PULSED SOURCE EXPERIMENTS WITH HEAVY WATER MODERATED NATURAL URANIUM LATTICES IN EXPO AND ECO

by

G. BIRKHOFF, L. BONDAR, W. HAGE and J. LEY

1972

Joint Nuclear Research Centre
Ispra Establishment – Italy
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October 1972

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The paper contains a survey of various experimental and analytical methods applied along with the results of reactivity and generation time measurements in the negative reactivity range between about 0.17 and 40 $. The results are confronted with two group calculations, and systematic deviations are
discussed. For the far subcritical cases a considerable deviation of about 5% of the theoretical reactivity values remains unexplained.

This paper is a publication on an internal working document from January 1968.
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ABSTRACT

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KEYWORDS

ECO MEASURED VALUES
SUBCRITICAL ASSEMBLIES ANALYTICAL SOLUTION
PULSED NEUTRON TECHNIQUES COMPUTER CALCULATIONS
REACTIVITY
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1. DEFINITION OF THE PROBLEM

The time behaviour of the fundamental mode of the neutron density in a bare reactor after the injection of a neutron burst (δ - pulse) may be described by a sum of exponentials

\[ n(t) = \sum_{i=0}^{m} A_i e^{-\alpha_i t} \]  

(1)

\[ a_i = \frac{\beta_0}{\lambda} + \frac{1}{\lambda} \sum_{j=0}^{m} \frac{\beta_j \lambda_j}{a_i - \lambda_j} = a_p + \frac{1}{\lambda} \sum_{j=0}^{m} \frac{\beta_j \lambda_j}{a_i - \lambda_j} \]  

(2)

\[ A_i = (1 + \frac{1}{\lambda} \frac{1}{(1+p)}) \sum_{j=0}^{m} \frac{\beta_j \lambda_j}{(a_i + \lambda_j)^2} - 1 \]  

(3)

This relation is obtained from the kinetic diffusion theory with the assumption of no retardation between the neutron populations of different energies, which is rather true for not too far subcriticality. The roots \( a_i \) of the so-called inhour equation (2) are all real and positive for the subcritical or delayed critical state (\( p < 0 \)). The numerically greatest root \( a_0 \) is determined mainly by the prompt neutron decay constant \( a_p \), while the remaining roots are closely related to the decay constants of the delayed neutron precursors.

In the case of a reactor of a long generation time, as we are dealing with, the contribution of the delayed neutrons to the decay constant \( a_0 \) is negligible only for reactivities greater than about 7 %. It becomes however, significant near critical.

The variation of the neutron density with time, as described by (1) (2) (3) is true only for a fundamental mode neutron source distribution. In practice we are using a point source, which might be represented by a linear superposition of the fundamental mode and an infinite number of harmonics.
The decay of the prompt neutron harmonics near critical is much faster than that of the fundamental mode. By a proper source-detector geometry it is possible to suppress certain important low order harmonics. After a certain waiting time only the fundamental mode of the prompt neutrons predominates. The situation becomes less favourable the more subcritical the reactor becomes.

In the case of a reflected reactor, as it is the ECO, the dynamic eigenfunctions differ from the static eigenfunctions. But the difference is rather small, as shown by FRAUDE (1), for a thin reflector like in ECO and might be neglected.

A special problem is the separation of the prompt neutrons from the delayed neutrons. The contribution of the delayed neutrons to the time variation of the neutron density is judged by the comparison of the amplitude and decay time ratios $A_0/A_1$ and $a_0/a_1$ respectively. For far subcriticality the ratios are great enough permitting to treat the delayed neutron contribution during the prompt decay as a constant. Near critical however, one must allow for the decay of the delayed neutrons during the prompt decay. The limit for the two situations can be set at the reactivity of about 5%.

In an actual pulsed source experiment it is necessary to repeat many times the pulsing of the reactor due to limitations in the source strength and the resolving time of the recording system. For a periodic pulsing at a repetition rate $R = 1/T$ with uniform $\delta$ bursts we get

$$n(t + N \cdot T) = \sum_{i=0}^{m} A_i \cdot e^{-\alpha_i t} \cdot \left( \frac{1 - \frac{1}{1-e^{-\alpha_i T}}}{1-e^{-\alpha_i T}} \right)^{(N-1)T}$$  \( (4) \)

The repetition rate is set as to allow for a complete decay of the prompt neutrons during one cycle.

With $T \gg \frac{1}{\alpha_0}$ equation (4) may be written
\[ n(t + N.T) = A_0 e^{-\alpha_0 t} + \sum_{i=1}^{m} A_i' e^{-\alpha_i t} \]  

(5)

with

\[ A_i' = A_i \frac{-\alpha_i (N-1)T}{1 - e^{-\alpha_i T}} \]  

(5a)

2. EXPERIMENTAL METHOD

A schematic of the pulsed source experiment is shown in Fig. 1, which represents a classical arrangement. As a pulsed neutron source the generator PHILIPS PM 5302/1 - with a maximum yield of about \(3 \times 10^{11}\) n/sec at a duty cycle of \(3.10^{-4}\) was used. Starting the timer \(T_i\), the multichannel scaler MCS (TMC, CN 110 A with plug in unit 212) stores in its first channel the background counts of the counting system. Next the neutron source generator is triggered. After a setable waiting time the registering of the counts per time intervall begins.

With the next trigger signal from the timer the cycle repeats and so on. The scaler SC serves for the counting of the inherent background. The following data recording procedure was generally followed.

1. Counting of the inherent background (neutron generator under high tension)
2. Start pulsing and recording
3. Stop pulsing after preset number of cycles but recording of the delayed neutron tail for about 15 min
4. Counting of the inherent background.

This data recording procedure delivers a counting rate distribution as described by equation (4) plus the higher harmonic contributions. It does not depend on the uniformity of the neutron bursts /2/7.

3. ANALYSIS METHODS

The analysis was generally restricted to the harmonic free part of the decay curve. Two types of analysis methods were applied. One is based on the determination of the prompt neutron decay constant \(\alpha_p\)-method)
the other on the evaluation of the prompt neutron and delayed neutron multiplication (integral method). The first method delivers

$$\frac{\rho}{\beta} = \alpha_p \frac{k \beta}{\lambda} - 1$$

where $\frac{k \beta}{\lambda}$ must be determined by a calibration measurement with known reactivity, for instance at $\rho = 0$.
The second method gives $\frac{\rho}{\beta}$ directly.

3.1 $\alpha_p$ Methods

3.1.1 Method I (SIMMONS, KING [37])

Conditioned by $\alpha >> \alpha_i$; $A >> A_i$; $A_o >> 1/T >> \alpha_i$

applied for $\frac{\rho}{\beta} \approx 7 \beta$ with $\lambda \approx 10^{-3}$ sec

The inhour equation is well approximated by

$$a_o \approx \frac{\beta + \rho}{\lambda} = \frac{1 - k (1-\beta)}{\lambda . k} = \alpha_p$$

$$A_o \approx \frac{A_p}{\lambda}$$

$$\sum_{i=1}^{m} A_i e^{\alpha_i t} = \text{constant for } 0 \leq t \leq T$$

$$n(t) \approx A_p e^{-\alpha t} + \text{const.}$$

$\alpha_p$ is derived from a least squares fit of (9) on the harmonic free part of the experimental decay curve ($t \approx t_w = \text{waiting time}$).
The check on the purity of prompt harmonics is made by varying the waiting time. In order to determine the reactivity in units of dollar a calibration measurement for the evaluation of the reduced generation time \( \frac{\rho}{\beta} \) is needed at known reactivity.

\[
\frac{\rho}{\beta} = \rho \cdot \frac{A}{\beta} - 1
\]  

(10)

3.1.2 Method II (FRAUDE / \( \int \int \int \))

Conditioned by \( \alpha T >> 1 \)

applied for \( \frac{\rho}{\beta} \leq 7 \) with \( \Lambda = 10^{-3} \) sec.

In this case the full expression for \( \alpha_0 \) must be taken

\[
\alpha_0 = \frac{\beta}{\Lambda} (1 + \frac{\rho}{\beta} + \frac{1}{\beta} \sum_{i=1}^{m} \frac{\beta_i - \lambda_i}{\alpha_i - \lambda_i}) = \rho + \delta
\]  

(11)

\[
\frac{\rho}{\beta} = \rho_0 \frac{\Lambda}{\beta} - 1 + \frac{1}{\beta} \sum_{i=1}^{m} \frac{\beta_i - \lambda_i}{\alpha_i - \lambda_i}
\]  

(12)

\( \frac{\rho}{\beta} \) is found by the following iteration process.

From the measurement we have

\[
n(t) = C \cdot \left\{ \frac{-\alpha_0 t}{1 - \alpha_0} + \sum_{i=1}^{m} A_i e^{-i\lambda_i t} \cdot \frac{l-e^{-\alpha_i(N-1)T}}{1-e^{-\alpha_i T}} \right\}
\]  

(13)

\[
n(T) = C \cdot \sum_{i=1}^{m} A_i e^{-\alpha_i T} \cdot \frac{l-e^{-\alpha_i(N-1)T}}{1-e^{-\alpha_i T}}
\]  

(14)

With an initial guess of \( \left( \frac{\rho}{\beta} \right)_0 \) the roots of the inhour equation \( \alpha_i \) and the \( A_i \) are calculated for \( i = 1, 2, \ldots, m \). The normalisation factor \( C \) is obtained from (14).
A least square fit of \( n_1(t) = C \left( A_0 e^{-\alpha_0 t} + \sum_{i=1}^{m} A_i e^{-\alpha_i t} \right) \), \( t > t_w \) gives a first improvement of \( \alpha_0 \).

With the new \( \alpha_0 \), a new \( \left( \frac{\beta}{\lambda} \right)^1 \) is calculated by the equation (12).

The iteration goes on up to no improvement of \( \alpha_0 \) and \( \left( \frac{\beta}{\lambda} \right)^1 \) is achieved by further steps. Near critical, the correction of \( \alpha_0 \) due to the delayed neutrons becomes important and the result is sensibly depending on the choice of the \( \lambda, \beta \), which are not well known mainly for the photo-neutrons.

The values actually chosen are taken from Ref. (47) corrected by the best fitting efficiency coefficient for the photoneutron yield (see Table 1). \( \frac{\beta}{\lambda} \) must be defined by a calibration measurement at known criticality.

A check on the good choice of the \( \beta, \lambda \) is to compare the normalization factor

\[
C = \frac{n(T)}{\sum_{i=1}^{m} A_i e^{-\alpha_i T}}
\]  

with the definition formula

\[
C \cdot \sum_{i=0}^{m} A_i = C \quad \text{because} \quad \sum_{i=0}^{m} A_i = 1
\]

taking the fitted \( C.A_0 \) value and the calculated \( C \) and \( A_i \) values \( (i = 1, 2, \ldots, m) \).

This check permits also to estimate the contamination of the decay curve by prompt and delayed neutron harmonics.
3.2 Integral Methods

3.2.1 Method III (SJÖSTRAND - GOZANI ∫₅/₇ ∫₆/₇)

Conditioned by

\[ a_o \gg a_i \quad i = 1, 2, \ldots, m \]

\[ A_o \gg A_i \]

\[ a_o \gg 1/T \gg a_i \]

applied for \( \frac{B}{B} \gg \frac{7}{7} \)

The total multiplication factor of a fundamental mode neutron is proportional to

\[ M_T = \frac{1}{1-k} \]  \hspace{1cm} (17)

The prompt multiplication is proportional to

\[ M_p = \frac{1}{1-k_p} = \frac{1}{1-k(1-B)} \] \hspace{1cm} (18)

From (17) and (18) it follows

\[ \frac{M_p}{M_T - M_p} = \frac{1-k}{kB} = \frac{B}{B} \] \hspace{1cm} (19)

SJÖSTRAND ∫₅/₇ has shown that for a reactor in the equilibrium state of the delayed neutrons, the integral over the fundamental mode prompt neutron density

\[ n_{o_p}(t) = A_p e^{\frac{-a_p t}{p}} \]

extended over the period T is proportional to the prompt multiplication \( M_p \).

The integral over the total fundamental mode density

\[ n_o(t) = \sum_{i=0}^{m} A_i e^{\frac{-a_i t}{1}} \]

is proportional to the total multiplication factor \( M_T \).
The separation of the prompt neutron density \( n_p(t) \) from the total neutron density \( n(t) \) is quite easy with the above condition \( \alpha_0 \gg \alpha_1 \) and \( A_0 \gg A_1 \) because the total neutron density may be taken as a sum of the prompt neutron density and a constant part \( B \) forming the delayed neutron contribution. We get from (20)

\[
\frac{\rho}{\beta} = \frac{A_p}{B_p} \left( \frac{1}{e^{\frac{\alpha_0}{T}} - e^{\frac{\alpha_1}{T}}} \right) \tag{21}
\]

Following the method of GOZANI the fundamental mode distribution \( n_{op}(t) \) is obtained from a least squares fit of the harmonic free part of the decay curve to

\[
n_{op}(t) = A_p e^{-\frac{\alpha_0}{T} t} \quad (t \gg t_w) \tag{22}
\]

In this method it is assumed a fundamental mode distribution of the delayed neutrons. With a reasonable source detector geometry this is a rather good approximation because the delayed neutron density harmonics are much less excited than the prompt neutron harmonics. On the other hand, they may play an important role if the source density distribution consists mainly of harmonics and the decay of harmonics is not much faster than the fundamental mode decay. With other words, if the total neutron density distribution during a pulsing period \( T \) is mainly consisting of higher harmonics. In such cases the results depend strongly on the source-detector geometry.

For a finite neutron burst duration \( d \) equation (19) must be replaced by
\[ \frac{\alpha}{\beta} = \left( \frac{A_p e^{-\alpha d/d_p}}{1-e^{-\alpha d/d_p}} \right) (B T)^{-1} \]  

(normalization to a \( \delta \)-pulse).

### 3.2.2 Method IV (BRONNER-DIO-SCHLOSSER \( \sqrt{7}J \))

Conditioned by \( \alpha \gg 1/T \)

\[ \frac{\alpha}{\beta} \ll 7 \] 

BRONNER et al. \( \sqrt{7}J \) have used a method which permits a simple separation of the prompt neutrons from the delayed neutrons even if the prompt decay is not much faster than the delayed neutron decay. The idea is to make a linear approximation of the exponential decay of the terms 

\[ A_i e^{-\alpha_i t}, \quad i = 1, 2, \ldots, m, \quad 0 \leq t \leq T \]

\[ n(t) = A e^{-\alpha t} + \sum_{i=1}^{m} \frac{A_i e^{-\alpha_i t}}{1-e^{-\alpha_i t}} + \frac{1}{\alpha} - \frac{1}{\beta} (1-e^{-\alpha t}) \]  

\[ n(t) \approx A e^{-\alpha t} + B(0) + \text{s.t} = A e^{-\alpha t} + B(0) - \frac{B(0) - B(T)}{T} t \]  

The quantities \( A, \alpha, \) and \( B(0) \) are found by a least squares fit of the harmonic free total neutron density to (24). \( (B(T) \) is measured).

\[ \alpha = \frac{\lambda+\rho}{\beta} = \alpha_0 - \frac{1}{\Lambda} \sum_{i=1}^{m} \frac{B_i \lambda_i}{\alpha_0 - \lambda_i} = \alpha_0 - \delta \]  

Because the difference between \( B(0) \) and \( B(T) \) is due to the jump of the prompt neutrons at time 0, we have

\[ A_p = A_0 + (B(0) - B(T)) \]
With (25) (24) (26) it follows from (20)

\[ \frac{\alpha}{\beta} = \frac{A_B}{A_P} \cdot \left( \frac{A_o + B_o - B(T)}{2} T - \frac{A_B}{A_P} \right)^{-1} \]  

(27)

In the case of a finite burst duration \( d \) equation (27) must be replaced by

\[
\frac{\alpha}{\beta} = \left\{ \begin{array}{l}
\frac{-\alpha d}{A_P} L_i d \\
1-e^{-\alpha d} \end{array} \right. \cdot \left[ \begin{array}{l}
\frac{-\alpha d}{A_o d} + \frac{B(o)-B(T)}{2} T - \frac{B(o)-B(T)}{2} d - \frac{A_e}{A_P} \frac{-\alpha d}{1-e^{-\alpha d}} \end{array} \right]^{-1}
\]

(normalization to a \( \delta \)-burst)  

(28)

3.2.3 Method \( V \) (GARELIS-RUSSEL-\( \frac{k_8}{\sigma} \) method \( \sqrt{8.7} \))

Conditioned by

\[ \frac{\alpha_o}{\alpha} >> 1 \]

\[ \frac{\alpha_o}{\alpha_p} \ll \frac{A_o}{A_p} \]

\[ \alpha_o >> \frac{1}{T} >> \frac{1}{\alpha_i} \]

applied for \( \frac{\alpha}{\beta} \gg 1 \$ \)

The so-called \( \frac{k_8}{\sigma} \) method, introduced by GARELIS et al. \( \sqrt{8.7} \) \( \sqrt{27} \) is derived from the one group bare reactor kinetic equations where

\[ S \equiv \frac{k_{\infty}}{\sigma} = \frac{k_8}{\sigma_{\infty}} = \frac{\beta}{\sigma_c} \]  

(29)

One finds the following relation

\[ \int_{0}^{\infty} n_p(t) e^{st} dt - \int_{0}^{\infty} n_p(t) dt = BT \]  

(30)

Equation (30) is valid also for the presence of prompt and delayed neutron harmonics.

The quantity \( \left( \frac{k_8}{\sigma} \right) \equiv S \) is found by an iteration process of equation (30)
where \( n_p(t) \) is the measured total neutron density minus the constant delayed neutron density \( B \).

In order to determine the reactivity \( \frac{\rho}{\beta} \) the fundamental mode prompt neutron decay constant \( \alpha_p \) must be determined too as described in 3.1.

\[
\frac{\rho}{\beta} = \frac{\alpha_p}{\beta} - 1
\]  

(31)

4. DELAYED NEUTRON DATA

The effective fission delayed neutron yield \( \beta_i \) has been evaluated by taking into account the fast fission in \( U^{238} \) through the formula

\[
\beta_i = \frac{\beta_i^8 \nu^8 \delta_8 + \beta_i^5}{\nu^5 \delta_8 + 1}
\]  

(32)

\( \delta_8 \) was determined experimentally by Ref.\( ^{15} \). Six fission delayed neutron groups are taken and the decay constants are those of the \( U^{235} \) fission neutrons. Efficiency coefficients of the photoneutrons stem from Ref.\( ^{16} \), but as pointed out in section 3.1.2 an empirical constant correction factor was applied for sake of best fitting the experimental decay curves near critical. Six photoneutron groups are regarded. A further correction due to the energy difference between prompt and delayed neutrons was applied

\[
\frac{k_p}{k_D} = e^{-\beta^2(t_p - t_D)}
\]

5. THE COMPUTER PROGRAM

The analysis methods as described in section 3. (methods I, II, III, IV and V) have been coded in FORTRAN IV language for the data processing at the IBM 7090 machine. A block diagram of this computer program is given in Fig. 2.
6. EXPERIMENTAL SET-UP

6.1 EXPO Experiments

EXPO is a far subcritical bare reactor $\sqrt{10^{-7}}$. A schematic diagram of the EXPO facility is shown in Fig. 3. The facility was fueled with ECO reference elements (see Fig. 7). With the source location at the wall of the tank in the core midplane $H/2$ ($H \Rightarrow D_2O$ level + extrapolation distances) only the odd numbered axial harmonics are excited as indicated in Fig. 4.

$$(-a_{0,m} e^{-a_{0,m}t} cos \frac{n\pi}{H} z)$$

The decay constants of the radial harmonics $A_{o,m}(t), j_o(\mu_m, r)$ with

$$A_{o,m}(t) = A_{o,m} e^{-a_{o,m}t}$$

are a priori much higher compared with the axial harmonics due to the geometry of the tank ($R << H$). If we place the detector at

- $Z = H/3$ we suppress the 3rd harmonic
- $Z = 2/5 H$ we suppress the 5th harmonic
- putting the detector in the axis of the tank we suppress moreover all azimuthal harmonics.

The neutron detector utilized was a Li$^6$ glass scintillator with a photomultiplier of 19 mm $\varnothing$ (type 152 AVP) which was mounted on a rigid extension of 13 mm $\varnothing$ Al tube. It could be inserted into a Al guiding tube of 28 mm $\varnothing$ and 1 mm wall thickness running into the tank. All desired axial and radial positions of the detector could be realized. A fast tunnel diode discriminator $^+$ was fed directly with the PMP pulses. The dead time of the total counting equipment is about $10^{-7}$ sec permitting a peak counting rate of about $5 \times 10^5$ c/sec without significant dead time correction. By this way it was possible to operate the pulsed source at its maximum yield and to achieve a high signal to inherent background ratio which is important for the accuracy of the integral methods (III, IV, V).

$^+$The electronic equipment was designed and constructed by the Electronics Service.
6.2 ECO Experiments

The critical facility ECO is described in Ref. 11. A schematic view is shown in Fig. 5. The only accessible positioning of the pulsed source was a horizontal channel in the bottom reflector as indicated in the Fig. 5. This position of the pulsed source is very unfavourable from the harmonics excitation point of view. It is of course possible to arrange for a better source position with certain modification on the pulsed source or reactor. But it was found rather inconvenient for the purpose of the experiments which was limited to the study of the method itself.

The detector used was partly a Li\textsuperscript{6} glass scintillator as described in 5.1 or a BF\textsubscript{3} proportional counter (28EB40 of 20th Century) depending on the neutron peak flux at the detector position.

7. RESULTS

7.1 EXPO Measurements

Two types of lattices were investigated in the EXPO facility.

- U-19-12/OMPH coolant fuel elements
- U/19-12/Diphyl coolant fuel elements.

With the U-19-12/OMPH fuel elements a series of measurements was made with a lattice of 26.6 cm square pitch completely filling the tank cross section (bare system).

Regarding the assumptions of the GARELIS (V) and the SJÖSTRAND (III) methods these are quite well fulfilled in that case. In particular have been checked the dependence of the results on the detector position and the duration of the neutron burst for two different core heights H of 260 cm and 230 cm.

In Table 2 the results of these measurements are summarized and compared with homogeneous two group calculations 12.

In Table 3 are given the results of the measurements with the U-19-12/diphyll fuel elements. The measurements have been done with two full core loadings of square lattices with 18.0 cm and 26.6 cm.

The theoretical \( \frac{D}{B} \) values are derived from the two group formula:
\[ \frac{\beta}{\beta} = \frac{(1 + L^2 \beta^2)(1 + \tau \beta^2) - k_{\infty}}{\beta k_{\infty}} \times \text{(33)} \]

From the formula:

\[ S = \frac{k}{\lambda} = \frac{k_{\infty}^{\beta}}{\lambda (1 + \tau \beta^2)} \times \text{(34)} \]

the generation time has been computed. \( L^2, \tau, k_{\infty} \) and \( \lambda_0 \) are calculated by the lattice code PINOCCHIO \( \text{\textregistered}12 \). The given error limits are due to the estimated error of \( \beta (\Delta \beta = \pm 5\%) \) and the uncertainty of the geometrical buckling \( \Delta \beta^2_{\rho_0} = \pm 0.1 \text{ m}^{-2} \).

The quoted error limits of the experimental results represent the linear term of the Taylor series of the \( \left( \frac{\beta}{\beta} \right) \)-perturbed formulae (21) and (31).

\[ \Delta \left( \frac{\beta}{\beta} \right)_{\text{III}} = \frac{1}{\frac{\alpha}{\beta} \rho_T} \left( \Delta \alpha_p + \frac{\Delta \beta}{\beta} \Delta \beta_p \right) \times \text{(35)} \]

\[ \Delta \left( \frac{\beta}{\beta} \right)_V = \frac{1}{S} \left( \Delta \alpha_p + \frac{\alpha}{S} \right) \Delta S \times \text{(36)} \]

\( \Delta A_p \) and \( \Delta \alpha_p \) are estimated from results of exponential fits on experimental curves with different waiting times \( t \).

\( \Delta B \) is the standard deviation of the measured delayed neutron density. The value of \( \Delta S \) is obtained by evaluating the quantity \( S = \frac{k_{\infty}^{\beta}}{\lambda} \) with the delayed neutron density varied within the error limits \( \pm \Delta B \). From the results we conclude:

a) there is no clear influence on the results by the pulse duration between 7 \( \mu \text{sec} \) and 50 \( \mu \text{sec} \).

b) the results of the GOZANI method (III) are more sensitive to the harmonics than the GARELIS method (V). Positive harmonics tend to reduce the reactivity value obtained from method III, while negative harmonics do the contrary. No clear effect was observed for the GARELIS method (V).
c) in the most favoured counter position which delivers the smallest harmonics effect, the results of both methods agree within their error limits but method III gives systematically little bit higher values.

d) the measured reactivity values deviate by about 15% from the calculated values using homogeneous two group theory with PINOCCHIO core parameters. This discrepancy, which appears for the ECO experiments too, is discussed below in section 7.3

7.2 ECO Measurements

Determination of prompt neutron decay constant at delayed critical

The prompt neutron decay constant at delayed critical \( \alpha_{pc} = \frac{6}{\Lambda} \) was determined from the measurement of \( \alpha_0 \) as a function of the \( D_2O \) level \( H \) with \( H < H_{crit} \). \( \alpha_p \) is obtained from eq. (25). As the correction term

\[
\frac{1}{\Lambda} \sum_{i=1}^{m} \frac{\beta_i \lambda_i}{\alpha_0 - \lambda_i}
\]

is rather small even near critical the precision of the \( \lambda, \beta_i, \lambda_i \) need not to be very high. \( \Lambda \) is defined with sufficient accuracy by the integral method (IV) (see 3.2).

In the one group approximation \( \alpha_p \) is a linear function of \( \frac{1}{\Lambda} \) while in the two group theory higher order terms of \( \frac{1}{\Lambda^2} \) appear. In all experimental cases, however, the higher order terms \( \frac{1}{\Lambda} \) are negligible.

The effective core height \( \bar{H} = H' + \Delta H \), measured \( D_2O \) level

\( \Delta H \rightarrow \) correction due to the calibration of the \( D_2O \) level meter, the linear extrapolation distances and the bottom reflector saving was evaluated from the relation

\[
\frac{1}{(H'_{crit} + \Delta H)^2} = \frac{\gamma^2}{\pi^2} = \frac{1}{H^2_{crit}}
\]

(36)

where \( \gamma^2 \) is the axial buckling measured in a flux mapping experiment \( \gamma_{13} \).
In Fig. 6 the prompt decay constant is plotted against \( \frac{1}{\tau} \) from which \( \alpha_{pc} \) is obtained. The reactivity values for different \( D_2O \) level are derived from the "two group corrected" formula.

\[
\frac{\beta}{\beta_{pc}} = \frac{\alpha}{\alpha_{pc}} - 1 \quad (37)
\]

\[
\alpha_{pc} = \alpha_{pc} \cdot \frac{1+\tau}{1+\tau} \frac{B_c^2}{B_{go}^2} = \frac{\beta}{\Lambda} \quad (38)
\]

with \( \frac{B_c^2}{B_{go}^2} = \frac{\pi^2}{\mu^2} + \mu^2 \), \( \frac{B_c^2}{B_{go}^2} = \frac{\pi^2}{H_{crit}^2} + \mu^2 \)

\( \mu^2 \) is the experimentally determined radial buckling \( \sqrt{13}/L \).

The reactivity values obtained for different \( D_2O \) levels are listed down in Table 4. The quoted error limits are derived in the same way as described under section 7.1. As pointed out above the accuracy of these measurements were hindered by the unfavourable source and detector positions. Nevertheless the methods I and II should in principle deliver meaningful results. On the other hand the integral methods failed completely at \( \frac{\beta}{\beta_{pc}} > 3 \) due to the strong excitation of higher harmonics. Near critical the inherent neutron background was very high as compared with the neutron density originating from the pulsed source (low signal to noise ratio) effecting in a negative sense the measurement of the delayed neutron density. The results of the integral methods are therefore meaningless, but they demonstrate the dependency of these methods on the source-detector geometry.

### 7.2.2 Reactivity of Safety Rods

The reactivity value of the two safety rods of ECO have been measured. After the reactor was balanced to criticality one of the safety rods or both together were dropped into the reactor. After the decay of the short living delayed neutron precursors the pulsed source was started.

Table 5 shows the results of the different measurements.

For comparison the reactivity values obtained by the rod drop method \( \sqrt{13}/L \) are included. The variation of the neutron generation time due to the safety rods has been neglected, because this effect should be rather small.
The agreement with the results of the method II is not very satisfactory but just at the error limits. In order to clarify the discrepancy the accuracy of the pulsed source must be improved by a proper source detector arrangement.

7.3 The Neutron Generation Time

The reduced neutron generation time \( \frac{\Lambda_c}{\beta} = \frac{1}{\alpha_p} \) at delayed critical was compared with the calculated one using homogeneous core parameters from the PINOCCHIO [12]. Actually the neutron generation time was calculated by the following two group formula for a reflected reactor (see [17]).

\[
\Lambda_c = \frac{\frac{1}{\nu_1} \int \phi_1^+ \phi_1 \, dV + \frac{1}{\nu_2} \int \phi_2^+ \phi_2 \, dV}{\frac{k_{\infty} E_B}{P} \int \phi_1 \phi_2 \, dV}
\]  

(39)

The flux times adjoint flux integrals were computed by the code EQUIPOISE 3 [14].

A comparison between the measured and the calculated generation time was made too for the cases of EXPO for subcritical experiments. The calculated values are computed by the bare reactor two group formula using PINOCCHIO homogeneous core parameters.

\[
\frac{\Lambda}{\beta} = \frac{2}{\beta} \cdot \frac{1 + \tau B_c^2}{1 + \tau B_c^2}
\]  

(40)

Table 6 shows the results.

The quoted error of the calculated reduced generation time takes into account an uncertainty of the effective delayed neutron fraction \( \beta \) and of the geometrical and critical buckling formula (40). For the experimental results the error propagation of the formulae

\[
\frac{\Lambda}{\beta} = \frac{1}{\alpha_p} \left( \frac{2}{\beta} + 1 \right)
\]  

(41)

or

\[
\frac{\Lambda}{\beta} = \frac{1}{k_{\infty} B/B} = \frac{1}{\beta}
\]  

(42)
was computed taking the uncertainty of the experimental quantities $\alpha_p$, $\frac{\rho}{\beta}$ and $S$ respectively.

For the ECO experiment the calculated reduced generation time differs by about 16%. Regarding the error limits in the optimistic case a deviation of +3% remains. It is thought that the difference is due to an overestimation of the calculated thermal neutron life time.

The tendency of the deviation between calculated reduced generation time becomes stronger for the cases of all EXPO experiments. This tendency may be explained partly by the influence of the leakage correction term of equation (40) which is incorrect at least due to the known underestimation of $B_c^2$ in PINOCCHIO by about 15%. Apart from this the discrepancy would suggest an increase of $\beta$ by at least 5% which is certainly unreasonable. Therefore it is thought on a systematic error of the measured reduced generation time of about -5%. As a consequence the reactivity of the EXPO measurements evaluated by the GOZANI and GARELIS methods are both underestimated by about 5%. No reason for this systematic error has been found, however.

8. CONCLUSIONS

Reactivity measurements in far subcritical bare heavy water assemblies are possible with a relative precision of about +3%. The neutron generation time seems, however, overestimated by about 5% corresponding to an underestimation of the absolute reactivity value by 5%. A heterogeneous kinetic eigenvalue calculation of the investigated strong heterogeneous lattices will be executed next, in order to study this discrepancy. A static heterogeneous code (3-group monopoles and dipoles) will be adapted to this calculus. Near critical, reactivity measurements are possible with good accuracy in the relative and absolute value provided the source-detector geometry is reasonable. Uncertainties in $\beta_{eff}$ do not enter into the directly measured reactivity in dollar units ($\rho^* = \rho/\beta_{eff}$).
The authors are indebted to the Electronics Service for the very good functioning of their electronic equipment utilized in the experiments especially to Mr. G. COLOMBO, A. PEDRINI, L. STANCHI. Thanks are given to the ECO operational group for their collaboration during the ECO experiments.
SYMBOLS USED

\[ \begin{align*}
\alpha_p & \quad \text{Prompt neutron decay constant} \\
\alpha_i & \quad \text{Decay constant} \\
A_i & \quad \text{Amplitude} \\
\beta_i & \quad \text{Delayed neutron fraction of group } i \\
\lambda_i & \quad \text{Delayed neutron precursors decay constant of group } i \\
\beta & \quad \text{Effective delayed neutron fraction} \\
k & \quad \text{Effective multiplication constant} \\
\Lambda & \quad \text{Effective thermal neutron life time} \\
p & \quad \text{Generation time} \\
\rho & \quad \text{Resonance escape probability} \\
\rho & \quad \text{Negative reactivity} \\
B & \quad \text{Equilibrium state delayed neutron density} \\
T & \quad \text{Pulsing period} \\
A_p & \quad \text{Prompt neutron amplitude} \\
t_w & \quad \text{Waiting time} \\
d & \quad \text{Duration of neutron burst} \\
k_n & \quad \text{Eff. multiplication of the } n\text{ th mode} \\
\tau_n & \quad \text{Eff. life time of the } n\text{ th mode neutron} \\
k_\infty & \quad \text{Infinite medium multiplication factor} \\
\tau_\infty & \quad \text{Infinite medium neutron life time} \\
\alpha_{pc} & \quad \text{Prompt neutron decay constant at delayed critical} \\
\beta & \quad \text{Prompt neutron life time at delayed critical} \\
B_n^2 & \quad \text{Geometrical buckling of the } n\text{ th mode} \\
B_c^2 & \quad \text{Critical buckling} \\
L^2 & \quad \text{Thermal neutron diffusion area} \\
\tau & \quad \text{Slowing down area} \\
\phi_{1/2} & \quad \text{Fast/slow neutron flux} \\
\phi_{1/2} & \quad \text{Fast/slow neutron adjoint flux} \\
\delta & \quad \text{Fast fission ratio} \\
\nu^5 / \beta & \quad U^{235} / U^{238} \text{ neutron yield per fission} \\
v_1 / v_2 & \quad \text{Fast/slow neutron velocity} \\
S & \quad \frac{k_\infty}{\tau} 
\end{align*} \]
<table>
<thead>
<tr>
<th>Group</th>
<th>$\lambda_i$ [sec$^{-1}$]</th>
<th>$\beta_i$</th>
<th>remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.87</td>
<td>2.54.10^-4</td>
<td>F</td>
</tr>
<tr>
<td>2</td>
<td>1.40</td>
<td>1.07.10^-3</td>
<td>F</td>
</tr>
<tr>
<td>3</td>
<td>3.11.10^-1</td>
<td>3.0.10^-3</td>
<td>F</td>
</tr>
<tr>
<td>4</td>
<td>2.77.10^-1</td>
<td>2.34.10^-4</td>
<td>P</td>
</tr>
<tr>
<td>5</td>
<td>1.15.10^-1</td>
<td>1.36.10^-3</td>
<td>F</td>
</tr>
<tr>
<td>6</td>
<td>3.17.10^-2</td>
<td>1.48.10^-3</td>
<td>F</td>
</tr>
<tr>
<td>7</td>
<td>1.69.10^-2</td>
<td>7.33.10^-5</td>
<td>P</td>
</tr>
<tr>
<td>8</td>
<td>1.27.10^-2</td>
<td>2.52.10^-9</td>
<td>F</td>
</tr>
<tr>
<td>9</td>
<td>4.81.10^-3</td>
<td>2.48.10^-5</td>
<td>P</td>
</tr>
<tr>
<td>10</td>
<td>1.50.10^-3</td>
<td>1.19.10^-3</td>
<td>P</td>
</tr>
<tr>
<td>11</td>
<td>4.28.10^-4</td>
<td>7.43.10^-6</td>
<td>P</td>
</tr>
<tr>
<td>12</td>
<td>1.17.10^-4</td>
<td>8.42.10^-6</td>
<td>P</td>
</tr>
</tbody>
</table>

$\beta$=7.77.10^-3

**TABLE 1:** Delayed neutron data (without fast leakage correction)

$$-B^2_i(t_p-t_d)$$

$F$ = fission neutrons

$P$ = photoneutrons
<table>
<thead>
<tr>
<th>No</th>
<th>Core height</th>
<th>burst duration</th>
<th>(see fig. 4) detector pos. Z</th>
<th>Method I</th>
<th>ρ/β (§)</th>
<th>Method V</th>
<th>PINOCCHIO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H(cm)</td>
<td>(µsec)</td>
<td>(cm)</td>
<td></td>
<td>(sec⁻¹)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>230.</td>
<td>7.</td>
<td>1</td>
<td>36.4</td>
<td>7.35</td>
<td>35.4</td>
<td>5.96</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+1.3</td>
<td>±0.8</td>
<td>±0.7</td>
<td>+0.30</td>
</tr>
<tr>
<td>2</td>
<td>230.</td>
<td>50.</td>
<td>1</td>
<td>36.6</td>
<td>7.23</td>
<td>35.8</td>
<td>7.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+1.0</td>
<td>±0.8</td>
<td>±0.6</td>
<td>±0.30</td>
</tr>
<tr>
<td>3</td>
<td>230.</td>
<td>7.</td>
<td>3</td>
<td>38.4</td>
<td>7.07</td>
<td>37.5</td>
<td>6.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+1.8</td>
<td>±0.8</td>
<td>±0.7</td>
<td>±0.31</td>
</tr>
<tr>
<td>4</td>
<td>230.</td>
<td>50.</td>
<td>3</td>
<td>37.1</td>
<td>7.24</td>
<td>36.3</td>
<td>40.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+1.2</td>
<td>±0.8</td>
<td>±0.7</td>
<td>±0.31</td>
</tr>
<tr>
<td>5</td>
<td>260.</td>
<td>7.</td>
<td>3</td>
<td>36.3</td>
<td>7.05</td>
<td>35.1</td>
<td>35.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+0.9</td>
<td>±0.8</td>
<td>±0.7</td>
<td>±0.31</td>
</tr>
<tr>
<td>6</td>
<td>260.</td>
<td>50.</td>
<td>3</td>
<td>35.4</td>
<td>7.17</td>
<td>34.4</td>
<td>2.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+0.8</td>
<td>±0.8</td>
<td>±0.6</td>
<td>±0.31</td>
</tr>
</tbody>
</table>

**TABLE 2: Results of pulsed source reactivity measurements in EXPO with U 19 12 OMPH fuel elements**
<table>
<thead>
<tr>
<th>№</th>
<th>Core height</th>
<th>burst duration</th>
<th>(see fig. 4) detector pos. Z</th>
<th>Method I</th>
<th>Method V</th>
<th>PINOCCHIO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H(cm)</td>
<td>(μsec)</td>
<td>(cm)</td>
<td>kβ/λ</td>
<td>kβ/λ</td>
<td>ρ/β</td>
</tr>
<tr>
<td>7</td>
<td>260.</td>
<td>7.</td>
<td>2</td>
<td>35.7</td>
<td>33.5</td>
<td>35.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>±1.1</td>
<td>± .8</td>
<td>± .7</td>
</tr>
<tr>
<td>8</td>
<td>260.</td>
<td>50.</td>
<td>2</td>
<td>37.5</td>
<td>35.4</td>
<td>35.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>±1.1</td>
<td>± .7</td>
<td>± 1.0</td>
</tr>
<tr>
<td>9</td>
<td>260.</td>
<td>7.</td>
<td>5</td>
<td>38.6</td>
<td>31.7</td>
<td>34.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>±1.5</td>
<td>± .7</td>
<td>± .7</td>
</tr>
<tr>
<td>10</td>
<td>260.</td>
<td>50.</td>
<td>5</td>
<td>42.4</td>
<td>35.3</td>
<td>34.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>±1.8</td>
<td>±1.0</td>
<td>± .7</td>
</tr>
<tr>
<td>11</td>
<td>260.</td>
<td>7.</td>
<td>1</td>
<td>33.8</td>
<td>34.4</td>
<td>34.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>±1.3</td>
<td>± .7</td>
<td>± .7</td>
</tr>
<tr>
<td>12</td>
<td>260.</td>
<td>50.</td>
<td>1</td>
<td>33.8</td>
<td>34.5</td>
<td>34.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>±1.0</td>
<td>±1.2</td>
<td>± 1.2</td>
</tr>
<tr>
<td>13</td>
<td>260.</td>
<td>7.</td>
<td>4</td>
<td>39.6</td>
<td>34.2</td>
<td>34.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>±2.3</td>
<td>±1.2</td>
<td>±1.2</td>
</tr>
</tbody>
</table>


**TABLE 3:** Results of pulsed source reactivity measurements in EXPO with U-19-12/Diphyl fuel elements

<table>
<thead>
<tr>
<th>pitch (cm)</th>
<th>el.N°</th>
<th>$A_p$ (counts/sec)</th>
<th>$\alpha_p$ (sec$^{-1}$)</th>
<th>B (counts/sec)</th>
<th>T (sec)</th>
<th>METHOD III</th>
<th>METHOD V</th>
<th>PINOCCHIO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\beta = (\frac{A_p}{\alpha_p})(B.T)^{-1}$</td>
<td>$\beta/\Lambda$ (sec$^{-1}$)</td>
<td>$\rho/\beta$ ($$)</td>
</tr>
<tr>
<td>23.0</td>
<td>32</td>
<td>$9.35 \times 10^{-4}$</td>
<td>$253 \pm 2$</td>
<td>$309 \pm 3$</td>
<td>0.05</td>
<td>23.8 (\pm 0.8)</td>
<td>10.87 (\pm 0.12)</td>
<td>22.2 (\pm 0.4)</td>
</tr>
<tr>
<td>26.6</td>
<td>24</td>
<td>$1.42 \times 10^5$</td>
<td>$245 \pm 2$</td>
<td>$356 \pm 3$</td>
<td>0.05</td>
<td>32.0 (\pm 0.9)</td>
<td>7.79 (\pm 0.09)</td>
<td>30.3 (\pm 0.6)</td>
</tr>
</tbody>
</table>

U-19-12 Diphyl coolant
$D_2^0$ title / 99.72%
$T = 22 \pm 2^\circ C$

$\beta = \beta_{eff} = 0.0079 \pm 5\%$
<table>
<thead>
<tr>
<th>$D_2O$ level (cm)</th>
<th>$\alpha_p$ (sec$^{-1}$)</th>
<th>$\rho/\beta$ ($)</th>
<th>remark see fig. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Method I</td>
<td>II</td>
<td>IV</td>
</tr>
<tr>
<td>110.14</td>
<td>77.2±1</td>
<td>32.7±1</td>
<td>32.2±1</td>
</tr>
<tr>
<td>116.14</td>
<td>66.0±2</td>
<td>20.5±1</td>
<td>20.1±1</td>
</tr>
<tr>
<td>114.57</td>
<td>30.8±1</td>
<td>15.7±1</td>
<td>14.1±1</td>
</tr>
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<td>153.55</td>
<td>18.4</td>
<td>11.7±1</td>
<td>10.6±1</td>
</tr>
<tr>
<td>160.0</td>
<td>13.0</td>
<td>11.7±1</td>
<td>10.6±1</td>
</tr>
<tr>
<td>164.55</td>
<td>-</td>
<td>10.0±0.5</td>
<td>-</td>
</tr>
<tr>
<td>167.6</td>
<td>extrapolated</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**TABLE 4:** Results of pulsed source reactivity measurement in ECO with U-19-12 diphyl fuel elements at different $D_2O$ water level

$D_2O$ title: 99.69

Core loading: pitch 23.5 cm, 119 fuel elements
### TABLE 5: Reactivity of ECO safety rods

<table>
<thead>
<tr>
<th>rods inserted</th>
<th>$\alpha_p$ (sec$^{-1}$)</th>
<th>$\rho/\beta$ ($%$)</th>
<th>remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Method II</td>
<td>IV</td>
<td>II</td>
</tr>
<tr>
<td>1</td>
<td>23.6±1</td>
<td>22.7±1</td>
<td>1.34±0.21</td>
</tr>
<tr>
<td>2</td>
<td>32.7±1</td>
<td>32.0±1</td>
<td>2.24±0.27</td>
</tr>
<tr>
<td>2</td>
<td>33.9±1</td>
<td>33.0±1</td>
<td>2.36±0.27</td>
</tr>
</tbody>
</table>

core loading: pitch 23.5 cm, 119 fuel elements

$D_2O$ title: 99.69

$\beta = \beta_{eff} = 0.777 \pm 5\%$
<table>
<thead>
<tr>
<th>Lattice pitch (cm)</th>
<th>fuel type</th>
<th>$\Lambda/\beta$ [sec$^{-1}$]</th>
<th></th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.5</td>
<td>U-19-12-diphyd</td>
<td>0.100 ± 0.005</td>
<td>0.116 ± 0.008</td>
<td>Method II, ECO critical</td>
</tr>
<tr>
<td>23.0</td>
<td></td>
<td>0.0980 ± 0.0012</td>
<td>0.110 ± 0.005</td>
<td>Method I, EXPO</td>
</tr>
<tr>
<td>23.0</td>
<td></td>
<td>0.0919 ± 0.0009</td>
<td></td>
<td>Method V, EXPO</td>
</tr>
<tr>
<td>26.6</td>
<td></td>
<td>0.131 ± 0.002</td>
<td>0.160 ± 0.009</td>
<td>Method I, EXPO</td>
</tr>
<tr>
<td>26.6</td>
<td></td>
<td>0.128 ± 0.001</td>
<td></td>
<td>Method V, EXPO</td>
</tr>
<tr>
<td>26.6</td>
<td>OMPH</td>
<td>0.146 ± 0.002</td>
<td>0.167 ± 0.009</td>
<td>Method I, EXPO</td>
</tr>
<tr>
<td>26.6</td>
<td></td>
<td>0.141 ± 0.002</td>
<td></td>
<td>Method V, EXPO</td>
</tr>
</tbody>
</table>

**TABLE 6:** Reduced generation time derived from ECO and EXPO measurements
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Fig. 1  Block diagram: pulsed source experiment

D  Detector
PA  Preamplifier
MA+Disc  Main amplifier and discriminator
HT  High tension unit
MCS  Multichannel scaler
TI  Timer
PS  Pulsed source
Input routine, measured data
\( \beta_i, \lambda_i \) initial guess \( \rho_o, \Lambda_o \)

Evaluation of \( n(t) \) and \( B^0 \) from raw data in first approximation

Least squares fit: \( n(t) - B^0 = A_p e^{-\alpha_p t} \)

\( t \geq t_w \)

IV
BROHNER-DIO-SCHLOSSER Method
Least squares fit
\[ n(t) = A_p e^{-\alpha_p t} + B(o) \frac{B(o) - B(T)}{T} t \]

evaluation of \( \rho \)

Solution of the inhour equation
\[ a_i = \frac{\beta_i - \rho}{\Lambda + \frac{1}{\Lambda}} + \frac{1}{\Lambda(1-\rho)} \sum_{j=1}^{m} \frac{\beta_j \lambda_j}{(\alpha_i - \lambda_j)^2} \]

\( \alpha_i = \left( 1 + \frac{1}{\Lambda(1-\rho)} \sum_{j=1}^{m} \frac{\beta_j \lambda_j}{(\alpha_i - \lambda_j)^2} \right)^{-1} \)

Least squares fit \( n(t) - C = \sum_{i=1}^{m} A_i e^{-\alpha_i t} \)

\[ \rho = \sum_{i=1}^{m} \frac{\beta_i \lambda_i}{(\alpha - \lambda_i)^2} + \beta - \alpha_o \Lambda \]

iteration on \( \rho \)

III
SJOSTRAND-GOZANI Method
evaluation of \( \frac{\rho}{\beta} \)

GARELIS-RUSSEL Methods

\[ \int_{0}^{T} n_p(t) (e^{st} - 1) dt = BT \]

\( 0 < t < T \)
iteration on \( S = k8 \)

evaluation of \( \frac{\rho}{\beta} \)

V

Fig. 2: Computer program-block-diagram-analysis of pulsed source experiment
Fig. 3 - Geometrical arrangement for pulsed-source experiments on Expo.

1. Guiding tube of Al
2. Detector
3. Pulsed source
Fig. 4 - Excitation of axial harmonics due to source arrangement at core midplane
Fig. 5 - Geometrical arrangement of pulsed source measurements on ECO

1. Pulsed source
2. Detector, Pos. 1
3. Detector, Pos. 2 (Azimuthally displaced by 90°)
4. Safety rod 1
5. Safety rod 2
Fig. 7 - "ORGEL"-type fuel element: cluster of 19 uranium metal rods (UM-19).

Fig. 6 - Prompt neutron decay constant as a function of geometrical buckling.
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Alfred Nobel
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