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THE SLUGFLOW EQUATION

by

L.NOBEL

1972

Joint Nuclear Research Centre
Ispra Establishment - Italy

Technology

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This conclusion is based on the assumption that the displaced liquid flowrate pro unit of bubble cross-section at the rear-end of the bubble, caused by the relative motion of the bubble is invariant with this cross-section and also with the total volume flow rate.

If this theory holds good the consequences are that the bubble diameter decreases with increasing total volume flow, for constant remaining bubble lengths.

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ABSTRACT

In the report is discussed the validity of the slugflow equation given, amongst others, by Nicklin et al. It appears that other types of equation are also possible probably due to the fact that the coefficient of the total volume flow rate taken by Nicklin et al. as a constant, in fact would be a function of bubble length and total volume flow rate.

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KEYWORDS

SLUG FLOW

FLOW RATE

BUBBLES

TUBES

EQUATIONS

Contents

THE SLUG FLOW EQUATION *)

If a co-current two-phase flow, in a vertical tube which commonly has a diameter greater than 10 mm, is characterised by bullet or piston shaped gas bubbles (also called G.I. Taylor bubbles), separated from each other by liquid slugs with a length of at least six tube diameters (a flow pattern known as fully developed slug flow), the velocity of the bubbles may be expressed according to NICKLIN, WILKES and DAVIDSON [1] by the equation:

$$V_G = C_1 V_M + C_2 \sqrt{g D} \qquad (1)$$

In this equation the meaning of the symbols is:

V_G = bubble velocity (m/sec)

V_M = volume flow density or throughput velocity of the gas-liquid fluid combination (m/sec)

$(V_M = V_{GA} + V_{LA} = \frac{Q_G}{A} + \frac{Q_L}{A})$

D = diameter of tube (m)

A = $\pi/4 . D^2 (m^2)$

Q_G and Q_L = volume flow of gas and liquid respectively.

For an air-water system C_2 = 0,35, as has been theoretically determined by DUMITRESCU [2], and experimentally confirmed by NICOLITSA and MURGATROYD [3] among others. Assuming C_2 to be

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invariant with V_M, Nicklin et al obtained for C_1 the value 1,2. In their experiments single bubbles were injected at the base of the test section with such low frequency that during the rise of the bubble $V_{GA} = 0$ could be maintained. The liquid volume flow density V_{LA} (superficial liquid velocity) was varied between 0,3 m/sec and 2,5 m/sec. The same technique for injecting single bubbles was employed by Nicolitsa and Murgatroyd.

These research teams obtained a value of 1,24 for C_1 in the range for V_{LA} between 0 - 0,25 m/sec, with a spread of $\pm$ 2%. Both series of experiments, i.e. those of Nicklin et al and those of Nicolitsa and Murgatroyd, were carried out in tubes of about one inch diameter and their results could be considered to be congruent.

However, GRACE et al [4], performing slug flow experiments in stationary water with a steady state air flow in a one inch tube, and with a superficial gas velocity of up to about 0,2 m/sec, arrived at the following slug flow equation:

$$V_G = 1{,}11\ V_{GA} + 0{,}359\sqrt{g\ D} \qquad (2)$$

So we have already three different values for C_1, i.e. 1,24, 1,2 and 1,11.

Still more diversity is caused by the results of work performed by MARRUCCI et al [5].

This team obtained, in five tubes of diameters 1,6: 2,1: 2,8: 4,2: and 5,3 cm respectively, and with an air-water system, results which can be described by the equation:

$$V_G = V_{LA} + 1.17\ V_{GA} + 0{,}35\sqrt{g\ D} \qquad (3)$$

Equations 2 and 3 could be written in a more general form, i.e.,

$$V_G = V_M + C_3 V_{GA} + 0{,}35 \sqrt{g D} \qquad (4)$$

and for C_3 we already have two values, i.e. 0,11 and 0,17.

These results are supported by observations made by VAN HEUVEN and BEEK[6].

Although no information on the water velocity has been provided, we can translate their results into our type of formula and express them as

$$V_G = V_M + 0{,}216 \; V_{GA}^{\;3/4} \qquad (5)$$

The experiments were performed in tubes with diameters of 0,48 cm and 0,238 cm, at superficial gas velocities of between 0,1 and 0,7 m/sec, and are in a certain sense beyond our scope. Equation 5 could be written in the form of equation 4, but because

$$E_o = \frac{\rho g D^2}{\sigma} = 3{,}13$$

is beneath the critical value of 4, according to WHITE and BEARDMORE [7], $C_2 = 0$.

In the same context, $C_3 = 0{,}216 \; V_{GA}^{\;1/3}$ and is thus no longer a constant. It is true, in spite of all these divergencies, $C_1 = 1$ and $C_3 \neq 0$.

For small values of V_{GA} only bubbly flow exists. Using a criterion of STEWART and DAVIDSON [8], that in stationary water slugging starts if

$$\frac{V_{GA}}{\sqrt{gD}} \geqslant 0{,}2$$

we may write for the boundary line indicating initial slug-flow for the 0,48 cm tube, based on equation 5,

$$V_{G_S} = 1{,}076\ V_M \tag{6}$$

If we now go over our findings, we may conclude that there is obviously no uniform, and perhaps not even a generally valid interpretation of the results.

If we plot the results of the group with V_G as a function of V_M, V_{GA} and $\sqrt{gD}$ according to equation 4, then we obtain the situation that has been illustrated in Fig. 1. In this Figure we have taken V_{LA} as being constant. At a certain value of V_{GA}, bullet shaped bubbles will be formed, and they will move at the velocity V_{G_1}.

Following equation 6, then $C_3 = 0{,}076$.

For higher air velocities, we follow curve $V_{G_1} - V_{G_2}$ and C_3 increases.

This curve may be approximated by the straight line A-B.

In Fig. 1 the mutual relationships between the functions are pronouncedly out of scale, so as better to express the meaning of the approximations. In Fig. 2 a more realistic presentation has been given.

0-0' represents the term $0{,}35\sqrt{gD}$.

The line O' - C is the "lowest" slug flow line.

The line $O'-V_{G_2}$ is here the upper slug flow line. The triangle between these lines represents all possible slug-flow situations.

For this sector an average inclination may be given, corresponding to the constant 1,2 in the equation of Nicklin et al.

Successively we obtain the following variations and modifications of the slug flow equation:

1. For V_{LA} = const, the slug flow equation may be written in the form:

$$V_G = V_M + C_2 \sqrt{gD} + C'_3 \; V^m{}_{GA}$$

(extension of eq. 5)

2. This relation may be sufficiently approximated by the relation

$$V_G = V_M + C_2\sqrt{gD} + C''_3 \; V_{GA}$$

(see eq. 4)

3. For different V_{LA} - values, the equation may be written as:

$$V_G = C_1 \; V_M + C_2\sqrt{gD} \qquad \text{(see eq. 1)}$$

here, C_1 is a variable.

4. For a rougher indication C_1 may be taken as the average of C_1 values, leading to the equation of Nicklin et al.

Assumptions 3 and 4 may be checked by comparison with the work of STREET and TEK [9]. In this work the spread in C_1 lies between $C_1 \approx 1,1$ and $C_1 \approx 1,4$.

Before continuing, we should perhaps discuss the accuracy of the Nicklin equation.

In this connection we can say that a bullet-shaped bubble acts on the liquid flowing ahead of it as a displacement device, due to the difference in velocity between bubble and liquid. The flowrate of this displaced liquid may be found to apply the continuity equation over the rear-end cross-section of the bubble. If we express the void fraction there by the symbol ξ_c , the "displacement" liquid flowrate $V_{LDA} = V_{LD} (1 - \xi_c)$ follows from the equation (see Fig. 3):

$$V_G \xi_c - V_{LD} (1 - \xi_c) = V_M \qquad (7)$$

and, solving for V_G, we obtain:

$$V_G = {}^1/\xi_c \, V_M + {}^1/\xi_c \, V_{LDA} \qquad (8)$$

The expression $\frac{V_{LDA}}{\xi_c}$ represents the specific "displaced liquid" flowrate due to the displacement action of the bubble. Now it can be imagined that this specific "displaced liquid" flowrate should be invariant with the rear-end cross-section of the bubble for a given tube diameter. Let us postulate that this would be the case.

The solution of equation 8 for $V_M = 0$ gives

$$\frac{V_{LDA}}{\xi_c} = C_2 \sqrt{gD} \qquad (9)$$

Assuming that this solution is invariant for V_M also, we obtain as the first equation

$$V_G = {}^1/\xi_c \; V_M + C_2\sqrt{gD} \qquad (10)$$

If our postulation is valid, comparison of equations 1 and 10 leads to

$$C_1 = {}^1/\,\xi_c \qquad (11)$$

Thus $C_1 = 1{,}2$ corresponds to $\xi_c = 0{,}833$, which is obviously the average value of ξ_c.

Nicklin et al explained the coefficient 1,2 by the maximum value of the liquid velocity at the center line in relation to the average liquid velocity equal to V_M.

For laminar flowing liquids, the velocity at the center line is equal to 2 V_M.

This was the explanation of GRACE et al for their experimental findings with an air-sugar solution system.

They obtained: $C_1 = 2{,}12$ and $C_2 = 0{,}194$.

C_2 was in very close agreement with the correlation presented by White and Beardmore and C_1 was very close to the factor 2.

However, COX[10] has shown that for viscous liquids the fractional amount of liquid left in the tube when a "bubble" has expelled the other fraction, is a function of $\frac{\mu_L U}{\sigma}$.

Here U is the velocity of the interface.

For high values of $\frac{\mu_L U}{\sigma}$, ξ_c tends to the asymptotical value of 0,4, hence the assumption $\xi_c = \frac{1}{2,12} = 0,471$, as would follow from the work of Grace et al., could be supported by the work of Cox.

The results of GOLDSMITH and MASON(11) also show that for a tube of diameter 0,8 cm, a film thickness of 1,08 mm will be obtained for an air-oil system with $\mu = 8,41$ P. This corresponds to a ξ_c-value of 0,533.

From the work of KOUREMENOS(12), a similar conclusion, though approximative in numerical value, may be drawn.

According to this work, the manometrical pressure height produced by a bubble is to be expressed by the relation

$$h_{bm} = \frac{V_b}{A} + h_s$$

h_s is the friction pressure height

V_b is the volume of a bubble.

Now we can write roughly $\ell = \frac{V_b}{\xi_c A}$

Assuming further that particularly for the longer bubbles $h_{bm} \sim \ell$, we obtain:

$$h_s = \ell \, (1 - \xi_c).$$

For a tube diameter of 4,5 cm, according to the relation given by Kouremenos,

$$h_s / \frac{V_b}{A} = 0,115 \left(\frac{\mu_L}{\mu_W} \right)^{0,222} + 0,0588 \left(\frac{\mu_L}{\mu_W} \right)^{0,115} \qquad (12)$$

here μ_L is the viscosity of any fluid other than water, μ_W is the viscosity of water.

With our approximations we obtain:

$$\frac{1-\xi_c}{\xi_c} \sim 0{,}115 \left(\frac{\mu_L}{\mu_W}\right)^{0{,}222} + 0{,}0588 \left(\frac{\mu_L}{\mu_W}\right)^{0{,}115} \qquad (13)$$

For μ= 8,41 P as in the case of (10), we obtain ξ_c = 0,60.

A calculation method is given for the liquid flow between bubble and wall in (1).

For V_M = 0, the flow balance gives

$$(1-\xi_c)(\sqrt{2g\ell} - 0{,}35\sqrt{gD}) = \xi_c \cdot 0{,}35\sqrt{gD} \qquad (14)$$

This equation is to be considered valid for <u>any</u> cross-section of the bubble, if we imagine that any bubble is a part of an ideal bubble. It now yields for the liquid hold-up around the bubble:

$$\frac{\lambda}{D} = \frac{1}{D}\int_0^\ell (1-\xi)\, d\ell = 0{,}495\sqrt{\frac{\ell}{D}} \qquad (15)$$

This solution holds to $\frac{\ell}{D} \sim 5$ (see (1), fig. 2). To "fit" the hold-up curve for $\frac{\ell}{D} > 5$, we write equation 14 as follows:

$$(1-\xi)\, C_\ell \sqrt{2g(\ell+\ell_p)} - 0{,}35\sqrt{gD} = \xi 0{,}35\sqrt{gD} \qquad (16)$$

In this formula, for ℓ = 0, and ξ= 0,

$$C_\ell \sqrt{2g\ell_p} = 0{,}35\sqrt{gD}$$

and thus, because in this case C_ℓ = 1.

$$\ell_p = 0{,}0612\ D \qquad (17)$$

being equal to $h_{bm} - \ell$

The factor C_ℓ is an adaptation factor, and from (16) follows:

$$C_\ell = \frac{0{,}35}{1-\xi}\sqrt{\frac{D}{2(\ell+\ell_p)}} \qquad (18)$$

From this equation follows for $\ell_p \ll \ell$, following formula (15)

$$C_\ell = 0{,}495\ \frac{d(\sqrt{\ell/d})}{d(\lambda/D)} \qquad (19)$$

From fig. 2, ref. (1), we derive our fig. 4; the relation between the liquid hold-up in multiples of tube diameter versus the square root of ℓ/D. Graphic differentiation of this function finally gives the coefficient C_λ .

The coefficient C_ℓ is plotted versus $\frac{10\ D}{\ell}$ in fig. 5 . It may easily have been found by using equation (18), in which, for a cylindrical bubble with a film thickness δ,

$$C_{\ell_{cyl}} \sim 0{,}0198\ \frac{D}{\delta}\sqrt{\frac{10\ D}{\ell}} \qquad (20)$$

According to the work of STREET and TEK[13] $\xi_{max} = 0{,}895$ and thus equation (18) becomes

$$C_{\ell_{cyl}} = 0{,}7524\sqrt{\frac{10\ D}{\ell}} \qquad (21)$$

For small values of $\frac{10\ D}{\ell}$ equation 21 may be used. For the bigger values of $\frac{10\ D}{\ell}$, i.e. for short bubbles, the real value of C_ℓ is somewhat below C_ℓ , as is demonstrated in fig. 5.

For $V_M > 0$, equation (16) may be extended as follows:

$$(1-\xi)\left[C_\ell\sqrt{2g(\ell+\ell_p)}-(V_G-V_M)-V_M\right] = V_G\xi - V_M \qquad (22)$$

in each cross-section of the bubble, equation (22) may be applied. Now,

$$\ell_p = \frac{(V_G-V_M)^2}{2g}$$

For slugflow $V_{M_{max}} \approx 2$ m/sec, and for a tube of about 2,5 - 3 cm, $\underline{\ell_p \approx 1 \text{ cm}}$.

Thus in most cases ℓ_p may be ignored. For $\xi = \xi_c$ equation (22) becomes:

$$(1-\xi_c)\left[C_\ell\sqrt{2g\ell} - 1/\xi_c V_M + 0{,}35\sqrt{gD}\right] = \xi_c\, 0{,}35\sqrt{gD} \qquad (23)$$

Writing now for $\frac{V_M}{\sqrt{2g\ell}} = N_{FR}$ we obtain as a general expression for slug flow:

$$\frac{N_{FR}}{c_\ell} = \left\{ \frac{\left[1 - \frac{0{,}35}{c\ell}\sqrt{\frac{D}{2\ell}}\right]\xi_c}{1-\xi_c} \right\}\xi_c \qquad (24)$$

Also in this formula C_ℓ is an adaptation coefficient equal to:

$$C_\ell = \frac{0{,}35}{1-\xi_c}\sqrt{\frac{D}{2\ell}} + \frac{N_{FR}}{\xi_c} \qquad (25)$$

Equation (25) may be compared with equation (18). The latter equation, with ℓ_p ignored, may be written as C_{ℓ_o} and we obtain, in general, writing for ξ at $V_M = 0, \xi_o$

$$C_\ell = C_{\ell_o}\left[1 + \frac{1-\xi_c}{\xi_c}\right]\frac{1-\xi_o}{1-\xi_c} \qquad (26)$$

Here $B = \frac{V_M}{0,35\sqrt{gD}}$

Now the relation between ξ_o and ξ_c is unknown. There are two possibilities for this relation, which are extremes of reality.

The first extreme is:

$C_\ell \equiv C_{\ell_o}$ and the relation between ξ_o and ξ_c may be written as

$$B = \frac{\xi_o - \xi_c}{1-\xi_c} \quad \frac{\xi_c}{1-\xi_o} \tag{27}$$

If this relation is plotted for $\xi_o = 0,895$, we obtain at $\xi_o \approx 0,7$, $B_{max} = 4,30$, see fig. 6 . For a one-inch tube this means $V_M \approx 0,75$ m/sec. This is certainly too low.

Another extreme is $\xi_c \equiv \xi_o$.
This gives

$$C_\ell = C_{\ell_o}\left(1 + \frac{1-\xi_o}{\xi_o} B\right) \tag{28}$$

The reality will probably lie between these two expressions, and we may expect an expression, such as:

$$C_\ell = C_{\ell_o}(1+\phi B) \tag{29}$$

In that case

$$B = \frac{\xi_o - \xi_c}{1-\xi_c} \cdot \frac{\xi_c}{(1-\xi_o) - \phi\xi_c} \tag{30}$$

In equation (29), ϕ will be a variable; for low values of $\frac{10\ D}{\ell}$, $\phi \to 0,117$, because $\phi = \frac{1-0,895}{0,895}$, and for high values of $\frac{10\ D}{\ell}$, $\phi \to 0$, because $C_\ell \to C_{\ell_o}$.

(compare equations (27) and (30))

It is thus clear that $0 \leq \phi \leq \frac{1-\xi_o}{\xi_o}$.

However, ϕ is also a function of B and for the moment only fictitious models are available for study, due to the lack of knowledge.

The extreme $\phi = 0$, or $C_\ell = C_{\ell_o}$ gives, for an increasing B, a decreasing ξ_c as follows from fig. 6. (The part left from the max. for B does not belong to the solution of equation (27)).

This kind of tendency may generally be expected with more realistic values for ϕ.

This can also be proved by means of equation (22).
Since $C_\ell = 1$ and $V_G - V_M = \sqrt{2g\,\ell_p}$, for short bubbles,

$$1 - \xi_c = \sqrt{\frac{\ell_p}{\ell + \ell_p}} \; .$$

Using the formula of Nicklin $\ell_p \approx \frac{(0,2\ V_M + 0,35\sqrt{gD})^2}{2g}$

and for increasing V_M, ℓ_p is therefore increasing and ξ_c accordingly decreasing.

Finally the following observations can be made.

Equation (21) suggests that for $\ell \to \infty$, $C_\ell = 0$. It may be imagined, however, that a limit value for ℓ/D exists.

From wave theory we know

$$c_W \geq \frac{1}{2\sqrt{\pi}} \sqrt{2g\ h_W} \quad \text{or} \quad c_W \geq 0,28 \sqrt{2\ gh_W}$$

/

In this formula C_W = velocity of the wave over shallow water

h_W = wave length.

In the case of slug flow, assuming the bubble is a wave, we obtain:

$$C_W = V_G + V_{LD}$$

$$h_W = \ell .$$

Thus
$$C_W = \frac{1}{\xi_c} V_M + 0{,}35\sqrt{gD} + \frac{\xi_c}{1-\xi_c}\, 0{,}35\sqrt{gD}$$

and applying equation (31), we obtain

$$\frac{N_{FR}}{0{,}28} = \left[1 - \frac{0{,}35}{0{,}28(1-\xi_c)}\sqrt{\frac{D}{2\ell}}\right]\xi_c \qquad (32)$$

Equations (24) and (32) are identical, if $C_\ell = 0{,}28$.

Substituting the value $C_\ell = 0{,}28$ in equation (21). the result is $\ell/D = 72$.

For this value, $F(\ell) = 1 - \frac{0{,}35}{0{,}28}\sqrt{\frac{1}{144}} = 0{,}896$ being the limit value for ξ_c.

No higher values can be found for ℓ/D in the literature at present available.

Ref.(1) reports as a maximum $\ell \sim 70$ D, and Ref.(9) gives for $\ell = 200$ cm, $\ell/D \simeq 70$.

Conclusions

1. There is a strong indication that in the Formula of Nicklin et al[1]

$$V_G = C_1 + C_2 \sqrt{gD},$$

C_1 is not a constant value.

This follows from the work of other researchers (4), (5), (6), and (7).

2. The factor C_1 is very probably equal to $1/\xi_c$, ξ_c being the local void fraction at the rear-end of the bubble. If this is true, then the "fixed" relation between C_1 and the ratio max. liquid velocity at the center line to the mean mixture velocity must be considered to be accidental.

3. From this theory it follows that with increasing V_M the bubble volume per unit length decreases, assuming the bubble length remains constant.

4. A maximum bubble length to tube diameter ratio could be explained, by analogy between bubble motion and wave velocity over shallow water.

Notation

A		pipe cross section (m^2)
B	=	$\frac{V_M}{0{,}35\sqrt{gD}}$
C	=	constant or coefficient
C_ℓ	=	adaptation factor of bubble
$C_{\ell cyl}$	=	idem for cylindrical bubble
C_{ℓ_o}	=	adaptation factor V_M = 0
C_W	=	wave velocity over shallow water (m/s)
D	=	tube diameter (m)
δ	=	film thickness (m)
E_o	=	Eötvös number = $\frac{\rho\, g\, D^2}{\sigma}$
$F(\ell)$	=	$1 - \frac{0{,}35}{C_\ell}\sqrt{\frac{D}{2\ell}}$
ϕ	=	$\frac{C_\ell - C_{\ell_o}}{B}$
g	=	gravitational acceleration (m/s^2)
h_{bm}	=	manometrical pressure height by bubble produced (m)
h_s	=	friction pressure height (m)
h_W	=	wave length (m)
ξ	=	local void fraction in cross-section of bubble
ξ_c	=	local void fraction at rear-end of bubble
ξ_o	=	local void fraction in cross-section of idealized bubble on length ℓ equal to local void fraction at rear-end of bubble with length ℓ, under flow conditions V_M = 0.
ℓ	=	bubble length (m)
ℓ_p	=	relative velocity head (m)
λ	=	liquid holdup around the bubble expressed in unit length of full tube
m	=	exponent
μ_L	=	viscosity of liquid other than water ($\frac{kg.s}{m^2}$)
μ_W	=	viscosity of water ($\frac{kg.s}{m^2}$)

N_{FR} = Bubble Fround Number = $\frac{V_M}{\sqrt{2\ g\ell}}$

Q_G = volume flow rate of gas (m^3/s)

Q_L = volume flow rate of liquid (m^3/s)

ρ = liquid density (kg/m^3)

σ = surface tension of liquid (kg/m)

U = velocity of interface (m/s)

V_b = bubble volume (m^3)

V_G = velocity of the gas phase (bubble) (m/s)

V_{GA} = volume flow density of gas phase (m/s)

V_{G_s} = gas velocity at the point of slug formation (m/s)

V_{LA} = volume flow density of liquid phase (m/s)

V_{LD} = liquid downflow velocity between bubble and wall (m/s)

V_{LDA} = volume flow density of downflow liquid (m/s)

V_M = volume flow density of gas-liquid mixture (m/s)

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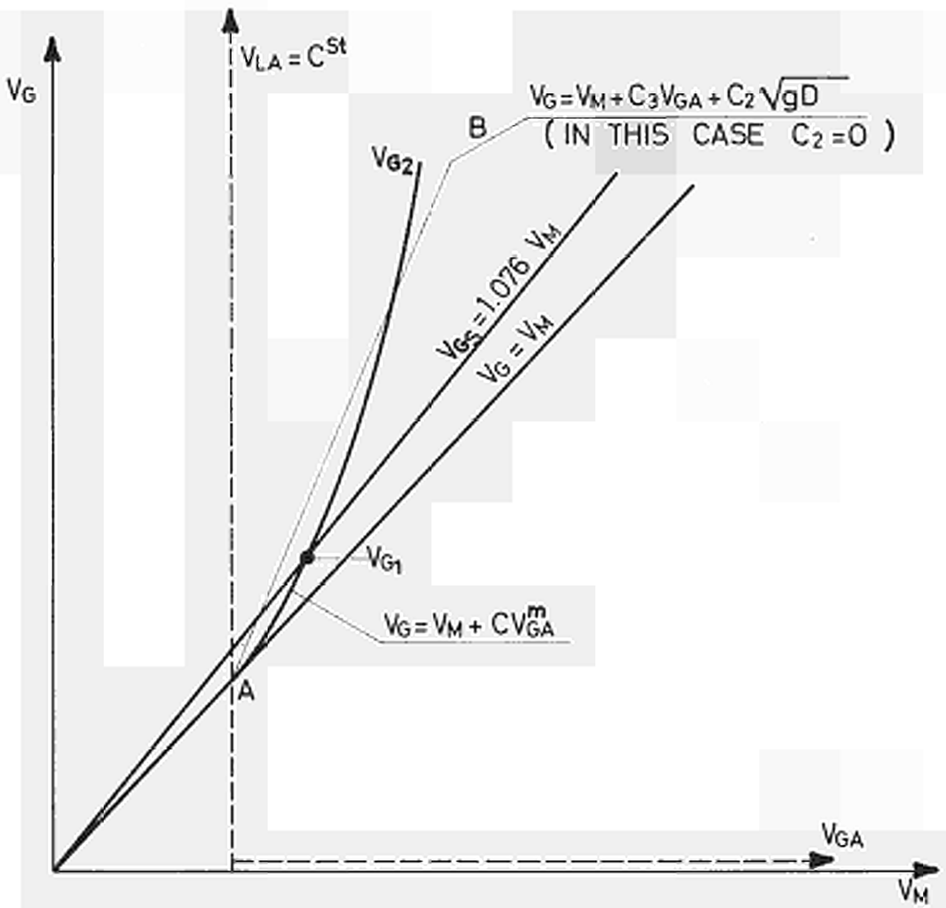

Fig.1 Representation of diverse types of slugflow-equations in a $V_G - V_M$ diagram (Representation out of proportions)

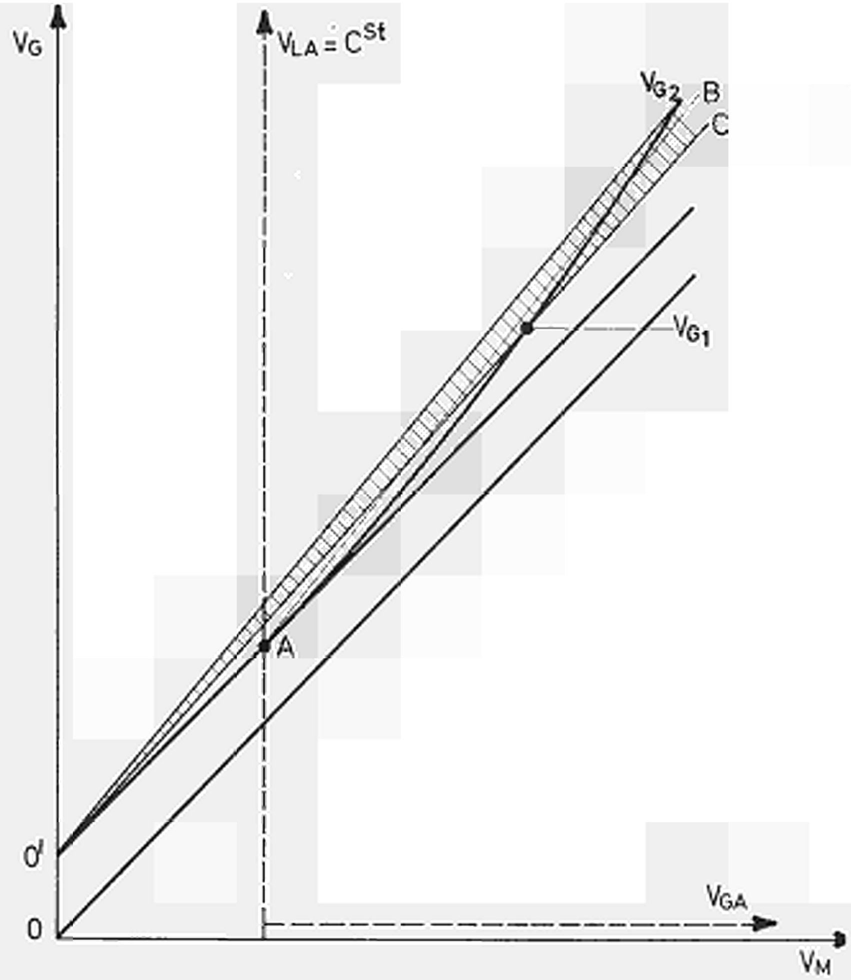

Fig. 2 Representation of diverse types of slugflow-equations in a $V_G - V_M$ diagram. (In realistic proportions)

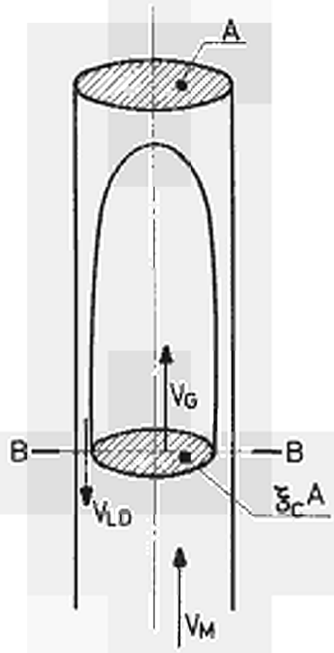

Fig. 3 Continuity equation over cross-section B-B for a slugflow twophase system in A vertical tube.

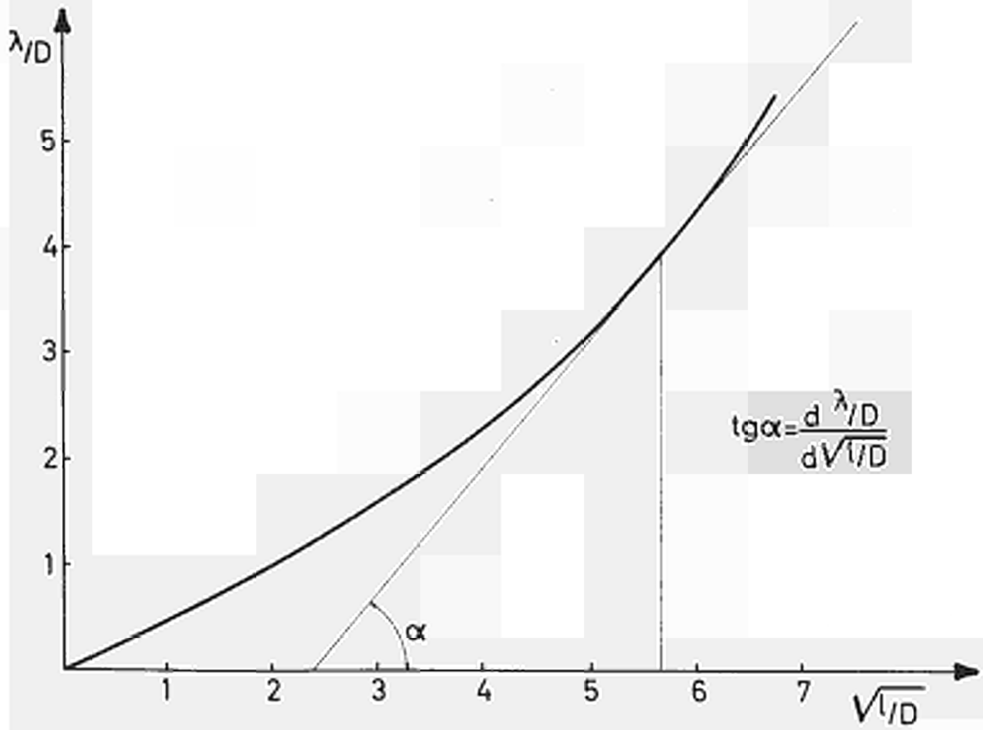

Fig. 4 Liquid holdup around a bulletshaped bubble in equivalent tube diameters versus the square root of bubble lenght in tube diameters.

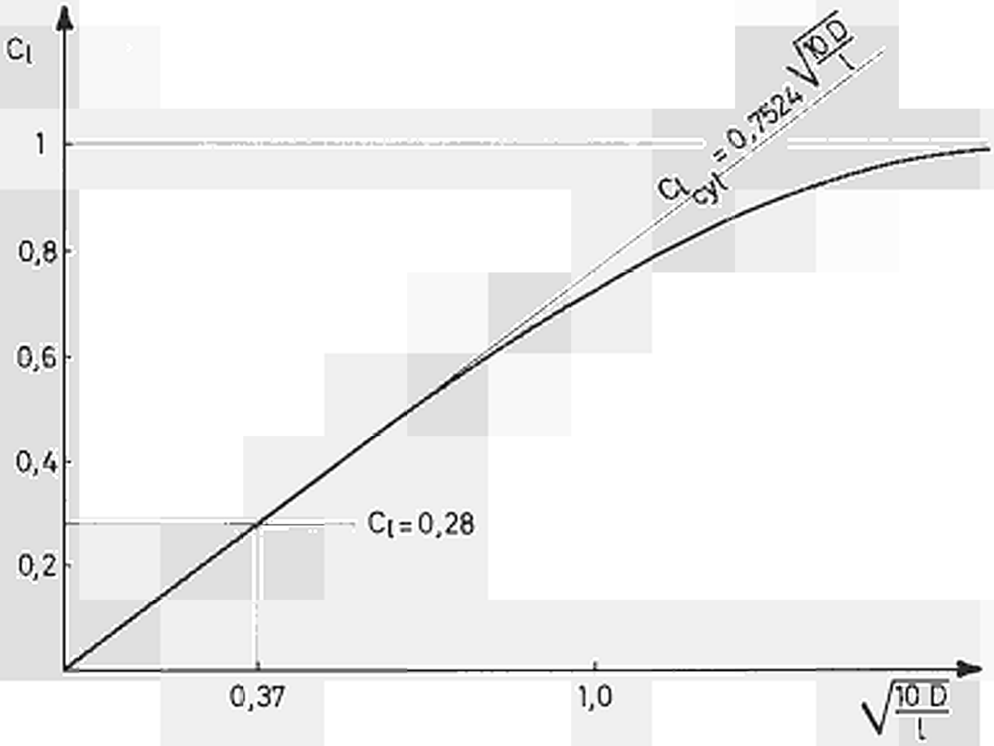

Fig. 5 The adaptation coefficient C_L versus $\sqrt{\frac{10\,D}{l}}$

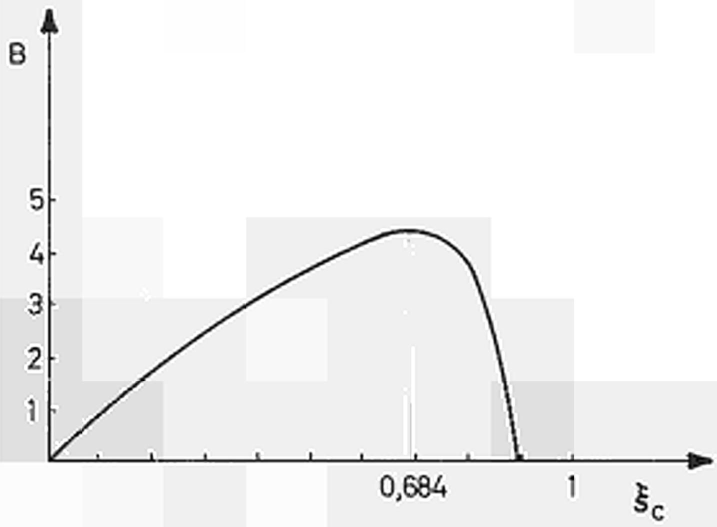

Fig. 6 Function B versus ξ_c with parameter $\xi_c = 0{,}895$ for the case $C_l = C_{l_0}$

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Alfred Nobel

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