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THE ANALYTICAL DETERMINATION OF THE HEAT TRANSFER COEFFICIENT BETWEEN THE FREE SURFACE OF A RIVER AND THE ATMOSPHERE

by

L. NOBEL

1971

Joint Nuclear Research Centre
Ispra Establishment - Italy

Engineering Department
Heat Transfer Division

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The data for these derivations have been taken from the work of Raphael.

By means of the known expression a minimum value has been calculated for the heat transfer coefficient, giving a simple basis for calculating the total heat that can be released on the river without exceeding its given maximum admissible temperature. For this the theory of Wemelsfelder was used.

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ABSTRACT

For a case where the temperature of a river lies above its equilibrium value with the atmosphere the heat transfer coefficient from the free surface of the river to the environment has been determined analytically.

The data for these derivations have been taken from the work of Raphael.

By means of the known expression a minimum value has been calculated for the heat transfer coefficient, giving a simple basis for calculating the total heat that can be released on the river without exceeding its given maximum admissible temperature. For this the theory of Wemelsfelder was used. The report also gives a calculation method for obtaining the natural daily temperature fluctuation of the river.

KEYWORDS

HEAT TRANSFER
SURFACES
RIVERS
ATMOSPHERE
TEMPERATURE

CONTENTS

1. INTRODUCTION *)

The discharge of large quantities of cooling water from electric power stations into rivers or lakes may give rise to a variety of problems, which can be assembled under the term thermal pollution.

In the very concise framework of our report, it will not be possible to go into a discussion of the biological aspects of thermal pollution, so we will therefore assume that the arbitrarily accepted admissible temperature of $30^{o}C$ for river water will in fact avoid all biological problems.

Having done this, the main problem to be solved is thus how to determine the nominal quantity of waste heat which can be permitted to discharge into the river, in order to ensure that even under the most unfavorable conditions the mean temperature of the river will never exceed a value of $30^{o}C$.

To this end the self cooling capacity of a river or lake has to be determined, as a function of the distance between two successive power stations situated along the river, and the volumetric flow capacity of the river.

It will be shown that in particular the heat transfer coefficient from the water surface to the environment must be determined very well, in order to avoid a large part of the cooling capacity of the river remaining unused.

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*) Manuscript received on December 15, 1970

2. THE BASIC EQUATION FOR THE COOLING CAPACITY OF A RIVER

The waste energy to be discharged by means of cooling water into the river amounts to about 0,3 Mcal/sec pro installed electrical Megawatt.

The total amount of waste heat for disposal can thus be calculated from the equation

$$Q_h = 0{,}3\ N \quad \text{Mcal/sec} \qquad (1)$$

if N is the electrical capacity of the powerstation in MW.

This amount of waste heat will be transported by a cooling water stream with a flowrate of K_W m^3/sec, an initial temperature θ_i, and a discharge temperature of θ_K °C.

Thus

$$Q_h = K_W \ \rho_W \ c_{p_W} \ (\theta_K - \theta_i) \qquad (2)$$

here ρ_W specific weight of cooling water in tons/m^3

and c_{p_W} specific heat of cooling water in Mcal/ton°C

After discharge into the river, the temperature of the river, after thorough mixing, will increase to a value of θ_e degrees C to be found by the equation:

$$\theta_e = \theta_i + \frac{K_W}{Q_W} (\theta_K - \theta_i)$$

$$= \theta_i + \frac{Q_h}{Q_W \ \rho_W \ c_{p_W}} \qquad (3)$$

as Q_W is the volumetric flowrate of the river in m^3/sec.

According to the theory of WEMELSFELDER [1] the temperature of the river θ_W decreases to the natural equilibrium temperature θ_N of the river following the equation:

$$\theta_W = \theta_N + (\theta_e - \theta_N)\, e^{-\frac{\bar{\alpha} F}{Q_W}} \qquad (4)$$

here $\bar{\alpha}$ is the heat exchange coefficient from the water surface to the environment taken as a constant during the cooling time in $\frac{\mathrm{Mcal}}{\sec\, m^2\, {}^oC}$, and F is the surface area of the river in m^2, situated between the positions where the temperature of the river amounts to θ_e and θ_W ^oC respectively (see also Fig. 1). If F_o is the surface area of the river between two power stations the temperature of the river must have been cooled down at least to the initial temperature θ_i.

Thus for $F = F_o$, $\theta_W \leq \theta_i$ (5)

This must be guaranteed for the minimum value of the heat transfer coefficient $\bar{\alpha}_{min}$ and the minimum flowrate of the river Q_{Wmin} and also at the maximal attainable natural equilibrium temperature $\theta_{N\,max}$.

According to Wemelsfelder, a value for $\frac{\bar{\alpha} F_o}{Q_W} = 1$ is a sound basis for the calculation of F_o.

Bearing in mind that $\theta_{e\,max} = 30^oC$ we obtain from equations 2 to 5

$$Q_h \leq (30 - \bar{\theta}_N) \left[\frac{e^{F_o \bar{\alpha}/Q_W} - 1}{e^{F_o \bar{\alpha}/Q_W}} \right] Q_W\, \rho_W\, cp_W \qquad (6)$$

Under the most unfavorable conditions $\bar{\alpha}_{min}$, $Q_{W\ min}$ and $\theta_{N\ max}$ is thus, assuming $F_o \bar{\alpha}_{min}/Q_{W\ min} = 1$,

$$Q_{h\ nominal} = (30 - \bar{\theta}_{N\ max})\left(\frac{e - 1}{e}\right) Q_{Wmin}\, \rho_W\, c_{PW} \quad (7)$$

Equation 7 will be called the basic equation for the cooling capacity of a river.

The following example illustrates the foregoing theory.

The following characteristics are given for a river

Min velocity of the river water	W_{min}	$= 0{,}02$ m/sec
Min width of the river	B_{min}	$= 100$ m
Min depth of the river	D_{min}	$= 6$ m
We calculate	$Q_{W\ min}$	$= 12\ m^3/sec$
In addition we know that	$\bar{\alpha}_{min}$	$= 6 \times 10^{-6}\ Mcal/m^2 sec\ ^oC$

From $F_o \bar{\alpha} min/Q_{W\ min} = 1$ we obtain $F_o = 20.000$ m

Now is $\bar{\theta}_{Nmax} = 24^oC$

Then

$$Q_{h\ Nom.} = 6\ \frac{e - 1}{e}\ 12 = 45\ Mcal/sec$$

and

$$N = \frac{45}{0{,}3} = 150\ Mw$$

From this example it becomes clear that the nominal electrical power of a powerstation is determined by the smallest temperature difference between the maximal admissable temperature of the river water and the maximal natural equilibrium temperature of the river and by the minimum throughput of the river.

The distance between stations along the river depends on the minimum value of the heat transfer coefficient and the minimum through put of the river.

The choise of a smaller or greater distance between the stations alters the permissible installed power of the station.

Also an important parameter is the installed power per Km cooling length of the river
With $l_o = 20$ Km, we found

$$\theta_e - \theta_i = 0{,}63\ (\theta_e - \theta_N),\ N = 150\text{Mw and } \frac{N}{l_o} = 7.5\ \frac{\text{Mw}}{\text{Km}}$$

For $l_o = 0{,}8 \times 20$ Km = 16 Km we find

$$O_e - \theta_i = 0{,}55\ (\theta_e - \theta_N),\ N = 131\ \text{Mw and } \frac{N}{l_o} = 8{,}2\ \frac{\text{Mw}}{\text{Km}}$$

For $l_o = 1{,}25 \times 20$ Km = 25 Km we obtain

$$\theta_e - \theta_i = 0{,}71(\theta_e - \theta_N),\ N = 169\text{Mw and } \frac{N}{l_o} = 6{,}8\ \frac{\text{Mw}}{\text{km}}$$

It is beyond the scope of this paper to study the economical merits of the different possibilities. It is however interesting to calculate the cooling time of the river. This is

$$t_d = \frac{l_o}{W_{min}} \times \frac{1}{3600} \times \frac{1}{24} \text{ days} = 12 \text{ days}$$

In our computation example we took a value of 6×10^{-6} Mcal/m^{2}sec^{o}C for $\bar{\alpha}_{min}$. The determination of a reliable value for $\bar{\alpha}_{min}$ is very important, given the relation to the cooling length of the river for one station.

The scope of the following paragraphs will be to find theoretically the best value for $\bar{\alpha}_{min}$.

3. THE NATURAL TEMPERATURE OF THE RIVER

Even without any thermal pollution the temperature of a river will fluctuate during the day due to mass transfer to and heat exchange with the environment (the influence of rain will be ignored). The physical processes responsable for this temperature fluctuation and for the nominal value of the natural temperature of the river are:

1) the short wave solar radiation heatflux (q_s)
2) the difference between the long wave back radiation of the water surface and the long wave atmospherical radiation ($\pm\ q_R$)
3) the energy transported by evaporation (q_E)
4) the heat exchange by air convection ($\pm\ q_C$)

The temperature changes of the river may now be found from the equation

$$d\,\theta_N = \frac{\Sigma q}{\rho_W\, c_{PW}\, Q_W}\, d\,F \qquad (8)$$

Because $dF = BWdt$ and $Q_W = WBD$, this equation can be modified into

$$d\theta_N = \frac{\sum q}{\rho_W \; c_{P_W} \; D} \; dt \qquad (9)$$

This equation can only be solved if q_s, q_r, q_e and q_c are known functions of t (t = given in seconds)

3.1. For the short wave solar radiation heat flux q_s only rough estimations can be made.
The maximum total effective insolation for a latitude of 46°45' averaged over a period of 24 hours can be approximated by the equation

$$\bar{q}_s = 4.25 + 3.25 \sin \frac{2\pi t_d}{365} \quad \frac{\mathrm{Mcal}}{m^2 \mathrm{day}} \qquad (10)$$

Here q_s is the value of q_s averaged over 24 hours
t_d is the time in days
$t_d = 0$ coincides with the 21st of March

Equation 10 gives a maximum on the 21st of Juin having $q_s = 7.5$ Mcal/m²day.
On this day the number of hours of sun shine at this latitude can reach up to 15-16 hours.
We assume that the solar radiation heat flux distribution over n hours of daylight can be given by the equation

$$q_s = \hat{q}_s \sin \frac{2\pi t_h}{2n} \quad \frac{\mathrm{Mcal}}{m^2 \sec} \qquad (11)$$

in which formula $\hat{q}_s$ is the daily maximum value of q_s, and t_h the time in hours.

$t_h = 0$ coincides with the moment that the sun rises above the horizon

Integration of equation 11 for $n = 16$, and determination of the averaged value $\bar{q}_s$ gives

$$\bar{q}_s = 0{,}422\ \hat{q}_s$$

or

$$\hat{q}_s = 2{,}37\ \bar{q}_s$$

With $\bar{q}_s = 7{,}5$ Mcal/m^{2}day, we arrive at

$$\hat{q}_s = 17{,}93 \text{ Mcal/m}^2\text{day} = 207 \times 10^{-6} \text{ Mcal/m}^2\text{sec}$$

The maximum solar altitude on the latitude of $46^{o}45'$ is on the 21st of juin about 66^{o}.

From the diagram given by RAPHAEL [2] and reproduced in our Fig. 2 for a cloudcover equal to zero the insolation amounts $210{,}84 \times 10^{-6}$ Mcal/m^{2}sec at a solar altitude of 66^{o}.

Equations 10 and 11 enable us to calculate for any day of the year the solar radiation heat flux distribution for an uncovered sky.

For any other latitude, however, we have to know the maximum solar altitude on the dates of the 21st of decembre and on the 21st of juin. Moreover we have to know the number of daylight hours on these days in this geographical position. With the aid of Fig. 2 an equation similar to equation 10 can then be found.

The minimum value of equation (10), occuring on the 21^{st} of Decembre amounts $1 \dfrac{Mcal}{m^2 day} = 11,57 \times 10^{-6} \dfrac{Mcal}{m^2 sec}$

On this date the maximum solar altitude at $46^{o}45'$ amounts to 20^{o} and the number of daylight hours is about 8.

The max. effective insolation $\hat{q}_s$ according to Fig. 2 amounts to 62×10^{-6} Mcal/m^2sec. The averaged total effective insolation may now be found by the formula

$$\bar{q}_s = \frac{n}{24} \times 0,63 \times \hat{q}_s = 13 \times 10^{-6} \frac{Mcal}{m^2 sec}$$

Comparison between this result and the value found with equation (10) gives good agreement (The value found with Fig. 2 lies about 12% higher).

For a cloud cover factor $0 \leq C \leq 1$ the solar radiation heat flux may be found from the formula also taken from [2], i.e.

$$\hat{q}_{s_c} = \hat{q}_s \; (1 - 0,71 \; C^2) \quad , \qquad (12)$$

3.2. Let us now consider the long wave radiation difference between the atmosphere and the surface of the river. For the effective long wave atmospheric radiation we can write

$$q_{RA} = 0,97 \, \sigma \beta \; (\theta_A + 273)^4 \; \frac{Mcal}{m^2 sec} \qquad (13)$$

In this formula σ = Stephan Boltzmann constant

$$= 5{,}5 \times 10^{-6} \frac{\text{Mcal}}{m^2 \sec \text{day}^4}$$

β = function of cloudcover factor and the partial pressure of the vapour in the atmosphere.

The back radiation of the water body can be found from the eqaution

$$q_{RW} = e\,\sigma \left(\frac{\theta_W + 273}{100}\right)^4 \frac{\text{Mcal}}{m^2 \sec} \qquad (14)$$

e is here the emissivity of the water surface = 0,97 Because normally the temperature of the water is somewhat higher than the air temperature. The net radiation

$$q_R = q_{RW} - q_{RA}$$

is then to be found from:

$$q_R = 0{,}97\,\sigma \left[\left(\frac{T_W}{100}\right)^4 - \beta \left(\frac{T_A}{100}\right)^4 \right]$$

$$= 1{,}335 \times 10^{-6} \left[\left(\frac{T_W}{100}\right)^4 - \beta \left(\frac{T_A}{100}\right)^4 \right] \frac{\text{Mcal}}{m^2 \sec} \qquad (15)$$

Writing for $\beta\, T_A^4 = (\varepsilon\, T_A)^4 = (T_W - \Delta T_\varepsilon)^4$

equation (15) may be transformed into the approximation

$$q_R = 5{,}34 \times 10^{-8} \left(\frac{T_W}{100}\right)^3 \left[T_W - \varepsilon\, T_A \right] \frac{\text{Mcal}}{m^2 \ \sec} \qquad (16)$$

The factor ε as a function of the partial vapour pressure in the air has been given in Fig. 3. The parameter in this diagram is the cloud cover factor C.

3.3. A large part of the energy transferred to the water surface will be transported back into the atmosphere by evaporation.

The physical condition of the air is of great importance in this process. This fact can be demonstrated clearly by means of the MOLLIER-diagram FIG. 4.
At the water surface the boundary layer formed by the air has the watertemperature θ_W.
This air layer will be saturated with a watercontent $X_W \frac{\text{kg water}}{\text{kg dry air}}$.
The air at far from the water surface has the temperature θ_A (dry air temperature) and a wet bulb temperature to be indicated by θ_{A_b}. Extrapolation of the θ_{A_b}- isotherm out of the supersaturated region in the sub saturated region, gives by intersection with the θ_A isotherm the actual water content X_{A_b} of the air.
The mass-flux due to evaporation may now be expressed by the equation:

$$q_{m_e} = \alpha_{m_e} (X_W - X_{A_b}) \qquad (17)$$

Here α_{m_e} = evaporation coefficient in ton/m^{2}sec.
α_{m_e} is strongly dependent on the windvelocity and for fairly high wind velocities it can be substituted

by

$$\alpha_{m_{\ell}} = 1{,}25 \quad 10^{-3} \quad \rho_A \quad V_A \qquad (18)$$

in this formula ρ_A is the density of the air in ton/m^3 and V_A the wind velocity in m/sec.

Equation (17) can be modified into a heatflux equation, bearing in mind that

$$q_{\ell} = q_{m\ell} \quad L \qquad (19)$$

in which formula L = latent heat of the water in Mcal/ton.

So by combining eq. (17), (18) and (19) we obtain

$$q_{\ell} = -1{,}25 \times 10^{-3} \rho_A \, V_A \quad L \, (X_W - X_{A_b}) \qquad (20)$$

The water content of the air will fluctuate between the limit values $X_{A_b\,max}$ and $X_{A_b\,min}$ corresponding to the air temperatures $\theta_{A\,max}$ and $\theta_{A\,min}$.

Let us assume that a relation of the following type would be realistic (see also Fig. 5)

$$X_{A_b} = X_{A_b\,max} - \frac{\theta_{Amax} - \theta_A}{\theta_{Amax} - \theta_{Amin}} (X_{Abmax} - X_{Abmin}) \qquad (21)$$

This relation may be transformed into:

$$X_W - X_{A_b} = \frac{X_{Abmax} - X_{A\,bmin}}{\theta_{A\,max} - \theta_{A\,min}} \left\{ \frac{(\theta_{Amax} - \theta_{Amin}) X_W + \theta_{Amin} X_{Abmax} - \theta_{Amax} X_{Abmin}}{X_{Abmax} - X_{Abmin}} - \theta_A \right\}$$

(22)

For small variations in θ_W this expression may be written as

$$X_W - X_{Ab} = f(\frac{X}{\theta}) \left\{ m\, \theta_W^2 - \theta_A \right\} \qquad (23)$$

In which formula m can be taken as a constant.
We found the averaged value for m is $m = 0{,}1535\ {}^{\circ}C^{-1}$
Combination with formula (20) gives

$$q_e = 1{,}25 \times 10^{-3}\, \rho_A\, V_A\, L\ \ f(\frac{X}{\theta}) \left\{ m\theta_W^2 - \theta_A \right\} \qquad (24)$$

$$f(\frac{X}{\theta}) = \frac{X_{Abmax} - X_{Ab\,min}}{\theta_{Amax} - \theta_{Amin}} \quad {}^{\circ}C^{-1}$$

$$m = \frac{(\theta_{Amax} - \theta_{Amin})X_W + \theta_{Amin} X_{Abmax} - \theta_{Amax} X_{Abmin}}{(X_{Abmax} - X_{Abmin})\ \theta_W^2}$$

In formula (24), θ_A will be a function of time e.g. for a mid summer day it is:

$$\theta_A = 25 + 9 \sin \frac{2\pi (t_h - 10)}{24} \qquad (25)$$

In this formula t_h is the time in hours.
The daily fluctuation of the natural water temperature roughly follows two sine curves which can be expressed by the formulas:

$$\theta_N = \bar{\theta}_{N_1} + \Delta\theta_{W_1} \sin 2\pi \frac{(t_h - a)}{2} \qquad (26)$$

between $t_h = a - \frac{n}{2}$ and $t_h = a + \frac{n}{2}$

and

$$\theta_N = \bar{\theta}_{N_2} + \Delta\theta_{W_2} \sin \frac{2\pi\ t_h + 12 - (a + n)}{2\ (24 - n)} \qquad (27)$$

between $t_h = a + \frac{n}{2}$ and $t_h = a - \frac{n}{2} + 24$

In these formulas is $\bar{\theta}_{N_1} = \bar{\theta}_N$ for $t_h = a$, $\bar{\theta}_{N_2} = \bar{\theta}_N$ for $t_h = a+12$

$2n$ = period of first sinecurve

$2(24-n)$ = period of second sine curve

$\Delta\theta_{W_1}$ and $\Delta\theta_{W_2}$ = amplitudes of the sine curves

See for definitions Fig. 6

As we have already stipulated, the temperature of the water will increase or decrease after 24 hours depending on equation (9), the integration of which gives

$$\Delta\theta_N = 3600 \frac{\int_0^{24} \sum q\ dt_h}{\rho_W\ c_{P_W}\ D} \qquad (28)$$

The daily temperature increase or decrease may also be found roughly by using mean temperature curves of the type

$$\bar{\theta}_N = 14 + 10 \sin \frac{2\pi\ t_d}{365} \qquad (29)$$

starting t_d at the 15th of april e.g.

3.4. The heat exchange by air convection can be found from an equation

$$q_C = \alpha_C \, (\theta_A - \theta_W) \, \frac{\text{Mcal}}{m^2 \sec} \qquad (30)$$

For α_C can be found

$$\alpha_C = 1{,}15 \, \rho_A \, v_A \, c_{PA} \, 10^{-3} \, \frac{\text{Mcal}}{m^2 \sec {}^{o}C} \qquad (31)$$

So the complete formula becomes

$$q_C = 1{,}15 \, \rho_A \, v_A \, c_{PA} \, 10^{-3} (\theta_A - \theta_W) \, \frac{\text{Mcal}}{m^2 \sec} \qquad (32)$$

3.5. In the following example we will try to calculate the temperature oscillation of a river assuming that $\bar{\theta}_N$ over the heating up period is equal to the $\bar{\theta}_N$ value over the period of cooling down. The further assumptions are made as follows:

$\bar{\theta}_N \; = \; 24^{o}C$

θ_A follows equation (25)

$$q_s = 100 \times 10^{-6} \sin \frac{2\pi (t_h - 4)}{32} \, \frac{\text{Mcal}}{m^2 \; \sec}$$

only for the positive values

$$q_R = - 1{,}44 \times 10^{-6} \left[T_W - 0{,}97 \, T_A \right] \frac{\text{Mcal}}{m^2 \sec}$$

θ_N follows equations (26) and (27)

$$q_E = - 1{,}25 \times 10^{-3} \; \rho_A \; v_A \, L \, f \, \left(\frac{X}{\theta}\right)(0{,}1535 \quad \theta_N^2 - \theta_A) \, \frac{\text{Mcal}}{m^2 \sec}$$

$$q_C = 1{,}15\rho_A\ V_A\ C_{p_A} x10^{-3}(\theta_A-\theta_N) \approx 0{,}36x10^{-6} V_A(\theta_A-\theta_N)\ \frac{\text{Mcal}}{\text{m}^2\text{sec}}$$

For the air conditions we assume $X_W = 0{,}019$

$X_{A_b\ max} = 0{,}01$

$X_{A_b\ min} = 0{,}007$

Then $f(\frac{X}{\theta}) = \frac{0{,}003}{18} = 16{,}7x10^{-5}\ {}^oC^{-1}$

Apart from $\Delta\theta_W$ we have also to determine V_A. Over a period of 24 hours $\Delta\bar{\theta}_W = 0$, thus

$$3600\left[\int_0^{24} q_s\,dt_h + \int_0^{24} q_R\,dt_h + \int_0^{24} q_e\,dt_h + \int_0^{24} q_c\,dt_h\right] = 0$$

Determining the terms of the above equations, we obtain

$$3600\int_0^{24} q_s dt_h = 0{,}633x100x10^{-6}x24x3600 = 3{,}64\ \frac{\text{Mcal}}{\text{m}^2}$$

$$\int_0^{24x3600} q_R dt = -1{,}44x10^{-6}x24x3600x8 = -\ 1.00\ \frac{\text{Mcal}}{\text{m}^2}$$

$$\int_0^{24x3600} q_e dt = -1{,}25x10^{-3}x1{,}3x10^{-3}x587x16{,}7x10^{-5}x24x3600xV_Ax(88-25)$$

$$= -\ 0{,}866\ V_A\ \frac{\text{Mcal}}{\text{m}^2}$$

$$\int_0^{24x3600} q_C dt = 0{,}36V_Ax10^{-6}x24x3600x[25-24] = 0{,}031V_A\ \frac{\text{Mcal}}{\text{m}^2}$$

Taking the summation of these terms gives

$$\int_0^{24\times3600} q\,dt = 0 \quad \text{or} \quad 2{,}64 - 0{,}835\, V_A = 0$$

This means that the air velocity amounts to V_A=3,16 m/sec which is certainly not an unrealistic value.

Now we have to calculate the half ripple value of the daily temperature fluctuation and the periods of the two sinecurves which compose the temperature ripple. Starting with the latter problem, we assume as a first approximation a value of 1 $^{\circ}$C for the amplitudes of the sinefluctuations. For the minimum value of the first sinecurve the summation of the actual heatfluxes is zero. So we obtain the relation:

$$100\ 10^{-6} \sin \frac{\pi}{16}(t_h-4)+1.44\ 10^{-6}\left\{0{,}97\left[298+\ \sin\frac{\pi}{12}(t_h-10)\right]-296\right\}$$
$$-0{,}503\ 10^{-6}\left\{81{,}2-25-\ \sin\frac{\pi}{12}(t_h-10)\right\}+1.14\ 10^{-6}\left\{25+9\sin\frac{\pi}{12}(t_h-10)-23\right\} = 0$$

This gives the solution $t_h \approx 6$ h 55'

For the maximum of the second sine curve we obtain:

$$100\ 10^{-6} \sin\frac{\pi}{16}(t_h-4)+1{,}44\ 10^{-6}\left\{0{,}97\left[298+\ \sin\frac{\pi}{12}(t_h-10)\right]-298\right\}$$
$$-0{,}503\ 10^{-6}\left\{96-25-9\sin\frac{\pi}{12}(t_h-10)\right\}+1.14\ 10^{-6}\left\{25+9\sin\frac{\pi}{12}(t_h-10)-25\right\}=0$$

This gives the solution $t_h \approx 18$ h 30'
In this particulare case of our example that $\Delta\bar{\theta} = 0$ is the input heat equal to the output heat, this means that we have only one temperature sine curve with the minimum temperature at t_h = 6h45' and the maximum temperature at t_h = 18h45'.

We are now able to calculate $\Delta\theta_W$. In this calculation however we still assume that $\Delta\,\theta_W \leqq 1^\circ C$ and the temperature sine curve of the water can be written as $\theta_N = 24 + \sin\,(t_h - 12{,}75)\,\frac{\pi}{12}$. The scope of the following calculation is to check whether the assumption that $\Delta\,\theta_W \leqq 1^\circ C$ is valid.
We obtain:

$$\theta_W = \frac{1}{2}\ \frac{3600}{6}\ 10^{-4}\Bigg[\int_{6,75}^{18,75} \sin\frac{\pi}{16}\,(t_h - 4)\,dt_h$$

$$+ 1.44x0{,}97x2{,}98x12 + 1{,}44x0{,}97x0{,}09\int_{6,75}^{18,75} \sin\frac{\pi}{12}(t_h - 10)dt_h$$

$$-1.44x2{,}97x12 - 1.44x0{,}01\int_{6,75}^{18,75} \sin\frac{\pi}{12}\,(t_h - 12{,}75)\,dt_h$$

$$-0{,}503x0{,}1535x0{,}01\int_{6,75}^{18,75}\Big[24 + \sin(t_h - 12{,}75)\,\frac{\pi}{12}\Big]^2 dt_h$$

$$+0{,}503x12x0{,}25 + 0{,}503x0{,}09\int_{6,75}^{18,75} \sin\frac{\pi}{12}(t_h - 10)dt_h$$

$$+1.14x12x0{,}01 + 1{,}14x0{,}09\int_{6,75}^{18,75} \sin\frac{\pi}{12}(t_h - 10)\ dt_h$$

$$-1.14x0{,}01\int_{6,75}^{18,75} \sin\frac{\pi}{12}\,(t_h - 12{,}75)dt_h\Bigg]$$

We can solve from this equation $\Delta\theta_W = 0{,}17\ ^{\circ}C$.

4. THE COOLING CAPACITY OF THE RIVER IN THE CASE OF THERMAL POLLUTION

In the foregoing paragraphs we have studied the natural cooling process of a river and we have found that the natural temperature of a river fluctuates with an amplitude reversed proportionally with the depth of the river.

For a not too shallow river the amplitude of the temperature fluctuation is of the order to one degree C. An initial temperature jump will decline rapidly but in fact follows the temperature fluctuations of the river.

We will now draw a comparison between the thermal balances for the heat exchange of the river surface and the environment, both for the case of thermal pollution of the river and for the case of the natural cooling process.

With thermal pollution we have:

$$q_p = q_s - 0{,}97 \times 1{,}38 \times 10^{-6} \left[\left(\frac{T_W}{100}\right)^4 - \beta\left(\frac{T_A}{100}\right)^4 \right] - 1{,}25 \times 10^{-3}\, \rho_A\, v_A\, L \left[x_W - x_{Ab} \right] - 1{,}15 \times 10^{-3}\, \rho_A\, v_A\, c_{PA} \left[\theta_W - \theta_A \right]$$

Without thermal pollution we obtain:

$$q_n = q_s - 0,97 \times 1,38 \times 10^{-6} \left[\left(\frac{T_N}{100}\right)^4 - \beta \left(\frac{T_A}{100}\right)^4 \right] -$$
$$1,25 \times 10^{-3} \, \rho_A \, v_A \, L \left[X_N - X_{Ab} \right] -$$
$$1,15 \times 10^{-3} \, \rho_A \, v_A \, c_{PA} \left[\theta_N - \theta_A \right]$$

The difference $q_p - q_n$ gives the extra cooling due to the temperature excess.

It yields:

$$q_p - q_n = - 1,38 \times 10^{-6} \left\{ \left[\frac{T_W}{100}\right]^4 - \left[\frac{T_N}{100}\right]^4 \right\} -$$
$$1,25 \times 10^{-3} \, \rho_A \, v_A \, L \, (X_W - X_N) -$$
$$1,15 \times 10^{-3} \, \rho_A \, v_A \, c_{PA} \, (\theta_W - \theta_A) \qquad (33)$$

The first term may again be written as

$$- 0,055 \times 10^{-6} \left[\frac{T_W}{100}\right]^3 (\theta_W - \theta_N)$$

In the second term we may substitute e.g. X_W by

$$X_W = 0,622 \, \frac{P_W}{P}$$

and $P_W - P_N = \dfrac{L}{T_N \, v''} \, (\theta_W - \theta_N)$

Writing P in bar we obtain for the second term

$$- \frac{3,24 \times 10^{-2} \, \rho_A \, v_A \, L^2}{P \, T_N \, v''} \, (\theta_W - \theta_N)$$

Deviding the terms by $\theta_W - \theta_N$ and substituting T_W by T_N we obtain

$$\frac{q_p - q_n}{\theta_W - \theta_N} = \alpha = 0{,}055 \times 10^{-6} \left[\frac{T_N}{100}\right]^3 + \frac{3{,}24 \times 10^{-2} \rho_A v_A\ L^2}{P\ \ T_N\ \nu''} +$$

$$1{,}15 \times 10^{-3} \rho_A\ v_A\ c_{PA}\ \ \frac{\text{Mcal}}{m^2\,{}^oC\ \text{sec}} \tag{34}$$

In the following calculation example we give an estimation of the nominal power of a block of power stations to be constructed in the Ludwigshafen agglomeration.

The cooling time of a river is normally more than 7 days.

Over this period we may take as an averaged heat transfer coefficient the value $\bar{\alpha} = 8 \times 10^{-6}\ \frac{\text{Mcal}}{m^2\,{}^oC\ \text{sec}}$

Furthermore we assume that $Q_{W_{min}} = 800\ m^3/\text{sec}$.

Then the heat capacity of the river is $800 \times 1 \times 1 = 800\ \frac{\text{Mcal}}{\text{sec}\,{}^oC}$

According to the criterium by Wemelsfelder is

$$F_o = \frac{Q_W\ \rho_W\ c_P}{\bar{\alpha}} = \frac{800}{8 \times 10^{-6}} = 10^8\ m^2$$

The width of the Rhine amounts as a mean value to about 400 m.

Thus the cooling length is $l_o = \frac{10}{4 \times 10^2} = 2{,}5 \times 10^5 m = \underline{250\ \text{km}}$

Assuming $\bar{\theta}_{N\ max} = 22^{o}C$, the maximum amount of heat to be stored by the river is

$$Q_{hNOM} = 8 \ \frac{e - 1}{e} \ 800 \ = \ 4000 \text{ Mcal/sec}$$

The nominal power of the block of power stations to be installed in the Ludwigshafen agglomeration is then

$$N = \frac{4000}{0,3} \quad 13.350 \text{ Mw}$$

A reduction of the cooling length up to 200 km, gives

$$Q_{h_{NOM}} = 8 \ \frac{e^{0,8} - 1}{e^{0,8}} \ 800 = 3520 \text{ Mcal/sec}$$

In this case the nominal power of the block is

$$N = \frac{3520}{0,3} = 11730 \text{ Mw}$$

After this cooling length the temperature of the Rhine is

$$\theta = 22 + (1 - 0,55)\ 8 = 25,6\ ^{o}C$$

In the case of a strongly increased debit of the Rhine the cooling time is in the order of one day, therefore we have to take for

e.g. Q_W=4000 m^3/sec a low α-value, say 5×10^{-6} $Mcal/m^2 sec\ ^{o}C$

Then the temperature of the Rhine will be

$$\theta = 22 + 8\, e^{-\frac{6 x 10^{-6} x 8 x 10^{7}}{4000}} = 29.24\ ^{o}C$$

However the heat discharged into the environment is still

$$Q_h = 4000 \times 0{,}76 \approx 3000 \text{ Mcal/sec}$$

and the power to be installed may be

$$N = \frac{3000}{0{,}3} = 10.000 \text{ Mw}$$

With the theory concerning the heat transfer to the environment we find with a $\overline{\theta_W - \theta_N} = 7.62\ ^{o}C$

$$Q_h = \alpha . F . \Delta\theta$$
$$= 5 \times 10^{-6}\ \ 8x10^{7} \times 7.62 = 3000 \text{ Mcal/sec}$$

From this example we see that for $Q_W \rightarrow \infty$, $\Delta\theta \rightarrow 8\ ^{o}C$ and

$$Q_h = 5 \times 10^{-6}\ \ 8x10^{7} x8 = 3200 \text{ Mcal/sec}$$

and $N = \frac{3200}{0{,}3} = \underline{10665 \text{ Mw}}$

Concluding the results of these calculations we may say that dividing the Rhine into cooling parts of about 200 km each we may install blocks of power stations with 10.000 Mw per cooling part.

Remark: In the foregoing view, we have not taken in consideration the temperature ripple due to the influences of the environment.

We have seen that this temperature ripple for $D > 6$ m

was smaller than $1^{o}C$.
Under these circumstances this temperature ripple may be accepted, since there is a growing tendency to allow a maximum temperature of $32^{o}C$.

5. CONCLUSION

In equilibrium with the natural heatbalance of the river (a heat balance due to atmospherical influences composed by the solar radiation, thc effective back radiation of the water surface, the evaporation cooling of the water and the heat exchange by air convection) the natural temperature of the river fluctuates during day and night and during the seasons of the year. The temperature ripple over 24 hours is in inverse proportion to the depth of the river. From an external heat source the river may be heated up several degrees C at the point of heat release.
However, a mean temperature of $30^{o}C$ may not be exceeded. Due to the temperature overshoot in comparison with the natural-equilibrium temperature of the river, the heat exchange from the free river surface to the environment takes place according to the formula
$q_h = \alpha F (\theta_W - \theta_N)$ in which

$$\alpha = 0{,}055 \times 10^{-6} \left(\frac{T_N}{100}\right)^3 + \left[\frac{3{,}24 \times 10^{-2} L^2 \rho''}{P \, T_N} + 1.15 \times 10^{-3} \, c_{PA}\right] \rho_A \, v_A \quad \frac{\mathrm{Mcal}}{m^2 \, {}^{o}C \, \mathrm{sec}}$$

The α -value may be taken as $\bar{\alpha} = 8 \times 10^{-6} \frac{Mcal}{m^2 \; sec \; ^\circ C}$ in cases where the cooling time has a duration of at least 7 days.

Where the throughput of the river is very high, and the cooling time reduced to about one day, we have to use in our calculations the value $\alpha_{min} = 5 \times 10^{-6} \frac{Mcal}{m^2 sec \; ^\circ C}$.

The determination of the amount of electrical power to be installed in a group of power stations distributed along the boarder of a river and erected at a given distance between each block has to be made by means of the formula

$$N = \frac{Q_{W\,min} \left[\dfrac{e^{\frac{\bar{\alpha} F_o}{Q_{W_m}}} - 1}{e^{\frac{\bar{\alpha} F_o}{Q_{W_m}}}} \right] (30 - \theta_{Nmax})}{0,3} \quad Mw$$

or from

$$N = \frac{\alpha_{min} \quad F_o \; (30 - \theta_{N_{max}})}{0,3} \qquad Mw$$

We will take the smallest value of the two.

Remarks F_o may be found from:

$$0,8 \leq \frac{\bar{\alpha} \; F_o}{Q_{W_{min}}} \leq 1.25$$

$F_o = l_o \cdot B$, l_o being the distance between the blocks of power stations.

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NOMENCLATURE

a	= value of t_h at which time $\Theta_N = \Theta_{N_1}$	hours
B	= width of the river	m
B_{min}	= minimum width of the river	m
C	= cloud cover factor	
c_{p_A}	= specific heat of air	$Mcal/ton^oC$
c_{p_W}	= specific heat of water	$Mcal/ton^oC$
D	= depth of the river	m
D_{min}	= minimum depth of the river	m
e	= emissivity of the water surface	
e	= basic number of Nep. log.	
F	= part of cooling surface of the river	m^2
F_o	= cooling surface of the river between two power stations	m^2
$f\ (\frac{X}{\Theta})$	= see definition on page 14	$^oC^{-1}$
K_W	= volumetric flowrate of cooling water	m^2/sec
L	= latent heat of water	Mcal/ton
l_o	= cooling length of the river between two power stations	m
m	= see definition on page 14	$^oC^{-1}$
N	= electrical capacity of a power station or of a block of power stations	Mw
n	= number of daylight hours	
P	= pressure of the atmosphere	bar
p_N	= vapour pressure at temperature Θ_N	N/m^2
p_W	= vapour pressure at temperature Θ_W	N/m^2
Q_h	= amount of waste heat to be rejected on the river	Mcal/sec

$Q_{h_{NOM}}$	= amount of waste heat based on installed electrical power	Mcal/sec
Q_W	= volumetric flowrate of the river	m^3/sec
$Q_{W_{min}}$	= minimum discharge of the river	m^3/sec
q_c	= heatflux by air convection	Mcal/m^2sec
q_e	= heatflux due to evaporation	Mcal/m^2sec
q_n	= total actual heatflux from the water surface to the environment under natural conditions	Mcal/m^2sec
q_p	= total actual heatflux from the water surface to the environment if the water is thermally polluted	Mcal/m^2sec
q_R	= net long wave radiation heat flux	Mcal/m^2sec
q_{RA}	= effective long wave radiation heat flux of the atmosphere	Mcal/m^2sec
q_{RW}	= back radiation heat flux of the water body	Mcal/m^2sec
q_s	= short wave solar radiation heat flux	Mcal/m^2sec
$\bar{q}_s$	= insolation heatflux averaged over 24 hours normally in	Mcal/m^2day
$\hat{q}_s$	= daily maximum value of q_s	Mcal/m^2sec
$\hat{q}_{s_C}$	= daily maximum insolation heat flux at a cloud cover C	Mcal/m^2sec
T_A	= Kelvin temperature of air	^oK
T_N	= natural Kelvin temperature of water	^oK
T_W	= Kelvin temperature of water	^oK
t	= time in seconds	sec
t_h	= time in hours	sec
t_d	= time in days	sec

V_A	= wind velocity	m/sec
W	= velocity of river water	m/sec
W_{min}	= minimum velocity of river water	m/sec
X_{Ab}	= actual water content of air	kg/kg
$X_{Ab\ max}$	= X_{Ab} at max. temperature of air	kg/kg
$X_{Ab\ min}$	= X_{Ab} at min. temperature of air	kg/kg
X_N	= max. water content in the air based on θ_N	kg/kg
X_W	= max. water content in the air based on θ_W	kg/kg
$\bar{X}_W$	= max. water content in the air based on $\bar{\theta}_N$	kg/kg
α	= heat transfer coefficient from the free river surface to the environment	$\frac{Mcal}{m^2\ {}^oC\ sec}$
$\bar{\alpha}$	= heat transfer coefficient averaged over the cooling time (at least 7 days)	$\frac{Mcal}{m^2\ {}^oC\ sec}$
α_{min}	= heat transfer coefficient averaged over a day under the most unfavorable conditions	$\frac{Mcal}{m^2\ {}^oC\ sec}$
α_C	= heat transfer coefficient due to convective heat transfer	$\frac{M\ cal}{m^2\ {}^oC\ sec}$
α_{m_e}	= evaporation coefficient	$\frac{ton}{m^2\ sec}$
β	= function of cloud cover factor and partial vapour pressure	
ε	= $\sqrt[4]{\beta}$	^oC
θ_A	= temperature of the air	^oC
θ_{Ab}	= wet bulb temperature of the air	^oC
$\theta_{A_{max}}$	= daily max. temp. of the air	^oC
$\theta_{A_{min}}$	= daily min. temp. of the air	^oC

Θ_e	= temperature of the river at a cooling length = 0	^oC
Θ_i	= temperature of the river after cooling length l_o	^oC
Θ_K	= discharge temperature of cooling water after condensor	^oC
Θ_N	= natural temp. of the water	^oC
$\bar{\Theta}_N$	= daily averaged natural temp. of the river	^oC
$\bar{\Theta}_{N_1}$	= mean water temperature over first half sine	^oC
$\bar{\Theta}_{N_2}$	= mean water temperature over second half sine	^oC
Θ_W	= water temp. of the river at a given point	^oC
$\Delta \Theta_{W_1}$	= amplitude of first half sine-curve	^oC
$\Delta \Theta_{W_2}$	= amplitude of second half sine-curve	^oC
σ	= Stephan Boltzmann constant	
ρ_A	= specific weight of air	ton/m^3
ρ_W	= specific weight of water	ton/m^3
$v'' = 1/\rho''$	specific volume of water vapour	m^3/ton

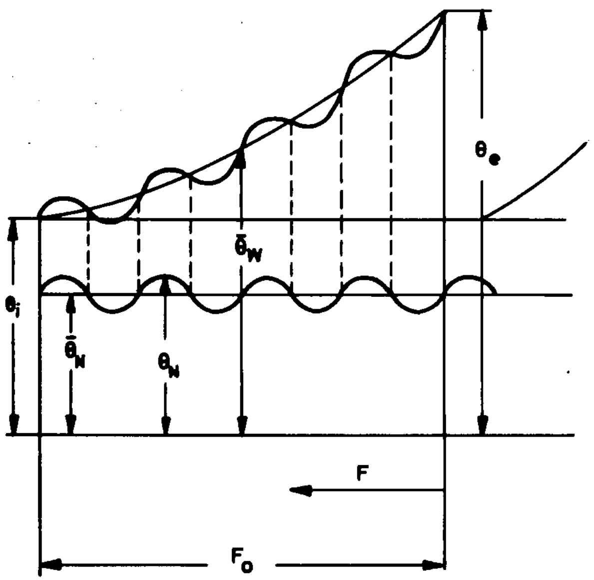

Fig.1

Temperature decay of the river water as a function of the cooling-surface of the river

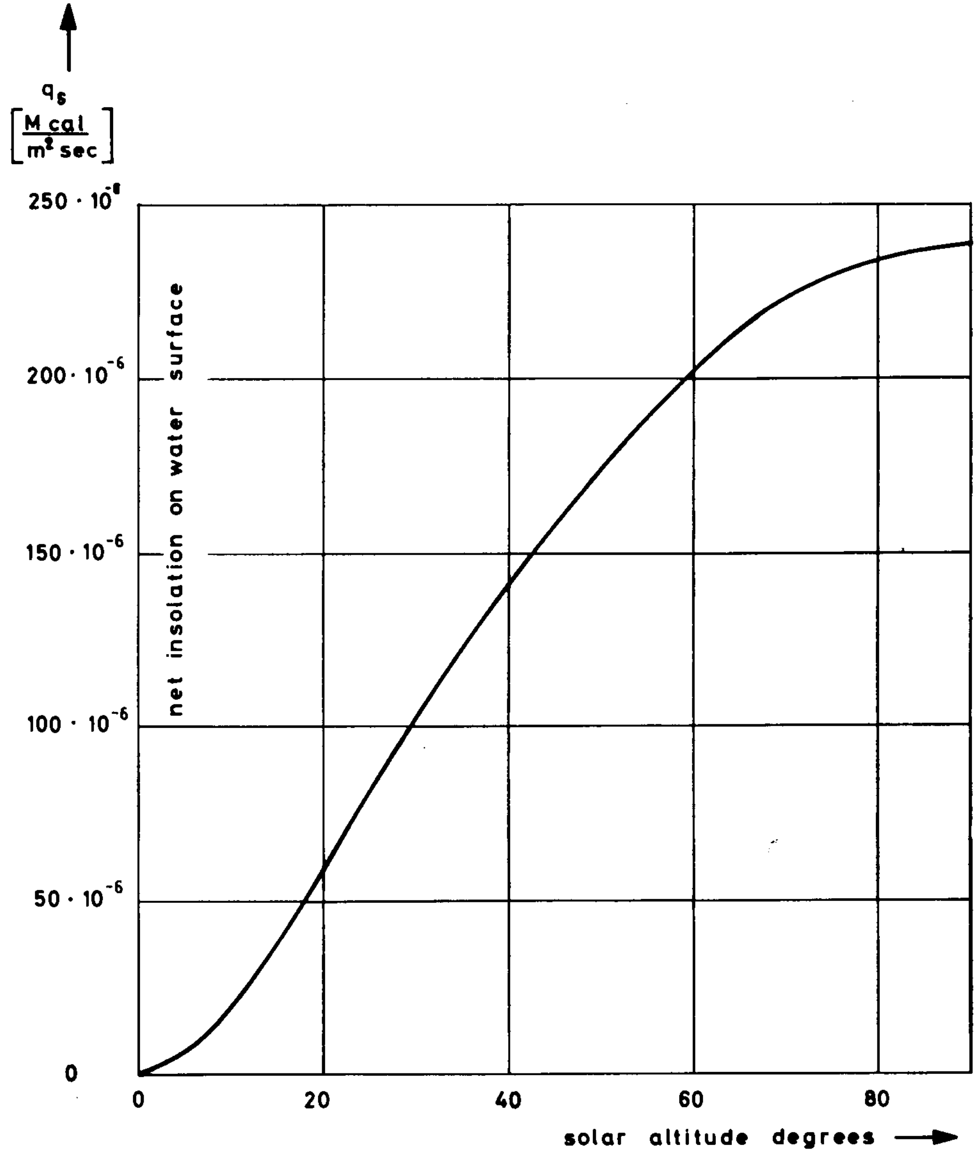

Fig. 2 The effective insolation heatflux as a function of the solar altitude, for a cloud cover factor = 0

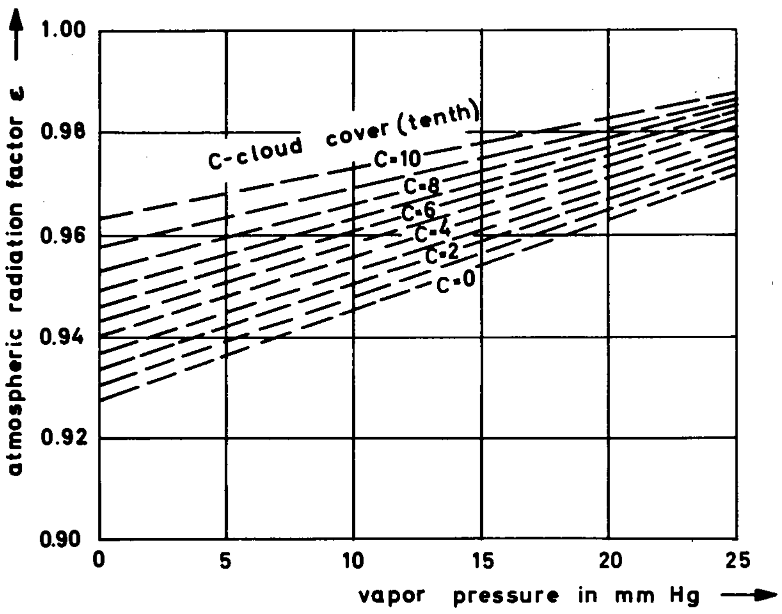

Fig. 3 Atmospheric radiation factor ε as a function of the vapor pressure and the cloud cover factor as a parameter

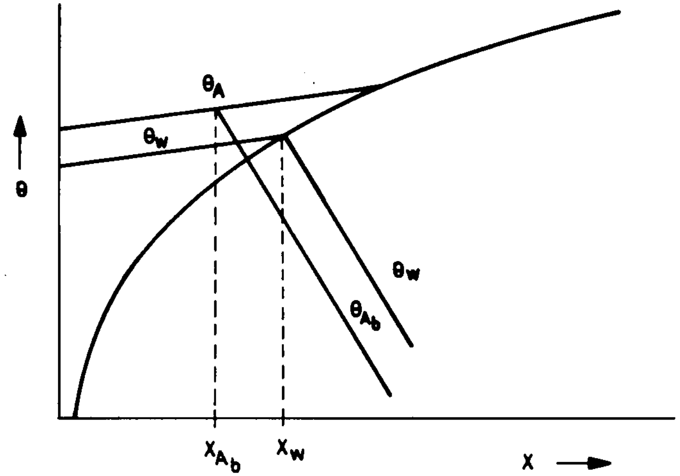

Fig. 4 Mollier - Diagram for humid air

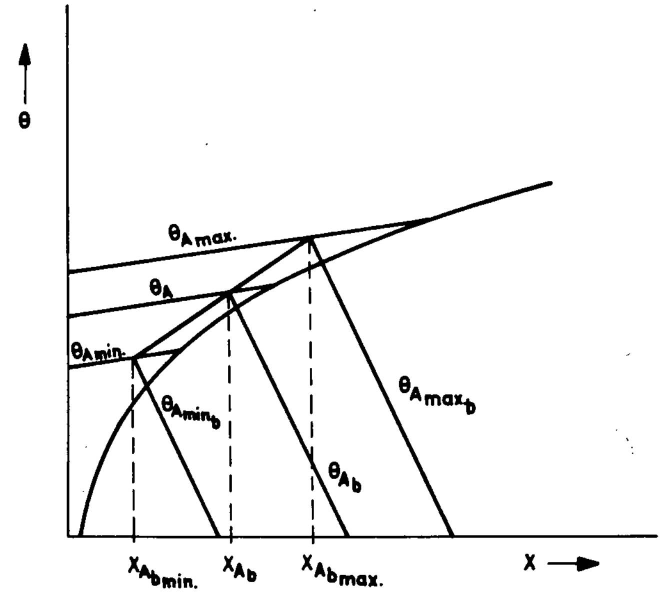

Fig. 5 Mollier - Diagram for humid air showing the approximative relation between X_{A_b}, $X_{A_{b\,min.}}$ and $X_{A_{b\,max.}}$

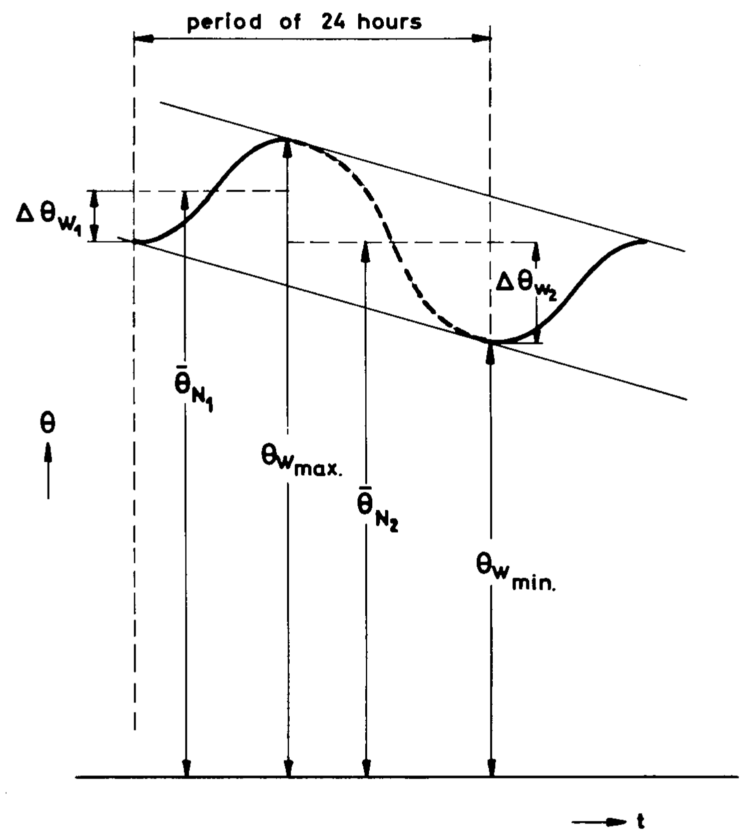

Fig 6 Temperature fluctuation of the river water described by two half-sine-functions

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Alfred Nobel

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