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PROPAGATION OF COHERENT ELECTROMAGNETIC RADIATION IN THE ATMOSPHERE

by

G. BLAESSER

1974

Joint Nuclear Research Centre
Ispra Establishment - Italy

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ABSTRACT

A summary of the present state of knowledge concerning the propagation of visible, infrared and microwave radiation in the atmosphere is given. After an introduction to the general characteristics of the three main wave fields, i.e. plane waves, spherical waves and Gaussian beams, and an outline of the main features of the scattering by impurities (aerosols etc.), the main part of this report is concerned with the effects of the stochastic fluctuations of the index of refraction which are caused by atmospheric turbulence, and which are responsible for amplitude and phase fluctuations in communication links, for losses in coherence, for scattering or even « beam steering » (deviation of the path of the whole beam).

PROPAGATION OF COHERENT ELECTROMAGNETIC RADIATION IN THE ATMOSPHERE

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Introduction

This report gives a summary of the present state of knowledge concerning the propagation of visible, infrared and microwave radiation in the atmosphere. Its scope is of introductory or "tutorial" character, so that particular theoretical details and calculations were omitted; the list of references is far from complete, but includes a number of review articles where more specific references can be found.

In the first chapter the characteristics of coherent wave fronts are discussed for the three special cases of plane waves, spherical waves and Gaussian beams. Since laser beams correspond well to the description by Gaussian beam waves and since, on the other hand, the properties of such beam waves are usually not familiar to the newcomer in this field, they are treated in more detail here.

The absorption bands of the main constituents of the atmosphere limit the transmission of electromagnetic energy essentially to a number of "windows" on the frequency scale. Within these transmission windows and in the absence of selective (resonance) absorption by impurities, the effect of absorption on the transmission of radiation is usually small compared to that of scattering, the main features of which are outlined in chapter II.

The index of refraction of the atmosphere exhibits stochastic fluctuations due to atmospheric turbulence, and also systematic variations caused by large-scale changes in the profiles of pressure, temperature and humidity. These variations of the index of refraction along the path of propagation are responsible for amplitude and phase fluctuations in communication links, for losses in coherence, for scattering or even "beam steering" (deviation of the path of the whole beam). These phenomena constitute the main topic of this report; they are discussed in chapters III, IV and V.

Finally, chapter VI summarizes experimental results of particular importance to the general understanding of these propagation effects.

Chapter I - ELECTROMAGNETIC WAVE FIELDS

In a source-free medium (div $\vec{E} = 0$) with constant index of refraction $n = \sqrt{\varepsilon}$ and unit permeability $\mu = 1$ Maxwell's equations for the electromagnetic field are equivalent to the wave equations for the electric ($\vec{E}$) and magnetic ($\vec{H}$) fields:

$$(1) \qquad \Delta\vec{E} = (n^2/c^2)\frac{\partial^2}{\partial t^2}\vec{E} \qquad \Delta\vec{H} = (n^2/c^2)\frac{\partial^2}{\partial t^2}\vec{H}$$

where Δ is the Laplace operator ($\Delta = \sum_{i=1}^{3} \frac{2}{x_i^2}$ in Cartesian coordinates x_1, x_2, x_3).

The fields $\vec{E}$ and $\vec{H}$, solutions of (1) are related to each other by

$$(1a) \qquad \text{rot}\,\vec{E} = -\frac{\partial}{c\,\partial t}\vec{H}$$

For monochromatic radiation of frequency ω, i.e. if

$$\vec{E} = \vec{E}_\omega(x_i).\exp(-j\omega t) \qquad \vec{H} = \vec{H}_\omega(x_i).\exp(-j\omega t)$$

the wave equations (1) and the condition (1a) become, respectively

$$(1') \qquad \Delta\vec{E}_\omega + k^2\vec{E}_\omega = 0 \qquad \Delta\vec{H}_\omega + k^2\vec{H}_\omega = 0$$

$$(1a') \qquad n.\,\text{rot}\,\vec{E}_\omega = jk\vec{H}_\omega$$

where the "wave number" k is defined by $k = n\omega/c = 2\pi/\lambda$.

Particular solutions of the equations (1'), (1a') are the plane wave, the field of a radiating dipole and the Gaussian beam:

a. Plane Wave

$$(2) \qquad E_\omega = E_o^{(\omega)}\exp(jk.r) \qquad \vec{H}_\omega = n\vec{e}\times\vec{E}_\omega$$

with a "wave vector" $\vec{k} = k\vec{e}$, where $\vec{e}$ is a unit vector in the direction of propagation, is obviously a solution of (1') and (1a'). The surfaces of constant phase are planes orthogonal to $\vec{k}$. The plane wave solution given by (2) corresponds to an ideal monochromatic wave with linear polarization (the direction of which is given by the direction of $\vec{E}_o^{\omega}$). More realistic wave fields can be constructed from superpositions of waves of this type. Radiation arriving from a star can be considered as a superposition $\int a(\omega)E_{\omega}\exp(-j\omega t)d\omega$ of such plane waves with the same direction of propagation $\vec{e}$.

If the direction of polarization is of no importance, we can write it in the scalar form

$$(2') \quad E = E_o \exp(jk.r)$$

where the scalar amplitude E is given by the expression $E = \vec{E}.\vec{E}_o/|\vec{E}_o|$.

b. Field of a Radiating Dipole

$$(3) \quad \vec{E}_{\omega} = -\vec{e}\times(\vec{e}\times\vec{p}_o)\frac{k^2}{rn}\exp(jkr) \qquad H_{\omega} = \vec{e}\times\vec{p}_o\frac{k^2}{r}\exp(jkr)$$

is the asymptotic solution (i.e. it satisfies the wave equation (1') if terms of order r^{-2} and higher can be neglected) for the case of an oscillating electric dipole of moment $n\vec{p} = n\vec{p}_o\exp(-j\omega t)$. $\vec{e} = \vec{r}/|\vec{r}|$ is the unit vector that points radially from the source to the observation point $\vec{r}$. If we are not interested in the polarization, we can rewrite eq. (3) in terms of the amplitude

$$(3') \quad E = (\frac{k^2 p}{n} o.\sin\vartheta)(\exp(jkr)/r) = A(\vartheta)\exp(jkr)/r$$

where ϑ is the angle between the directions of $\vec{p}$ and of $\vec{e}$. The intensity $I = (c/8\pi)n|E|^2$ of this wave exhibits the well known angular dependence on $\sin^2\vartheta$ and the total energy radiated by the dipole per unit time, dW/dt, is obtained integrating the intensity over the surface of a large sphere centered at the dipole

$$(4) \quad dW/dt = r^2\int I d\Omega = \frac{ck^4}{3}p_o^2 = \frac{16\pi^4}{3}\lambda^{-4}cp_o^2$$

This is the λ^{-4}-relationship for the energy emission by an electric dipole which is always valid if the linear dimensions of the dipole are small compared with the wavelength of the emitted radiation.

In communication systems, the angular relationship between "transmitter" and "receiver" is usually fixed, i.e. one investigates the dependence of the field strength on distance r for points on the same "radial" ϑ = const, so that the factor $A(\vartheta)$ in (3') can be considered as a constant. This means that instead of the dipole solution one supposes a "spherical wave" which is - strictly speaking - never a topologically valid solution of the wave equations of electrodynamics (1') and (1a') but satisfies the scalar wave equation

$$(5) \qquad \Delta u + k^2 u = 0 .$$

c) Gaussian Beam Waves

Choosing the direction of propagation $\vec{e}$ of a beam of limited cross section as the z-axis we may try to describe it by expressions of the form(1)

$$(6) \qquad \vec{E}_\omega = \vec{E}_o(x, y, z)\exp(jkz) \qquad \vec{H}_\omega = n\vec{e}\times\vec{E}_\omega$$

where $\vec{E}_o(x, y, z)$ can be considered as a slowly varying function of the z-coordinate (since its main z-dependence is already included in the exponential factor), so that the second derivative $\partial^2 E_o/\partial z^2$ can be neglected in comparison with the quantities $\partial^2\vec{E}_o/\partial x^2$ and $\partial^2\vec{E}_o/\partial y^2$. Thus we obtain from eq. (1') the following approximate equation for $\vec{E}_o$:

$$(7) \qquad \frac{\partial^2\vec{E}_o}{\partial x^2} + \frac{\partial^2\vec{E}_o}{\partial y^2} + 2jk\frac{\partial\vec{E}_o}{\partial z} = 0$$

This equation admits a solution of the Gaussian form

$$(8) \quad \vec{E}_o = \vec{A} \exp\left[j(P(z) + \frac{k.r^2}{2q(z)}) \right]$$

where $\vec{A}$ is a constant vector, $r^2 = x^2 + y^2$, and $P(z)$, $q(z)$ are functions to be determined. The quantity $P(z)$ represents a complex phase-shift relative to the plane wave solution, while the complex "beam parameter" q can be expressed in the form

$$(9) \quad \frac{1}{q(z)} = \frac{1}{R(z)} + j\frac{2}{kw(z)^2}$$

with real functions $R(z)$ and $w(z)$;$R(z)$ can be interpreted as the radius of curvature of the wavefront that intersects the axis at z, and $w(z)$ as the "width" of the Gaussian beam cross section, i.e. as the distance r at which the amplitude is decreased to a value 1/e times that on the z-axis.

Inserting the expression (8) into equation (7) and comparing terms of equal power in r, one finds

$$(10a) \quad \frac{dq}{dz} = 1 \qquad (10b) \quad \frac{dP}{dz} = -\frac{i}{q}$$

Integration of (10a) gives $q(z) = q_o + z$, where q_o is the value of the complex beam parameter in the "reference plane" $z = 0$. It is convenient to choose the reference plane $z = 0$ at the "beam waist" where the phase front is plane ($R = \infty$) so that $q_o = -jkw_o^2/2$ is purely imaginary. We then have

$$(11) \quad q(z) = -j\frac{kw_o^2}{2} + z$$

This implies (c.f. equation (9)) the relations

$$(12a) \quad w(z)^2 = w_o^2(1 + (2z/kw_o^2)^2)$$

$$(12b) \quad R(z) = z(1 + (kw_o^2/2z)^2)$$

i.e. the Gaussian beam contracts to the minimum diameter $2w_o$ at the waist; the beam contour $w(z)$ is a hyperbola with asymptotes that form an angle $\vartheta \approx \tan\vartheta = 2/kw_o$ with the axis of propagation. This angle is the well known far-field diffraction angle of a Gaussian amplitude distribution.

From equation (10b) for the complex phase shift $P(z)$ at a distance z from the waist, we obtain

$$(13)\quad P(z) = \int_o^z \frac{j}{z-j(2w_o^2/k)} = j.\ln(1 + j\frac{2z}{kw_o^2})$$

$$= j.\ln\sqrt{1 + (2z/kw_o^2)^2} - \arctan(2z/kw_o^2)$$

$$= j.\ln\frac{w(z)}{w_o} - \Phi(z)$$

with $w(z)$ given by (12a) and $\Phi(z) = \arctan(2z/kw_o^2)$. Thus we can finally write the Gaussian beam in the form

$$(14)\quad E = A(w_o/w(z))\exp(-r^2/w(z)^2)\exp\left[j(kz-\Phi(z) + \frac{r^2k}{2R(z)})\right]$$

According to STROHBEHN[2] this equation describes correctly Gaussian beams as long as $z/w_o \ll (kw_o/2)^3$, which is not a strong limitation since it gives even for values of w_o of the order of only a millimeter upper limits for z of the order of 10^8 m.

Gaussian beams are produced by many lasers that oscillate in the fundamental mode; their parameters w_o and $R(z)$ can be varied experimentally by suitable stops and focussing devices (lenses or spherical mirrors), respectively. The methods of paraxial ray optics can be extended to deal with Gaussian beams (c.f. references (1) and (3)).

Chapter II - ABSORPTION AND SCATTERING OF ELECTROMAGNETIC RADIATION BY ATMOSPHERIC CONSTITUENTS AND IMPURITIES

Except in situations of extreme pollution at the wavelengths of specific absorption of the pollutants, the absorption in the atmosphere is due to its principal constituants which are N_2, O_2, water vapour and CO_2 (at higher altitudes also O_3). Since N_2 and O_2 have no dipole moment, they show no molecular absorption bands. Thus the main absorption effects that influence low altitude atmospheric communications are due to water vapour and CO_2 while high altitude links such as ground-to-satellite communications, can also strongly be affected by O_3 absorption.

This absorption precludes the practical use of the part of the spectrum corresponding to wavelengths above 15μ and up to several millimeters for the transmission of information. The "near infrared" region (wavelengths below 15μ) is divided by some strong absorption lines into a series of "windows" in which the absorption is relatively small and can be taken into account by multiplying the intensity of the transmitted radiation by an averaged transmission factor of the form

$$(15) \quad T_a(\lambda, x) = \frac{1}{\Delta\lambda} \int_{\lambda-\Delta\lambda}^{\lambda+\Delta\lambda} d\lambda' \exp(-\alpha(\lambda')\bar{n}_a x)$$

where $\Delta\lambda$ is the wavelength interval used in the transmission and centered at λ, $\alpha(\lambda)$ the absorption coefficient per unit concentration of the absorber, $\bar{n}_a$ its mean concentration along the path x. Values for $\alpha(\lambda)$ for the main atmospheric absorbers and recipes for the estimation of transmission factors as proposed first by LANGER can be found in textbooks on infrared communications(4). However, it appears from such evaluations that for a suitable choice of the radiated wavelength inside the infrared transmission windows and in particular in the visible part of the spectrum the atmospheric absorption can usually be neglected (i.e. $T_a \approx 1$) while the total transmission factor $T = T_a T_s$ becomes equal to the scattering transmission factor T_s which takes into account the reduction of the transmitted intensity, due to scattering by the molecular constituents of the air and by aerosol particles suspended in it.

Elastic scattering by molecules, also called "Rayleigh scattering", varies with wavelength as λ^{-4}, i.e. blue light is scattered more strongly than red light - a fact which accounts for the blue colour of clear sky. This λ^{-4}-law is, however, typical for all scattering processes involving scatterers of linear dimensions much smaller than the wavelength of the scattered radiation. It follows simply from the fact that the incident radiation gives rise to a fluctuating dipole moment which radiates (i.e. scatters) energy according to eq. (4).

The scattering of electromagnetic waves by dielectric spheres of arbitrary size was treated rigorously by MIE[5]. His methods were extended by others to particles of other shapes; the main results of this theory are summarized in the monograph of VAN DE HULST[6]. Within this theoretical framework one can discuss the main characteristics of the scattering by aerosols, which is usually referred to as "Mie scattering". Of course, if the dimensions of the aerosol particles become much smaller than the wavelength of the radiation, the Mie theory yields the λ^{-4}-law. Thus Rayleigh scattering - in the mathematical sense - is a limiting case (and not something different) from Mie scattering. For example, it applies to the extinction of cm-waves by scattering on raindrops in strong showers. The most dramatic features in Mie scattering occur, however, when the linear dimensions of the scatterers are of the same order as the wavelength; in such situations one may encounter pronounced resonance scattering. Finally, when the particle size is much larger than the wavelength, the scattering becomes rather independent of the wavelength (c.f. the white colour of vapours and clouds consisting of liquid drops of dimensions $>>\lambda$); in these situations we deal with ordinary reflection of light by macroscopic bodies. The angular distribution of the reflected light depends on the surface properties of the reflector; for optically rough surfaces Lambert's law ($I_s \propto \cos\vartheta$, where ϑ is the angle between the surface normal and the direction in which one observes the scattered intensity I_s) gives a rather good description of the observed scattering.

A rigorous mathematical calculation of the transmission factor for scattering under real atmospheric conditions is impossible mainly for two reasons:

1) In aerosols the size distribution of the scatterers is not uniform (and often not even known). Also, their geometry might not be known; in the case of suspended liquid droplets they can be assumed to be spherical because of the effect of surface tension, but dust particles, ice crystals etc. can have about any shape. Thus, the effective cross section, i.e. the result of the superposition of different particle sizes and shapes, as a function of wavelength is usually not known. (On the other hand, if the particles are known to be of spherical form, their size distribution can be inferred from experimental scattering results at different wavelengths; c.f. references (7), (8), (9)).

2) Since the scattered radiation is not lost but re-emitted in a different direction, multiple scattering contributes to the radiation intensity if the optical path in the medium exceeds the extinction length for scattering. To deal with multiple scattering effects the integro-differential equations of the theory of radiation transport have to be solved as in the related problems of radiation shielding in reactors[10] or in the theory of the photosphere of stars[11]. Such transfer of electromagnetic radiation in the atmosphere has been considered in particular for nuclear weapons assessment and civil defense studies concern-

ing the effects of the infrared radiation of a nuclear blast[12], and Monte Carlo codes (LITE) have been developed for such investigations[13].

Fortunately, for the design of communication links it is usually sufficient to use some approximate semi-empirical formulae for the estimation of the transmission factor[4]. Since the visual range V - also called "meteorological range" - is determined quasi exclusively by the concentration of aerosol particles in the air, it is a convenient parameter to use in such a semi-empirical formula. Indeed, the results of many observations are rather well represented by the expression

$$(16)\quad T_s(\lambda, x) = \exp\left(-\frac{3.91}{V}(\lambda/0.55)^{-q}x\right)$$

with

$$(17)\quad q = 0.585\ V^{1/3}$$

In these relations V and x are to be expressed in kilometers and λ in microns. Formula (16) contains a wavelength dependence of the "effective" atmospheric extinction coefficient of the form λ^{-q} - in contrast to the Rayleigh part λ^{-4} - which depends on the meteorological situation. For extremely good visibilities (i.e. meteorological ranges of the order of 300 km) q approaches the Rayleigh exponent, $q \rightarrow 4$, i.e. the formula (16) predicts blue sky conditions in this limit, whereas for poor visibilities - below 1 km - q will be of the order of only 0.5 corresponding to a nonselective scattering typical for the white and grey colour of the clouds. The wavelength enters into the expression (16) relative to the wavelength of 0.55 μ at which the standard measurements of the visual range are performed. The standard definition of the visual range is such that for a distance x = V the intensity of the 0.55μ radiation is reduced by the scattering (thus apart from a geometric r^{-2}-reduction due to beam divergence) to 2% of its original value, so that $T(0.55\mu, V) = 0.02$. The factor $3.91 = -\ln(0.02)$ in (16) corresponds to this definition.

Chapter III - MICROSTRUCTURE OF ATMOSPHERIC TURBULENCE

We shall discuss in this chapter some aspects of the theory of atmospheric turbulence that are important for the propagation of electromagnetic waves in the air. A more detailed discussion can be found in the book of TATARSKI(14), while the general concepts of the theory of turbulence are summarized by BACHELOR(15); recent summaries of the theory of atmospheric turbulence in the planetary boundary layer are also found in ref. (16) and (17).

Air turbulence is primarily observed in the form of fluctuating wind velocities $\vec{U}(\vec{r}, t)$ which depend on the position $\vec{r}$ of the observation point and on the time t. Let $\vec{U}_m(\vec{r})$ be the mean value of the wind velocity at the point r, i.e. the result of a time averaging over a certain interval of the order of 30 sec to 1 min. This value $\vec{U}_m(\vec{r})$ depends on the meteorological situation and changes rather slowly with time. Subtracting $\vec{U}_m(\vec{r})$ from the instantaneous values of the wind velocity we obtain the turbulent component $\vec{u}(\vec{r}, t)$ such that $\vec{U}(\vec{r}, t) = \vec{U}_m(\vec{r}) + \vec{u}(\vec{r}, t)$ where the mean value of $\vec{u}(\vec{r}, t)$ is zero. Sometimes, as on a calm sunny day, $\vec{U}_m(\vec{r})$ may be completely zero, but $\vec{u}(\vec{r}, t)$ will be finite due to thermal convection.

In order to gain some insight into the statistical structure of the stochastic velocity field $\vec{u}(\vec{r}, t)$ we have to correlate the velocity fluctuations in different points $\vec{r}_1$, $\vec{r}_2$. This can be done by the study of the "structure functions"

$$(18) \quad D_{ik}(\vec{r}_1, \vec{r}_2) = \overline{(u_i(\vec{r}_1) - u_i(\vec{r}_2))(u_k(\vec{r}_1) - u_k(\vec{r}_2))}$$

or the "correlation functions"

$$(19) \quad B_{ik}(\vec{r}_1, \vec{r}_2) = \overline{u_i(\vec{r}_1)u_k(\vec{r}_2)}$$

The bar over these products denotes averaging, as usual, and the indices ($i = 1,2,3$ and $k = 1,2,3$) enumerate the velocity components in a cartesian system of coordinates.

The field of turbulent velocity fluctuations is called "homogeneous" if the functions $D_{ik}(\vec{r}_1, \vec{r}_2)$ and $B_{ik}(\vec{r}_1, \vec{r}_2)$ do not change if we shift the positions of the two anemometers that measure $u_i(\vec{r}_1)$ and $u_k(\vec{r}_2)$ by the same distance $\vec{d}$, i. g. if $D_{ik}(\vec{r}_1 + \vec{d}, \vec{r}_2 + \vec{d}) = D_{ik}(\vec{r}_1, \vec{r}_2)$ for arbitrary $\vec{d}$. It is evident that the air turbulence in the atmosphere is not completely homogeneous, since the atmosphere is always bounded by the surface of the earth; however, in many problems involving scales of turbulent motion which are small compared to the distance from the observation points to the surface of the earth, the assumption of homogeneity introduces only a very small error. In the case of homogeneous turbulence the functions D_{ik} and B_{ik} depend only on the difference $\vec{r}_1 - \vec{r}_2$, and if

$$B_{ik}(0) = \sigma^2_{ik} = \overline{u_i(\vec{r})u_k(\vec{r})}$$

is bounded, we may express the structure functions in terms of the correlation functions and vice versa, since then

$$(20) \quad D_{ik}(\vec{r}_1 - \vec{r}_2) = 2\, B_{ik}(\vec{r}_1 - \vec{r}_2) - 2\,\sigma^2_{ik} \; .$$

Note that the diagonal values σ^2_{ii} ($i=1, 2, 3$) of the tensor σ^2_{ik} are the mean square values of the velocity fluctuations in the turbulent flow field along the three coordinate axes.

As we shall see in more detail later, the turbulent air motion takes place in the form of eddies of different sizes, and it is clear that the difference $\vec{u}(\vec{r}_1) - \vec{u}(\vec{r}_2)$ is mainly correlated with the properties of eddies of dimension $s = |\vec{r}_1 - \vec{r}_2|$. Eddies of size s are approximately isotropic, if their distance from flow boundaries is large compared to s. We will call the velocity field "locally isotropic" in the neighbourhood of the point $\vec{r}$ if there exists a length L such that for distances $s < L$ the structure functions D_{ik} (and the correlation functions B_{ik}) depend there only on the distance s and on the relative orientation of the two anemometers but not on their absolute orientation in space. In such a case $D_{ik}(s)$ can have only two independent components, since the relative orientation of the two detectors permits an invariant decomposition of any wind velocity u into a component $\vec{u}_{\parallel} = (\vec{u}.\vec{e})\vec{e}$ parallel to the direction given by the unit vector $\vec{e} = (\vec{r}_1 - \vec{r}_2)/s$ (the direction that joins the two anemometers) and a component $\vec{u}_{\perp} = \vec{u} - (\vec{u}.\vec{e})\vec{e}$ orthogonal to it, so that D_{ik} must be given by the expression

$$(21) \quad D_{ik}(s) = D_{\parallel}(s)e_i e_k + D_{\perp}(s)(\delta_{ik} - e_i e_k)$$

involving the projection operators $\pi^{\parallel}_{ik} = (\vec{e}\vec{e})_{ik} = e_i e_k$ on the direction s and $\pi^{\perp}_{ik} = (1 - \vec{e}\vec{e})_{ik} = \delta_{ik} - e_i e_k$ on the plane orthogonal to it, and the invariants

$$(22) \quad D_{\parallel}(s) = \overline{(\vec{u}_{\parallel}(\vec{r}_1) - \vec{u}_{\parallel}(\vec{r}_2))^2}$$

and

$$(23) \quad D_{\perp}(s) = \overline{(\vec{u}_{\perp}(\vec{r}_1) - \vec{u}_{\perp}(\vec{r}_2))^2}$$

which can be formed with the vectors $\vec{u}_{\parallel}(\vec{r}_1) - \vec{u}_{\parallel}(\vec{r}_2)$ and $\vec{u}_{\perp}(\vec{r}_1) -$ $- \vec{u}_{\perp}(\vec{r}_2)$ in the two subspaces. Similarly, it is

$$(24) \quad B_{ik}(s) = B_{\parallel}(s)e_i e_k + B_{\perp}(s)(\delta_{ik} - e_i e_k) .$$

The flow of the air in the atmosphere can be considered as incompressible, since all wind speeds are well below sonic velocities. The condition of incompressibility, div $\vec{u} = 0$, leads to a relation between the two functions $D_{\parallel}$ and $D_{\perp}$, so that finally only one independent function is sufficient to characterize the turbulent flow. In fact, we then have

$$(25) \quad \sum_{i=1}^{3} \partial D_{ik}/\partial x_i = 0$$

We will derive from eq. (25) a relation for the Fourier transform $\Phi_{ik}(\vec{\varkappa})$ of the correlation function $B_{ik}(\vec{r})$, defined by

$$(26) \quad B_{ik}(\vec{r}) = \iiint d^3\varkappa \; \Phi_{ik}(\vec{\varkappa})\exp(j\vec{\varkappa}\vec{r}) .$$

From equation (20) we find that $2(1-\exp(j\vec{\varkappa}\vec{r}))\Phi_{ik}(\vec{\varkappa})$ is the Fourier transform of D_{ik}:

$$(27) \quad D_{ik}(\vec{r}) = 2\iiint d^3\varkappa \; (1-\exp(j\vec{\varkappa}\vec{r}))\Phi_{ik}(\vec{\varkappa}) .$$

The incompressibility condition (25) then leads to

$$(28) \quad \sum_{i=1}^{3} \varkappa_i \Phi_{ik}(\vec{\varkappa}) = 0$$

In the case of local isotropy we obtain by Fourier transform of equation (24) the following form for the function $\Phi_{ik}(\vec{\varkappa})$:

$$(29) \quad \Phi_{ik}(\vec{\varkappa}) = F(\varkappa)\delta_{ik} + G(\varkappa)\varkappa_i\varkappa_k/\varkappa^2$$

where the scalar functions $F(\varkappa)$ and $G(\varkappa)$ are the Fourier transforms of $B_{\parallel} - B_{\perp}$ and of $B_{\perp}$, respectively. Equation (28) then leads to

$$G(\varkappa) + F(\varkappa) = 0$$

or

(30) $\Phi_{ik}(\vec{\varkappa}) = (\delta_{ik} - \varkappa_i \varkappa_k / \varkappa^2)\, F(\varkappa)$.

Up to now we have only defined the structure and correlation functions and studied the limitations imposed on them by the conditions of homogeneity, local isotropy and incompressibility. In order to arrive at an explicit expression of the structure function, we have to use a model of turbulent flow. A model which is sufficiently accurate for our purposes was developed by KOLMOGOROV[18]. It starts from the fact that a laminar flow of air of flow velocity U becomes unstable and changes into turbulent flow whenever its Reynolds number $Re = UL/\nu$ exceeds a critical Reynolds number Re_{crit}. ν is the specific viscosity, $\nu = \mu/\rho$, of the air and L some characteristic length, determined in meteorological applications by the characteristics of the wind profile. The eddies of dimension l which are formed in the turbulence - with a turbulent velocity u_l - are again unstable if their Reynolds number $Re_l = l u_l/\nu$ exceeds Re_{crit}; they break up into a next generation of smaller eddies, and so on. This process of transfer of flow energy from larger eddies to smaller ones continues until eddies of size l_o are formed for which $Re_{l_o} \approx Re_{crit}$. At that level, the energy is dissipated into heat by laminar viscous flow, at a rate ε (per unit mass) which is proportional to $\nu u_o^2/l_o^2$. Under stationary conditions this rate of energy dissipation must be equal to the rate of energy transfer (per unit mass) $\sim u_l^3/l$ from eddies of size l to smaller ones (the flow energy per unit mass in the eddies of size l is proportional to u_l^2, and the rate of transfer must be proportional to u_l^2/τ_l where $\tau_l \sim l/u_l$ is the lifetime of these eddies). Thus the turbulent velocity u_l decreases with the size l of the eddies according to

(31) $u_l \sim (\varepsilon l)^{1/3}$

Since the structure functions $D_{ik}(s)$ are mainly determined by the eddies of size $l = s$ and since D_{ik} has the dimensions of the square of a velocity, it is reasonable to assume that

$$(32) \quad D_{ik} \sim (\varepsilon s)^{2/3}$$

as long as $L > s > l_o$. The two linear dimensions L and l_o which limit this so-called "inertial subrange" are called the "outer scale" and the "inner scale" of turbulence, respectively.

While L ranges from meters to hundreds of meters, l_o is of the order to some millimeters. A detailed discussion of the boundaries l_o and L of the inertial subrange can be found in ref. (19).

By fourier transformation we obtain from eq. (32) that $F(\varkappa)$ is proportional to $\varkappa^{-11/3}$ in the range $2\pi/L < \varkappa < 2\pi/l_o$, i.e. with some suitable constant of proportionality A:

$$(33) \quad F(\varkappa) = A\,\varepsilon^{2/3}\,\varkappa^{-11/3}$$

A consequence of the turbulent motion of the air are fluctuations in temperature, humidity and other characteristic parameters of the air for which macroscopic gradients exist, since the turbulent flow mixes continuously air of different characteristics and creates in this way statistical deviations from the local mean values of these parameters. On the other hand, diffusive forces tend to re-establish the local equilibrium, so that in the steady state a balance will be established between the forces that create the local fluctuations and those that dissipate them.

We consider here in particular the fluctuations in temperature. At a point $\vec{r}$ the local temperature is given by $T(\vec{r}) = \overline{T}(\vec{r}) + T'(\vec{r}, t)$. The dependence of $\overline{T}(\vec{r})$ on $\vec{r}$ is of macroscopic scale while that of

$T'(\vec{r},t)$ is a microscopic one, i.e. grad $\bar{T} <<$ grad T'. Conduction of heat by molecular transport tends to establish the local equilibrium, i.e. tends to reduce T', giving rise to a molecular heat flow $\vec{q}_m = -D\,\mathrm{grad}\,T'$ (D = coefficient of molecular heat transport). Since the thermodynamic driving force of this process is given by grad T', the mean dissipation of the local temperature inhomogeneities per unit time is then proportional to the average of the product "current times driving force", i.e. to

$$(34)\quad \bar{N} = \overline{D(\mathrm{grad}\,T')^2}$$

Meanwhile, local fluctuations are continuously created by the turbulent flow that transports heat from the (macroscopically) warmer parts of the atmosphere to the colder ones, i.e. that tends to reduce the macroscopic heat gradient by a transport phenomenon which is similar to molecular heat transport in its mathematical aspects, but takes place on a macroscopic scale. With a coefficient K of turbulent heat transport which exceeds the molecular one by orders of magnitude (K>>D) one can write the turbulent heat current in the form

$$(35)\quad \vec{q}_t = -K\,\mathrm{grad}\,\bar{T}$$

Strictly speaking, the temperature $\bar{T}$ in eq. (35) should be replaced by the "potential temperature" $\bar{T} + \Gamma z$, where $\Gamma = 9.8 \cdot 10^{-3}$ degree/m is the adiabatic temperature gradient, since the moving air parcels adjust themselves adiabatically to the pressure of the surrounding air, and this effect alters the original temperature of the moving parcel, while its potential temperature remains constant. As a consequence, we see that convective flow in a neutral atmosphere - where $\bar{T} = T_o - \Gamma z$ - produces no turbulent heat current.

The rate of transfer of heat from the macroscopic temperature inhomogeneities to the microscopic ones - which is equal to the rate of creation of the microscopic inhomogeneities - is then proportional to the product of the macroscopic current times its driving force, i.e. to $K(\mathrm{grad}(\bar{T} + \Gamma z))^2$. For local stationarity, the rate of creation of microscopic temperature fluctuations must be equal to the rate of their distruction, so that

$$(36) \quad K(\mathrm{grad}(\bar{T} + \Gamma z))^2 = \overline{D(\mathrm{grad}\, T')^2} = \bar{N}$$

This "heat dissipivity" $\bar{N}$ is, like ε, a parameter characteristic for the turbulent state and the temperature profile of the atmosphere. Thus we expect that the temperature structure function $D_T(s) = \overline{(T'(\vec{r}_1) - T'(\vec{r}_2))^2}$ or the correlation function of temperature $B_T(s) = \overline{T(r_1)T(r_2)}$ will depend on s, $\bar{N}$ and ε. Since $\bar{N}$ has the dimensions (temperature)^2/time and D_T the dimensions of (temperature)^2, while $s/u_s \sim s/(\varepsilon s)^{1/3}$ has the dimensions of time, we can write

$$(37) \quad D_T = a^2 \bar{N} \varepsilon^{-1/3} s^{2/3} = C_T^2 s^{2/3}$$

with some constant a and C_T defined by

$$(38) \quad C_T = a\bar{N}^{1/2} \varepsilon^{-1/6}$$

C_T is called the "structure parameter" of temperature fluctuations; it is a measure of the intensity of the turbulent temperature fluctuations. The values of C_T usually lie between 0.01 degree/m$^{1/3}$ for weak turbulence and 0.5 degree/m$^{1/3}$ for strong turbulence.

By Fourier transformation we obtain from eq. (37) the spectrum of the temperature fluctuations

$$(39) \quad \Phi_T = 0.033\, C_T^2\, \varkappa^{-11/3} \, .$$

The interaction between atmospheric turbulence and electromagnetic radiation is due to turbulence induced variations of the index of refraction n. The index of refraction of the air is very close to unity; so one usually writes

(40) $n = 1 + N \cdot 10^{-6}$

The quantity N depends on temperature, pressure and humidity. For visible and infrared radiation the influence of humidity can be neglected and it is[2]

(41) $N = N_{dry} = 77.6\ P/T$ (optical frequencies)

where P is the atmospheric pressure in millibars and T the temperature in degree Kelvin.
For microwaves the strong dipole moment of the water molecules gives an appreciable dependence of N on the vapour pressure $e_s f$ (f = relative humidity, $e_s(T)$ = saturation pressure of water vapour); it is[20]

(42) $N = N_{dry} + N_{wet}$ (microwaves)

with N_{dry} given by the same expression (41) and

(43) $N_{wet} = 3.73 \cdot 10^5\ e_s f/T^2$

For simplicity we consider in the following only the dry term.

Since air turbulence involves only velocities which are small compared to the speed of sound, we can assume that in all convective processes the moving air parcels are always in pressure equilibrium, i.e. at the same pressure as the surrounding air. Therefore there are practically no pressure fluctuations due to turbulence at the observation point $\vec{r}$ (which is fixed in space), so that n' - the fluctuating part of n -

is determined by temperature fluctuations alone, and we find

$$(44) \quad n'(\vec{r},t) = 77.6 \cdot 10^{-6} \frac{P(\vec{r})}{\bar{T}^2(\vec{r})} T'(\vec{r},t)$$

Thus, the structure function of the refractive index fluctuations is given by

$$(45) \quad D_n(s) = \overline{(n'(\vec{r}_1) - n'(\vec{r}_2))^2} = C_n^2 \, s^{2/3}$$

with

$$(46) \quad C_n = (77.6 \cdot 10^{-6} \, P/T^2) C_T$$

For $T = 288.2^\circ K$ and $P = 1013$ mb this gives $C_n = 0.95 \cdot 10^{-6} \, C_T$. For the spectral function $\Phi_n(\varkappa)$, the Fourier transform of $B_n(s) = \overline{n'(\vec{r}_1) n'(\vec{r}_2)}$, we obtain from (45) by Fourier transformation the expression

$$(47) \quad \Phi_n(\varkappa) = 0.033 \, C_n^2 \varkappa^{-11/3}$$

These relations are valid within the inertial subrange, i.e. for $L > s > l_o$ or $2\pi/L < \varkappa < 2\pi/l_o$ respectively. Outside of this intervall scattering due to turbulence is relatively small; so it is sometimes convenient mathematically to provide suitable cutoff factors and to use the following expressions[19] for Φ_n:

$$(48) \quad \Phi_n(\varkappa) = 0.033 \, C_n^2 \varkappa^{-11/3} \exp(-\varkappa^2/\varkappa_m^2)$$

with $\varkappa_m = 5.92/l_o$ which is valid for all $\varkappa$ larger than $2\pi/L$, or

$$(49) \quad \Phi_n(\varkappa) = 0.063 \, \sigma_n^2 L_o^3 \exp(-\varkappa^2/\varkappa_m^2)/(1 + \varkappa^2 L_o^2)^{11/6}$$

which can be used for all values of $\varkappa$. σ_n^2 denotes the variance of the refractive index fluctuations.

Typical values for the structure parameter C_n of refractive index fluctuations are[21]:

weak turbulence:	$C_n = 8 \cdot 10^{-9}\ m^{-1/3}$
intermediate turbulence:	$C_n = 4 \cdot 10^{-8}\ m^{-1/3}$
strong turbulence:	$C_n = 5 \cdot 10^{-7}\ m^{-1/3}$

Chapter IV - PROPAGATION OF OPTICAL WAVES IN THE TURBULENT ATMOSPHERE

The turbulence-induced fluctuations of the index of refraction are inhomogeneities that scatter electromagnetic radiation. The physical aspects of this scattering depend essentially on the ratio l/λ where l is the dimension of the scattering inhomogeneity and λ the wavelength of the incident radiation. In fact, the scattering by an inhomogeneity of size l can be calculated from the general expressions of the Mie theory - which can be simplified considerably in this particular case where the index of refraction of the inhomogeneity differs only by a very small amount from the mean index of refraction (which we assume to be unity) -, and it is well known that the Mie theory predicts particularly strong scattering if l/λ is of order of unity. This case occurs in the scattering of microwaves, since the atmospheric turbulence contains eddies of all sizes from a few millimeters to some meters or even tens or hundreds of meters. The eddies of the size of the wavelength are distributed at random among eddies of other size in the general turbulence, so that the scattering of the microwave by the individual eddies will add up incoherently. Scattering is effective in all directions, even backward. This is of practical importance for the application of microwave scattering for the detection and study of clear air turbulence (CAT) by remote radar sensing[22][23][24] and for establishing microwave communication beyond the optical horizon by "scatter links"[20]. The physical aspects of this scattering will be considered in the next chapter.

Here, we will be concerned with the effect of turbulence on the optical frequencies, where the wavelengths are always small compared to even the smallest eddies, so that the scattering is predominantly forward, i.e. it is confined to a cone of angular aperture $\vartheta = \lambda/l$; it is coherent and can be thought of as a random refraction of the incident wave. Though it is mathematically straightforward to derive the formulae that apply to this case from the general scattering results by a limit operation (c.f. TATARSKI for plane and spherical waves(14) and ISHIMARU for beam waves(25)), it seems more intuitive to treat this case by a discussion of the refraction of waves by successive random phase screens as first proposed by LEE and HARP(26). For some situations geometrical optics (which are essentially the WKB-approximation to the wave equation) give already the correct answer(14)(27); this is plausible, since the dimensions of the inhomogeneities are so large compared to the wavelength, that in many instances diffraction effects can be neglected. However, it is well known that at long distances Z from the region where the interaction took place, diffraction effects can be observed. In fact, diffraction effects become important whenever the dimension of the eddies l are smaller than the diameter of the first Fresnel zone, i.e. whenever $l < \sqrt{\lambda Z}$. For visible light of 0.5µm and smallest eddies of the size of, say 5 mm, this leads to a distance of 50 m, which is not even very large for most practical applications.

Since the velocities involved in turbulent motion are extremely small compared to the speed of light, we can neglect the time dependence of $n'(\vec{r},t)$, during the transit of the electromagnetic wave. We consider a plane air slab of thickness Δz extending in the x- and y-directions, situated at $z = \zeta$, and a plane wave $E_o(z) = \exp(jkz)$ impinging normally on it. The distribution $n'(r)_{z=\zeta} = n'(x, y, \zeta)$ of the index of refraction fluctuation in the slab at the moment of incidence can be written in terms of the spatial Fourier components in the x- and y-directions:

(50) $n'(r)_{z=\zeta} = \iint du_1 du_2 N(\zeta, u_1, u_2) \exp(ju_1 x + ju_2 y)$

We make the assumption - discussed below - that terms of order n'^2 and higher can be neglected because of the smallness of n', so that all equations governing the wave propagation are linear in n'; thus we can consider the contribution of each Fourier component separately and add the results. A Fourier component $a.\exp(jux)$ gives rise to a periodic phase modulation

$$\Delta\Phi = a\,\Delta z k \exp(jux) \qquad \text{where } a\Delta z \ll 1$$

of the wave leaving the slab, which becomes

(51) $\exp[j(k\zeta + \Delta\Phi)] \approx \exp(jk\zeta)(1 + j\Delta\Phi) = E_o(\zeta) + E_s(\zeta, x)$

where

(52) $E_s(\zeta, x) = ja\Delta z k \exp[j(k\zeta + ux)]$

is a plane wave inclined with respect to the z-axis by an angle ϑ such that $\mathrm{tg}\,\vartheta = u/k$. Since the smallest eddies are of dimensions $l \gg \lambda$, the spatial frequencies u of n' must all be $\ll k$, so that the angle $\vartheta \approx u/k = \lambda/l$ is very small.

It is now assumed that no multiple scattering takes place, since this would involve terms of order $(n')^2$ and higher. This assumption - which is equivalent to the Born-approximation in the usual scattering theory - is really the most serious limitation of all existing theories of wave propagation in turbulent media. It leads to notable discrepancies with experimental results when the total scattering contribution becomes large. Attempts to use some well known procedures of theoretical physics for higher scattering approximations - such as diagram expansions and renormalization techniques(28), moment methods(29) or the integro-differential equations of radiation transport[30] - so far have not led to generally accepted results for the case that the Born-approximation breaks down. Fortunately, experiments and also some of the calculations indicate that at least the amplitude fluctuations tend to an asymptotic constant limit ("saturation") and that the Born-approximation holds as long as it predicts fluctuations well below that limit (c.f. Chapter VI).

Thus, here we assume that the two waves E_o and E_s propagate with the wave number k of the unperturbed medium in slightly different directions to the receiver plane z = Z where they superimpose to give the total field E(Z) which we write in the form

$$(53) \quad E(Z) = E_o(Z) + E_o(Z)\Delta\psi_1(x, Z)$$

i.e. we express the scattering by a term $\Delta\psi_1(x, Z) = E_s/E_o$ which gives the scattered field relative to the unperturbed field. We find, with $\eta = Z - \zeta$ and $k' = k\cos\vartheta = \sqrt{k^2 - u^2} \approx k - \frac{u^2}{2k}$

$$(54) \quad \Delta\psi_1(x, Z) = ja\,\Delta z k \exp(jux)\exp\left[j(k'- k)\right]$$

If we sum coherently the contributions of all Fourier components of $n'(r)$ according to eq. (50) and also sum over all planes Δz from 0 to Z (or integrate over $\eta = Z - z$ from 0 to Z) we can find the complete scattered wave $\psi_1 = \Sigma\Delta\psi_1$, such that the total field at (x, y, Z) is given by

$$(55) \quad E(x, y, Z) = E_o(Z) + E_o(Z)\psi_1(x, y, Z)$$

Since the total scattered contribution ψ_1 has still to be small in order that the Born-approximation be valid, we can write eq. (55) also in the form

$$(56) \quad E(x, y, Z) = E_o(Z) \exp\left[\psi_1(x, y, Z)\right]$$

Thus $\chi = \mathrm{Re}\{\psi_1\}$ is the fluctuation of "log-amplitude" with respect to the unperturbed solution, and $S = \mathrm{Im}\{\psi_1\}$ that of the phase.

We are not so much interested in the fluctuating field ψ_1 itself as in its correlation or structure functions which relate the statistics of signals received simultaneously in two detectors in the plane $z = Z$, separated by a distance $\vec{a} = (x, y)$. We first calculate the amplitude correlation $B_\chi(\vec{a})$. Since different spatial frequencies $\vec{u} = (u_1, u_2)$ can be considered as uncorrelated, we can sum the contributions $dB_\chi(\vec{a}, \vec{u})$ to obtain $B_\chi(\vec{a})$, where the $dB_\chi(\vec{a}, \vec{u})$ are the amplitude correlation functions for the spatial frequency $\vec{u}$. Noting that the reality of $n'(\vec{r})$ implies $N(Z-\eta, -u) = N^*(Z-\eta, u)$ we find

$$(57) \quad dB_\chi(\vec{a}, \vec{u}) = du_1 du_2 2k^2 \int_0^Z d\eta_1 \sin(k'-k)\eta_1 \int_0^Z d\eta_2 \sin(k'-k)\eta_2 \cos(\vec{u}.\vec{a}) \cdot$$

$$\cdot \; F(u, \eta_1 - \eta_2)$$

with

$$(58) \quad F(u, \eta_1 - \eta_2) = \overline{N(\eta_1, u) N(\eta_2, u) du_1 du_2} = \int_{-\infty}^{\infty} dK \cos(K(\eta_1 - \eta_2)) \Phi_n(\varkappa)$$

where
$\varkappa = \sqrt{u^2 + K^2}$ and $\Phi_n(\varkappa)$ is given by eqs. (47) to (49).
Introduction of the new variables of integration

$$(59) \quad s = (\eta_1 + \eta_2)/2 \qquad\qquad w = \eta_1 - \eta_2$$

one integration in equation (57) can be carried out so that[14]:

$$(60) \quad dB_\chi(\vec{a}, \vec{u}) = du_1 du_2 2\pi k^2 \int_0^Z \Phi_n(u) \sin^2(u^2 s/2k) \cos(\vec{u}.\vec{a}) ds$$

In this form it is still possible to include a smooth variation of the statistical properties of the refractive index fluctuations along the path by assuming a weak dependence on s of the structure parameter C_n so that

(61) $\Phi_n(u) = C_n^2(s)\,\Phi_o(u)$

with

(62) $\Phi_o(u) = 0.033\, u^{-11/3} \qquad (u_{min} < u < u_m)$

so that finally

(63) $$dB_\chi(\vec{a},\vec{u}) = du_1 du_2 2\pi k^2 \Phi_o(u) \int_o^L C_n^2(s) \sin^2(u^2 s/2k)\, ds \cos(\vec{u}.\vec{a})$$

The contribution of the spatial frequency $\vec{u}$ to the phase perturbation correlation function is obtained from the imaginary part of ψ_1 which involves the cosine instead of the sine, so that

(64) $$dB_S(\vec{a},\vec{u}) = du_1 du_2 2\pi k^2 \Phi_o(u) \int_o^z C_n^2(s) \cos^2(u^2 s/2k) ds \cos(\vec{u}.\vec{a})$$

In particular, if C_n^2 is independent of s (homogeneous turbulence), these expressions can be integrated so that we obtain

(65) $$\left.\begin{array}{l} dB_\chi(a,u) \\ dB_S(a,u) \end{array}\right\} = du_1 du_2 k^2 Z(1 \pm (k/Zu^2)\sin(u^2 Z/k))\, \Phi_n(u)\cos(\vec{u}.\vec{a})$$

From eqs. (63) and (64) the correlation functions B_χ and B_S are obtained by integrating over $du_1 du_2 = u du d\varphi \quad (\varphi = \sphericalangle(\vec{u},\vec{a}))$

(66) $$\left.\begin{array}{l} B_\chi(a) \\ B_S(a) \end{array}\right\} = 4\pi^2 k^2 \int_{u_{min}}^{u_m} du \int_0^Z ds\, u\, \Phi_n(u) J_o(u.a) \left\{\begin{array}{l} \sin^2(u^2 s/2k) \\ \cos^2(u^2 s/2k) \end{array}\right.$$

or in the case that C_n^2 is independent of s,

(67) $$\left.\begin{array}{l} B(a) \\ B_S(a) \end{array}\right\} = 2\pi^2 k^2 Z \int_{u_{min}}^{u_m} J_o(u.a) \left\{1 \pm (k/Zu^2)\right\} \sin(u^2 Z/k) u \Phi_n(u) du$$

The integration variables u_{min} and u_m can be replaced by 0 and ∞

if expressions of $\Phi_n(u)$ with suitable cutoff-factors are used. Because of the vanishing of the integrand for $u \to 0$ the quantity $B_\chi(a)$ is insensitive to the exact form of $\Phi_n(u)$ for small u. Unfortunately, this is not the case with B_S. It turns out that the structure function of the phase fluctuations $D_S(a) = (S(0)-S(a))^2 = 2(B_S(0)-B_S(a))$ does not present this problem, since it involves the factor $1-J_o(ua)$ instead of J_o which vanishes as u^2 and suppresses the dependence of the integral on its lower limit.

The case of a spherical wave, originating at the origin of the coordinate system and normalized to an amplitude 1 at the receiver (z = Z), i.e.

$$(68) \quad E_o(x, y, z) = (Z/z)\exp(jk\sqrt{x^2 + y^2 + z^2})$$

can be treated by the same method. The slab of turbulent air of thickness z at $z = \zeta$ leads to a modification of the transmitted wave, which can be written for each spatial frequency $\pm\vec{u}$ contained in $n'(x, y, \zeta)$ as the original spherical wave plus two additional spherical waves differing in amplitude from the original wave by the factors $k\Delta zN(\pm u, \zeta)$ and in phase by $j\exp(ju^2\zeta/2k)$ and originating from the points $(u\zeta/k, 0, 0)$ and $(-u\zeta/k, 0, 0)$[(26)]. Here the (x, y)-coordinates have been chosen in such a way that the x-direction coincides with the direction of $\vec{u}$; also it is assumed that the field will be observed near the z-axis, so that $x, y \ll \sqrt{2\zeta\sqrt{\lambda\zeta}}$. These waves combine at the receiver plane to give

$$(69) \quad \Delta\psi_1 = k\Delta z\exp\left[j\frac{u^2\zeta(Z-\zeta)}{2kZ}\right]\left\{N(\vec{u}, \zeta)\exp(j\frac{ux\zeta}{Z}) + N^*(\vec{u}, \zeta)\exp(-j\frac{ux\zeta}{Z})\right\}$$

The further mathematical development then follows the same lines as in the plane-wave case and leads to

$$(70)\quad \left.\begin{matrix} dB_\chi(\vec{a}) \\ \\ dB_S(\vec{a}) \end{matrix}\right\} = 2\pi k^2 \int_o^z \Phi_n(u) \left\{ \begin{matrix} \sin^2\left(\frac{u^2\zeta(Z-\zeta)}{2kZ}\right) \\ \\ \cos^2\left(\frac{u^2\zeta(Z-\zeta)}{2kZ}\right) \end{matrix} \right. \cos(\vec{u}.\vec{a}\frac{\zeta}{Z})d\zeta du_1 du_2$$

and

$$(71)\quad \left.\begin{matrix} B_\chi(\vec{a}) \\ \\ B_S(\vec{a}) \end{matrix}\right\} = 4\pi^2k^2 \int_{u_{min}}^{u_m} du \int_o^Z d\zeta\ u\Phi_n(u)J_o\left(\frac{u\zeta a}{Z}\right) \left\{ \begin{matrix} \sin^2\left(\frac{u^2\zeta(Z-\zeta)}{2kZ}\right) \\ \\ \cos^2\left(\frac{u^2\zeta(Z-\zeta)}{2kZ}\right) \end{matrix} \right.$$

These results agree with those found by SCHMELTZER(31) and FRIED(32). If the refractive index fluctuations are finite only over the range $Z - H < \zeta < Z$ (zero elsewhere) and if Z is made arbitrarily large, so that $\zeta/Z \rightarrow 1$ over the range of integration, eq. (71) goes over to

$$\left.\begin{matrix} B_\chi(\vec{a}) \\ \\ B_S(\vec{a}) \end{matrix}\right\} = 4\pi^2k^2 \int_{u_{min}}^{u_m} du \int_{Z-H}^{Z} d\zeta\ u\,\Phi_n(u)J_o(ua) \left\{ \begin{matrix} \sin^2\left(\frac{u^2(Z-\zeta)}{2k}\right) \\ \\ \cos^2\left(\frac{u^2(Z-\zeta)}{2k}\right) \end{matrix} \right.$$

which is the plane wave result, eq. (66), as it should be.

LEE and HARP(26) derived by the same phase screen method also the results for a Gaussian beam, which agree with the expressions found by SCHMELTZER(31) and ISHIMARU(25) by a direct solution of the wave equation. We do not quote these results here as no further physical insight can be gained from the resulting rather awkward formulae, which, however, are a valuable starting point for a numerical analysis of the beam wave case which is discussed by ISHIMARU (loc. cit.).

We have seen that the final expressions for the correlation functions of the quantity i (where the index i stands for χ or S) have the form:

$$(72)\quad B_i(a) = 2\,\pi^2 k^2 Z \int_o^\infty u\,du\; F_i(u)\; \Phi_n(u)$$

One factor of the integrand is simply the power spectrum $\Phi_n(u)$ of the refractive index fluctuations, while the other, $F_i(u)$, is called the "filter function" for the quantity i, since it weights selectively the spectral term. It is a measure of the scattering efficiency of the perturbations of different sizes $l = 2\,\pi/u$, and depends also on the geometrical variables Z (distance receiver-source), a (receiver separation in the receiver plane), k (wavenumber of the incident radiation) and in the case of Gaussian beams on the beam width w. "The importance of filter functions becomes clear when attempts are made to interpret experimental measurements in terms of atmospheric parameters. A single measurement obviously cannot uniquely determine a number of parameters, and it is necessary to determine those parameters to which the measurement is most sensitive."(26)

As an example, we note that the filter functions for amplitude and phase in the plane-wave homogeneous-turbulence case are given by (c.f. eq. (67))

$$(73)\quad \left.\begin{array}{l} F_\chi(u) \\ F_S(u) \end{array}\right\} = J_o(ua)\left(1 \pm \frac{k}{u^2 Z}\sin\frac{u^2 Z}{k}\right) = J_o(ua)\left(1 \pm \frac{N_F}{2\pi}\sin\frac{2\pi}{N_F}\right)$$

Apart from the factor $J_o(ua)$ which describes the effect of the separation of the two receivers, the spatial frequency response is determined by a filter function which depends only on the Fresnel number

$$(74)\quad N_F = (l/W_F)^2 = \frac{4\pi^2}{u^2 W_F^2}$$

which is the ratio of the geometrical cross section of the obstacle to the area of the first Fresnel zone; W_F is the width of the first Fresnel zone: $W_F = \sqrt{\lambda Z} = \sqrt{2\pi Z/k}$. From the form of the filter functions we conclude that small wave numbers (large inhomogeneities) are weighted strongly for phase fluctuations, and large wavenumbers (small inhomogeneities) contribute strongly to fluctuations in log-amplitude.

While the (spectral) filter functions F_i just introduced, give the weighting of the different spatial frequencies in the case of a known spatial distribution of the intensity of turbulence along the path of observation s - usually under the hypothesis, as in eq. (73), of an independence of the turbulence characteristics from the spatial positions - we can also define "spatial filter functions" G_i under the assumption, that the turbulence spectrum can be written in the form of eq. (61) with a known analytical form of $\Phi_o(u)$. We then write the correlation function for the observable i in the form

$$(75) \quad B_i(a) = B_i^o(a) \int_o^Z ds\, C_n^2(s) G_i(s)$$

These spatial filter functions are especially interesting for applications in "atmospheric probing", since in such problems one usually assumes a spectrum of the form (61) with $\Phi_o(u) \sim u^{-11/3}$ as given by the Kolmogorov theory (c.f. eq. (62)), and one tries to determine $C_n^2(s)$ by a best fit to the measurements. The form of the spatial filter functions $G_i(s)$ gives an indication on the parts of the path which are most sensitive to the values of $C_n^2(s)$. For example, it can be seen from eqs. (63) and (64) that amplitude fluctuations are more sensitive to the turbulence around the emitter, while phase fluctuations depend evenly on the conditions along the whole path.

Detailed discussions and plots of spectral and spatial filter functions can be found in the paper of LEE and HARP[26] for plane and spherical waves and in that of ISHIMARU[25] for some types of Gaussian beams.

We will turn now to the discussion of some practical consequences of turbulence effects on optical communication systems. We have seen that atmospheric turbulence leads to fluctuations in amplitude and phase of the signal arriving at the receiver. The amplitude fluctuations are analogous to "fading" in radio transmission; they are called "beam scintillation" in our case of optical communication links, and are variations in the spatial power density at the receiver caused by interference within the beam cross section. The fluctuations in log-amplitude have probably a Gaussian probability distribution[33][34] with a variance given by $\sigma_\chi^2 = B_\chi(0)$.

The phase fluctuations lead to spatial coherence degradation, image dancing and beam spreading. It is convenient to define a "lateral phase coherence length" ρ_o by the condition[21] that the root mean square phase difference between the signals at two points in the receiver plane which are the distance ρ_o apart will be equal to π:

$$(76) \quad \Delta\varphi(\rho_o) = \sqrt{D_S(\rho_o)} = \sqrt{\overline{[S(\rho_o) - S(0)]^2}} = \pi$$

In other words, an interfereometer with the two slits apart a distance larger than ρ_o will detect no phase correlation. This spatial coherence loss is especially important for heterodyne receiving systems[35][36] since it limits the maximum receiving antenna aperture to dimensions of the order ρ_o. On the other hand, a phase insensitive (intensity) detector of linear dimensions $>>\rho_o$ (i.e. a direct detection receiver with an entrance pupil of that size) measures an intensity equal to the spatial average over its aperture, and since the fluctuations are uncorrelated over most of this region, their effects tend to be smoothed out(37); this is called the "aperture averaging effect".

Within each coherence region the phase fluctuations cause "image dancing"(38)(39). If the phase, as a result of the fluctuations, varies linearly by an amount $\Delta\varphi$ over a distance b in the plane of the wave, the wave front (plane of equal phase) is effectively tilted by an angle $\alpha \approx \Delta\varphi/kb$. The variance of these "angle-of-arrival fluctuations" is therefore given by

$$(77) \quad \overline{\alpha^2} = \sigma_\alpha^2 = D_S(b)/k^2b^2 \qquad\qquad (b << \rho_o)$$

These variations in the arrival angle α at the entrance of the receiver cause the image point to wander in the focal plane by the distance $f\alpha$, where f is the focal length of the receiver optics.

As the different coherent subregions of the beam undergo independent angle-of-arrival fluctuations, the beam as a total is spread out, i.e. the beam divergence increases and the power density on its axis decreases. Beam spreading can be considered as a typical small angle scattering effect; its magnitude is automatically taken into account by the correct beam wave solution, but since the resulting mathematical expressions are of a rather unconvenient form, some authors(21) prefer to use the simple plane wave solution and to correct for beam spreading afterwards by assuming that the width of the spread beam is given approximately by $w_{spr} = 2Z\sigma_\alpha$ with σ_α as in eq. (77). Beam spreading has to be taken into account when w_{spr} becomes large compared to the width w of the unperturbed Gaussian beam at the receiver.

In the case of very strong coherent phase fluctuations across the beam, the whole beam can be deviated from its path ("beam steering" or "beam wander") and eventually miss the receiver completely. It is clear that such an effect cannot be described correctly even by beam wave theory since the perturbation approximation fails for such a strong deviation from the unperturbed solution. It can be treated, however, within the limits of geometrical optics[40]; this leads essentially to the same formula for the mean square deviation σ_ρ^2 of the arrival point $\vec{\rho}$ from its mean value $\vec{\rho} = 0$, as the one given in the case of beam spreading for w_{spr}^2, i.e. $\sigma_\rho^2 \sim Z^2 \sigma_\alpha^2$, so that beam steering and beam spreading can be considered as almost the same phenomenon; indeed, if the intensity in the receiver plane is averaged over longer observation times, beam wander gives just an additional contribution to beam spreading.

Beam steering has to be distinguished from "beam bending" which is not really a turbulent effect, but is due to a gradual systematic variation of the index of refraction along the propagation path. Beam bending can have dramatic effects such as the formation of mirages in some rather rare occasions, but it can be distinguished from turbulence effects by its slow variation with time (timescales of minutes or even hours, instead of seconds or less). Since in the case of beam bending, the variation of the index of refraction is very gradual compared with the wavelength of the radiation, "ray tracing" by the methods of geometrical optics can be performed to compute the beam trajectories provided that the index of refraction or - equivalently - the temperature distribution in the neighbourhood of the path is known; unfortunately, this is rarely the case.

Chapter V - SCATTERING OF MICROWAVES BY ATMOSPHERIC TURBULENCE

If the index of refraction $n = \sqrt{\varepsilon}$ becomes - due to the turbulent fluctuations $n'(r)$ - a function of position, Maxwell's equations lead to a wave equation for the electric field $\vec{E}$ of the following form:

$$(78) \quad \Delta\vec{E} + k^2 n^2(\vec{r})\vec{E} + 2\,\mathrm{grad}\{\vec{E}.\,\mathrm{grad}\,[\ln n(r)]\} = 0$$

The last term - which gives rise to a turbulent depolarization - is usually small and can be neglected, so that

$$(79) \quad \Delta\vec{E} + k^2 n^2(\vec{r})\vec{E} = 0$$

Assuming $\bar{n} = 1$ and noting that for small n' we have $(1 + n')^2 \approx 1 + 2n'$, we can write eq. (79) in the form

$$(80) \quad \Delta\vec{E} + k^2\vec{E} = -2k^2 n'\vec{E}$$

We try again the solution in the form of the first Born-approximation (c.f. the discussion in the preceding chapter) and write the field $\vec{E}$ in the form

$$(81) \quad \vec{E} = \vec{E}_o + \vec{E}_s$$

where the scattered wave $\vec{E}_s$ is assumed to be - relative to the unperturbed incident wave $\vec{E}_o$ - of the order of smallness n', so that second order products such as $n'\vec{E}_s$ can be neglected.
Since $\vec{E}_o$ is a solution of the homogeneous wave equation, $\Delta\vec{E}_o + k^2\vec{E}_o = 0$, we obtain for $\vec{E}_s$ the equation

$$(82) \quad \Delta\vec{E}_s + k^2\vec{E}_s = -2k^2 n'\vec{E}_o$$

which indicates that the waves $\vec{E}_s$ originate from sources of strength $-2k^2 n'(\vec{r})\vec{E}_o$. Eq. (82) has the well known solution

$$(83)\quad \vec{E}_s(\vec{r}_p) = -\frac{k^2}{2\pi}\int_V d^3r \frac{n'(\vec{r})\vec{E}_o(\vec{r})}{|\vec{r}_p - \vec{r}|} \exp(jk|\vec{r}_p - \vec{r}|)$$

where the integration extends over the scattering volume V and $\vec{r}_p$ denotes the location of the observation point.

If the observation point is far remote from the volume V where the scattering takes place - such that $\lambda r_p >> V^{1/3}$ - we can make the approximation $|\vec{r}_p - \vec{r}| \approx r_p - (\vec{r}.\vec{r}_p)/r_p$ in the exponential and replace the denominator by r_p. This gives a spherical wave in the scattering direction $\vec{m} = \vec{r}_p/r_p$.

$$(84)\quad \vec{E}_s(\vec{r}_p) = \vec{\tau}_{\vec{m}} \frac{e^{jkr_p}}{r_p}$$

with the "scattering amplitude" $\vec{\tau}_{\vec{m}}$

$$(85)\quad \vec{\tau}_{\vec{m}} = -\frac{k^2}{2\pi}\int_V d^3r\, n'(\vec{r})\vec{E}_o(\vec{r})\exp(-jk(\vec{r}.\vec{m}))$$

As incident wave we can assume a linearly polarized plane wave propagating in a direction given by the unit vector $\vec{u}$,

$$\vec{E}_o = \vec{p}\exp(j\vec{k}.\vec{r}) = \vec{p}\exp(jk\vec{u}.\vec{r})$$

where $\vec{p}$ is the unit vector in the direction of polarization. Actually, it is now unimportant whether we consider beam waves or plane waves, since the most effective scattering is caused now by eddies of the order of a wavelength which scatter incoherently and are insensitive to the macroscopic distribution of the incoming wave field. Thus we obtain

$$(86)\quad \vec{\tau}_{\vec{m}} = \vec{p}\tau(\vec{\varkappa}) = -\frac{k^2}{2\pi}\int_V d^3r\, n'(\vec{r})\exp(j\vec{\varkappa}.\vec{r})$$

where the "scattering vector" is given by

(87) $\vec{\varkappa} = k(\vec{u} - \vec{m})$

Its magnitude is given by $\varkappa = 2k \sin(\vartheta/2)$, where the "scattering angle" ϑ is the angle between the directions $\vec{u}$ and $\vec{m}$ of the incident and scattered waves, respectively, such that $\vec{u}.\vec{m} = \cos\vartheta$. The mean intensity at the observation point $\vec{r}_p$ is given by the mean value of the component of energy flow in the outward direction $\vec{m}$ at that point, i.e. by definition of the corresponding component $\vec{m}.\vec{S}_s$ of the Pointing vector

(88) $\vec{S}_s = (c/8\pi)\mathrm{Re}\left\{\vec{E}_s \times \vec{H}_s\right\} = (c/8\pi)\vec{E}_s \times (\vec{m} \times \vec{E}_s)$

so that

(89) $\overline{\vec{m}.\vec{S}_s} = (c/8\pi)\,\overline{|\vec{E}_s|^2}\sin^2\chi \sim \overline{|\tau(\vec{\varkappa})|^2}\sin^2\chi$

Here we have denoted the angle between the direction of polarization $\vec{p}$ and the direction of observation $\vec{m}$ by χ, so that $\vec{m}.\vec{p} = \cos\chi$. According to eq. (89) the intensity is determined essentially by the "differential scattering cross section"

(90) $Q = \overline{|\tau(\vec{\varkappa})|^2}\sin^2\chi = \dfrac{k^4 V}{4\pi^2}\sin^2\chi \displaystyle\int_V d^3r\, B_n(\vec{r}) \exp(j\vec{\varkappa}.\vec{r})$

where

$B_n(\vec{r}) = \overline{n'(\vec{r}_o + \vec{r})n'(\vec{r}_o)}$ is the correlation function of the refractive index fluctuations. If V is larger than the correlation volume (which is of the order of l^3 where l is the size of the scattering eddies) the integrand vanishes outside V and we can replace the integral over V by an integral over the whole space; the integral is then equal to $(2\pi)^3$ times the Fourier transform $\Phi_n(\vec{\varkappa})$ of $B_n(\vec{r})$, i.e.

(91) $Q = 2\pi k^4 V \sin^2\chi\, \Phi_n(\vec{\varkappa})$

In particular, with the Kolmogorov-expression (47) for $\Phi_n(\vec{\varkappa})$ we obtain

$$(92) \quad Q(\varkappa) = Q(2k\sin\vartheta/2) = 0.016\, k^{1/3} V\, C_n^2 (\sin\vartheta/2)^{-11/3} \sin^2\chi$$

This expression is valid within the limits $(2\pi/L) < 2k\sin\vartheta/2 < (2\pi/l_o)$. For backward scattering ϑ is equal to 180^o, so that $\sin(\vartheta/2) = 1$ and $2k = 4\pi/\lambda < 2\pi/l_o$. Thus backscattering is possible only if $\lambda > 2\, l_o$. This condition shows that only radio waves (and no optical radiation) can be used for monostatic remote probing of atmospheric turbulence. Since Q depends only on the cube root of k, it appears that microwaves with wavelength of the order of one centimeter might have an advantage in the relative scattering intensity, but that the k-dependence of the cross section is weak enough to permit the use even of decimeter waves for such purposes[24].

We conclude this chapter by the remark that for optical frequencies the expressions derived here remain applicable as long as the scattering is confined to a cone in the forward direction, subtended by the maximum scattering angle ϑ_m; this angle is defined by $2k\sin(\vartheta_m/2) \approx k\,\vartheta_m = 2\pi/l_o$ or $\vartheta_m = \lambda/l_o$. This condition was already mentioned at the beginning of the preceding chapter. Indeed, we can obtain the results of that chapter, writing $\vec{E}_s = \vec{E}_o\psi_1$ and using - for the plane wave case - equation (83) with the approximations $k|\vec{r}_p - \vec{r}| \approx k\eta + \frac{k(x^2+y^2)}{2\eta}$ in the exponential and $|\vec{r}_p - \vec{r}| \approx \eta$ in the denominator.

For $l_o = 5$ mm and $\lambda = 0.5$ μm we find an angular aperture of the forward scattering cone of 0.1 milliradians.

Chapter VI - SOME EXPERIMENTAL RESULTS ON THE PROPAGATION OF ELECTROMAGNETIC WAVES IN THE ATMOSPHERE

Experiments done under conditions that allow a direct check of the theoretical predictions require simultaneous measurements of the pertinent meteorological parameters such as C_T. Since such measurements are not easy to perform, the first real test of the theory was made only in 1965 by GRACHEVA and GURVICH[41], who measured the variance of the log-intensity fluctuations over a flat horizontal path up to 2 000 m. They encountered the phenomenon of saturation: while for shorter distances L the variance of log-intensity $\sigma_{\ln I}^2 = 4\overline{\chi^2}$ was proportional to $L^{11/3}$ as predicted by the theory, the $\sigma_{\ln I}$ -vs-L-curve "saturated" (i.e. became horizontal) at

$\sigma_{\ln I} \approx 1.6 (\overline{\chi^2} \approx 0.64)$.

The same phenomenon was later verified by others[42][43][44], though with a certain spread of the reported saturation values. DABBERT[45][46] investigated the scintillation of log-amplitude on ground paths up to 7.5 km and slanted paths (from or to a 465 m tower) from 0.6 km to 10 km and fitted the saturating results to expressions of the form $\sigma_{meas.} = \sigma_{theor.}/(1 + \alpha\sigma^{\beta}_{theor.})$ where the fitting parameters α, β showed a slight frequency dependence and also a dependence on the class of atmospheric stability. According to his results the measured scintillation magnitude $\sigma_{meas.}$ saturates as range and turbulence (i.e. C_n) increase, but finally even decreases with further increase in range and/or turbulence (supersaturation region). The maximum of $\sigma_{meas.}$ (at the saturation point) is proportional to $\lambda^{-7/12}$ (as it is in the unsaturated region) while in the supersaturation region there is little dependence on frequency.

Saturation indicates the point where the perturbation theory ceases to be applicable. Over short distances (10 km) the validity of the perturbation theory was also confirmed by an excellent agreement with the measurements by GRAY and WATERMAN[47] of the log-amplitude covariance function. OCHS et al.[48] have measured the normalized covariance function over longer distances and got a good agreement with the theoretical predictions up to 15 km. However, KERR[49] reported cases where even at moderate turbulence and short path lengths substantial disagreement with the theory was observed. The reason for these discrepancies is not known, but it is possible that in some meteorological situations the Kolmogorov model does not correspond to the main mechanism of turbulent energy dissipation.

Measurements of the phase covariance function by BOURICIUS et al.[50][51] showed good agreement with the theoretical predictions. The first group of experiments[50] used a 25 m return path, i.e. a total distance transmitter-receiver of 50 m, and the spatial correlation measurements were replaced by temporal autocorrelation measurements, assuming the validity of Taylor's hypothesis that the turbulence field of temperature fluctuations does not change during a short time interval τ, so that it is merely transported across the light path by the mean transverse wind $\vec{v}_n$. Under this assumption the random field at $\vec{d}$ and time $t+\tau$ coincides with the field at $\vec{d} - \vec{v}_n\tau$ at time t, so that $D_S(\vec{d}) = D_S(\vec{v}_n\tau)$. The second group of experiments[51] used a 70 m direct path and an interfereometric array[52] of two receiving slits at 4 different spacings $d \leq 30$ cm for a direct verification of Taylor's hypothesis. In both experiments simultaneous C_T-measurements were done with two 2 μm Pt-wire resistance thermometers (10 cm distant from each other) located near the center of the light path, and the mean wind was recorded with a conventional anemometer.

Taylor's hypothesis gave an excellent agreement between the corresponding temporal and spatial spectra, and the results for $D_S(d)$

agreed with those of the TATARSKI theory for all values of d (≤ 30 cm) in the second case, while in the first case this agreement could be verified even for values of $d = v_n \tau$ of the order of 1 m. Since the path was at a height of only 1.6 m, the upper scale of turbulence, L_o, should have been also of the order of 1 m, so that the TATARSKI theory turned out to be correct within the whole inertial subrange.

While the knowledge of the mean wind along the transmission path permits - with Taylor's hypothesis - the use of only one receiver and the substitution of temporal autocorrelation measurements for spatial correlation measurements (which necessitate at least two receiving antennas or slits etc. separated by a distance d), the determination of an unknown mean wind velocity by time cross correlation measurements of the signals at two receivers separated by a distance d has been shown to be also feasible[53] (remote sensing of the mean wind).

A comparison by BUFTON of the theoretical value of $\overline{\chi^2}$ computed with the help of C_T-profiles, obtained by radiosonde balloons for heights up to 25 km with the observed scintillations of stellar objects agreed within a factor of 2[54]. For these vertical paths saturation does not occur since turbulence is usually restricted to several rather shallow layers. These results show the applicability of expressions of the form given in eq. (64) with a variation of C_n along the light path. Comparing the scintillation characteristics of 0.488 μm radiation reflected from the GEOS-II satellite at a height of 1250 km with stellar data MINOTT[55] found values of log-amplitude variance and normalized power spectral density within the limits measured for stellar scintillation.
Assuming the validity of the theory and a smooth variation of the structure constant C_n as a function of altitude h, expressible in parabolic form $C_n(h) = a_o + a_1 h + a_2 h^2$, SUBRAMANIAN[56] determined the coef-

ficients a_i from a best fit to the observed data for the scintillations of a 0.6328 μm-laser beam reflected from a retroreflector mounted on a thethered balloon which was successively flown at several altitudes. The consistency of the results indicated a certain potential of this method for remote determinations of the structure constant, though its assumptions would not apply to the case of a layered atmosphere where the turbulence structure exhibits pronounced jumps in magnitude as function of altitude.

All these experiments were done at optical frequencies (mostly with the 0.6328 μm HeNe-laser). The observations of LEE and WATERMAN[57] showed a good agreement with the TATARSKI theory also for mm-waves (35 GHz) for a path of 28 km.

Concerning other propagation effects such as beam steering, a direct comparison between the predictions of geometrical optics with the help of measured C_T-values and observations along a 480 m path (240 m return path) and a 1380 m path, both at 20 m height, is discussed by CHIBA[40]. The agreement between his theory and the experiments was satisfactory. The strongest observed fluctuations correspond to angular fluctuations $\sigma_\alpha = \sigma_\rho / z$ of the order of about 40 μrad. Similar results are given by KURIGER[58] for a distance of 7226 m (3613 m return path).

LAWRENCE and STROHBEHN[59] report on experiments by OCHS and LAWRENCE on beam bending over paths from 5 to 45 km: A typical 24-hour measurement showed a diurnal variation of about 30 μ rad/km on a 15 km path. These variations were usually slow enough that a servotracking system with a maximum angular rate of 7.5 μrad/sec could compensate for them.

GRUSS[60] reports that on a typical communications relay link, with distances up to 20 km, no beam loss due to atmospheric beam bending was observed (during an observation period of about two years), and that beam deviations due to mechanical fluctuations of the supporting structures in response to wind pressure had much more serious effects.

MASON and LINDBERG(61) studied the behaviour of an HeNe-laser beam over an 80 km long, 1.3 km high path between two mountains, thus free from surface effects, and found - as should be expected - that in this case beam bending (which is due to the vertical refractive index gradient) is primarily a function of lapse rate (vertical gradient of mean temperature) as measured by radiosonde soundings.

In a study of the spreading of a 0.9050 μm Ga-As-laser beam of an initial angular aperture of 66 x 5 μrad at distances from 130 m to 4.5 km RAIDT and HÖHN(62) observed that it is mainly due to aerosol scattering and not to atmospheric turbulence, even for medium visibilities. With increasing distance and decreasing meteorological range (visibility) beam spreading increases.

This point is also investigated in the work of DOWLING and LIVINGSTON(63) who measured the spread of a laser beam along path lengths up to 1.75 km (height 1.8 m or 2.4 m above ground) at 0.63 μm and at 10.6 μm. The observed beam spread can be separated into short-term (Θ) and long-term (Θ') rms averages that differ by beam wander Φ (beam steering), so that $\Theta' = (\Theta^2 + \Phi^2)^{1/2}$. Φ has been found to be essentially independent of the wavelength and is adequately described by geometrical optics. The short-term average beam spread is strongly wavelength dependent. The measurements at 10.6 μm are nearly diffraction limited, whereas the corresponding data for 0.63 μm are strongly influenced by variations of C_n. The experimental data are presented by empirical fits of the type $\Theta = a + bC_n^{\beta} Z^{\gamma} k^{\delta}$, with experimentally determined constants a, b, β, γ, δ which do not agree too well with the predictions of the TATARSKI theory.

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