EUR 5049 e

COMMISSION OF THE EUROPEAN COMMUNITIES

CODE TAFEST
NUMERICAL SOLUTION TO TRANSIENT HEAT-CONDUCTION PROBLEMS USING FINITE ELEMENTS IN SPACE AND TIME

by

J. DONEA and S. GIULIANI

1974

Joint Nuclear Research Centre
Ispra Establishment - Italy

Materials Division

LEGAL NOTICE

This report is on sale at the addresses listed on cover page 4

at the price of B.Fr. 60.—

Commission of the
European Communities
D.G. XIII - C.I.D.
29, rue Aldringen
Luxembourg

February 1974

EUR 5049 e

CODE TAFEST - NUMERICAL SOLUTION TO TRANSIENT HEAT-CONDUCTION PROBLEMS USING FINITE ELEMENTS IN SPACE AND TIME by J. DONEA and S. GIULIANI

Commission of the European Communities
Joint Nuclear Research Centre - Ispra Establishment (Italy)
Materials Division
Luxembourg, February 1974 - 42 Pages - 9 Figures - B.Fr. 60.—

The present report describes the computer code TAFEST that has been developed for the purpose of solving two-dimensional transient heat-conduction problems. The concept of finite elements in space and time is used as a means of obtaining numerical responses.

EUR 5049 e

CODE TAFEST - NUMERICAL SOLUTION TO TRANSIENT HEAT-CONDUCTION PROBLEMS USING FINITE ELEMENTS IN SPACE AND TIME by J. DONEA and S. GIULIANI

Commission of the European Communities
Joint Nuclear Research Centre - Ispra Establishment (Italy)
Materials Division
Luxembourg, February 1974 - 42 Pages - 9 Figures - B.Fr. 60.—

The present report describes the computer code TAFEST that has been developed for the purpose of solving two-dimensional transient heat-conduction problems. The concept of finite elements in space and time is used as a means of obtaining numerical responses.

EUR 5049 e

CODE TAFEST - NUMERICAL SOLUTION TO TRANSIENT HEAT-CONDUCTION PROBLEMS USING FINITE ELEMENTS IN SPACE AND TIME by J. DONEA and S. GIULIANI

Commission of the European Communities
Joint Nuclear Research Centre - Ispra Establishment (Italy)
Materials Division
Luxembourg, February 1974 - 42 Pages - 9 Figures - B.Fr. 60.—

The present report describes the computer code TAFEST that has been developed for the purpose of solving two-dimensional transient heat-conduction problems. The concept of finite elements in space and time is used as a means of obtaining numerical responses.

EUR 5049 e

COMMISSION OF THE EUROPEAN COMMUNITIES

CODE TAFEST
NUMERICAL SOLUTION TO TRANSIENT HEAT-CONDUCTION PROBLEMS USING FINITE ELEMENTS IN SPACE AND TIME

by

J. DONEA and S. GIULIANI

1974

Joint Nuclear Research Centre
Ispra Establishment - Italy

Materials Division

ABSTRACT

The present report describes the computer code TAFEST that has been developed for the purpose of solving two-dimensional transient heat-conduction problems. The concept of finite elements in space and time is used as a means of obtaining numerical responses.

KEYWORDS

T-CODES
IBM COMPUTERS
FORTRAN
FINITE ELEMENT METHOD
TRANSIENTS
THERMAL CONDUCTION
TIME DEPENDENCE
SPACE
TWO-DIMENSIONAL CALCULATIONS
ACCURACY
USES

CONTENTS

1. Introduction. 5

2. Basic variational equation. 6

3. A right triangular prism in the (x, y, t) domain. 7

3.1. Choice of the local temperature field. 7

3.2. Element characteristics. 7

3.3. Nodal loads due to the boundary conditions. 10

4. Achievable accuracy with respect to the Crank-Nicholson scheme.

5. The computer code TAFEST. 12

5.1. Description of input data. 13

5.2. Input data sheets. 19

5.3. Description of the printed output. 25

6. A particular problem 34

References. 35

1. INTRODUCTION

Within the frame of the finite element method, solutions to the transient heat-conduction equation are governed by a system of first-order linear differential equations of the form[1] :

$$[K]\ \{T(t)\} + [C]\{\dot{T}(t)\} = \{F(t)\} \qquad (1.a)$$

$$\{T(0)\} = \{T_0\} \qquad (1.b)$$

where $\{T\ (t)\}$ denotes the temperature vector, $\{F\ (t)\}$ is the 'load' vector, $[K]$ is the conductivity matrix and $[C]$ the heat-capacity matrix.
The vector $\{To\}$ specifies the initial values of the temperature.

The differential system (1) can be integrated numerically with the aid of a digital computer. The most critical step is, of course, to choose an integration method that combines efficiency and accuracy.

In this report, the concept of finite elements in space and time is used as a means of integrating the differential system (1). The basic variational formulation involving both time and space variables is described by reference to the Galerkin process.

Although various elements of the space-time domain can easily be derived[2], the only element described here is a right triangular prism of the (x, y, t) domain. This element is shown to lead to a better short-time accuracy than the Crank-Nicholson scheme used by Wilson-Nickell[3], Zienkiewicz-Parekh[4] and Fullard[5].

The main features of the computer code TAFEST are described in the last part of the report. This code was developed for the purpose of solving two-dimensional transient heat-conduction problems by means of the indicated space-time element. A typical example has been included in order to show the type of results that can be obtained on using the code.

2. BASIC VARIATIONAL EQUATION

Let it be required to solve the transient heat-conduction equation

$$k \ \mathrm{div}(\mathrm{grad}\ T) + Q(x,y,z,t) - \rho c \frac{\partial T}{\partial t} = 0 \qquad (2)$$

in a domain V bounded by a surface S .

In order to formulate a variational problem associated with eq. (2), we multiply it by an arbitrary admissible temperature variation δT and use the property

$$\mathrm{div}(a \vec{B}) = a \ \mathrm{div} \vec{B} + \vec{B} \ \mathrm{grad}\ a \qquad (3)$$

Such a manipulation indicates that

$$\mathrm{div}(k \ \mathrm{grad}\ T \ \delta T) - k \ \mathrm{grad}\ T \ \mathrm{grad}\ \delta T + Q \delta T - \rho c \frac{\partial T}{\partial t} \delta T = 0 \qquad (4)$$

We now integrate eq. (4) over the domain V and the time t and transform the volume integral for the first term by means of the divergence theorem. This enables the order of the partial derivatives to be reduced and yields :

$$-\int_t \oint_S k \frac{\partial T}{\partial n} \delta T \, dS dt + \int_t \int_V k \ \mathrm{grad}\ T \ \mathrm{grad}\ \delta T \, dV dt -$$

$$- \int_t \int_V Q \, \delta T \, dV dt + \int_t \int_V \rho c \frac{\partial T}{\partial t} \delta T \, dV \, dt = 0 \qquad (5)$$

with n denoting the outward unit normal to S .

Since eq. (5) holds for an arbitrary temperature-variation δT , eq. (1) is also satisfied. The variational equation (5) can thus be used as a basis for a numerical solution of transient-conduction problems.
The main problem in solving eq. (5) consists in the definition of suitable finite elements in the space-time domain. For any such element, the local field will be represented in the form

$$T(x,y,z,t) = \sum_{i=1}^{M} N_i(x,y,z,t) \; T_i \qquad (6)$$

where the modes N_i depend on space and time, while the M nodal values T_i are indèpendent on the coordinates x, y,z,t. The characteristic equations for the element are obtained by introduction of the local representation (6) into the basic variational equation (5).
The assembly of the various elements appearing in the discretization of the space-time domain follows the usual rules of the finite element method.

3. A RIGHT TRIANGULAR PRISM IN THE (x, y, t) DOMAIN

Although various elements in space and time can easily be derived[2], we shall concentrate on the particular element that has been choosen for the computer code TAFEST to be described in section 5.

3.1. Choice of the local temperature field

The right triangular prism represented in Fig. 1 has six degrees of freedom. In order to ensure the continuity of the temperature on the interfaces between the various elements, the local temperature field is choosen in the form :

$$T^e(x,y,t) = a + bx + cy + dt + ext + fyt \tag{7}$$

In function of the nodal parameters (Fig. 1)

$$\{T^e\}^* = (T_i, T_j, T_k, T_l, T_m, T_n) \tag{8}$$

the local field can be written as

$$T(x,y,t) = [N_i, N_j, N_k, N_l, N_m, N_n] \{T^e\} \tag{9}$$

with

$$N_i = M_i(t_l - t) \quad ; \quad N_j = M_j(t_l - t) \quad ; \quad N_k = M_k(t_l - t)$$

$$N_l = M_i(t - t_i) \quad ; \quad N_m = M_j(t - t_i) \quad ; \quad N_n = M_k(t - t_i)$$

$$M_i = \frac{a_i + b_i x + c_i y}{2V} \quad ; \quad V = \text{Volume of the prismatic element.}$$

$$a_i = x_j y_k - x_k y_j \quad ; \quad b_i = y_j - y_k \quad ; \quad c_i = x_k - x_j$$

The modes M_j and M_k are obtained by cyclic permutation of the indexes in the order i, j, k. The parameters t_i and t_l define the time interval spanned by the element.

3.2. Element characteristics

The governing equations for the element are obtained through introduction in eq. (5) of independent variations on the six nodal parameters (8). Such an operation yields the following relationship :

$$[H^e] \{T^e\} = \{F^e\} \tag{10}$$

The matrix $\left[H^e\right]$ is the sum of a conductivity matrix

$$\left[K^e\right] = \frac{kV}{12A^2}\begin{bmatrix} ([B]+[C]) & \frac{1}{2}([B]+[C]) \\ \frac{1}{2}([B]+[C]) & ([B]+[C]) \end{bmatrix} \quad (11)$$

and a heat-capacity matrix

$$\left[\mathcal{P}^e\right] = -\frac{\rho c}{8A^2}\begin{bmatrix} [P] & -[P] \\ [P] & -[P] \end{bmatrix} \quad (12)$$

where

$$[B]+[C] = \begin{bmatrix} (b_i b_i + c_i c_i) & (b_i b_j + c_i c_j) & (b_i b_k + c_i c_k) \\ & (b_j b_j + c_j c_j) & (b_j b_k + c_j c_k) \\ \text{Symmetric} & & (b_k b_k + c_k c_k) \end{bmatrix} \quad (13)$$

$$[P] = \begin{bmatrix} P_{ii} & P_{ij} & P_{ik} \\ P_{ij} & P_{jj} & P_{jk} \\ P_{ik} & P_{jk} & P_{kk} \end{bmatrix} \quad (14)$$

$$P_{ij} = \int_A (a_i + b_i x + c_i y)(a_j + b_j x + c_j y)\, dx\, dy$$

A = Area of triangle i,j,k.

$$\left\{F^e\right\}^* = (F_i, F_j, F_k, F_l, F_m, F_n) \quad (15)$$

If the internal heat-generation Q is independent of space but varies linearly from Q_i to Q_l during the time interval $t_l - t_i$, the contributed nodal loads are easily shown to be

$$\left\{F_Q^e\right\}^* = \frac{V}{9}\left(F_Q^1, F_Q^1, F_Q^1, F_Q^2, F_Q^2, F_Q^2\right) \tag{16}$$

where

$$F_Q^1 = Q_i + \frac{1}{2}Q_l \quad ; \quad F_Q^2 = \frac{1}{2}Q_i + Q_l$$

3.3. Nodal loads due to the boundary conditions

Prescribed normal heat-flux

Suppose we impose between nodal points i and j (Fig. 1) a uniform normal heat-flux which varies linearly from $\bar{\varphi}_i$ to $\bar{\varphi}_l$ during the time interval $t_l - t_i$. The first term in eq. (5) shows that such a condition induces the nodal loads

$$\left\{F_{\bar{\varphi}}^e\right\}^* = -\frac{L(t_l - t_i)}{6}\left(F_\varphi^1, F_\varphi^1, 0, F_\varphi^2, F_\varphi^2, 0\right) \tag{17}$$

where

$$F_\varphi^1 = \bar{\varphi}_i + \frac{1}{2}\bar{\varphi}_l \qquad F_\varphi^2 = \frac{1}{2}\bar{\varphi}_i + \bar{\varphi}_l$$

L = Length of side i-j

Convective heat-transfer

Suppose now we have a convective heat-transfer between nodes i and j. The heat-transfer coefficient h as well as the fluid temperature T_f vary linearly during the time interval $t_l - t_i$. This type of boundary condition yields a contribution to both the matrix $[H^e]$ and the nodal loads $\{F^e\}$.

The additional terms in the matrix $[H^e]$ are

$$[H^e_{conv}] = \frac{L(t_l - t_i)}{36} \begin{bmatrix} T_1 & \frac{1}{2}T_1 & 0 & T_2 & \frac{1}{2}T_2 & 0 \\ & T_1 & 0 & \frac{1}{2}T_2 & T_2 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & T_3 & \frac{1}{2}T_3 & 0 \\ \text{Symmetric} & & & & T_3 & 0 \\ & & & & & 0 \end{bmatrix} \qquad (18)$$

where

$$T_1 = 3h_i + h_l \quad ; \quad T_2 = h_i + h_l \quad ; \quad T_3 = h_i + 3h_l$$

$$h_i = h(t_i) \quad ; \quad h_l = h(t_l)$$

The nodal loads contributed by the condition of convection are

$$\{F^e_{conv}\}^* = \frac{L(t_l - t_i)}{24} \left(S_1, S_1, 0, S_2, S_2, 0\right) \qquad (19)$$

where

$$S_1 = h_i(3T_f^i + T_f^l) + h_l(T_f^i + T_f^l)$$

$$S_2 = h_l(3T_f^l + T_f^i) + h_i(T_f^i + T_f^l)$$

T_f^i, T_f^l = Fluid temperature at times t_i and t_l.

4. ACHIEVABLE ACCURACY WITH RESPECT TO THE CRANK-NICHOLSON SCHEME

The one-dimensional example of a constant heat-flux applied to a semi-infinite solid has been analyzed in order to illustrate the achievable accuracy with the space-time element previously described.
A finite element solution for this problem is given by Wilson and Nickell [3] using a regular mesh with $\Delta x = 0.2$. The time integration is performed on the basis of a recurrence relation which can be shown to be a generalization of the Crank-Nicholson scheme[2]. Constant time steps $\Delta t = 0.1$ are used.
We solved the same problem by means of finite elements in space and time, i.e. with eq. (10) as the integration formula.
Fig. 2 compares both numerical solutions to the exact one.
As can be seen, a much better short-time accuracy is achieved with the space-time element. The reasons for this better behaviour with respect to the Crank-Nicholson scheme are fully explained elsewhere[6].

5. THE COMPUTER CODE TAFEST

In this section we describe the main features of the computer code TAFEST. This code was developed for the purpose of solving two-dimensional transient heat-conduction problems by means of finite elements in space and time. Starting from known temperatures at time t, the last three equations in relation (10) are used as an integration scheme to yield the temperatures at time $t + \Delta t$. The assembled equations are solved by means of Choleski's method. A general flow chart of the programme is given on Fig. 3.
TAFEST has been written in Fortran IV language and compiled on the IBM 370/165 computer of CETIS (EURATOM C.C.R. - Ispra).
In the present version, the code has a size of about 200K bytes, so that no auxiliary storage space is needed.

5.1. Description of input data

The input data required by TAFEST are defined here in the sequence in which they occur. References to card numbers will be found in the listing of data and formats which follows this section.

CARD (1)

TIT	The problem title in 72 alpha-numeric characters. This information is used to identify the problem in the printed output.

CARD (2)

NUMEL	The number of triangular-shaped elements in the structure (max. 700)
NUMNP	The number of nodal points (max. 400)
NUMTM	Number of points used in the discretization of the time (Initial time included) (max. 50)
N1	Option to define the coordinate system used for input 0 means Cartesian 1 means Polar
N2	Option to define the type of the heat flow 0 means plane 1 means axisymmetric

CARD (3)

N3 (I)	Option to punch temperature cards at time TM (I) (I = 1, NUMTM)

0 Print nodal temperatures but do not punch;
1 Print nodal point and element temperatures
Punch the element temperatures;
2 Print and punch nodal point temperatures;
3 Print and punch element and nodal point temperatures.

CARD (4)

NTI Number of nodal points with prescribed temperatures (max. 100)

NTB Number of elements with one side subject to convection (max. 100)

NTF Number of elements with a non-zero normal heat-flux prescribed on one side (max. 100)

NTQ Number of groups of elements with internal heat-generation (max. 100)

CARD (5)

COND Main thermal conductivity (w/cm-°C)

CAPA Main heat capacity ρc (Joule/cm^3 - °C)

CARD (6)

TM (I) Location (expressed in seconds) of the various points used in the discretisation of the time. (I = 1, NUMTM) (TM (1) is the initial time for the transient problem).

Six time stations are given per card.

CARD (7)

One card is required for each element (N = 1, NUMEL)

N	The element index number
NPI (N) NPJ (N) NPK (N)	Index numbers of the element nodal points
CT (N)	The effective thermal conductivity of element N is COND - CT (N) (See card (5))
CP (N)	The effective heat-capacity of element N is CAPA - CP (N) (See card (5))

CARD (8)

One card is required to describe each nodal point (M = 1 , NUMNP)

M	The nodal point index number
XORD(M)	The x or r-coordinate of nodal point M (mm)
YORD(M)	The y or θ-coordinate of nodal point M (angles are given in degrees)

CARD (9)

J	Non processed index that may be used to number the nodal points if J = I.
T (I)	Initial temperature (°C) at nodal point I (I = 1, NUMNP ; 4 nodal point temperatures are given per card)

Cards (10) and (11) are repeated NTI times (I = 1, NTI).

CARD (10)

NTT (I)	Index number of a node with prescribed temperature
NTMI (I)	Number of points in time which are given to describe the evolution of the prescribed temperature. (Piecewise linearization of the effective temperature)

CARD (11)

TI (I, N)	Nodal point temperature (°C) at time TIMI(I,N)
TIMI (I,N)	Time in seconds

Three groups TI, TIMI are given per card (N = 1, NTMI (I))

Cards (12) and (13) are repeated NTB times (I = 1, NTB)

CARD (12)

M	Index number of an element subject to convection heat-transfer on one side.
NTMB (I)	Number of points in time which are given to describe the evolution of the convective heat-transfer.
LI (I)	Nodal points defining the element side
LJ (I)	Subject to convection.

CARD (13)

H (I, N)	Value of the heat-transfer coefficient (W/cm^2 - °C) at time TIMB (I, N)

TF (I,N)	Temperature of the reference fluid (°C) at time TIMB (I, N)
TIMB (I,N)	Time in seconds

Two groups H, TF, TIMB are given per card (N = 1, NTMB (I))
Cards (14) and (15) are repeated NTF times (I = 1, NTF).

CARD (14)

M	Index number of an element with a prescribed normal heat-flux on one side.
NTMX (I)	Number of points in time which are given to describe the evolution of the prescribed heat-flux.
MI (I) MJ (I)	Nodal points defining the element side with prescribed heat-flux.

CARD (15

FLUX (I,N)	Prescribed heat-flux (W/cm^2) at time TIMX (I,N)
TIMX (I,N)	Time in seconds

Three groups FLUX, TIMX are given per card (N = 1, NTMX (I)).
Cards (16) and (17) are repeated NTQ times (I = 1, NTQ).

CARD (16)

IFIRST (I)	Index number of the first element in a group with internal heat-generation.
ILAST (I)	Index number of the last element in a group with internal heat-generation.

NTMQ (I) Number of points in time which are given to describe the evolution of the internal heat generation.

CARD(17)

Q (I,N) Heat generation (W/cm^3) at time TIMQ (I,N)

TIMQ (I,N) Time in seconds

Three groups Q-TIMQ are given per card (N = 1, NTMQ (I))

5.2. Input Data Sheets

Card 1		
Column	1 – 72	
Format	18 A4	
Symbol	TIT	

Card 2						
Column	1 – 6	7 – 12	13 – 18	19 – 24	25 – 30	
Format	I 6	I 6	I 6	I 6	I 6	
Symbol	NUMEL	NUMNP	NUMTM	N 1	N 2	

Card 3							
Column	1	2	3	4	– – – – – – – – – – – –	79	80
Format	I 1	I 1	I 1	I 1	– – – – – – – – – – – –	I 1	I 1
Symbol	N3 (1)	N3 (2)	N3 (3)	N3 (4)	– – – – – – – – – – – –	N3(79)	N3(80)

Card 4					
Column	1 – 6	7 – 12	13 – 18	19 – 24	
Format	I 6	I 6	I 6	I 6	
Symbol	NTI	NTB	NTF	NTQ	

TAFEST

Card 5			
	Column	1 – 12	13 – 24
	Format	E12.5	E12.5
	Symbol	COND	CAPA

Card 6							
	Column	1 – 12	13 – 24	25 – 36	37 – 48	49 – 60	61 – 72
	Format	E12.5	E12.5	E12.5	E12.5	E12.5	E12.5
	Symbol	TM (I)	TM (I)	TM (I)	TM (I)	TM (I)	TM (I)

Card 7							
	Column	1 – 6	7 – 12	13 – 18	19 – 24	25 – 36	37 – 48
	Format	I 6	I 6	I 6	I 6	E12.5	E12.5
	Symbol	N	NPI (N)	NPJ (N)	NPK (N)	CT (N)	CP (N)

Card 8					
	Column	1 – 4	5 – 6	7 – 18	19 – 30
	Format	I 4	2X	E12.5	E12.5
	Symbol	M	—	XORD (M)	YORD (M)

TAFEST

Card 9									
	Column	1 - 6	7 - 18	19 - 24	25 - 36	37 - 42	43 - 54	55 - 60	61 - 72
	Format	I 6	E12.5	I 6	E12.5	I 6	E12.5	I 6	E12.5
	Symbol	J	T (I)	J	T (I)	J	T (I)	J	T (I)

Card 10			
	Column	1 - 6	7 - 12
	Format	I 6	I 6
	Symbol	NTT (I)	NTMI (I)

Card 11							
	Column	1 - 12	13 - 24	25 - 36	37 - 48	49 - 60	61 - 72
	Format	E12.5	E12.5	E12.5	E12.5	E12.5	E12.5
	Symbol	TI (I,N)	TIMI (I,N)	TI (I,N)	TIMI (I,N)	TI (I,N)	TIMI (I,N)

Card 12					
	Column	1 - 6	7 - 12	13 - 18	19 - 24
	Format	I 6	I 6	I 6	I 6
	Symbol	M	NTMB (I)	LI (I)	LJ (I)

TAFEST

Card 13							
	Column	1 – 12	13 – 24	25 – 36	37 – 48	49 – 60	61 – 72
	Format	E12.5	E12.5	E12.5	E12.5	E12.5	E12.5
	Symbol	H (I,N)	TF(I,N)	TIMB(I,N)	H (I,N)	TF(I,N)	TIMB(I,N)

Card 14					
	Column	1 – 6	7 – 12	13 – 18	19 – 24
	Format	I 6	I 6	I 6	I 6
	Symbol	M	NTMX(I)	MI (I)	MJ (I)

Card 15							
	Column	1 – 12	13 – 24	25 – 36	37 – 48	49 – 60	61 – 72
	Format	E12.5	E12.5	E12.5	E12.5	E12.5	E12.5
	Symbol	FLUX (I,N)	TIMX(I,N)	FLUX (I,N)	TIMX (I,N)	FLUX (I,N)	TIMX (I,N)

Card 16				
	Column	1 – 6	7 – 12	13 – 18
	Format	I 6	I 6	I 6
	Symbol	IFIRST(I)	ILAST(I)	NTMQ (I)

TAFEST

Card								
Card 17	Column	1 – 12	13 – 24	25 – 36	37 – 48	49 – 60	61 – 72	
	Format	E12.5	E12.5	E12.5	E12.5	E12.5	E12.5	
	Symbol	Q(I,N)	TIMQ(I,N)	Q(I,N)	TIMQ(I,N)	Q(I,N)	TIMQ(I,N)	

TAFEST

5.3. Description of the printed output

As an illustration of the printed output of TAFEST, we reproduce hereafter the results for the one-dimensional problem of a constant heat-flux applied to a semi-infinite solid (see section 4).

*** TEST TAFEST - CONSTANT HEAT FLUX APPLIED TO A SEMI-INFINITE SOLID ********

THIS PROBLEM IS SOLVED UNDER PLANE CONDITIONS

NUMBER OF ELEMENTS	=	38
NUMBER OF NODAL POINTS	=	40
NUMBER OF TIME POINTS	=	11
THERMAL CONDUCTIVITY (W/CMC)	=	1.0000E 00
THERMAL CAPACITY (J/CM3C)	=	1.0000E 00
NUMBER OF NODES WITH PRESCRIBED TEMPERATURE	=	0
NUMBER OF ELEMENTS WITH CONVECTION	=	0
NUMBER OF ELEMENTS WITH PRESCRIBED HEAT FLUX	=	1
NUMBER OF GROUPS OF ELEMENTS WITH HEAT GENERATION	=	0
N1	=	0
N2	=	0

POINTS IN THE TIME DOMAIN

TIME (SEC)	N3	TIME (SEC)	N3	TIME (SEC)	N3	TIME (SEC)	N3	TIME (SEC)	N3	TIME (SEC)	N3
0.0	0	1.000E-01	0	2.000E-01	0	3.000E-01	0	4.000E-01	0	5.000E-01	0
6.000E-01	0	7.000E-01	0	8.000E-01	0	9.000E-01	0	1.000E 00	0		

NODE	X-ORD (MM)	Y-ORD (MM)	NODE	X-ORD (MM)	Y-ORD (MM)	NODE	X-ORD (MM)	Y-ORD (MM)
1	0.0	0.0	2	0.0	4.00000E 00	3	2.00000E 00	0.0
4	2.00000E 00	4.00000E 00	5	4.00000E 00	0.0	6	4.00000E 00	4.00000E 00
7	6.00000E 00	0.0	8	6.00000E 00	4.00000E 00	9	8.00000E 00	0.0
10	8.00000E 00	4.00000E 00	11	1.00000E 01	0.0	12	1.00000E 01	4.00000E 00
13	1.20000E 01	0.0	14	1.20000E 01	4.00000E 00	15	1.40000E 01	0.0
16	1.40000E 01	4.00000E 00	17	1.60000E 01	0.0	18	1.60000E 01	4.00000E 00
19	1.80000E 01	0.0	20	1.80000E 01	4.00000E 00	21	2.00000E 01	0.0
22	2.00000E 01	4.00000E 00	23	2.20000E 01	0.0	24	2.20000E 01	4.00000E 00
25	2.40000E 01	0.0	26	2.40000E 01	4.00000E 00	27	2.60000E 01	0.0
28	2.60000E 01	4.00000E 00	29	2.80000E 01	0.0	30	2.80000E 01	4.00000E 00
31	3.00000E 01	0.0	32	3.00000E 01	4.00000E 00	33	3.20000E 01	0.0
34	3.20000E 01	4.00000E 00	35	3.40000E 01	0.0	36	3.40000E 01	4.00000E 00
37	3.60000E 01	0.0	38	3.60000E 01	4.00000E 00	39	3.80000E 01	0.0
40	3.80000E 01	4.00000E 00						

ELEM.	I	J	K	CT (W/CM C)	CP (J/CM3 C)	ELEM.	I	J	K	CT (W/CM C)	CP (J/CM3 C)
1	1	3	2	0.0	0.0	2	3	4	2	0.0	0.0
3	3	6	4	0.0	0.0	4	3	5	6	0.0	0.0
5	5	7	6	0.0	0.0	6	7	8	6	0.0	0.0
7	7	10	8	0.0	0.0	8	7	9	10	0.0	0.0
9	9	11	10	0.0	0.0	10	11	12	10	0.0	0.0
11	11	14	12	0.0	0.0	12	11	13	14	0.0	0.0
13	13	15	14	0.0	0.0	14	15	16	14	0.0	0.0
15	15	18	16	0.0	0.0	16	15	17	18	0.0	0.0
17	17	19	18	0.0	0.0	18	19	20	18	0.0	0.0
19	19	22	20	0.0	0.0	20	19	21	22	0.0	0.0
21	21	23	22	0.0	0.0	22	23	24	22	0.0	0.0
23	23	26	24	0.0	0.0	24	23	25	26	0.0	0.0
25	25	27	26	0.0	0.0	26	27	28	26	0.0	0.0
27	27	30	28	0.0	0.0	28	27	29	30	0.0	0.0
29	29	31	30	0.0	0.0	30	31	32	30	0.0	0.0
31	31	34	32	0.0	0.0	32	31	33	34	0.0	0.0
33	33	35	34	0.0	0.0	34	35	36	34	0.0	0.0
35	35	38	36	0.0	0.0	36	35	37	38	0.0	0.0
37	37	39	38	0.0	0.0	38	39	40	38	0.0	0.0

TOTAL AREA (CM 2) = 1.51999E 00

ELEM 1 TIME POINTS 2 NODES 1 2

FLUX (W/CM2)	TIME (SEC)	FLUX (W/CM2)	TIME (SEC)	FLUX (W/CM2)	TIME (SEC)
-1.000E 00	0.0	-1.000E 00	1.000E 02		

INITIAL TEMPERATURES - TIME (SEC) = 0.0

NODE	TEMPERATURE (C)	NODE	TEMPERATURE (C)	NODE	TEMPERATURE (C)	NODE	TEMPERATURE (C)
1	0.0	2	0.0	3	0.0	4	0.0
5	0.0	6	0.0	7	0.0	8	0.0
9	0.0	10	0.0	11	0.0	12	0.0
13	0.0	14	0.0	15	0.0	16	0.0
17	0.0	18	0.0	19	0.0	20	0.0
21	0.0	22	0.0	23	0.0	24	0.0
25	0.0	26	0.0	27	0.0	28	0.0
29	0.0	30	0.0	31	0.0	32	0.0
33	0.0	34	0.0	35	0.0	36	0.0
37	0.0	38	0.0	39	0.0	40	0.0

DIMENSIONS OF THE MATRIX S = 40* 4

TIME (SEC) = 1.000E-01

NODE	TEMPERATURE (C)	NODE	TEMPERATURE (C)	NODE	TEMPERATURE (C)	NODE	TEMPERATURE (C)
1	0.39	2	0.37	3	0.16	4	0.18
5	0.08	6	0.08	7	0.03	8	0.04
9	0.02	10	0.02	11	0.01	12	0.01
13	0.00	14	0.00	15	0.00	16	0.00
17	0.00	18	0.00	19	0.00	20	0.00
21	0.00	22	0.00	23	0.00	24	0.00
25	0.00	26	0.00	27	0.00	28	0.00
29	0.00	30	0.00	31	0.00	32	0.00
33	0.00	34	0.00	35	0.00	36	0.00
37	0.00	38	0.00	39	0.00	40	0.00

TIME (SEC) = 2.000E-01

NODE	TEMPERATURE (C)	NODE	TEMPERATURE (C)	NODE	TEMPERATURE (C)	NODE	TEMPERATURE (C)
1	0.49	2	0.49	3	0.33	4	0.33
5	0.20	6	0.19	7	0.11	8	0.11
9	0.06	10	0.06	11	0.03	12	0.03
13	0.02	14	0.02	15	0.01	16	0.01
17	0.00	18	0.00	19	0.00	20	0.00
21	0.00	22	0.00	23	0.00	24	0.00
25	0.00	26	0.00	27	0.00	28	0.00
29	0.00	30	0.00	31	0.00	32	0.00
33	0.00	34	0.00	35	0.00	36	0.00
37	0.00	38	0.00	39	0.00	40	0.00

TIME (SEC) = 3.000E-01

NODE	TEMPERATURE (C)	NODE	TEMPERATURE (C)	NODE	TEMPERATURE (C)	NODE	TEMPERATURE (C)
1	0.62	2	0.61	3	0.43	4	0.43
5	0.29	6	0.29	7	0.19	8	0.19
9	0.12	10	0.12	11	0.07	12	0.07
13	0.04	14	0.04	15	0.02	16	0.02
17	0.01	18	0.01	19	0.01	20	0.01
21	0.00	22	0.00	23	0.00	24	0.00
25	0.00	26	0.00	27	0.00	28	0.00
29	0.00	30	0.00	31	0.00	32	0.00
33	0.00	34	0.00	35	0.00	36	0.00
37	0.00	38	0.00	39	0.00	40	0.00

6. A PRACTICAL PROBLEM

We analyzed a transient heat-flow in the graphite matrix of a HTGR fuel element. As indicated by Fig. 4, the analysis is limited to the symmetric portion of the graphite matrix. Fig. 5 shows the finite element grid that has been used. The transient heat-flow is due to a power increase of the reactor. Along the interface between fuel and graphite, we prescribe a uniform normal heat-flux φ which increases with time as indicated on Fig. 6.
The same figure gives the evolution of the heat transfer coefficient h between graphite and coolant. The thermal properties of graphite are

$$k = 0.2 \ \text{W/cm °C} \quad ; \quad \rho c = 4.5 \ \text{Joules/cm}^3 - \text{°C}$$

while the coolant temperature is 600°C.

The upper curves in Fig. 6 show the temperature evolution at points A and B of the symmetric cell. It can be noted that if the temperature at point B decreases as soon as the heat transfer coefficient h is increased, the temperature at point A reacts with a small phase-difference due to the effect of the heat capacity. Figures 7 - 8 - 9 show the isothermal curves in the graphite matrix at three typical instants of the transient problem.

REFERENCES

1. O.C. Zienkiewicz, "The Finite Element Method in Engineering Science", McGraw-Hill, London (1971).

2. J. Donéa, "Méthodes variationnelles appliquées à l'analyse de problèmes mécaniques et thermiques posés par la technologie nucléaire", Thèse de Doctorat, Université de Liège, Belgique (1973).

3. E.L. Wilson, R.E. Nickell, "Application of the finite element method to heat conduction analysis", Nucl. Eng. and Des., Vol. 4, 276-286, (1966).

4. O.C. Zienkiewicz, C.J.Parekh , "Transient Field Problems: Two-dimensional and Three-dimensional Analysis by Isoparametric Finite Elements", International J. Num. Meth. in Engng." Vol. 2, 61-71, (1970).

5. K. Fullard, "FLHE, a Finite Element Programme for the Calculation of Temperatures in arbitrary Structures, Part I: User's Guide", CEGB Rep. RD/B/N 1849.

6. J. Donéa, "On the accuracy of finite element solutions to the transient heat-conduction equation", (to be published).

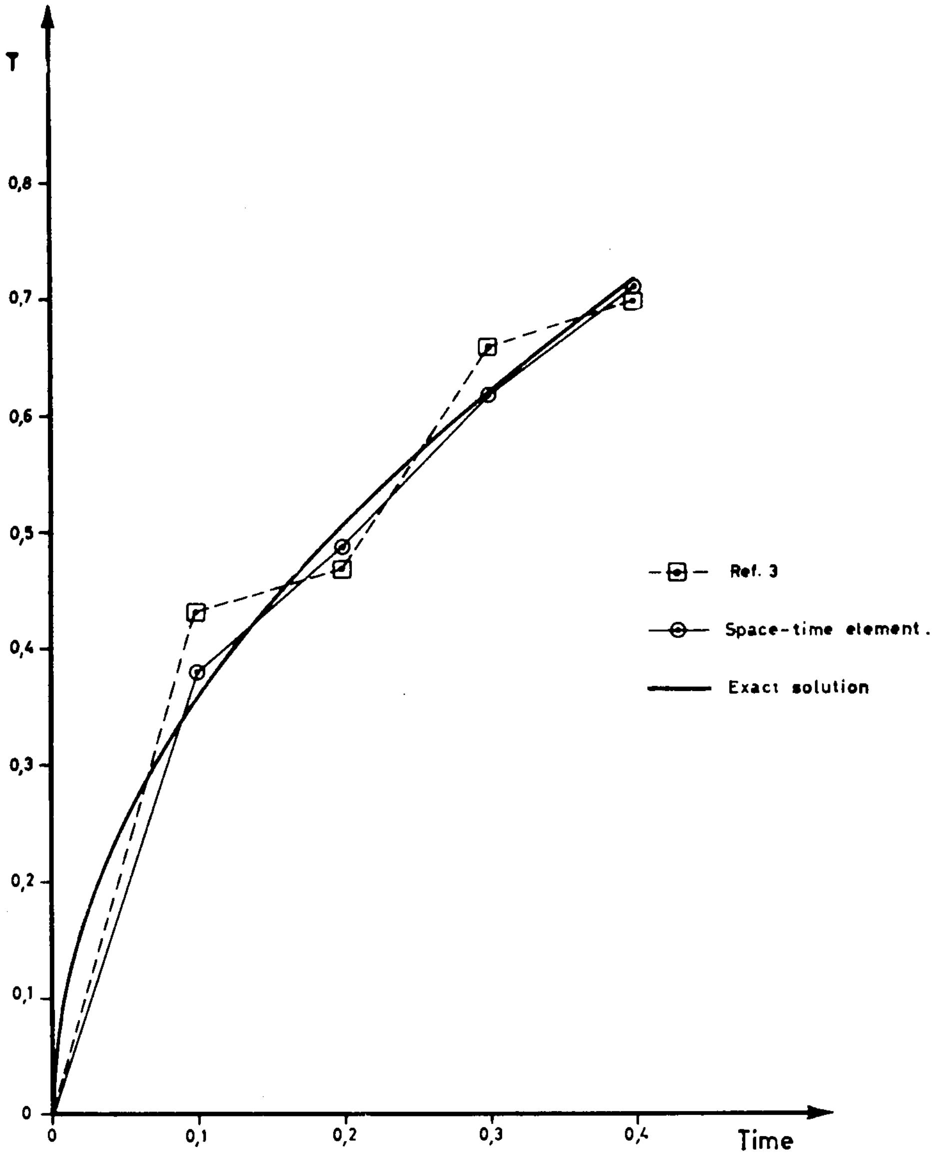

Fig. 2 Temperature distribution versus time at the surface of a semi-infinite solid.

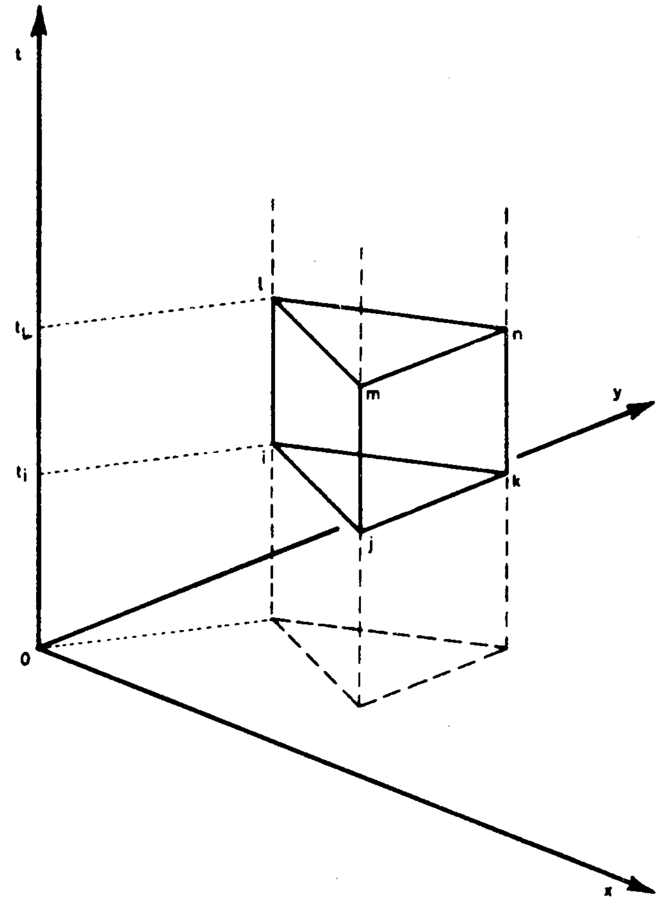

Fig. 1 A right triangular prism in the (x,y,t) domain.

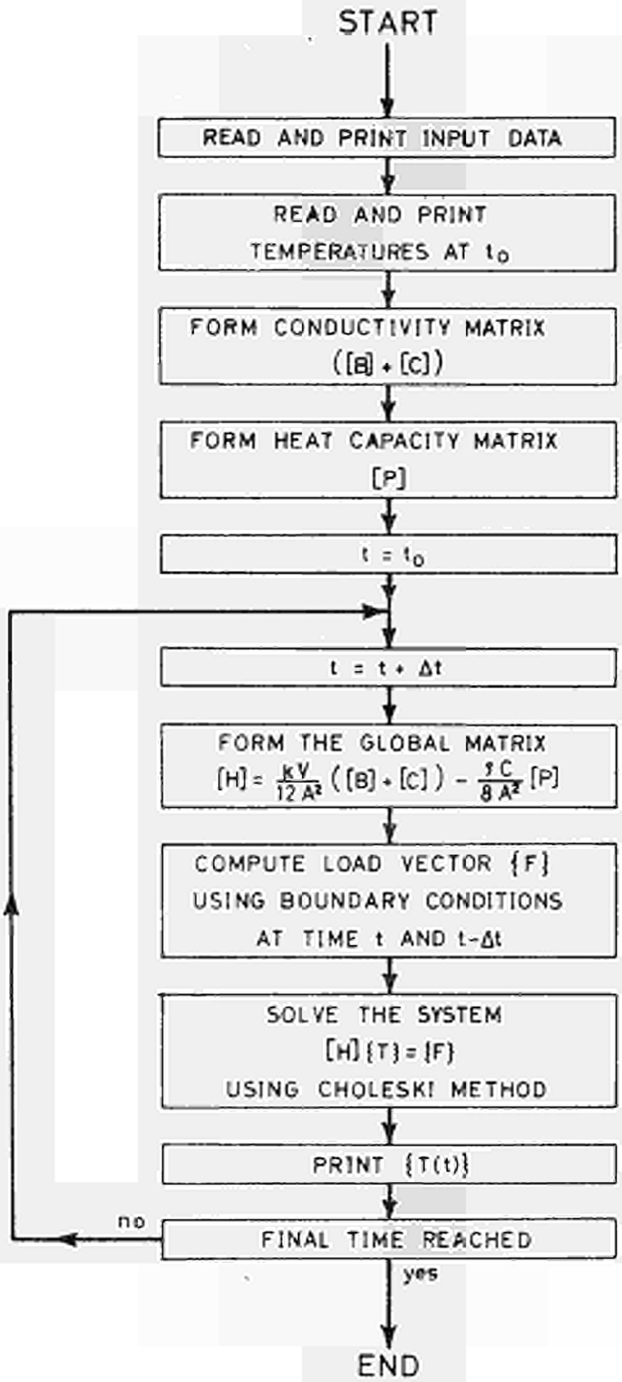

Fig. 3 General flow chart of TAFEST.

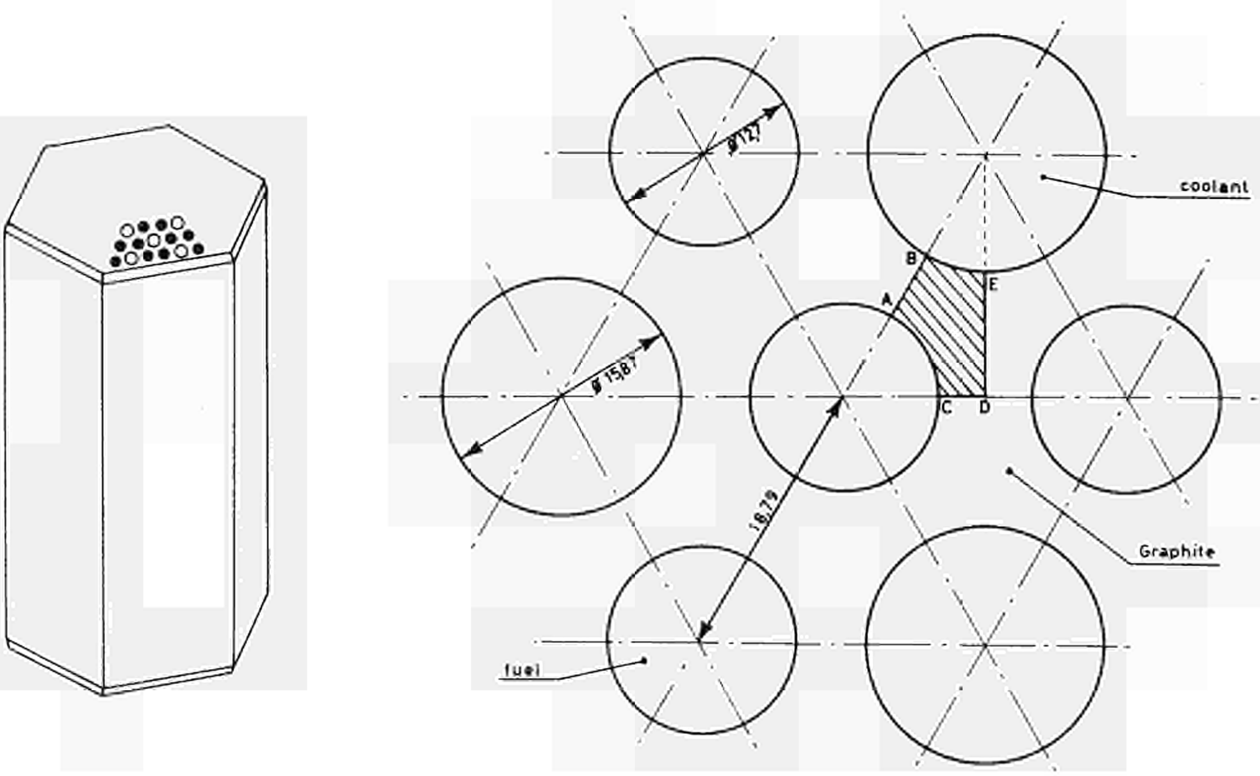

Fig. 4 Graphite matrix of a HTGR fuel element and symmetric cell.

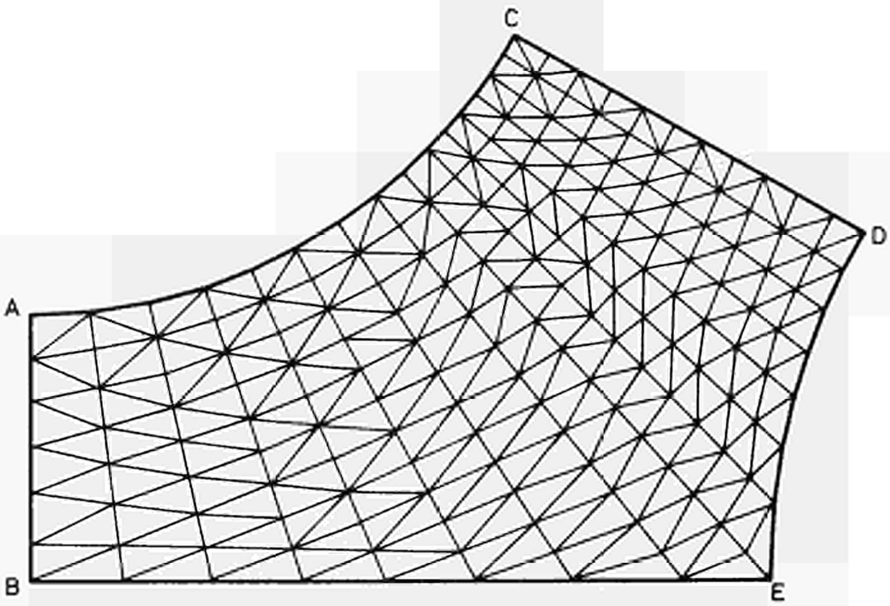

Fig. 5 Finite element grid.

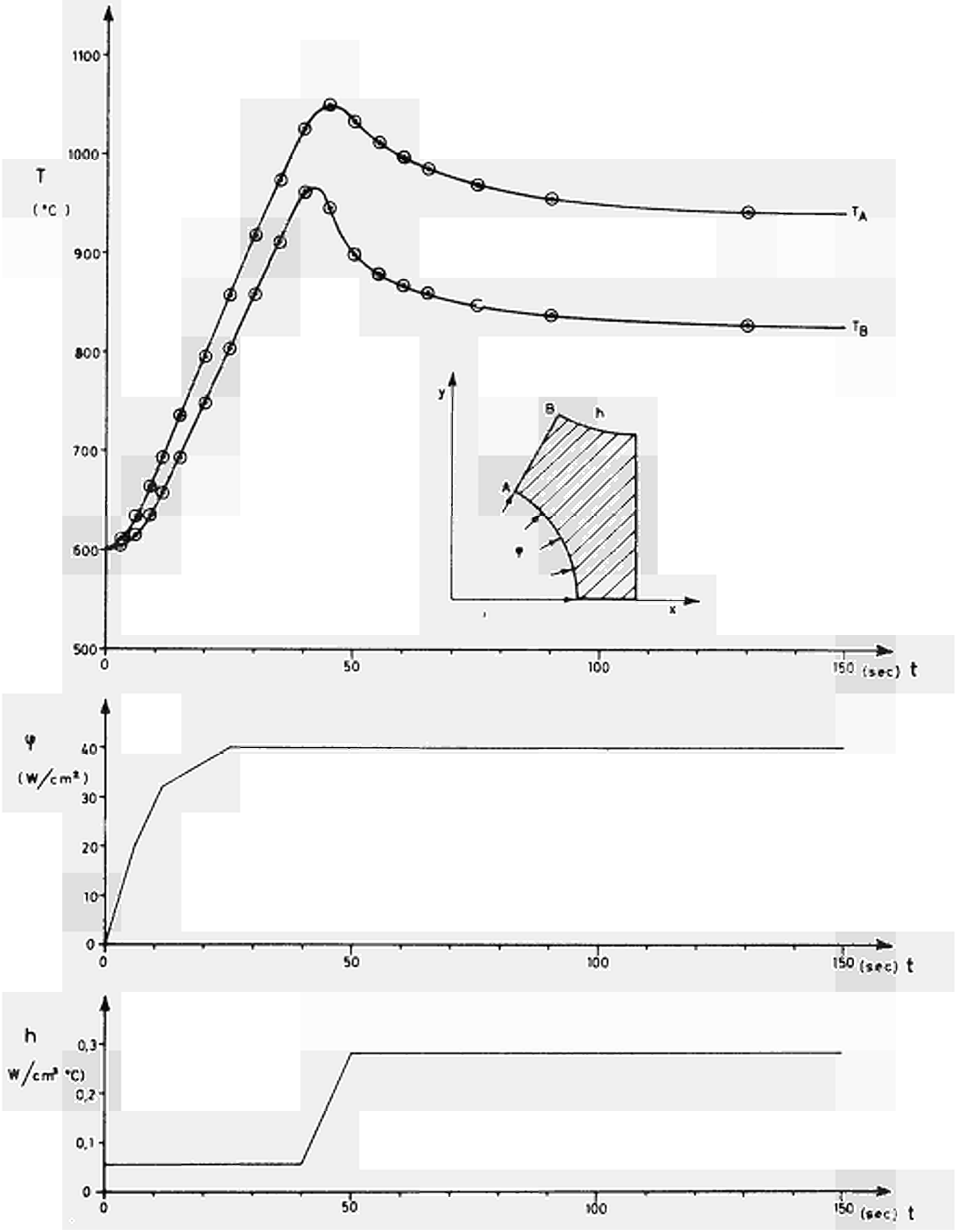

Fig. 6 Boundary conditions and temperature evolution at points A and B.

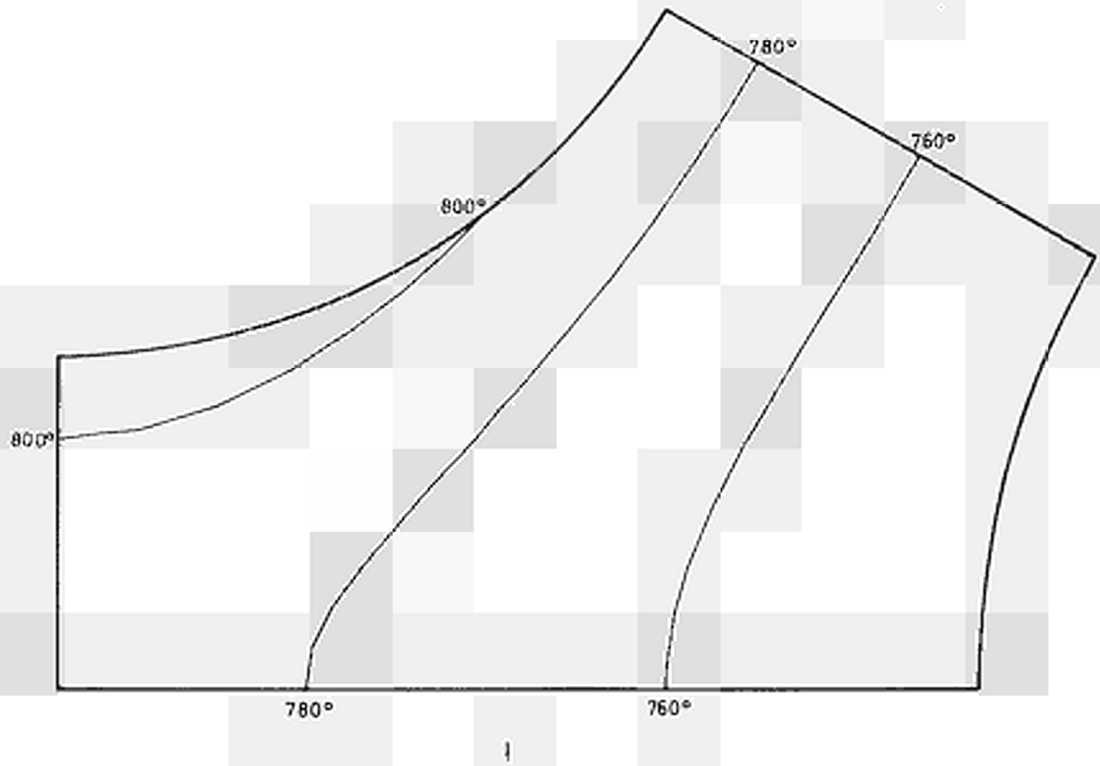

Fig. 7 Isothermal curves at t = 20 sec.

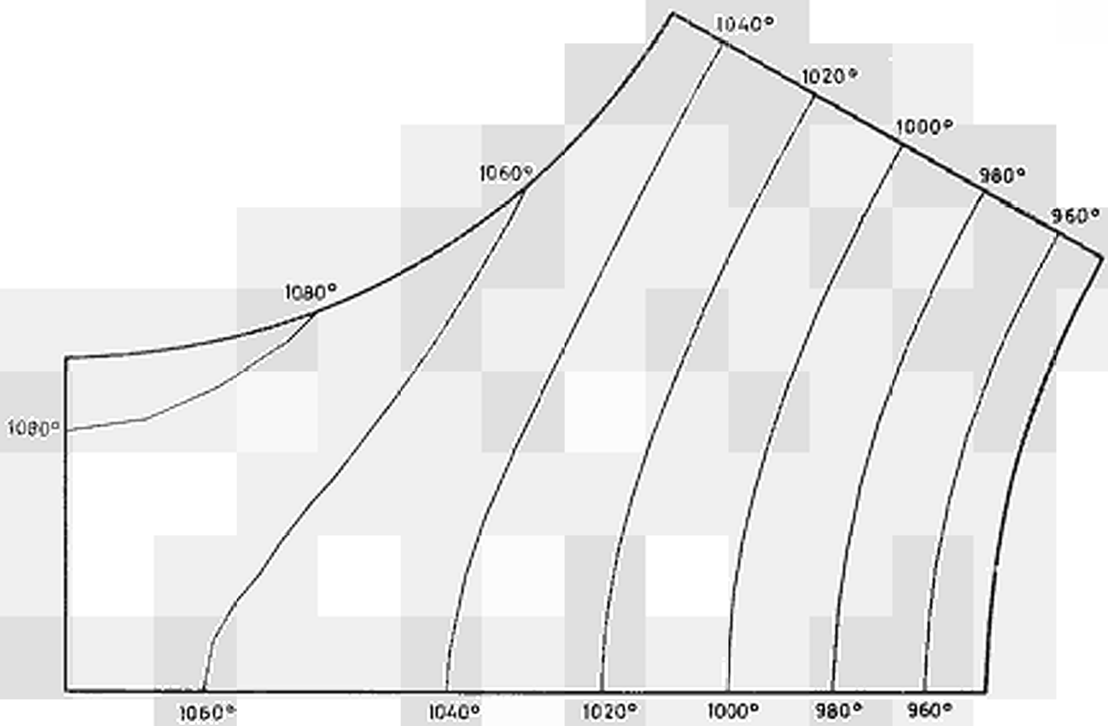

Fig. 8 Isothermal curves at t = 45 sec.

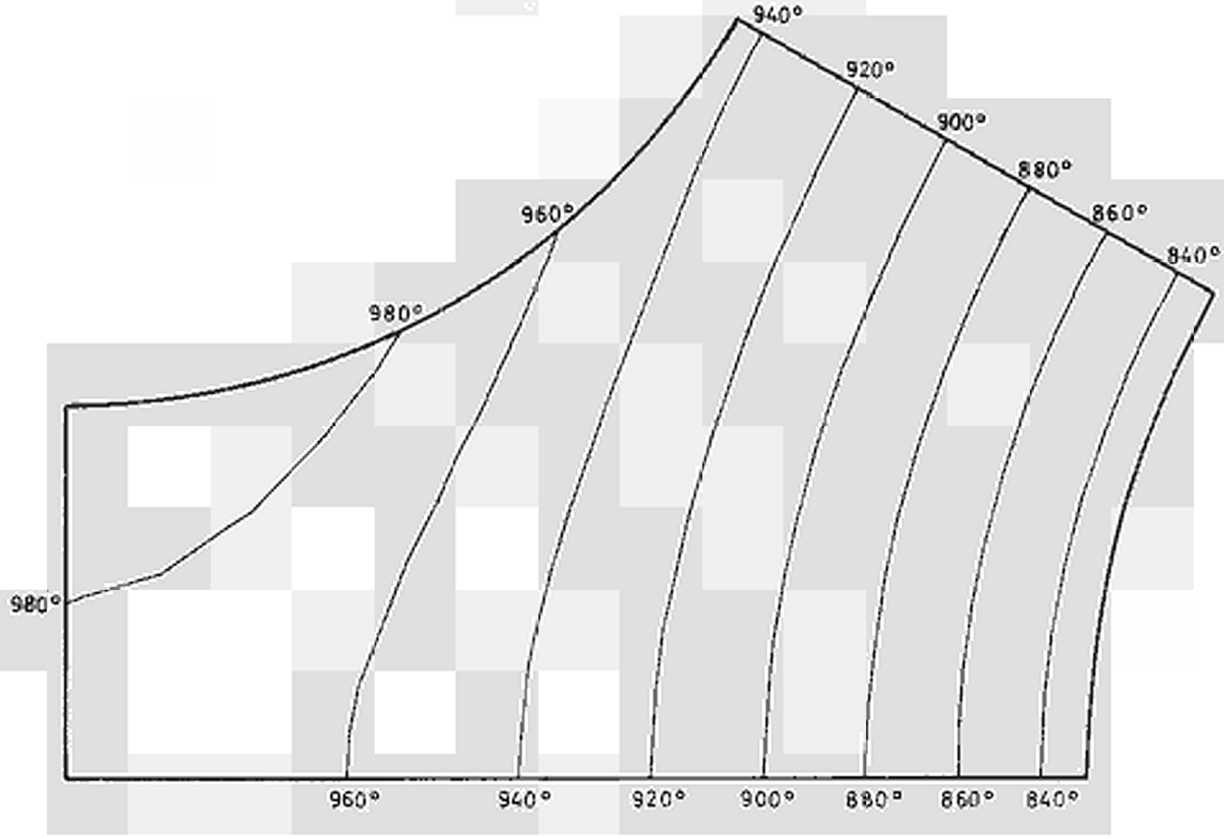

Fig. 9 Isothermal curves at t = 180 sec.

NOTICE TO THE READER

All scientific and technical reports published by the Commission of the European Communities are announced in the monthly periodical **"euro-abstracts"**. For subscription (1 year: B.Fr. 1 025,—) or free specimen copies please write to:

Office for Official Publications
of the European Communities
Boîte postale 1003
Luxembourg
(Grand-Duchy of Luxembourg)

To disseminate knowledge is to disseminate prosperity — I mean general prosperity and not individual riches — and with prosperity disappears the greater part of the evil which is our heritage from darker times.

Alfred Nobel

SALES OFFICES

The Office for Official Publications sells all documents published by the Commission of the European Communities at the addresses listed below, at the price given on cover. When ordering, specify clearly the exact reference and the title of the document.

UNITED KINGDOM

H.M. Stationery Office
P.O. Box 569
London S.E. 1 — Tel. 01-928 69 77, ext. 365

BELGIUM

Moniteur belge — Belgisch Staatsblad
Rue de Louvain 40-42 — Leuvenseweg 40-42
1000 Bruxelles — 1000 Brussel — Tel. 12 00 26
CCP 50-80 — Postgiro 50-80

Agency :
Librairie européenne — Europese Boekhandel
Rue de la Loi 244 — Wetstraat 244
1040 Bruxelles — 1040 Brussel

DENMARK

J.H. Schultz — Boghandel
Møntergade 19
DK 1116 København K — Tel. 14 11 95

FRANCE

Service de vente en France des publications des Communautés européennes — Journal officiel
26, rue Desaix — 75 732 Paris - Cédex 15e
Tel. (1) 306 51 00 — CCP Paris 23-96

GERMANY (FR)

Verlag Bundesanzeiger
5 Köln 1 — Postfach 108 006
Tel. (0221) 21 03 48
Telex: Anzeiger Bonn 08 882 595
Postscheckkonto 834 00 Köln

GRAND DUCHY OF LUXEMBOURG

Office for Official Publications of the European Communities
Boîte postale 1003 — Luxembourg
Tel. 4 79 41 — CCP 191-90
Compte courant bancaire: BIL 8-109/6003/200

IRELAND

Stationery Office — The Controlle
Beggar's Bush
Dublin 4 — Tel. 6 54 01

ITALY

Libreria dello Stato
Piazza G. Verdi 10
00198 Roma — Tel. (6) 85 08
CCP 1/2640

NETHERLANDS

Staatsdrukkerij- en uitgeverijbedrijf
Christoffel Plantijnstraat
's-Gravenhage — Tel. (070) 81 45 11
Postgiro 42 53 00

UNITED STATES OF AMERICA

European Community Information Service
2100 M Street, N.W.
Suite 707
Washington, D.C., 20 037 — Tel. 296 51 31

SWITZERLAND

Librairie Payot
6, rue Grenus
1211 Genève — Tel. 31 89 50
CCP 12-236 Genève

SWEDEN

Librairie C.E. Fritze
2, Fredsgatan
Stockholm 16
Post Giro 193, Bank Giro 73/4015

SPAIN

Libreria Mundi-Prensa
Castello 37
Madrid 1 — Tel. 275 51 31

OTHER COUNTRIES

Office for Official Publications of the European Communities
Boîte postale 1003 — Luxembourg
Tel. 4 79 41 — CCP 191-90
Compte courant bancaire: BIL 8-109/6003/200

OFFICE FOR OFFICIAL PUBLICATIONS OF THE EUROPEAN COMMUNITIES
Boîte postale 1003 — Luxembourg 6302