF CONCENTRATION, (FROM Ref. 53)

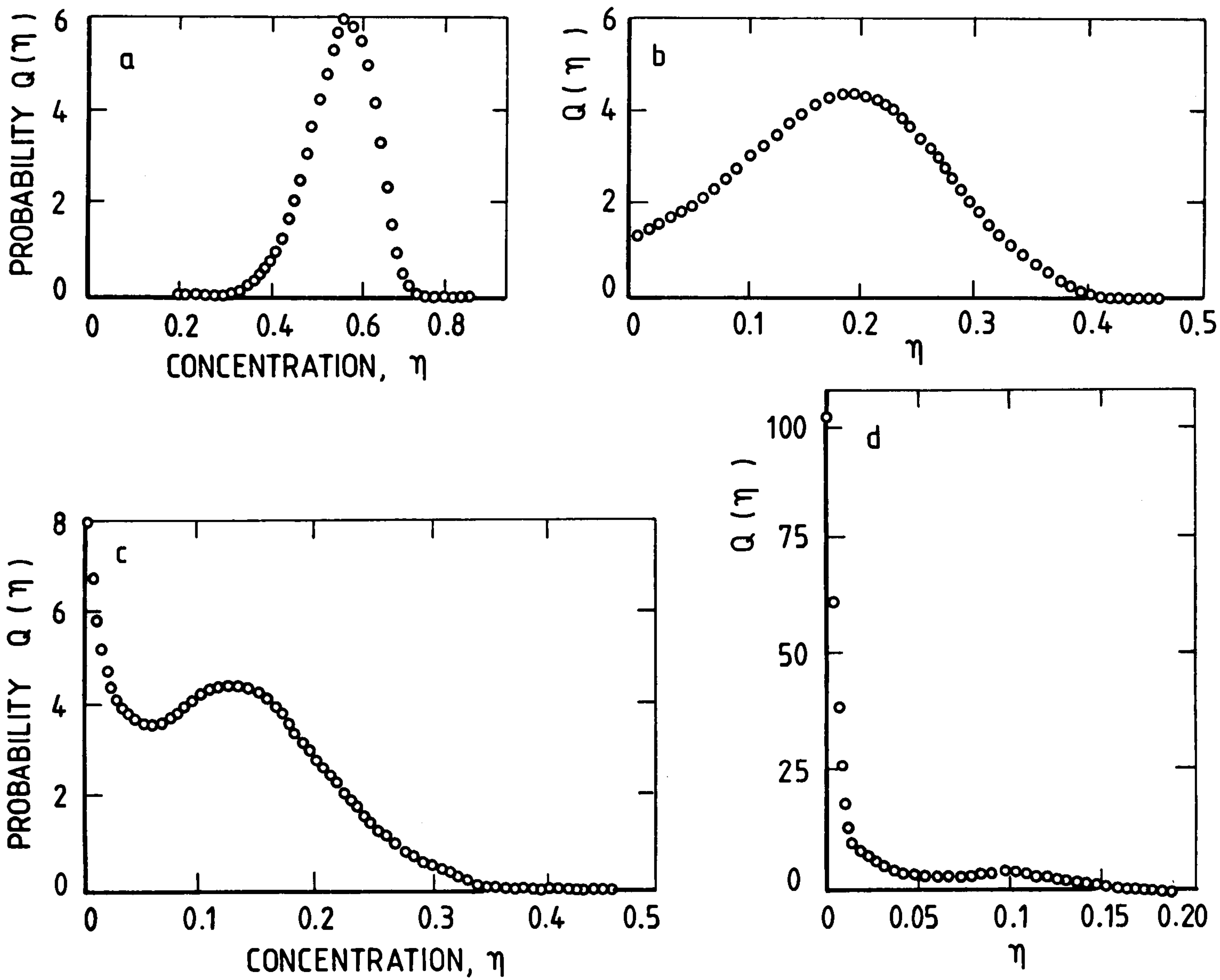


FIG. 27 - P.D.F. OF CONCENTRATION ,(FROM Ref. 54)

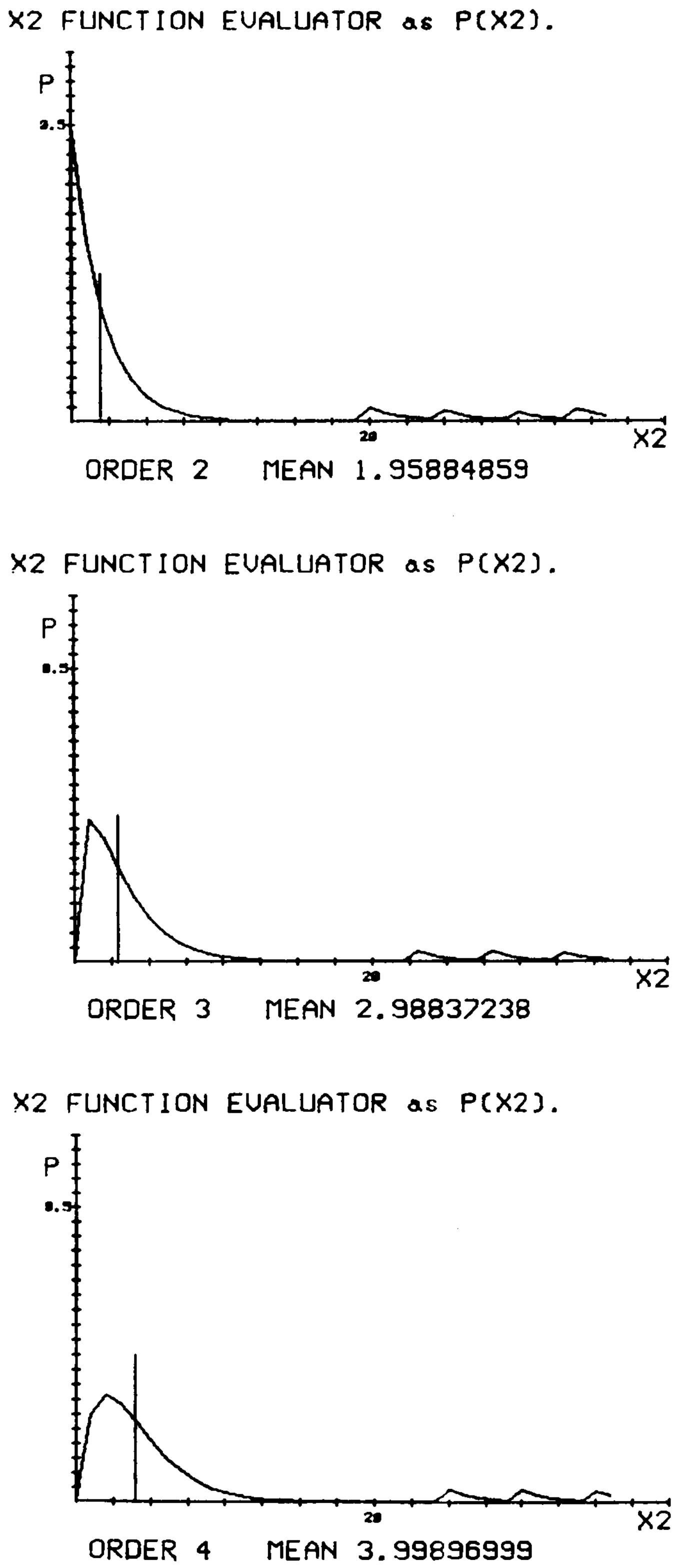
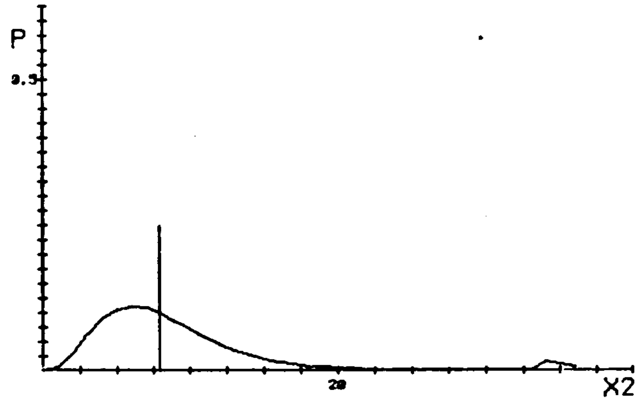


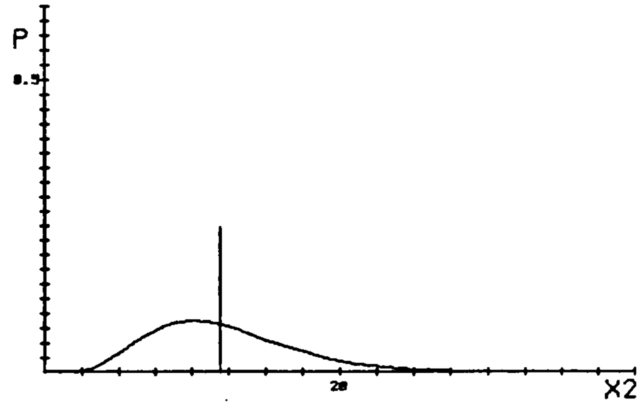
Fig. 28 - PLOT OF X^2 FUNCTION AS $P(X^2) - \alpha$.-

X2 FUNCTION EVALUATOR as P(X2).



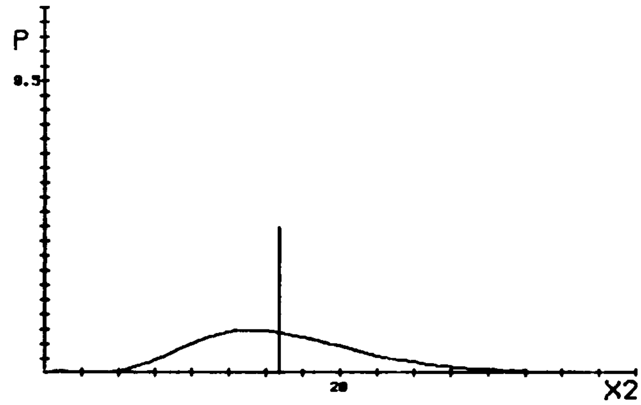
ORDER 8 MEAN 7.99947099

X2 FUNCTION EVALUATOR as P(X2).



ORDER 12 MEAN 11.9895171

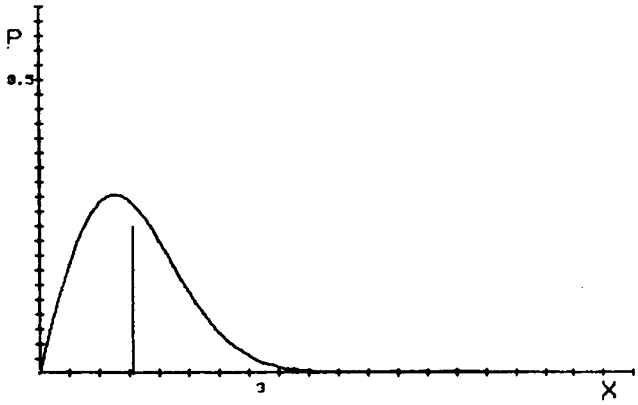
X2 FUNCTION EVALUATOR as P(X2).



ORDER 16 MEAN 15.9029839

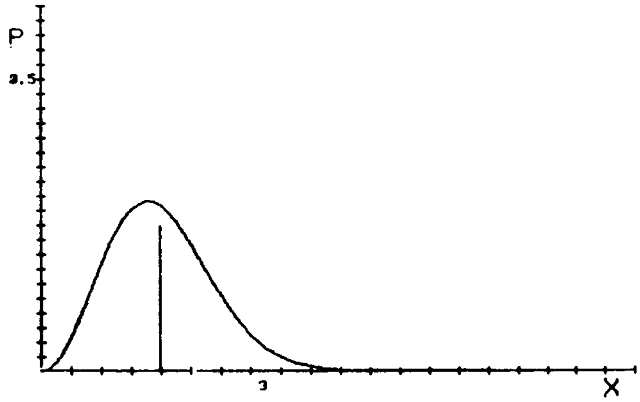
Fig. 28 - PLOT OF X^2 FUNCTION AS $P(X^2)$- b.-.

X2 FUNCTION EVALUATOR as P(X).



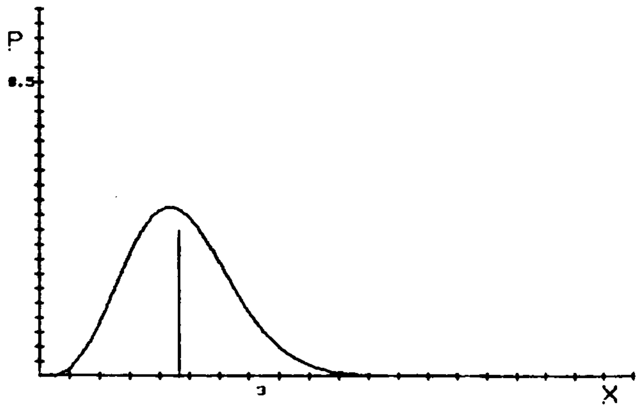
ORDER 2   MEAN 1.25331407

X2 FUNCTION EVALUATOR as P(X).



ORDER 3   MEAN 1.59576758

X2 FUNCTION EVALUATOR as P(X).



ORDER 4   MEAN 1.87996937

Fig. 29 - PLOT OF X^2 FUNCTION AS P(X)-α.-.

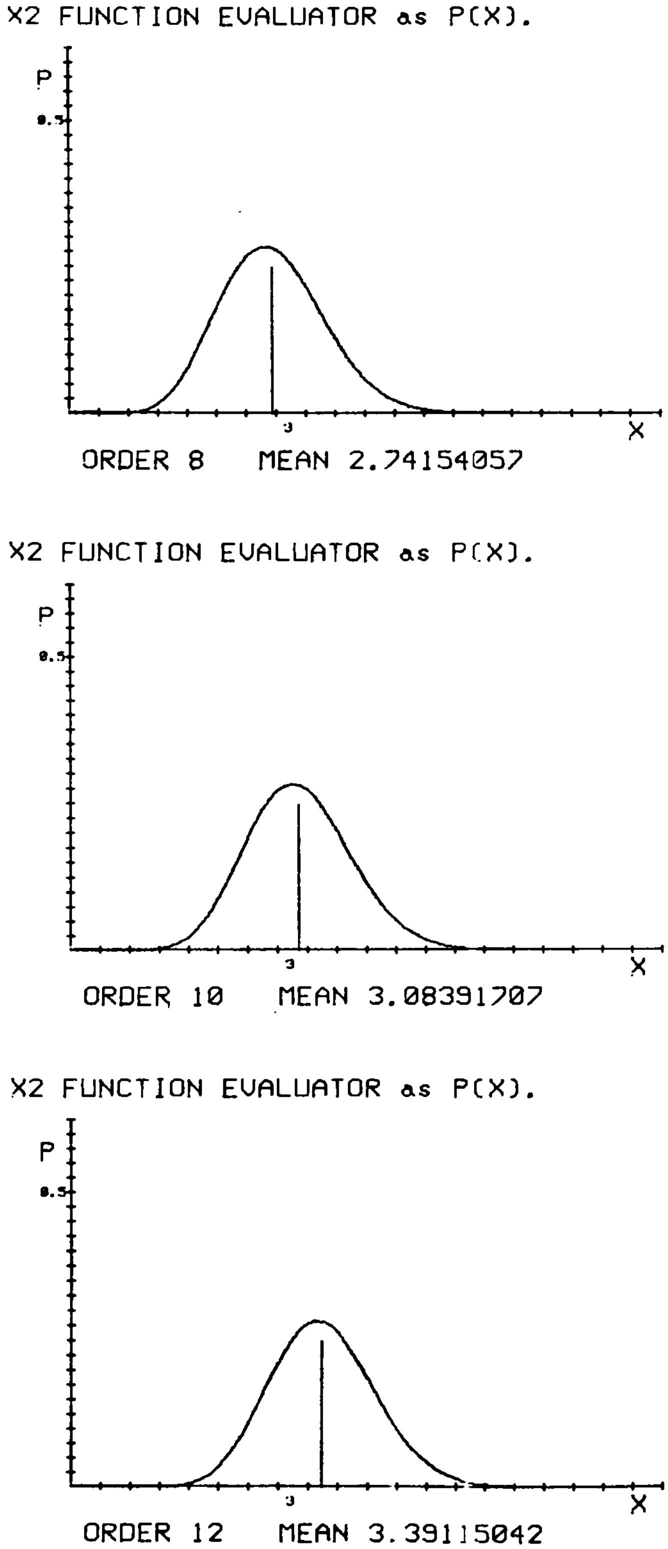


Fig. 29 - PLOT OF X^2 FUNCTION AS P(X)-6-.

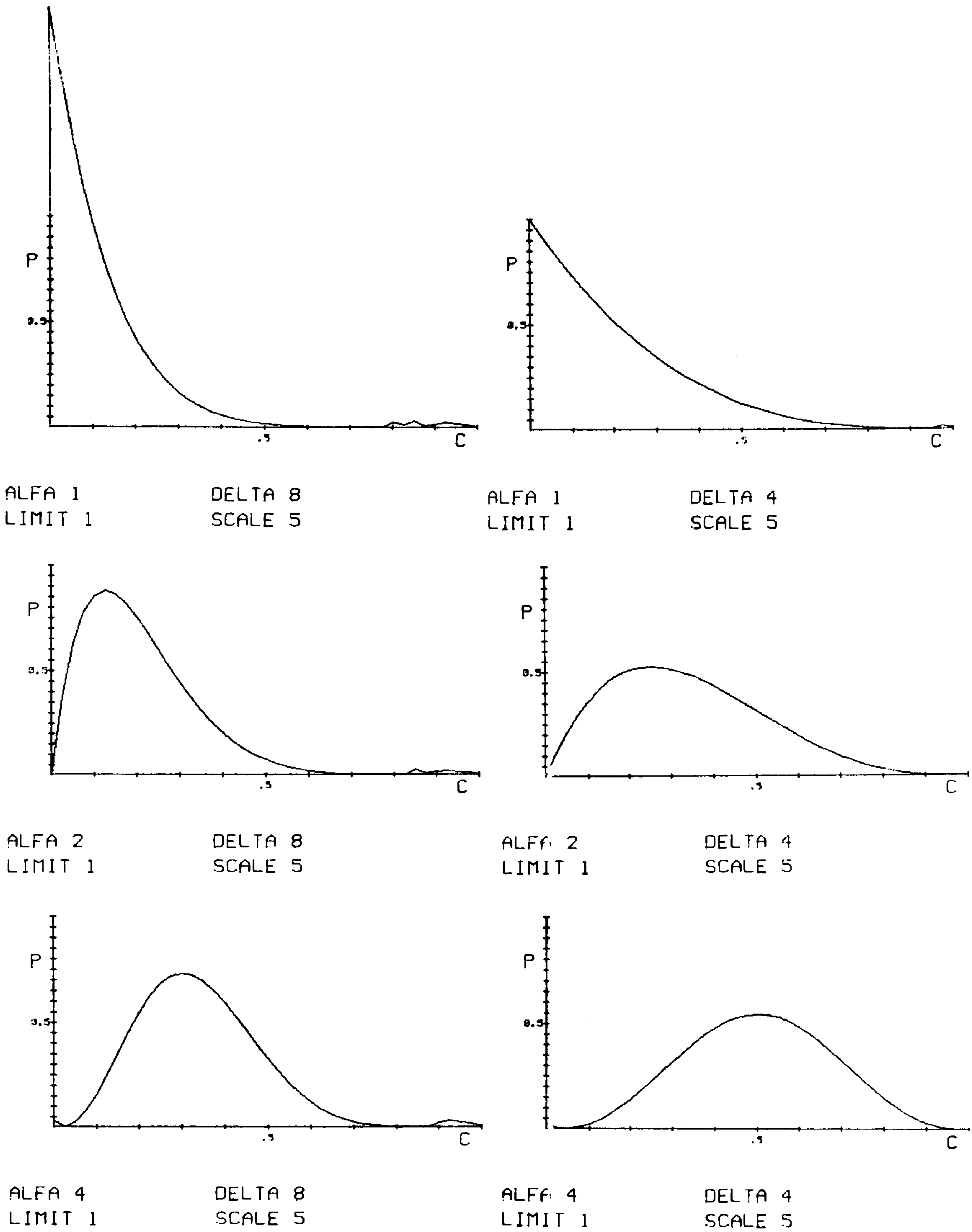
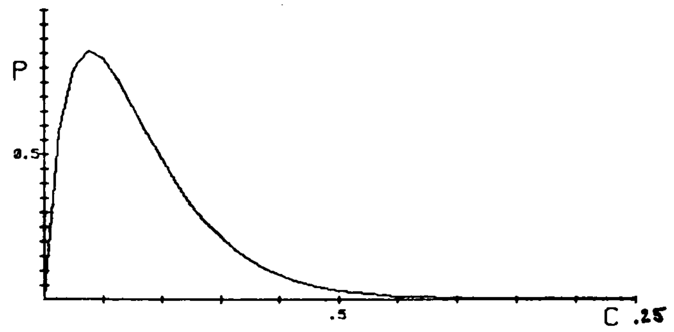
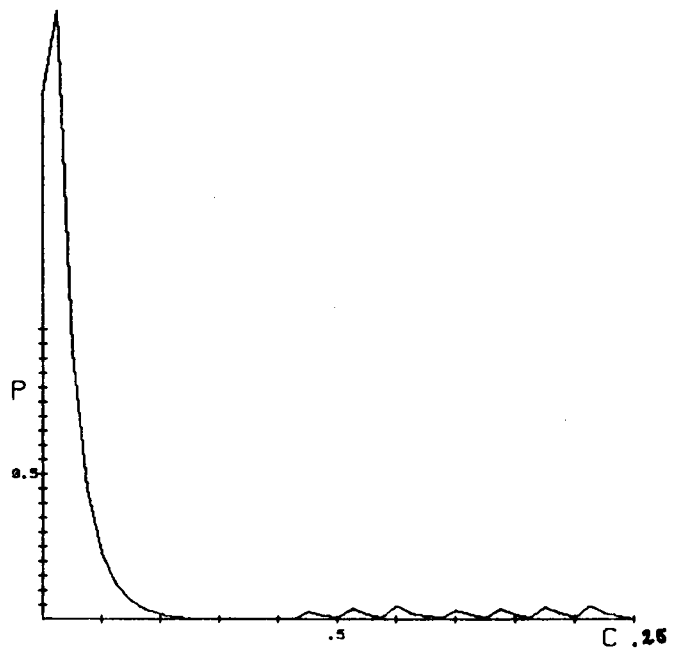


Fig. 30 - EVOLUTION OF BETA FUNCTION AS FUNCTION OF EXPONENT'S VALUES.



MEAN .0396 SIGMA .028116
ALFA 1.86557754 DELTA 45.2449664
LIMIT .25 SCALE 1



MEAN 6.9E-03 SIGMA 8.5836E-03
ALFA .634829821 DELTA 91.369492
LIMIT .25 SCALE .5

Fig 31 - APPLICATION OF BETA FUNCTION P.D.F. TO THE RESULTS OF REF. 52 AT X/D = 0.75

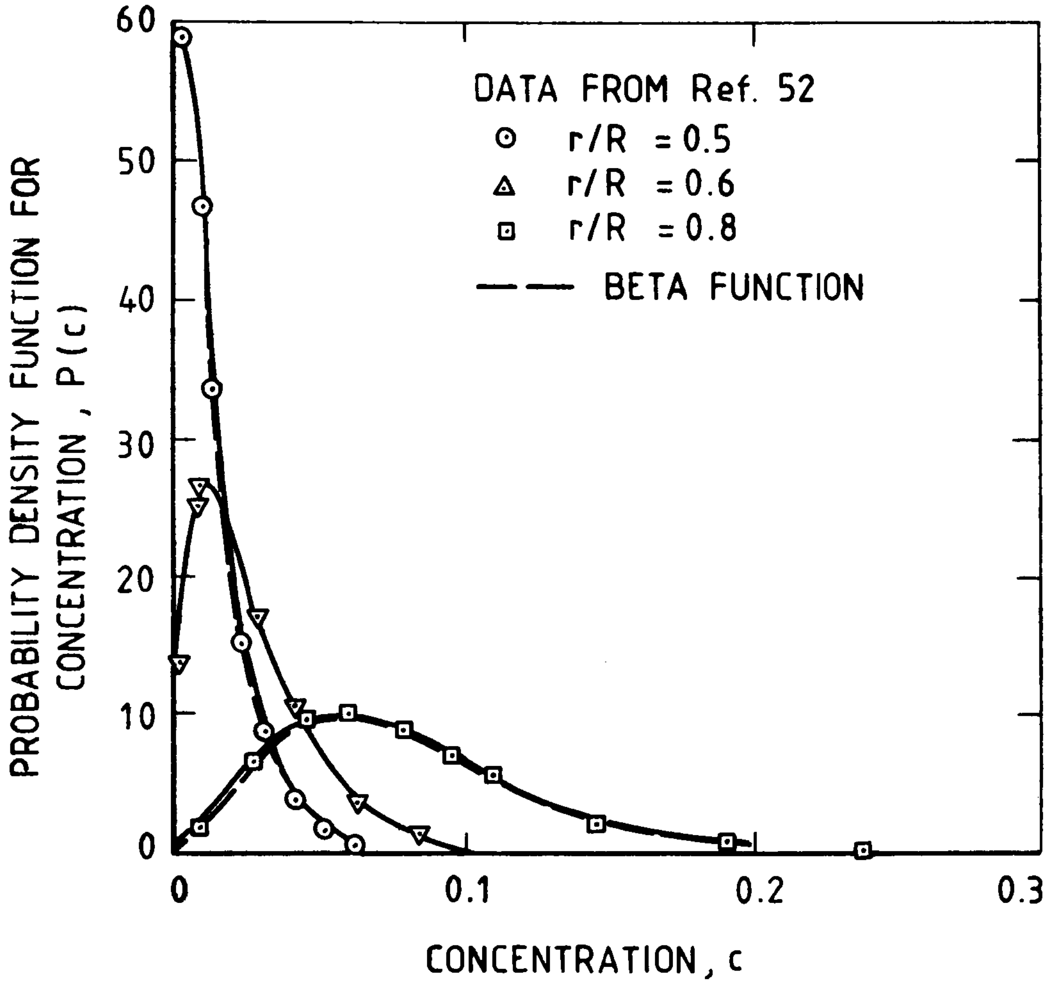


FIG. 32 - COMPARISON OF THE BETA FUNCTION WITH PROBABILITY DISTRIBUTION FUNCTIONS FOR CONCENTRATION (FROM Ref. 55)

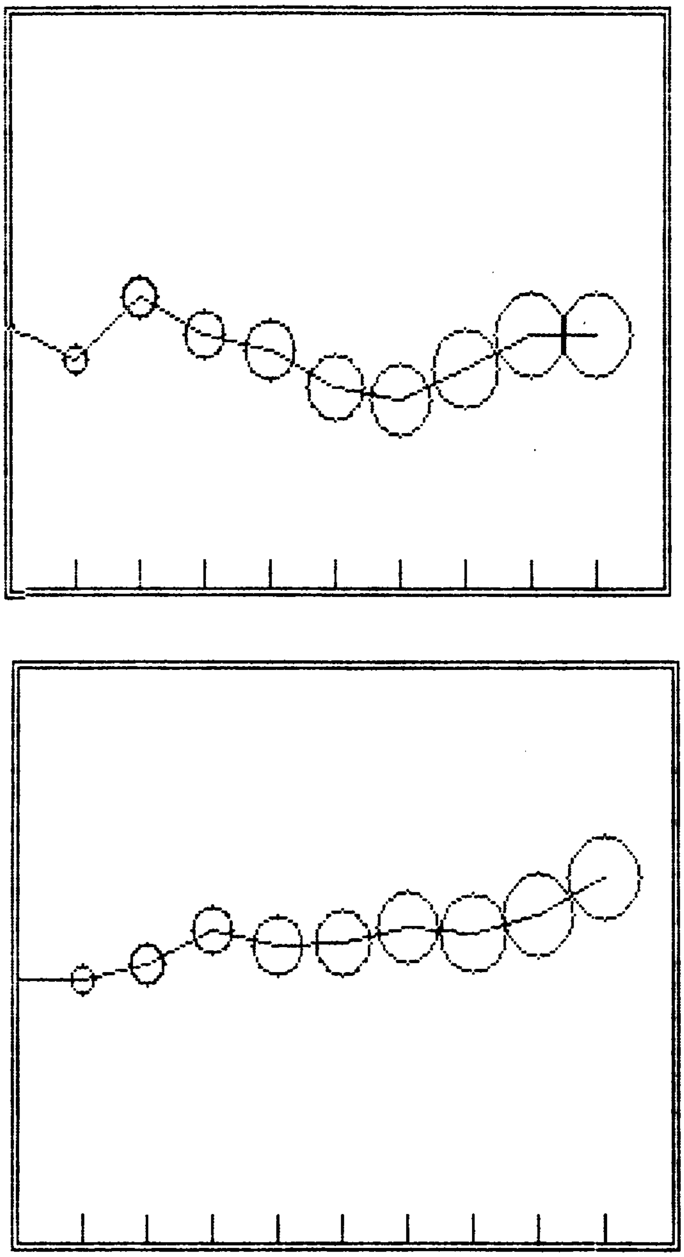


FIG. 33 -SCHEMATIC ILLUSTRATION OF CLOUD TRAJECTORY
IN A FLOW WITH RAMDOM TRANSVERSAL VELOCITY.
TWO INDEPENDENT EVENTS.

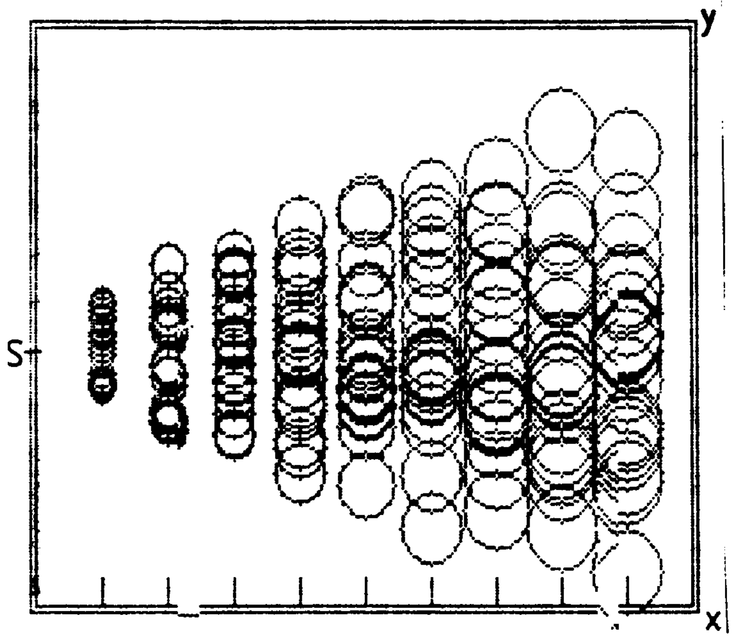


FIG. 34 - CLOUD TRAJECTORIES WITH RAMDOM TRANSVERSAL VELOCITY COMPONENTS. SCHEMATIC ILLUSTRATION OF CUMULATIVE RESULTS.

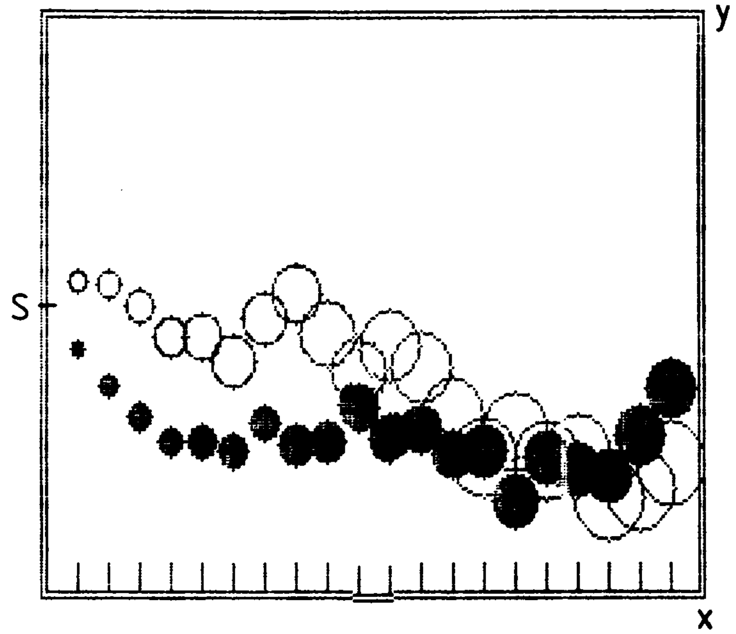


FIG. 35 - SCHEMATIC ILLUSTRATION OF POSSIBLE EFFECTS OF CLOUD MEANDERING IN A NON UNIFORM DIFFUSIVITY FLOW.

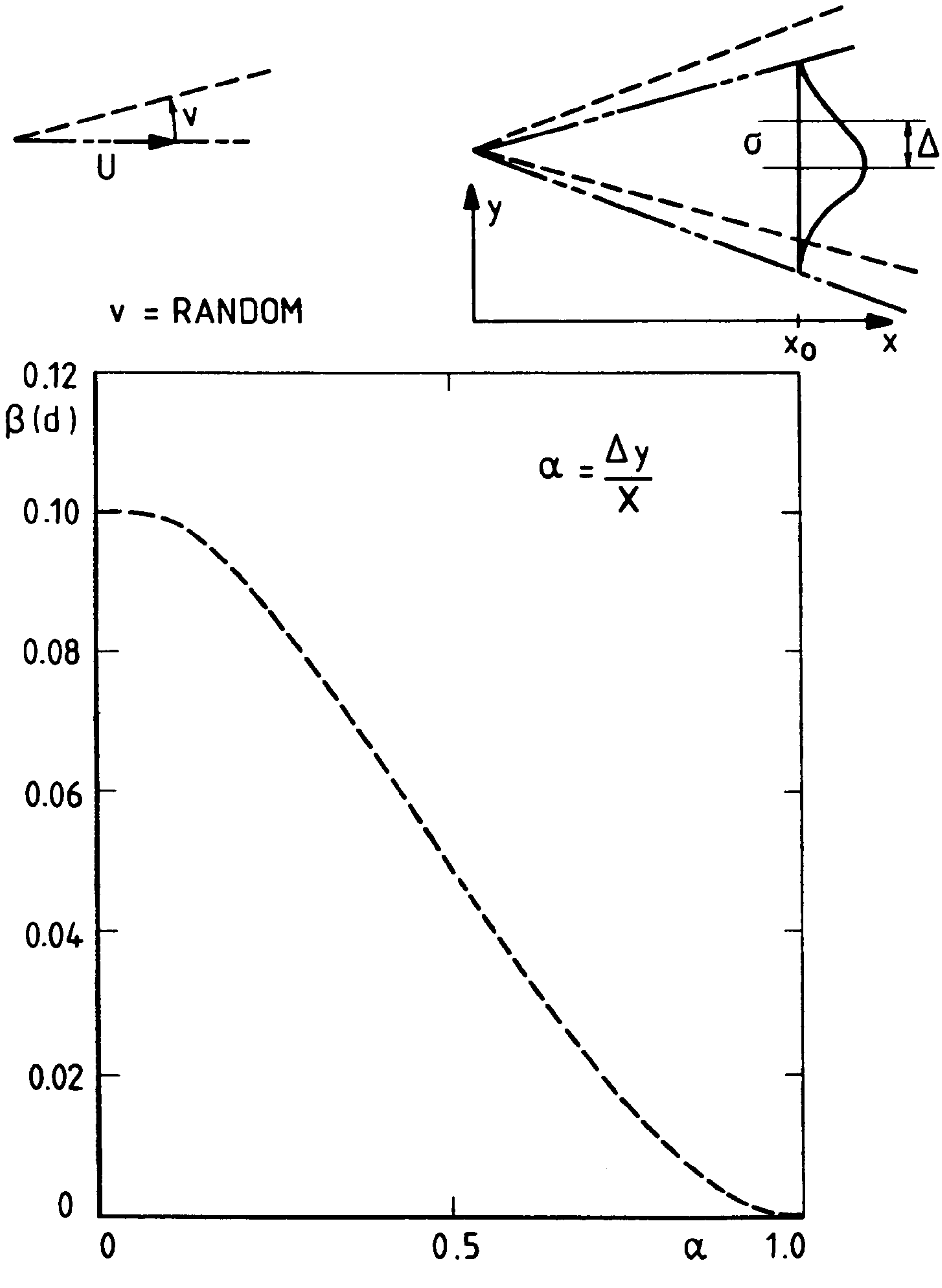


FIG. 36 - THE P.D.F. OF THE LATERAL WIND.

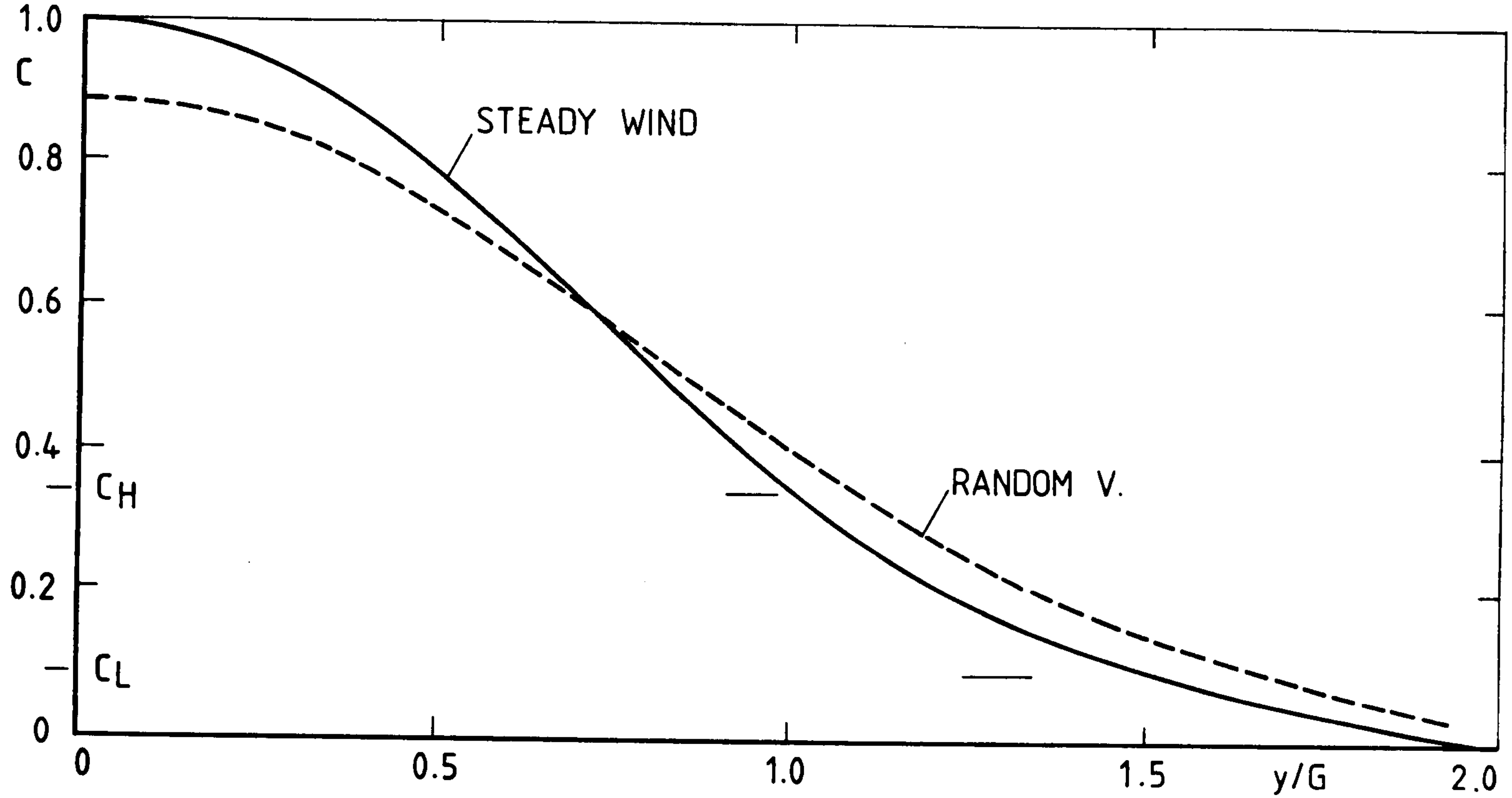


FIG. 37 - MEAN CONCENTRATION PROFILES.

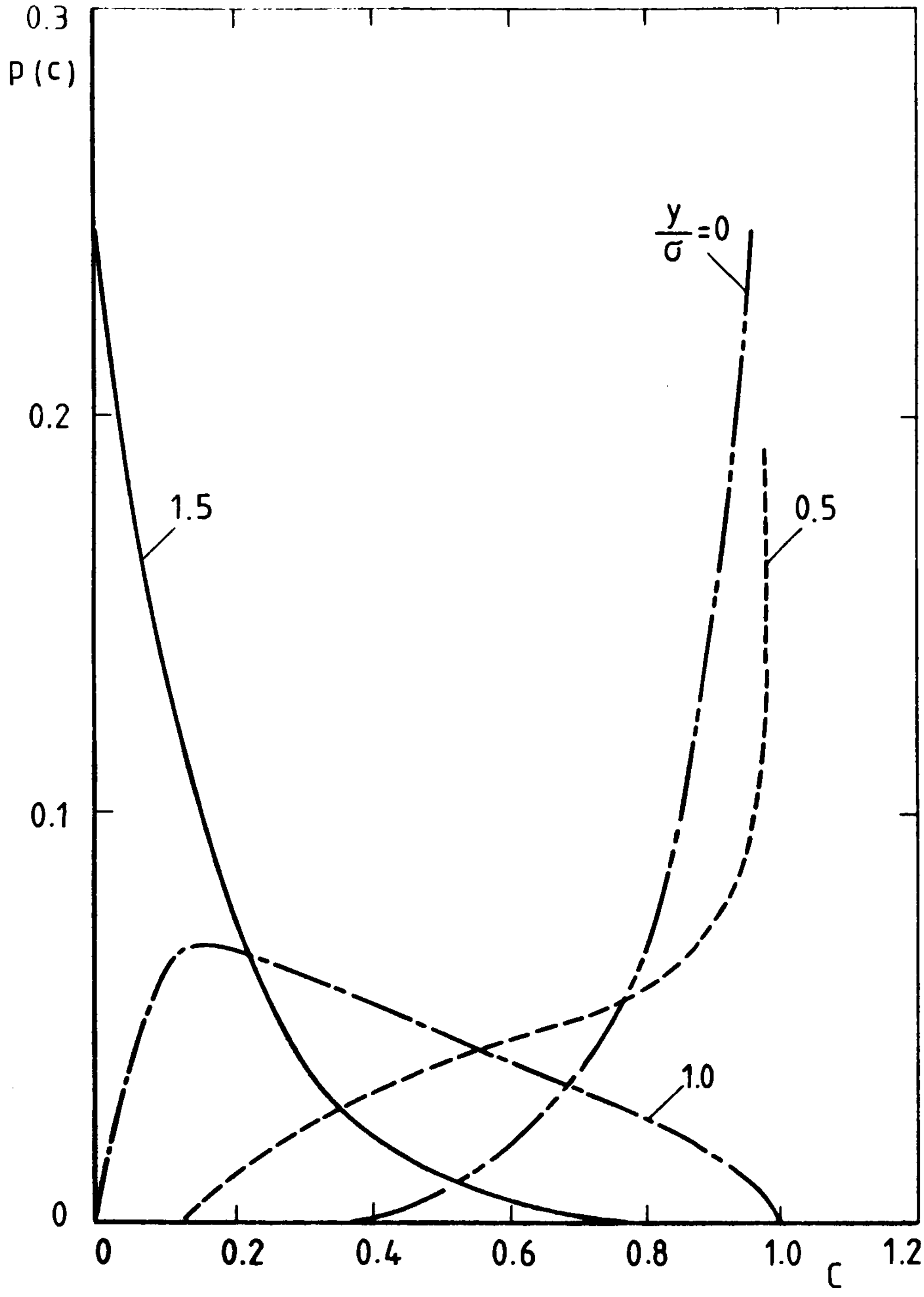


FIG. 38 - P.D.F. OF C AS FUNCTION OF DISTANCE FROM MEAN CENTER