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THE LONG-TIME BEHAVIOUR OF FAST POWER REACTORS WITH PLUTONIUM RECYLING*

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1. INTRODUCTION

The long-time behaviour of fast power reactors with plutonium recycling generally includes the overall time behaviour caused by the fuel burn-up in the reactor and the internal and external fuel management. It is useful to distinguish between two types of problems:

(a) The behaviour of the reactor between two loading events which refers to the time dependence of neutron flux, reactivity, power distribution, fuel composition etc.; usually it is the object of the conventional fuel burn-up studies.

(b) The behaviour of the reactor during many reloading cycles, which refers to the time dependence of the isotopic composition of plutonium, the uranium and plutonium content of the fuel, the breeding gain etc.

In this paper we will confine ourselves to the second class of problems only. We consider a fast power reactor in which only fissionable material is consumed which is bred in the same reactor, except for the plutonium amount to start up the reactor and for a certain period afterwards. Therefore, we have to consider a closed fuel cycle including the core and the external plants for reprocessing the irradiated fuel and refabricating the fuel elements. The blanket plutonium is introduced into the closed fuel cycle, the breeding gain is drawn off this cycle. The fuel cycle for common reprocessing of the core and blanket elements is shown in Fig. 1.

The time variations of the isotopic composition of the plutonium in the fuel cycle system are caused by the neutron irradiation in the reactor and by the internal and external fuel management[*)]. The composition of the refabricated fuel elements is determined by that uranium-to-plutonium ratio which maintains the reactor critical, assuming the total number of the heavy isotopes in the fuel elements to be constant.

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*) Internal fuel management is always done to improve the burn-up behaviour of the reactor; external fuel management refers to reprocessing and refabrication.

The problems which we are going to investigate were studied first in [1] on the basis of an idealized zero-dimensional model without taking into account the internal and external fuel management. Loading and unloading the core and blanket elements were regarded as continuous processes. In this paper we will investigate the long-time behaviour of the plutonium composition in the fuel and the physical aspects of the fuel cycle economy under more realistic conditions. Especially loading and unloading the core and blanket elements will be treated as discontinuous processes; the external fuel management and the times for reprocessing and refabrication will be taken into account.

As we will not look at the first type of problems of long-time behaviour, we can make some simplifications which refer essentially to the secondary time variations of neutron flux and spectrum caused by the fuel burn-up. By these simplifications we are able to solve the long-time problems with sufficient accuracy by tolerable numerical efforts. The formalism developed below is applied to a sodium cooled 1 000 MW(e) power reactor [2]. The main numerical results are given in the last section of this paper.

2. THE MATHEMATICAL FORMALISM FOR CALCULATING THE LONG-TIME BEHAVIOUR

2.1. Conditions for Reactor Operation

We consider a cylindrical reactor with few core zones, an axial and a radial blanket (Fig. 2). Exchanging the core and blanket elements takes place at each of the discrete time values

(1) $$t_s = s \cdot \Delta t_C , \quad s = 1,2,3,\ldots ;$$

Δt_C is a given constant correlated to the thermal reactor power Q and the given maximum burn-up of the fuel elements as shown in equation (6).

The core elements are loaded and unloaded according to the cyclic burn-up scheme described in [3]. The core is subdivided into radial regions; the elements of the same radial region are combined into groups of n elements (e.g. n=3). At each point of time t_s only one element of each group is exchanged for a fresh fuel element. During a period of time $n\Delta t_C$ each element of a group is exchanged once and only once. The life time of the core elements is $\delta = n\Delta t_C$, i.e. the total irradiation time of a fuel element in the core. At each time t_s only the n-th part of the core elements is exchanged for fresh elements. If V_C is the total fuel element volume of the core, the exchanged fuel element volume at t_s is

$$(2) \qquad \Delta V_C = \frac{1}{n} V_C.$$

The operation of the axial and radial blankets must be in agreement with the rhythm given by (1); i.e. loading and unloading the blanket elements must succeed in intervals $m\Delta t_C$, m may be a positive integer. The elements of the axial blanket are coupled with the core elements; therefore both are managed jointly. Contrary to this the management of the radial blanket is largely independent of the core management. We assume the radial blanket to be divided into N_B radial zones, which are exchanged as a whole. The life-time of the j-th zone of the radial blanket may be

$$(3) \qquad \delta_{Bj} = m_j \cdot \Delta t_C , \quad j = 1,2,\dots,N_B;$$

the integer m_j is related to the maximum burn-up in the j-th zone.

The amount of uranium and plutonium discharged out of the core and the axial and radial blankets is calculated in two steps: a) the calculation of the space dependent uranium and plutonium concentrations of the discharged elements by solving the fuel burn-up equations; b) the calculation of the uranium and plutonium content of these elements by volume integration. The fuel burn-up calculations and the volume integration for the core and blanket elements are discussed in the following sections.

2.2. The Fuel Burn-up Equations

The time behaviour of the fuel composition during neutron irradiation in the reactor is determined by the well-known fuel burn-up equations. As the main components of the fuel we regard the two uranium isotopes ^{235}U and ^{238}U and the four plutonium isotopes ^{239}Pu, ^{240}Pu, ^{241}Pu, and ^{242}Pu. Therefore, we have to solve a system of six first-order differential equations in time and flux time, respectively. The coefficients of these equations are the one group space dependent neutron flux and microscopic cross sections for neutron absorption and capture, which are defined in the multigroup picture as

$$\phi(r,z) = \sum_i \phi_i(r,z) \ , \qquad \sigma(r,z) = \frac{\sum_i \sigma_i \phi_i(r,z)}{\phi(r,z)} \ ; \tag{4}$$

$\phi_i(r,z)$ is the neutron flux in the i-th neutron energy group at r,z; the $\phi_i(r,z)$ are determined by multigroup calculations. The σ_i are the microscopic cross sections of the heavy nuclei in the fuel for neutron absorption and capture in the i-th energy group.

In general, ϕ and also the σ are time dependent as a consequence of the variation of the fuel composition in the reactor during neutron irradiation. But it will be possible to calculate the uranium and plutonium concentrations in the irradiated elements at the end of their life time by solving the fuel burn-up equations with sufficient accuracy, if we take into account time averaged ϕ and σ instead of the exact time dependent quantities. The latter, in general, would require the simultaneous solution of the burn-up equations and the criticality equation of the reactor, e.g. multigroup-diffusion equations. In our case the time averaged one group quantities are calculated according to equation (4) with time averaged neutron spectra, which are determined by multigroup calculations for an average burn-up state of the reactor assuming a homogeneous distribution of the fuel. The one group neutron flux for the average burn-up state is normalized to the given reactor power Q which is regarded as constant.

2.3. The Uranium and Plutonium Output of the Core and the Axial Blanket

We suppose that the core is divided into N radial zones of different fuel compositions. The $N_{j,k}(s,o)$ may be the time dependent initial concentrations of the nuclei $k = 1,2,..,6$ [1)] in the fuel elements which are

1) The indices $k = 1,2,...,6$ refer to the isotopes ^{235}U, ^{238}U, ^{239}Pu,...,^{242}P respectively, which are the components of the fuel mixture.

loaded into the core zone $j = 1,2,\ldots,N$ at the time t_s. We presume

$$N_{j,k}(s,o) \equiv N_{j,k}(o,o) \text{ for all } s<s_o \ ; \quad j = 1,2,\ldots,N \ ; \quad k = 1,2,\ldots,6.$$

s_o designates the time t_{s_o} at which refabricated fuel elements are loaded into the core for the first time, i.e. for closing the fuel cycle. s_o is determined by the external fuel management as shown below. The $N_{j,k}(o,o)$ denote the j-th initial composition of the fuel elements at $t_o = 0$. For all $s \geq s_o$ the $N_{j,k}(s,o)$ are to be calculated as functions of s, as shown below.

Now we have to calculate the fuel output of the core at the time t_s. The $N^*_{j,k}(s,\tau_{maxj})$ may be the concentrations of the heavy nuclei in the fuel elements unloaded from the j-th zone at t_s. The corresponding initial concentrations may be given by $N^*_{j,k}(s,o)$. The $N^*_{j,k}(s,\tau_{maxj})$ are the solutions of the fuel burn-up equations with these initial values and the space dependent flux times

$$(5) \qquad \tau_{maxj} = n \cdot \Delta t_C \cdot \phi(r,z) \ , \quad j = 1,2,\ldots,N$$

r,z is any point of the core zone j. τ_{maxj} is related to the maximum fuel burn-up for the elements of the core zone j in MWD/tonne. From equation (5) follows the relation between Δt_C, the maximum burn-up for the inner core zone j = 1 expressed by the corresponding maximum flux time, and the neutron flux at the core center $\phi(o,o)$:

$$(6) \qquad \Delta t_C = \frac{1}{n} \ \frac{\tau_{max1}}{\phi(o,o)} \ ;$$

$\phi(o,o)$ is proportional to the reactor power Q.

Obviously, the following relation holds:

$$(7) \qquad N^*_{j,k}(s,o) = \begin{cases} N_{j,k}(o,o) & \text{for } s-n<s_o, \\ N_{j,k}(s-n,o) & \text{for } s-n\geq s_o, \end{cases} \qquad j = 1,2,\ldots,N; \ k = 1,2,\ldots,6$$

The total number $\Delta N^{*}_{j,k}(s)$ of the nucleus k which is unloaded from the core zone j at t_s follows by integrating $N^{*}_{j,k}(s,\tau_{maxj})$ over the volume of the unloaded fuel elements. We obtain

$$(8) \qquad \Delta N^{*}_{j,k}(s) = \frac{2\pi}{n} \int_{-\frac{H+D}{2}}^{\frac{H+D}{2}} \int_{R_j}^{R_{j+1}} N^{*}_{j,k}(s,\tau_{maxj}) r dr dz, \qquad k = 1,2,\ldots,6$$

where the discharged nuclei of the axial blanket are included. H denotes the height of the core, D the total thickness of the axial blanket, R_j the inner and R_{j+1} the outer radial boundary of the j-th zone. The division by n is necessary, for only the n-th part of the elements of the zone j is discharged. In the core range we have to take into account the time dependence of the initial composition of the fuel; in the axial blanket region we take the initial composition of the elements constant.

The total output of the core and the axial blanket expressed as the total number of the discharged nuclei at the time t_s, denoted as $\Delta N_{Ck}(s)$, is

$$(9) \qquad \Delta N^{*}_{Ck}(s) = \sum_{j=1}^{N} \Delta N^{*}_{j,k}(s) , \qquad k = 1,2,\ldots,6.$$

2.4. The Uranium and Plutonium Output of the Radial Blanket

The uranium and plutonium content of the radial blanket zone j at the time of discharging, which may be denoted as $\Delta N^{*}_{Bj,k}$, $j = 1,2,\ldots,N_B$, $k = 1,2,\ldots,6$, is also calculated by a volume integral in analogy to equation (8). But in the case of the radial blanket n is equal to 1 and the initial values of the elements are time independent. The flux time distribution in the j-th zone for solving the burn-up equation is

$$\tau_{maxj} = m_j \cdot \Delta t_C \cdot \phi(r,z) , \qquad j = 1,2,\ldots,N_B ,$$

r,z is any point in the radial blanket zone j.

For a given value of s only that zone j of the radial blanket is unloaded for which

$$(10) \qquad \frac{t_s}{m_j \cdot \Delta t_C} = \frac{s}{m_j} = \text{an integer.}$$

If no value of j exists for a given s, the total output $\Delta N^*_{Bk}(s)$, $k = 1,2,\ldots,6$ of the radial blanket is zero:

$$(11a) \qquad \Delta N^*_{Bk}(s) = 0 \quad \text{for all } k.$$

If there are ℓ special values of j for a given s, denoted as $j(\ell')$, $\ell' = 1,2,\ldots,\ell$, for which equation (10) is valid, all the designated zones are unloaded at t_s. Thus, the uranium and plutonium output of the radial blanket at t_s is

$$(11b) \qquad \Delta N^*_{Bk}(s) = \sum_{\ell'=1}^{\ell} \Delta N^*_{Bj(\ell'),k}$$

2.5. <u>The Composition of the Refabricated Fuel Elements and the Breeding Gain</u>

In the case of joint reprocessing of core and blanket elements the total number of nuclei, which are unloaded from the reactor at t_s and which are to be reprocessed, is

$$\Delta N^*_k(s) = \Delta N^*_{Ck}(s) + \Delta N^*_{Bk}(s), \quad k = 1,2,\ldots,6, \quad s = 1,2,\ldots;$$

the $\Delta N^*_{Bk}(s)$ may vanish for certain values of s as shown before. The total numbers of uranium and plutonium nuclei are

$$(12) \qquad \Delta N^*_U(s) = \Delta N^*_1(s) + \Delta N^*_2(s) \;, \quad \Delta N^*_{Pu}(s) = \sum_{k=3}^{6} \Delta N^*_k(s).$$

Losses during reprocessing reduce the number of nuclei by a factor of p_U and p_{Pu}, respectively. Thus, after reprocessing we have

$$\Delta N^*_U(s) \Longleftarrow p_U \cdot \Delta N^*_U(s)$$
$$\Delta N^+_{Pu}(s) \Longleftarrow p_{Pu} \cdot \Delta N^*_{Pu}(s)$$

using the same symbols for the reduced numbers. The enrichment of the uranium available after reprocessing is

$$(13) \qquad \Gamma(s) = \frac{\Delta N_1^*(s)}{\Delta N_U^*(s)}$$

and the relative composition of the plutonium mixture available for the refabrication of the fuel elements is given by

$$(14) \qquad Q_k(s) = \frac{\Delta N_k^*(s)}{\Delta N_{Pu}^*(s)}, \qquad k = 3,4,5,6.$$

The initial uranium and plutonium concentrations of the fuel elements for the j-th core zone, which are refabricated from plutonium of the s-th discharge, i.e. from $\Delta N_{Pu}^*(s)$, may be denoted as $n_{j,U}(s)$ and $n_{j,Pu}(s)$, respectively. The $n_{j,k}(s)$, $k = 1,2,\dots,6$ may be the initial concentrations of the uranium and plutonium isotopes in the refabricated fuel elements. Thus, the following relations hold:

$$(15) \qquad \begin{aligned} n_{j,U}(s) &= n_{j,1}(s) + n_{j,2}(s) \\ n_{j,Pu}(s) &= \sum_{k=3}^{6} n_{j,k}(s) \end{aligned} \qquad j = 1,2,\dots,N$$

The $n_{j,U}$, $n_{j,Pu}$ and $n_{j,k}$ are space independent.

The basis for calculating the initial compositions of the refabricated fuel elements are the 2N linear algebraic equations

$$(16) \qquad \begin{aligned} \sum_{k=1}^{6} C_{j,k} \cdot n_{j,k}(s) &= K_j(o) \\ \sum_{k=1}^{6} n_{j,k}(s) &= Z_j(o) \end{aligned} \qquad j = 1,2,\dots,N$$

with the abbreviations

$$(16a) \qquad \begin{aligned} C_{j,k} &= (\nu\sigma^f)_{j,k} - \sigma^a_{j,k}, \qquad k = 1,2,\dots,6; \\ K_j(o) &= \sum_{k=1}^{6} C_{j,k}\, n_{j,k}(o), \quad n_{j,k}(o) \equiv N_{j,k}(o,o) \\ Z_j(o) &= \sum_{k=1}^{6} n_{j,k}(o) \end{aligned}$$

The one group quantities $\sigma^{a}_{j,k}$ and $(\nu\sigma^{f})_{j,k}$ refer to any reference point in the j-th zone (e.g. $r = R_j$, $z = o$). As mentioned above, they are calculated for the average fuel burn-up state of the reactor assuming a homogeneous fuel distribution. The $\nu\sigma^{f}$ are defined by analogy to equation (4).

The equations (16) are used to calculate the 2N unknown initial concentrations $n_{j,U}$ and $n_{j,Pu}$ with a presupposed uranium enrichment and the isotopic composition of the plutonium given by (14). The first equations (16) are given to maintain criticality. By these equations the reactivity worth of the refabricated fuel elements is fixed approximately at a constant value for each zone by adjusting the fuel composition to a constant difference between neutrons produced and absorbed in the fresh fuel at a given point of each zone. The constant differences are defined by the fresh fuel of the initial reactor at $t_o = 0$.

The second equations (16) take the total number of heavy nuclei in the fresh fuel elements for each zone to be constant. The constants are given again by the initial conditions at $t_o = 0$. For simplification we assume that the enrichment of the uranium used for fuel element refabrication is constant and equal to the initial enrichment γ at $t_o = 0$. Thus, we have the relations

$$(17) \qquad n_{j,1}(s) = \gamma \cdot n_{j,U}(s), \quad n_{j,2}(s) = (1-\gamma) \cdot n_{j,U}(s).$$

If only the plutonium mixture from the s-th discharge of the reactor is used for refabricating, i. e. in the case of positive breeding gain, the concentrations of the plutonium isotopes are

$$(18) \qquad n_{j,k}(s) = Q_k(s) \cdot n_{j,Pu}, \quad k = 3,4,5,6.$$

With the relations (17)and (18) the equations (16) are rearranged to

$$(19) \qquad \begin{aligned} n_{j,U}(s) \cdot C_{j,U} + n_{j,Pu}(s)\, C_{j,Pu}(s) &= K_j(o) \\ n_{j,U}(s) \qquad\quad + n_{j,Pu}(s) \qquad\qquad &= Z_j(o) \end{aligned}$$

with

(19a) $$C_{j,U} = C_{j,1}\,\gamma + C_{j,2}\,(1-\gamma),\quad C_{j,Pu}(s) = \sum_{k=3}^{6} C_{j,k}\,Q_k(s)$$

If $C_{j,Pu}(s) \neq C_{j,U}$ the equations are solved by

(20) $$n_{j,U}(s) = \frac{Z_j(o)C_{j,Pu}(s)-K_j(o)}{C_{j,Pu}(s)-C_{j,U}},\qquad n_{j,Pu}(s) = \frac{-Z_j(o)C_{j,U}+K_j(o)}{C_{j,Pu}(s)-C_{j,U}},\qquad j = 1,2,\dots,N.$$

For physical reasons $n_{j,U}(s)$ and $n_{j,Pu}(s)$ must be positive. If

(21) $$C_{j,U} \leq 0,$$

i.e. $\gamma \leq 1 / (1-C_{j,1}/C_{j,2})$, $n_{j,Pu}(s)$ is positive. If (21) is true, the numerator of the first equation (20) must be positive, i.e.

(22) $$C_{j,Pu}(s) \geq \frac{K_j(o)}{Z_j(o)},\quad j = 1,2,\dots,N.$$

In the most interesting cases of fast power breeders with natural or depleted uranium as the fertile material the two conditions (21) and (22) are always true and the solutions (20) are positive [2]. In the following we will restrict ourselves to these cases. But the initial compositions for the refabricated fuel elements are only determined by equation (20), if the number $\Delta N^*_{Pu}(s)$ of the available plutonium nuclei is enough for manufacturing the fuel element volume ΔV_C, i.e. in the case of positive breeding gain. The total number of plutonium nuclei from the s-th unloading of the reactor which is required for refabricating the fuel element volume ΔV_C is given by

[2] In the cases of higher enrichments of the uranium in the fuel the equations (16) continue to be true. But the time dependence of the uranium enrichment is to be taken into account which is caused by the fuel burn-up and perhaps by the fuel management. Especially in the case of starting up the reactor as a converter with ^{235}U as the fissionable material we have to distinguish the time dependent enrichment of the uranium in the closed fuel cycle according to (13) and the presupposed enrichment of the uranium which is introduced into the fuel cycle from the outside to compensate for the consumed fertile material. By these facts the equations (17) and (19) are modified. But we will not discuss these more general equations and the conditions by which the solutions are positive here.

$$N_{Pu}(s) = \frac{1}{n} \sum_{j=1}^{N} \Delta v_j \cdot n_{j,Pu}(s).$$

Δv_j is the total volume of the fuel elements of the core zone j; thus, $\Delta v_j/n$ is the exchanged fuel element volume. Losses during the fuel element fabrication reduce the number of plutonium nuclei by a factor q_{Pu}, therefore, the number of the required plutonium nuclei is $N_{Pu}(s)/q_{Pu}$, instead of $N_{Pu}(s)$. If

$$(23) \qquad N_{Pu}(s) \frac{1}{q_{Pu}} \leq \Delta N^*_{Pu}(s) ,$$

the initial compositions of the refabricated fuel elements, are determined by the equations (20).

The breeding gain of the s-th fuel cycle is defined by

$$(24) \qquad N_{BG}(s) = \Delta N^*_{Pu}(s) - \frac{1}{q_{Pu}} N_{Pu}(s) ;$$

it is positive in accordance with (23).

2.6. The Parameter s_o

The time required for the external fuel management is $(s_o-1)\Delta t_C$ with the above definition of s_o. Thus, the plutonium unloaded from the reactor at t_s is reloaded into the core at $t_{s+s_o-1} = (s+s_o-1)\Delta t_C$. Therefore, we have the relation

$$(25) \qquad N_{j,k}(s+s_o-1,0) = n_{j,k}(s) \quad \text{for all values of k and j.}$$

s_o is determined by the characteristics of the reprocessing and refabricating plants. The external fuel cycle is characterized essentially by the parameters δ_c, δ_r, and δ_f, which are the minimum cooling time and the times for reprocessing and refabricating, respectively. These quantities define a new parameter

$$\delta_{ex} = \delta_c + \delta_r + \delta_f ;$$

furthermore, we define

$$p = \left[\frac{\delta_{ex}}{\Delta t_c} \right] , \qquad [x] \text{ is the maximum integer below x.}$$

For simplification we will assume in the following that the refabricated fuel element volume ΔV_C of any fuel cycle is loaded into the core as a whole, i.e. it should not be distributed over different loading events. Now, we will express the time for loading the fuel element volume ΔV_C of the s-th fuel cycle into the core in terms of p.

The time at which the first elements of the s-th fuel cycle are refabricated lies between $t_s + p\Delta t_C$ and $t_s + (p+1)\Delta t_C$ as the case may be, whether or not $\delta_{ex}/\Delta t_C$ is an integer. For economical reasons we assume that the time after which the complete volume ΔV_C is refabricated will not exceed Δt_C, i.e. the last elements of the s-th cycle may be refabricated not later than $t_s + (p+2)\Delta t_C$. This condition avoids unnecessary accumulations of plutonium in the external fuel cycle. In general, three values of s_o are possible:

$$s_o = p+\varepsilon \quad , \quad \varepsilon = 1,2 \text{ or } 3, \text{ respectively.}$$

$\varepsilon=1$: if $\delta_{ex}/\Delta t_C=p$ and if the fuel element volume ΔV_C is managed as only one batch. $\varepsilon=2$: (1) if $\delta_{ex}/\Delta t_C \geq p$ and if the fuel element volume ΔV_C is divided into partial batches and the last batch is refabricated not later than $t_s + (p+1)\Delta t_C$; (2) if $\delta_{ex}/\Delta t_C > p$ and if ΔV_C is managed as a whole. $\varepsilon=3$: only if $\delta_{ex}/\Delta t_C > p$ and if ΔV_C is divided into partial batches and the last batch is refabricated later than $t_s + (p+1)\Delta t_C$

3. NUMERICAL RESULTS

The numerical results refer to the sodium cooled reactor Na1 |2|. The availability of the reactor may be 80 per cent; thus, the thermal power is assumed to be

$$Q = 2.000 \text{ MW}$$

on the average. Furthermore, we suppose

$$n = 3, \quad \Delta t_C = 200 \text{ d}$$

for the fuel burn-up cycles in the core. The corresponding maximum burn-up in r=o is 86.000 MWD/tonne averaged in the z-direction. The geometrical data are given in Fig. 2. The initial composition of the core and blanket elements at t_o=o is given in TABLE I:

TABLE I Initial composition of the core and blanket elements at $t_o = 0$ in 10^{21} nuclei per cm^3 reactor volume [5]

	Core Zone 1	Core Zone 2	Axial Blanket	Radial Blanket
^{235}U	0	0	0	0
^{238}U	5.642	5.299	7.07	10.5
^{239}Pu	0.842	1.107	0	0
^{240}Pu	0.247	0.325	0	0
^{241}Pu	0.028	0.037	0	0
^{242}Pu	0.006	0.007	0	0

The relative isotopic composition of the initial plutonium mixture is: ^{239}Pu: 75 %, ^{240}Pu: 22 %, ^{241}Pu: 2,5 %, ^{242}Pu: 0,5 %, it is the so called Pu^{∞} calculated for the reactor Na1 by the method given in [1].

TABLE II Fuel compositions for the average burn-up state of the reactor which are used for calculating the one group neutron flux and cross sections (in 10^{21} nuclei per cm^3 reactor volume)

Reactor Zone	Core Zone 1	Core Zone 2	Axial Blanket	Radial Blanket			
Fuel mixture	1	2	3	4	5	6	7
Average Burn-up MWD/tonne	50.000	40.000	2.000	5.000	300	2.000	200
^{235}U	0	0	0	0	0	0	0
^{238}U	5.287	5.078	6.928	10.179	10.421	10.314	10.449
^{239}Pu	0.796	0.997	0.125	0.263	0.075	0.159	0.049
^{240}Pu	0.273	0.342	0.003	0.006	0.001	0.003	0
^{241}Pu	0.040	0.047	0	0	0	0	0
^{242}Pu	0.008	0.009	0	0	0	0	0
Fission Product Pairs	0.358	0.299	0.015	0.051	0.003	0.024	0.002

The 26-multigroup calcualtions are carried out for the average burn-up state of the reactor specified in TABLE II. The fuel compositions given in this Table are taken to be constant in the subzones of the reactor designated in Fig. 2. The Karlsruhe 26-group set [4] is used for the multigroup calculations.

The radial blanket is divided into two radial zones, i.e. $N_B = 2$. The radial boundary between the two zones (Fig. 2) is R = 164 cm. The inner zone is always exchanged after the irradiation time $3\Delta t_C$, the outer zone always after $12\Delta t_C$, i.e. $m_1 = 3$, $m_2 = 12$. The maximum burn-up of the radial blanket in the plane z = 0 is 10.000 MWD/tonne in the inner and 9.500 MWD/tonne in the outer zone. The ^{238}U and ^{239}Pu output of the radial blanket as functions of time are given in Fig. 3. The unloaded plutonium consists of about 97 % ^{239}Pu and 4 % ^{240}Pu.

The parameters for the external fuel management are δ_c = 100 days for cooling, δ_r = 30 days for reprocessing, δ_f = 70 days for refabricating. Dividing the discharged fuel element volume ΔV_C into partial batches, we get $s_o = 3$. Thus, the external fuel inventory, which corresponds to the fuel element volume of $(s_o-1)\Delta V_C = 2\Delta V_C$, amounts to 2.020 kg plutonium of the initial composition and 8.460 kg uranium.

Fig. 4 shows the time behaviour of the relative plutonium composition of the fuel elements in the core. As a consequence of the group structure of the fuel elements in the core with n = 3, there are three lines for each plutonium isotope. Each of these refers to those fuel elements which are loaded and unloaded jointly. The diagram shows the continuous behaviour of the plutonium mixture in the fuel during neutron irradiation (idealized as linear to simplify plotting) and the discontinuous behaviour at the times t_s caused by exchanging the fuel elements. The initial composition of the plutonium is plotted as $\overset{\infty}{Pu}$.

TABLE III shows the most important quantities of the reactor Na1 which are changed by the time variations of the plutonium composition in the fuel mixture.

The numerical results reveal the dependence of the time behaviour of the plutonium composition in the fuel elements on the fuel burn-up and the internal and external fuel management. In particular, there is a strong coupling between the radial blanket operation and the closed fuel cycle in the case of joint reprocessing of the core and blanket elements, which increases the variations of the plutonium composition by exchanging the fuel elements as well as during burn-up (Fig. 4). Furthermore, it follows from the numerical results that the fuel cycle economy of the reactor system depends on the time variations of the plutonium composition in the fuel (TABLE III).

Finally, it follows from the results of this paper what importance must be attached to the long-time behaviour of the plutonium composition in fast power reactors with plutonium recycling. It may be the purpose of further investigations of these problems to find out conditions of the internal and external fuel management by which the variations of the plutonium composition are caused essentially only by the fuel burn-up in the reactor. Thus, the time variations of the plutonium composition may be constant during the long-time operation of the reactor as far as possible, i.e. a so-called quasi-stationary behaviour of the plutonium composition. Finding out conditions of such type requires additional numerical investigations to be carried out by the method given in this paper, varying the parameters of the problem. In addition, the method may be extended to the cases of partial plutonium recycling by which plutonium is fed into the system from the outside in a given composition.

TABLE III The most important quantities of the reactor system which are changed by plutonium recycling as functions of the time parameter s

plutonium loaded into the core at time t_s: $\Delta M_{Pu}(s)$

plutonium unloaded from the core at time t_s: $\Delta M^*_{Pu}(s)$

uranium loaded into the core at the time t_s: $\Delta M_U(s)$

net uranium introduced into the system from the outside: $M_U(s)$

breeding gain of the $(s-s_o+1)$-th fuel cycle: $\Delta M_{BG}(s-s_o+1)$, $s_o=3$

s	$\Delta M_{Pu}(s)$ [kg]	$\Delta M^*_{Pu}(s)$ [kg]	$\Delta M_U(s)$ [kg]	$M_U(s)$ [kg]	$\Delta M_{BG}(s-s_o+1)$ [kg]	relative Pu composition of $\Delta M_{BG}(s-s_o+1)$ and $\Delta M_{Pu}(s)$			
						^{239}Pu	^{240}Pu	^{241}Pu	^{242}Pu
1	1010	954	4228	456	0	-	-	-	-
2	1010	954	4228	456	0	-	-	-	-
3	1016	954	4222	747	33	0.730	0.229	0.034	0.007
4	1016	954	4222	451	33	0.730	0.229	0.034	0.007
5	971	954	4267	495	333	0.779	0.188	0.027	0.006
6	1016	959	4222	752	33	0.730	0.229	0.034	0.007
7	1016	959	4222	456	33	0.730	0.229	0.034	0.007
8	975	921	4262	455	332	0.770	0.192	0.030	0.008
9	1022	959	4216	746	31	0.718	0.234	0.037	0.010
10	990	959	4248	482	26	0.755	0.205	0.031	0.008
11	975	925	4262	459	332	0.770	0.192	0.030	0.008
12	1022	964	4216	1336	31	0.718	0.234	0.037	0.010
13	994	937	4244	454	26	0.749	0.208	0.033	0.010
14	935	925	4302	444	878	0.817	0.152	0.023	0.007
15	1004	964	4234	769	27	0.738	0.217	0.034	0.010
16	994	940	4244	457	26	0.749	0.208	0.033	0.010
17	979	892	4258	418	333	0.763	0.145	0.032	0.010
18	1007	949	4231	749	27	0.734	0.219	0.035	0.012
19	965	940	4273	486	22	0.784	0.180	0.027	0.009
20	970	928	4268	468	328	0.776	0.185	0.030	0.010
21	1007	951	4231	753	27	0.734	0.219	0.035	0.012
22	997	916	4240	428	25	0.745	0.210	0.034	0.012
23	972	920	4266	457	329	0.773	0.186	0.030	0.011
24	986	951	4252	1358	25	0.760	0.200	0.031	0.010
25	990	943	4248	464	25	0.754	0.203	0.032	0.011
26	930	922	4307	501	871	0.825	0.145	0.022	0.008
27	1010	934	4228	730	27	0.731	0.220	0.036	0.013
28	992	937	4246	456	25	0.752	0.204	0.032	0.012
29	959	887	4278	433	324	0.789	0.175	0.027	0.009
30	1005	954	4233	758	27	0.737	0.215	0.035	0.013
31	961	939	4276	488	21	0.790	0.176	0.026	0.009
32	973	912	4264	446	329	0.771	0.187	0.030	0.011
33	1006	949	4232	751	27	0.736	0.216	0.035	0.014
34	983	913	4255	439	24	0.763	0.196	0.030	0.011
35	970	923	4267	462	328	0.775	0.184	0.030	0.012

REFERENCES

[1] K. OTT, A. JANSEN, Long-time behaviour of fast breeders, Proceedings of the Seminar on the Physics of Fast and Intermediate Reactors, Vienna, SM-18/2 (1962) 213-228.

[2] D. SMIDT, A. MÜLLER et al., Referenzstudie für den Na-gekühlten schnellen Brutreaktor (Na1), KFK-299, GfK Karlsruhe, 1964.

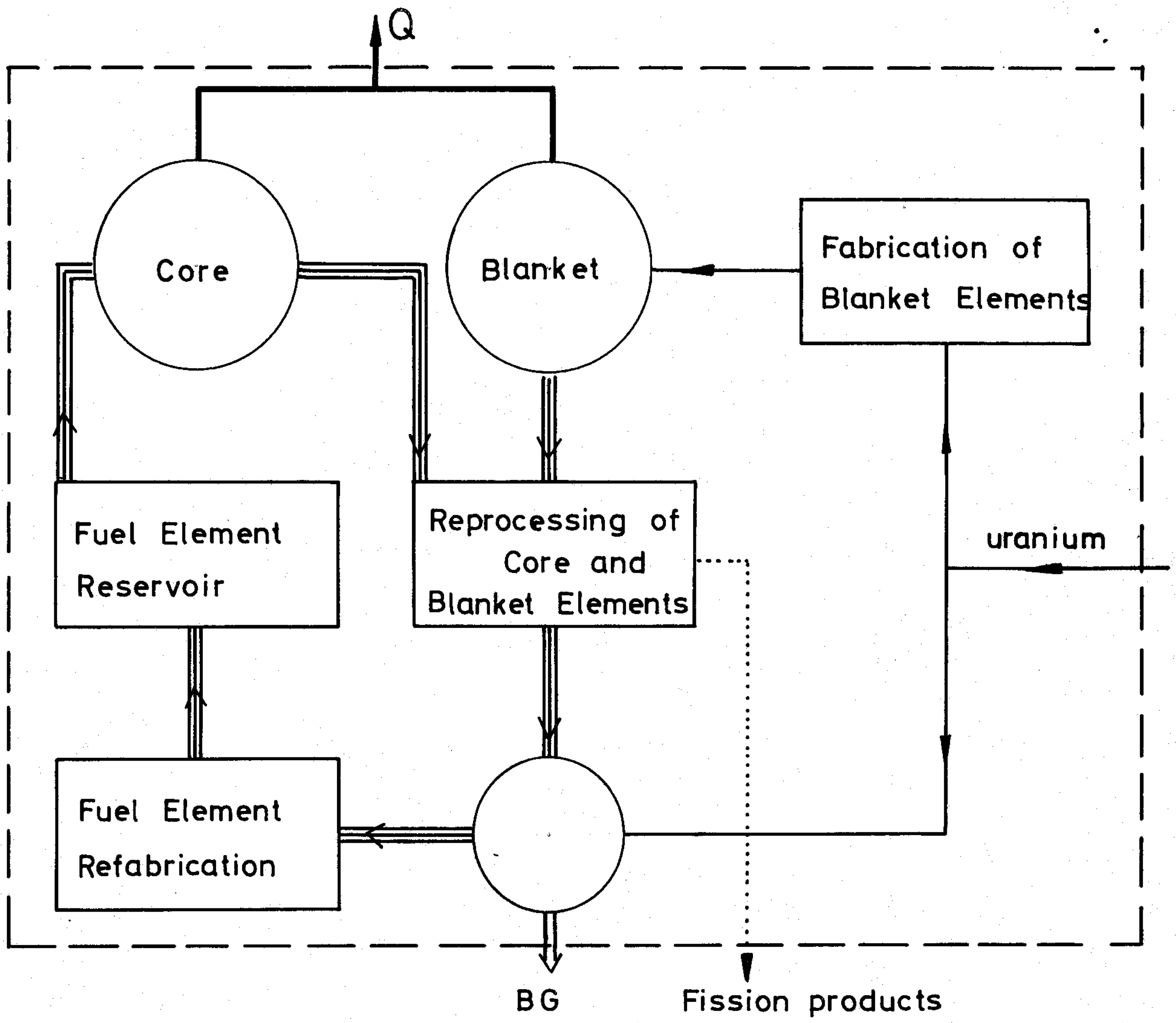
[3] A. JANSEN, K. OTT, K.E. SCHROETER, Zum Abbrandverhalten schneller Reaktoren, PSB-report 121/64.

[4] 26-group set KFK 26-10, Karlsruhe (1965), for first information: K. KÜSTERS, M. METZENROTH, The influence of some important group constants on integral fast reactor quantities, ANL-7120 (1965) 423-433.

[5] K.E. SCHROETER, Zweidimensionale 26-Gruppen-Diffusions-Rechnungen für den natriumgekühlten 1000 MW(e)-Reaktor Na1 (to be published).

Fig. 1

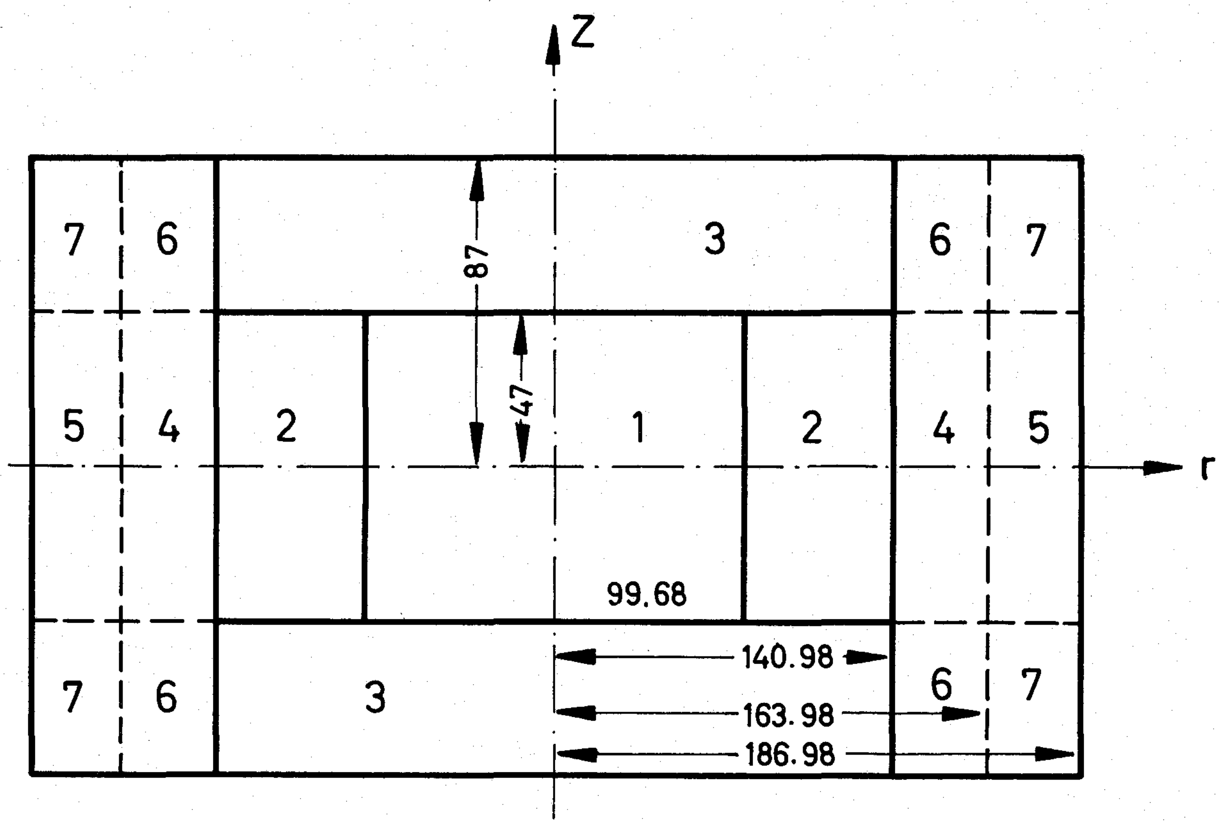
Fuel Cycle of a Fast Power Reactor with Plutonium Recycling



- thermal power
- uranium introduced from the outside
- excess plutonium (breeding gain BG)
- fission products drawn off to the outside
- fuel cycle (uranium-plutonium mixture)

Fig. 2

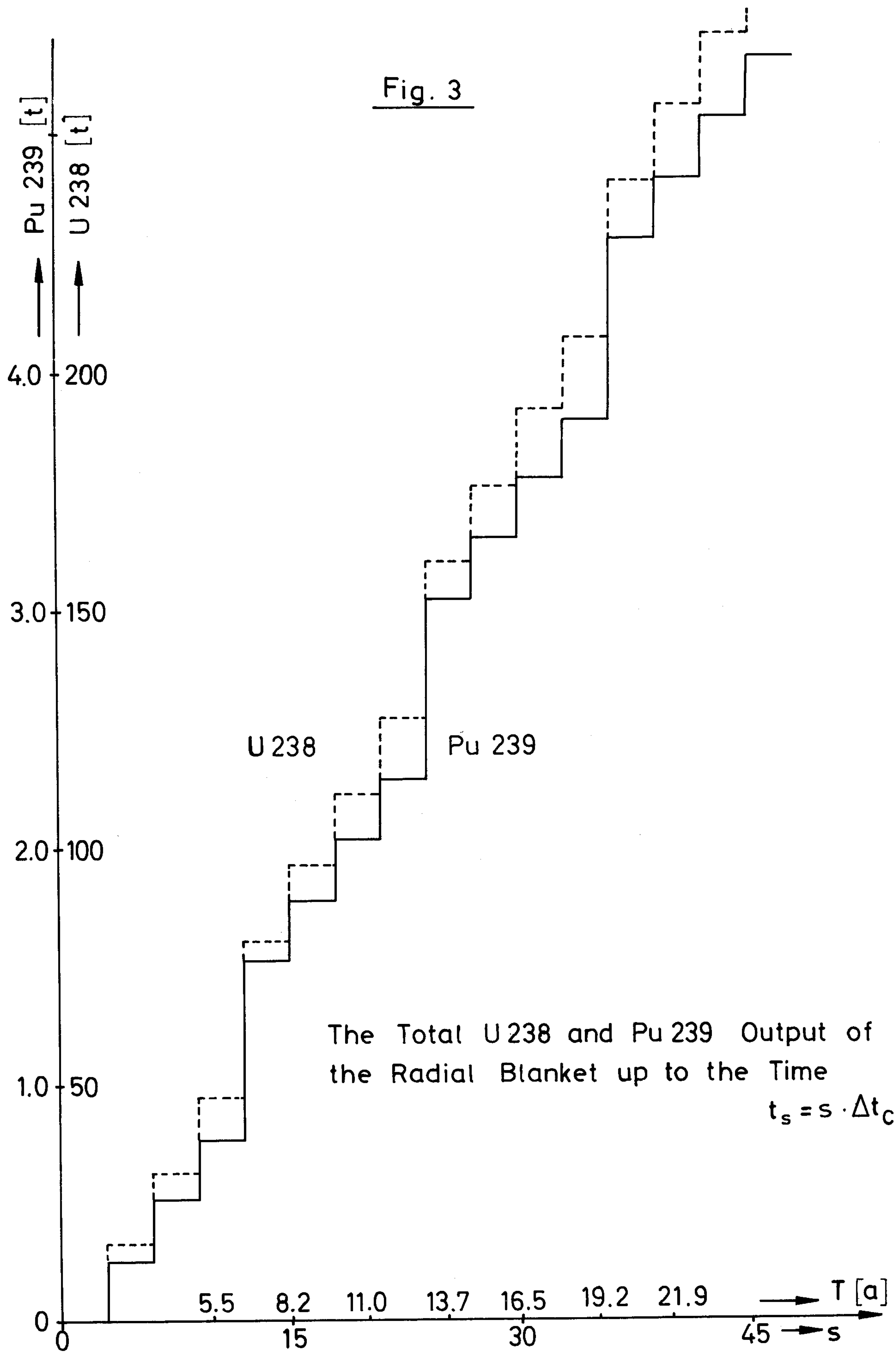
Cross-Sectional View of the Reactor Na 1
(measures are given in cm)



1 Corezone 1
2 Corezone 2
3 Axial Blanket
4, 5, 6, 7 Radial Blanket

The numbers 1 to 7 design the fuel mixtures of the average burn-up state of the reactor given in Table 2

Fig. 3



The Total U 238 and Pu 239 Output of the Radial Blanket up to the Time $t_s = s \cdot \Delta t_c$

Fig. 4 Relative Pu-Composition of the Fuel Elements in the Core of the Reactor Na1 (n = 3, s_0 = 3)