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In the standardisation of radioactive solutions the size of the sample is determined by weighing. These mass determinations must be very accurate in order not to be the limiting factor in the standardisation accuracy.

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The best accuracy which may be obtained with commercial balances is ± 2 to 3 μg on solid samples, when the substitution method with external calibrated weights is applied.
PERFORMANCE OF MICROBALANCES

by

J. BRULMANS, W. VAN DER EIJK and H. MORET

1972

Joint Nuclear Research Centre
Geel Establishment - Belgium
Central Bureau for Nuclear Measurements - CBNM
ABSTRACT

In the standardisation of radioactive solutions the size of the sample is determined by weighing. These mass determinations must be very accurate in order not to be the limiting factor in the standardisation accuracy.

Upon instigation of a working group on micro-weighings, created by the Bureau International des Poids et Mesures, several microbalances were tested on their performance when applying different weighing methods.

The best accuracy which may be obtained with commercial balances is ± 2 to 3 μg on solid samples, when the substitution method with external calibrated weights is applied.

KEYWORDS

MICROBALANCES
WEIGHT
ACCURACY
ERRORS

RADIOACTIVITY
SOLUTIONS
SAMPLING
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1. **Introduction**

In the calibration of radioactive solutions the mass of drops of about 20 mg must be weighed. In order to achieve the desired accuracy in the calibration, the weighing error should be kept smaller than a few microgrammes. As a result of discussions (1,2,3,4) by radionuclides metrologists on the attainable accuracy on weighings with micro-balances, a working group "Microweighings" has been founded by the Bureau International des Poids et Mesures. In this group it was decided that the CBNM laboratories should investigate the performances of some microbalances.

The performance of a balance is determined by its sensitivity, its precision of weighing, the accuracy of the mass of the built-in balance weights and the linearity of the optical scale. Furthermore, the attainable accuracy will be a function of the weighing procedures, the conditions, e.g. stability of reading, and the accuracy of weights used to calibrate the balance.

2. **Apparatus and experimental conditions**

The performances of the following micro-balances have been determined: two M5 Mettler, one M5SA Mettler, one MPR5 Sartorius.

All balances are installed in air-conditioned rooms, with the temperature kept at \((20 \pm 0.5)\)°C and the relative humidity at \((50 \pm 10)\)%. In one room the balances (one of the M5 and the M5SA balance) are placed on stone slabs supported by concrete pillars, which have no mechanical contact with the building. In the other room the balances are placed on special balance tables, designed to eliminate vibrations.

The weighings have been done with sets of weights calibrated against the CBNM standard kilogram, which itself has been
calibrated against the international kilogram in September 1968. Some of the sets of weights, used in these experiments, have been prepared at CBNM from Vachromium (80/20 Ni-Cr) with a density of 8.3447 g cm\(^{-3}\) at 20°C, as determined at CBNM. All weights of these sets are made of one piece and do not have screw threads (to avoid collection of dust).

3. Description of experiments

3.1. Calibration of the dial weights

The balance weights corresponding to the dial positions 3g (maximum weight of filled pycnometers, foil mounts etc.) to 0.01 g for the Mettler M5 balances and of 0.9 to 0.001g for the Sartorius MPR5 balance, have been calibrated 2 times by different operators using different sets of calibrated weights. Each calibration was repeated 4 times with a zero reading before and after every weighing.

3.2. Calibration of the optical scale

For mass determinations with highest accuracy it is necessary to know the accuracy of the optical scale. As the optical scale is used to interpolate between successive dial weight values, the linearity of the scale is an important factor in the final accuracy of the weighing. The linearity of the optical scales of the M5 and M5SA microbalances has been determined in all or one of the three following ways:

(a) A first method consists of a straightforward calibration by using a set of reference weights. For every mg division 5 readings were taken, with a zero reading before and after

x Vakuumschmelze, Hanau, Germany
every weighing. With each weighing the sensitivity of the optical scale between 0 and 10 mg was determined and all scale readings were corrected with this sensitivity.

(b) The second method is to measure the scale length between the mg divisions with one and the same mg weight. The advantage of this method is, that only one weight is needed, and that calibrated weights are not required. It is sufficient to adjust a piece of wire of a mass-stable material in such a way, that its mass corresponds with the scale value of 1 mg. After adjustment of the zero point, this weight is placed on the balance and the reading (which is close to the first mg-division) is taken. The weight is removed and with tare weights (e.g. short lengths of metal wire) the reading is adjusted, near to the first mg-division. By putting the weight on the pan again the distance between the first and the second mg-division is determined, etc. Each reading is repeated 4 times.

In this way the distances between divisions are obtained in arbitrary units. When the sensitivity of the scale is also determined with a 10 mg calibrated weight, the sum of all distances can be normalised and the distances expressed in mass units.

(c) A third method consists of the measurement of the distance between successive scale divisions of 100 μg with the vernier of the balance. The scale divisions are brought into view in the same way as in b. The method is very simple. As no calibrated or reference weight is involved, it provides no mass calibration. Effects, relative to, e.g., the quality of the knife-edges, do not enter into the measurement as the beam does not change its position. This method is equivalent to taking the optical scale out of the balance and performing
length calibration with mechanical-optical means, but without all the implications this would imply. Every scale division has been measured 5 times.

3.3. Precision (reproducibility) as a function of the weighing method

There are several procedures to determine the mass of an object. A generally used method consists of comparing the mass of the object with the mass of the dial weights of the balance. Before and after every weighing the zero reading of the balance has to be taken. This is called here the direct reading method.

Another practice is that of substitution weighing, when external reference weights are used. In this method the object is placed on the pan of the balance and counterbalanced with the dial weights, which are used as tare weights. After reading of the optical scale the object is removed and replaced by a set of calibrated weights, which are chosen such that the reading differs by no more than 0.5 mg. The weight of the object is then equal to the calibrated weight plus the difference in reading on the optical scale expressed in mass units (leaving the buoyancy effect out of consideration). This method does not need any zero reading, nor that the dial weights be calibrated. But a set of calibrated weights is needed.

The precision of the substitution weighing has been determined by comparing two 10 g standard weights 10 times. The dial weights have been used as tare weights only and were not changed during the successive weighings.

The precision of the direct weighing method, using the dial weights, has been determined, weighing an object of 9.999 ... g (requiring maximum manipulation of the dial weights) 10 times. Between weighings the dial weights were
set to zero in order to take the zero reading.
Values for the precision of both methods have also been obtained in the calibration of the dial weights (see 3.1.).

3.4. Sensitivity of the balances

Sensitivity has been determined with a calibrated 1 mg (substitution method) or a calibrated 10 mg weight (direct reading method).

3.5. **TABLE 1**

Survey of experiments

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration of dial weights</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Calibration of optical scale</td>
<td>a</td>
<td>a</td>
<td>xb</td>
<td>xb</td>
</tr>
<tr>
<td></td>
<td>xb</td>
<td>xb</td>
<td>c</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_x$ substitution method</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_x$ direct weighing</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1) a relative to standard weights
   b moving 1 mg along the scale
   c with vernier of balance

4. **Results**

4.1. Calibration of the dial weights of the balances

From the experiments described in 3.1. and 3.5. the corrections on the dial weights have been calculated with the method of least squares (5, 6). In the appendix the calculation of the
corrections for the M5SA balance is given as an example. The results are collected in table 2.

**TABLE 2**

*Correction (in /ug) to dial weights of balances from nominal value*

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>+ 15.6</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>+ 15.7</td>
</tr>
<tr>
<td>3</td>
<td>+ 7.7</td>
<td>+ 7.1</td>
<td>+ 7.8</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>+ 1.4</td>
<td>+ 7.5</td>
<td>- 2.5</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>+ 8.6</td>
<td>+ 4.8</td>
<td>+ 1.9</td>
<td></td>
</tr>
<tr>
<td>.9</td>
<td>- 0.5</td>
<td>0.0</td>
<td>0.0</td>
<td>+ 47.1</td>
</tr>
<tr>
<td>.8</td>
<td>+ 2.8</td>
<td>- 1.2</td>
<td>- 1.9</td>
<td>+ 36.7</td>
</tr>
<tr>
<td>.7</td>
<td>+ 1.1</td>
<td>- 4.8</td>
<td>+ 1.2</td>
<td>+ 27.0</td>
</tr>
<tr>
<td>.6</td>
<td>+ 2.7</td>
<td>- 0.3</td>
<td>- 6.7</td>
<td>+ 16.6</td>
</tr>
<tr>
<td>.5</td>
<td>+ 1.0</td>
<td>- 3.9</td>
<td>- 3.6</td>
<td>+ 6.0</td>
</tr>
<tr>
<td>.4</td>
<td>- 1.5</td>
<td>+ 3.9</td>
<td>+ 3.6</td>
<td>+ 41.1</td>
</tr>
<tr>
<td>.3</td>
<td>+ 1.8</td>
<td>+ 2.7</td>
<td>+ 1.7</td>
<td>+ 30.7</td>
</tr>
<tr>
<td>.2</td>
<td>+ 0.1</td>
<td>- 0.9</td>
<td>+ 4.8</td>
<td>+ 21.0</td>
</tr>
<tr>
<td>.1</td>
<td>+ 1.7</td>
<td>+ 3.6</td>
<td>- 3.1</td>
<td>+ 10.6</td>
</tr>
<tr>
<td>.09</td>
<td>+ 2.1</td>
<td>+ 0.2</td>
<td>0.0</td>
<td>+ 48.9</td>
</tr>
<tr>
<td>.08</td>
<td>- 1.1</td>
<td>- 1.1</td>
<td>- 7.1</td>
<td>+ 30.5</td>
</tr>
<tr>
<td>.07</td>
<td>+ 0.8</td>
<td>- 1.5</td>
<td>- 3.9</td>
<td>+ 40.5</td>
</tr>
<tr>
<td>.06</td>
<td>- 1.2</td>
<td>- 1.0</td>
<td>- 5.1</td>
<td>+ 22.5</td>
</tr>
<tr>
<td>.05</td>
<td>+ 0.7</td>
<td>- 1.4</td>
<td>- 1.2</td>
<td>+ 3.5</td>
</tr>
<tr>
<td>.04</td>
<td>+ 1.4</td>
<td>+ 1.6</td>
<td>+ 1.9</td>
<td>+ 45.4</td>
</tr>
<tr>
<td>.03</td>
<td>- 1.8</td>
<td>+ 0.3</td>
<td>- 5.2</td>
<td>+ 27.4</td>
</tr>
<tr>
<td>.02</td>
<td>+ 0.1</td>
<td>- 0.1</td>
<td>- 2.0</td>
<td>+ 37.0</td>
</tr>
<tr>
<td>.01</td>
<td>- 1.9</td>
<td>+ 0.4</td>
<td>- 3.2</td>
<td>+ 19.0</td>
</tr>
<tr>
<td>.009</td>
<td></td>
<td></td>
<td></td>
<td>+ 16.5</td>
</tr>
<tr>
<td>.008</td>
<td></td>
<td></td>
<td></td>
<td>+ 10.6</td>
</tr>
<tr>
<td>.007</td>
<td></td>
<td></td>
<td></td>
<td>+ 10.3</td>
</tr>
<tr>
<td>.006</td>
<td></td>
<td></td>
<td></td>
<td>+ 4.4</td>
</tr>
<tr>
<td>.005</td>
<td></td>
<td></td>
<td></td>
<td>+ 7.4</td>
</tr>
<tr>
<td>.004</td>
<td></td>
<td></td>
<td></td>
<td>+ 9.1</td>
</tr>
<tr>
<td>.003</td>
<td></td>
<td></td>
<td></td>
<td>+ 3.2</td>
</tr>
<tr>
<td>.002</td>
<td></td>
<td></td>
<td></td>
<td>+ 2.9</td>
</tr>
<tr>
<td>.001</td>
<td></td>
<td></td>
<td></td>
<td>+ 3.0</td>
</tr>
</tbody>
</table>

As each dial position corresponds to a combination of maximum 4 incorporated weights per decade (5, 2, 1, 1\(x\)) and as from the appendix it follows that per incorporated weight a standard deviation of about 1 \(\mu g\) must be assigned, the standard deviations on the above values vary from 1 to 2 \(\mu g\).
From table 2 the values of the maximum corrections and the corresponding readings are collected in table 3.

**TABLE 3**

Maximum corrections when using nominal values of dial weights

<table>
<thead>
<tr>
<th>Balances</th>
<th>Maximum corrections (µg)</th>
<th>Dial weight combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>M5SA (1971)</td>
<td>+ 13.5</td>
<td>1.89 g</td>
</tr>
<tr>
<td>M5 (1962)</td>
<td>+ 13.0</td>
<td>2.44 g</td>
</tr>
<tr>
<td>M5 (1963)</td>
<td>+ 22.4</td>
<td>4.24 g</td>
</tr>
<tr>
<td>MPR5 (1959)</td>
<td>+ 112.5</td>
<td>0.999 g</td>
</tr>
</tbody>
</table>

Especially in the last case appreciable errors can be made.

4.2. **Calibration of the optical scale**

In table 1 are indicated the balances of which the linearity of the optical scale has been determined and what methods have been used.

The results are given in figure 1.

For the M5SA (1971) balance it is found with method a (where the scale is compared to standard weights) that the maximum corrections are + 3 /µg and - 4 /µg. According to method b (where a 1 mg weight is moved along the scale) the biggest deviations, are found at the end of the scale, between 17 mg and 20 mg. As with method a, the uncertainty of the weighing at every point is ± 2 /µg. But in method b all errors on the single measurements will be added, whereas in method a it will be the same for every point. This explains why the shapes are not identical. The normalisation of the
distance between the scale divisions (method b) by a sensitivity determination with 10 mg introduces an uncertainty of \( \frac{1}{10} \) th of the error involved in this sensitivity determination. The measurement of the linearity of the optical scale by comparing the scale lengths with the vernier of the balance (method c) gives a similar shape as in methods b and a.

The M5 (1962) microbalance has, according to method a, maximum corrections of +12 \( \mu \text{g} \) and -13 \( \mu \text{g} \), and according to method b +11 \( \mu \text{g} \) and -15 \( \mu \text{g} \).

The linearity of the optical scale of the M5 (1963) has been determined 20 times with method b, at different times. In each series only simple readings, without sensitivity determinations, have been taken. The scale values \( R_i \) thus obtained, are reduced by a factor \( k \) to arrive at unit sensitivity at the 10 mg point:

\[
R_{ic} = k R_i \\
k = \frac{10}{\sum_{i=1}^{10} R_i}
\]

with \( R_i = \) average scale value observed between divisions \((i - 1)\) and \( i \),

\( R_{ic} = \) corrected average scale value.

As the determinations were repeated 20 times, the data points in fig. 1 (M5 1963) are reliable to within \( \pm 1.0 \) \( \mu \text{g} \). Here the maximum corrections of the optical scale are +2.5 \( \mu \text{g} \) and -4.7 \( \mu \text{g} \). As with both the M5SA (1971) and the M5 (1962) balances, the shapes obtained with methods b and c are similar it may be concluded that the errors at the different scale divisions of the balances are due to imperfections of the optical scale itself, with the exception of range 0...10 mg of balance M5 (1962), where the
deviation is probably due to wear of knife edges.

4.3. Comparison of precisions of the weighing methods

Precision tests have been done with the M5SA (1971) and M5 (1962) balances according to the procedure described in 3.3.

In table 4 the standard deviation, defined as

\[ s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}} \]

has been given

<table>
<thead>
<tr>
<th>TABLE 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precision of weighing methods</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Balance</th>
<th>Standard deviation (s_{x/\mu g})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Substitution</td>
</tr>
<tr>
<td>M5 (1962)</td>
<td>2.0</td>
</tr>
<tr>
<td>M5SA (1971)</td>
<td>1.9</td>
</tr>
</tbody>
</table>

These figures demonstrate clearly that a lower precision in weighing is obtained in direct weighings with the dial weights. The same conclusion can be drawn from weighings formally done with the M5 (1963) and the MPR5 (1959) balance, where the values found for the precisions are of the same magnitude.

4.4. Sensitivity

Throughout all experiments the sensitivities of the balances have been determined, by observing the deflection of the balance when a calibrated weight of 1 or 10 mg is added
to (or removed from) the load on the pan.
The sensitivity was not adjusted as it varies from day to day. An adjustment upsets the thermal equilibrium inside the balance case to such an extent that reliable readings are not obtainable during a considerable length of time. This period might be even one or two days.
Instead of using an external weight, also a dial weight of 1 or 10 mg may be used provided its mass has been calibrated.

5. Discussion of results

What accuracies can be expected from weighings on microbalances? It will already be clear, that the accuracy obtained depends on the method of weighing that has been used.
Therefore several cases will be treated. In table 5 the data necessary for an analysis are collected.
### TABLE 5

Summary of precisions and maximum corrections found

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Correction dial</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>weights maximum</td>
<td>+13.5 ± 2.8</td>
<td>+13.0 ± 2.6</td>
<td>+22.4 ± 2.6</td>
<td>+112.5 ± 3.5</td>
</tr>
<tr>
<td>positive (/ug)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correction dial</td>
<td>-3.4 ± 2.0</td>
<td>-6.3 ± 2.0</td>
<td>-16.3 ± 2.4</td>
<td>-3.0 ± 1.0</td>
</tr>
<tr>
<td>weights, maximum</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>negative (/ug)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correction optical scale, maximum</td>
<td>+3.0 ± 2.0</td>
<td>+12.0 ± 2.0</td>
<td>+3 ± 1</td>
<td></td>
</tr>
<tr>
<td>positive (/ug)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correction optical scale, maximum</td>
<td>-4.0 ± 2.0</td>
<td>-13.0 ± 2.0</td>
<td>-5 ± 1</td>
<td></td>
</tr>
<tr>
<td>negative (/ug)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>s direct weighing</td>
<td>3.5</td>
<td>3.9</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>χ (ug)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>s substitution</td>
<td>1.9</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>χ method (/ug)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Results from earlier experiments

From the maximum positive and maximum negative corrections, the maximum error on a differential weighing can be calculated. This has been done for the following cases.

a. Direct weighing without any corrections

As an example we assume a differential weighing on the M5SA (1971) balance at the dial weight readings 1.89 and 0.41, corresponding to the maximum deviations indicated in table 5. This would give an error of \(- (13.5 + 3.4) = - 16.9 /ug.\) From fig. 1 we take the maximum corrections on the optical scale, i.e. \(+ 3 /ug at 13 mg\) indication and \(- 4 at the reading 5 mg, resulting in an extra.*
possible error of - 7 /ug.
In total - 23.9 /ug.
The standard deviation of the corrections follows from $3 + 2$
dial weights (1.89 and 0.41) comprising in total 12 incorporated
weights in the balance. This is a standard deviation of $\sqrt{12} /ug = 3.5 /ug$.
Together with two times a standard deviation of 2 /ug on the optical
scale calibration makes a total standard deviation of $\sqrt{20} /ug = 4.5 /ug$.
The total correction is thus + 4.5 /ug, standard deviation 4.5 /ug.
In the same way the maximum corrections for a differential
weighing have been calculated for the other balances.
Evidently, if weighing without corrections is applied, errors
of equal size as the corrections in table 6 are made.

TABLE 6

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>correction (/ug)</td>
<td>23.9</td>
<td>44.3</td>
<td>46.7</td>
<td>115.5</td>
</tr>
<tr>
<td>standard deviations on corrections (/ug)</td>
<td>4.5</td>
<td>4.4</td>
<td>3.9</td>
<td>3.6</td>
</tr>
</tbody>
</table>

From table 5 it follows that on M5SA a direct weighing implies
a standard deviation of 3.5 /ug. In a differential weighing this
becomes $s = 5 /ug$. For the other balances it is rather 6 /ug.

b. Direct weighing with corrections for dial weights and optical scale.
The errors indicated in table 6 under "corrections" are not
committed, but it should be realised that the corrections have
uncertainties of about 4 to 5 /ug standard deviation.

c. Differential weighing of samples of less than 20 mg weight
(reading of the optical scale only). In any case the sensitivity
must be determined with a weight of 10 or 20 mg, otherwise gross errors are to be expected. 
But still, with an uncalibrated optical scale maximum errors ranging from 7 to 25 /μg can be committed (fig. 1). With a calibrated scale, standard deviations of 2 /μg in the calibration and 2 /μg in the actual readings (single observations) are applicable, resulting in a total standard deviation of about 3 /μg for a single weighing and about 4 /μg for differential weighing.

d. Substitution weighing without calibration of the optical scale. Here the weight of the sample can always be approached to within 0.5 mg with combinations of calibrated weights. So the error due to scale errors can be kept below half of the difference in scale errors between two successive mg-divisions. For the Mettler balances tested this corresponds to a maximum of $1/2 \times 3.5 /\mu g = 2 /\mu g$. The standard deviation of a single reading is 2 /μg. As the dial weights are used a tare weights, their calibration is not relevant. The total standard deviation is 3 /μg for a single weighing and 4 /μg for a differential weighing.

e. Substitution weighing with a calibrated optical scale. Calibration of the optical scale would eliminate the error mentioned in d, but the uncertainty on the calibration introduces a standard deviation of the same size.
6. **Conclusions**

When the balances are used as proposed by the manufacturer (i.e. direct weighing) the standard deviation is 3 to 4 \( \mu g \) in the worst case. This standard deviation applies to a single observation including zero reading.

Non-linearity of the optical scales and the adjustment of the built-in weights are such that errors far exceeding the standard deviation are introduced. Calibration of optical scales and weights considerably improves the accuracy but the calibration process itself introduces several \( \mu g \) uncertainty, such that accuracies of 4 \( \mu g \) (expressed as standard deviation) are about the best which can be obtained in direct weighing, if more than 4 readings are taken.

The best results are obtained if the substitution method is applied, i.e. comparison of the weight of the sample with the weight of a set of external calibrated weights. The dial weights are used as tare weights. The set of calibrated weights is chosen such that both readings are within 0.5 mg.

Mass determinations are then affected by a statistical error of 2 \( \mu g \) standard deviation (single observation). By repeated measurements this is easily reduced to about 1 \( \mu g \) (4 observations). To this error an uncertainty due to imperfections of the optical scale must be added. This error is 2 \( \mu g \) in the very worst case. A total error of 2 to 3 \( \mu g \) is typical for the substitution weighing.

However, the method requires a set of carefully calibrated (external) weights. The calibration error of the set of reference weights is not considered here as it applies to both weighing methods. The authors are fully aware of the fact that the availability and the reliability of a set of reference weights are problems which are not easily solved.

They feel, however, that dealing with this subject is outside the scope of this report, but will be dealt with in a subsequent paper.
Acknowledgements

The authors express their appreciation for the painstaking care with which the experiments have been done by Mrs. Dijckmans and Messrs. Hendrickx and Zehner.

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2. Y. Le Gallic, La micropesée, source importante d'erreur en métrologie d'activité, CEA R-4169, CEN-Saclay, (1971).


Appendix

Direction calibration of the dialweights - corrections in /ug

METTLER M5SA - 1972

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<th>DIAL (in g)</th>
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<th>2.</th>
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### Calculated corrections for the (.5), (.2), (.1), (.1*) dialweights

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<td>.4 = d =</td>
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<td>- 1.5</td>
</tr>
<tr>
<td>.5 = e =</td>
<td>+ 1.0</td>
<td>+ 1.0</td>
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<td>.6 = f =</td>
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<td>+ 2.7</td>
</tr>
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<td>.7 = g =</td>
<td>+ 1.4</td>
<td>+ 1.1</td>
</tr>
<tr>
<td>.8 = h =</td>
<td>+ 2.5</td>
<td>+ 2.8</td>
</tr>
<tr>
<td>.9 = i =</td>
<td>- 0.6</td>
<td>- 0.5</td>
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</table>

\[
.5 = \frac{1}{19} \left( -2a - 2b - 4c - 3d + 6e + 4f + 4g + 2h + 3i \right) = + 1.0
\]
\[
.2 = \frac{1}{38} \left( -5a + 14b + 9c + 2d - 4e - 9f + 10g + 5h - 2i \right) = + 0.1
\]
\[
.1 = \frac{1}{38} \left( +14a - 5b + 9c + 2d - 4e - 9f - 9g + 5h - 2i \right) = + 1.7
\]
\[
.1* = \frac{1}{38} \left( -7a - 7b - 14c + 18d + 2e - 5f - 5g - 12h + 20i \right) = - 3.3
\]

### Calculated corrections for the (.05), (.02), (.01), (.01*) dialweights

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<td>.06 = f =</td>
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<td>.07 = g =</td>
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<tr>
<td>.08 = h =</td>
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<td>- 1.1</td>
</tr>
<tr>
<td>.09 = i =</td>
<td>+ 2.0</td>
<td>+ 2.1</td>
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</table>

\[
.05 = \frac{1}{19} \left( -2a - 2b - 4c - 3d + 6e + 4f + 4g + 2h + 3i \right) = + 0.7
\]
\[
.02 = \frac{1}{38} \left( -5a + 14b + 9c + 2d - 4e - 9f + 10g + 5h - 2i \right) = + 0.1
\]
\[
.01* = \frac{1}{38} \left( +14a - 5b + 9c + 2d - 4e - 9f - 9g + 5h - 2i \right) = - 1.9
\]
\[
.01* = \frac{1}{38} \left( -7a - 7b - 14c + 18d + 2e - 5f - 5g - 12h + 20i \right) = + 3.2
\]

\[
s = \pm \sqrt{\frac{\sum r^2_0.01}{10} + \frac{r^2_0.1}{10}} = \pm 0.6 \text{ /ug}
\]
Fig. 1 Corrections on optical scale

- method a, using reference weights
- method b, moving a 1 mg weight along the scale
- method c, using vernier of balance
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