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COMMISSION OF THE EUROPEAN COMMUNITIES

THE THREE-DIMENSIONAL PLOTTING PROGRAM TRICE

by

G. NASTRI and C. CERVINI

1970

Joint Nuclear Research Center
Geel Establishment - Belgium
Central Bureau for Nuclear Measurements - CBNM

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ABSTRACT

The program TRICE (FORTRAN IV IBM 1800) projects a surface on a plane from a given observation point and plots the projected surface without hidden points.

KEYWORDS

FORTRAN
IBM
T-CODES
DIAGRAMS
RECORDING SYSTEMS
SURFACES

CONTENTS

Introduction *)

TRICE is a FORTRAN IV (IBM 1800) program performing the perspective transformation of the coordinates of a surface σ viewed from a given observation point C and projected into a plane π. No line connecting C with a point of σ can be parallel to π. The resulting projection is then plotted without hidden points. The set of routines under the name of TRICE includes a main program TRICE and three subroutines TRIFN, TRIVJ, XRYR. Additional subroutines used for the plot (MXMN, PLOT) are described in reference [1].

The main program TRICE reads as input data cards:
a) the coordinates of the observation point C;
b) the angles (in degrees) made by the line of sight (normal from C to the projection plane π) with the three cartesian axes x, y, z;
c) the distance of the projection plane π from the observation point C.
Then TRICE plots the projected points which are visible from the observation point C.
Two points of the projected surface σ are connected by a continuous line only if they are both visible.

The perspective transformation of the coordinates of the points of σ is executed by the subroutine XRYR.

The subroutine TRIFN reads as input data the coordinates of the points of the surface σ (to be projected and plotted) from cards, tape or disk. The user must write the program to generate the surface σ in a suitable form for TRIFN.
Examples follow in sections 9. 1., 9. 2., 9. 3.

The subroutine TRIVJ determines whether a point is hidden or not and builds a matrix where hidden points are marked by zero and visible point by 1.
If requested, this matrix is printed out.
If ISEV = 0 no analysis of hidden points is done and all of the points are plotted.

1. 1. The Algorithm of the Projection (Main Program TRICE and Subroutine XRYR)

In order to perform the projection of a point $P \equiv (x, y, z)$ on the plane π we need the following data:

C_x, C_y, C_z, the coordinates of the observation point C;

α, β, γ, the angles made by the line of sight (normal to π from C) with the axes x, y, z;

d, the distance of the projection plane π from C.

*) Manuscript received on 13 March 1970

The formulas used are the following [2] :

$$q_x = C_x + d \cos\alpha$$
$$q_y = C_y + d \cos\beta$$
$$q_z = C_z + d \cos\gamma$$

$$K = d/[(x-C_x)\cos\alpha + (y - C_y)\cos\beta + (z - C_z)\cos\gamma] \quad (1.1)$$

$$\xi = C_x + K(x - C_x)$$
$$\eta = C_y + K(y - C_y)$$
$$\zeta = C_z + K(z - C_z)$$

$$XR = [(\xi - q_x) \cos\beta - (\eta - q_y) \cos\alpha] / \sin\gamma \quad (1.2)$$

$$YR = (\zeta - q_z) / \sin\gamma \quad (1.3)$$

XR, YR are the coordinates of the projection of P in the plane π.
If $\sin\gamma = 0$ the perspective transformation is

$$XR = [-(\xi - q_x)\cos\gamma + (\zeta - q_z) \cos\alpha] / \sin\beta \quad (1.4)$$

$$YR = (\eta - q_y) / \sin\beta \quad (1.5)$$

The angles α, β, γ, must satisfy the condition

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

1.2. Why the Subroutine XRYR is Called Three Times

In order to save memory space, the coordinates of the projected points are calculated three times.

I. One for the search of the maximum and minimum values of XR, YR.
Such extreme values are required for the determination of the scale of the plot.

II. Then the values of XR, YR are again evaluated to plot the curves containing points with the same coordinate x (in the original cartesian reference). Each time a new curve (defined by the same x) must be drawn, the new values forming the vectors (XR(I), YR(I) destroy the old ones.

III. We must evaluate a third time the whole set of vectors XR(I), YR(I) for tracing the curves defined by the same value of y (in the original cartesian reference).
This way we may deal with a surface with up to 69 x 69 = 4761 mesh points.

2. Subroutine TRIFN for Reading the Coordinates of the Surface σ to be Plotted.

The surface σ to be plotted is given by points. The coordinates x, y of such points are defined by the mesh points of a grid (see fig; 1) and the coordinates z correspond to them through the indices I, J. The coordinates X(I), Y(J) of the grid as well as the corresponding Z(I, J) of the surface σ to be plotted may be taken from one of the following sources:
a) cards
b) tape (unit specified by input data NTAPE)
c) disk files TRIXY, TRIZ.
In the case a) it is even possible to read only the coordinates z(I, J) from the cards and to calculate in the program the grid elements starting from the extreme values XMIN, XMAX; YMIN, YMAX.

In the cases b) and c) a previous program is supposed to have prepared the tape or disk.
Examples are given in sections 9. 1., 9. 2., 9. 3.
The coordinates defining the grid and σ can be optionally printed out.

3. 1. The Visibility Tests Performed by TRIVJ

We develop our criterion of visibility by studying the line segment $\overline{CP}$ which joins the observation point C with a variable point P of the surface σ to be plotted (cf. fig. 3).
If $\overline{CP}$ has no intersections with the surface σ the point P is visible, otherwise it is hidden.
In order to determine the existence of such intersections we select suitable points P_k of $\overline{CP}$ (k = 1, 2. . . .) identified by the coordinates x_k, y_k and calculate the corresponding values of z for $\overline{CP}$ and for the surface σ ; z_k and z^* are such values of the coordinate z at x_k, y_k, respectively for $\overline{CP}$ and for the surface σ .
If the sign of the difference

$$\delta_k = z_k - z^* \qquad (k = 1, 2. . . .) \qquad (3. 1.)$$

does not change over the whole range of index k, no intersection exists between $\overline{CP}$ and σ: as a consequence the point P is visible. The opposite conclusion is reached if δ_k changes its sign.

TRIVJ examinates all of the points P of σ whose coordinates are defined by TRIFN. For each point P TRIVJ creates one or two sets of points P_k depending on the index ISEV given as input data.

There are one or two corresponding visibility tests.

a) The first set of points P_k is defined by the abscisses

$$x_k = X(K) \qquad (3. 2.)$$

received from TRIFN: only the range is limited to the extension of the line segment $\overline{CP}$.
The first visibility test assumes such set of points P_k.

b) The second set P_k has the ordinates

$$y_k = Y(K) \qquad (3.3.)$$

as defined in TRIFN. While the visibility test associated with the first choice of P_k is performed always for ISEV > 0 the second set P_k as well as the associated test are considered only if ISEV $\geq$ 3, 4.

The index ISEV, called severity degree, specifies the requested severity in doing the visibility test. To the severity corresponds the time required. If ISEV = 3, 4 the time of execution is practically doubled.

3.2. The First Visibility Test

For each point P we assume at the beginning that it is visible by defining

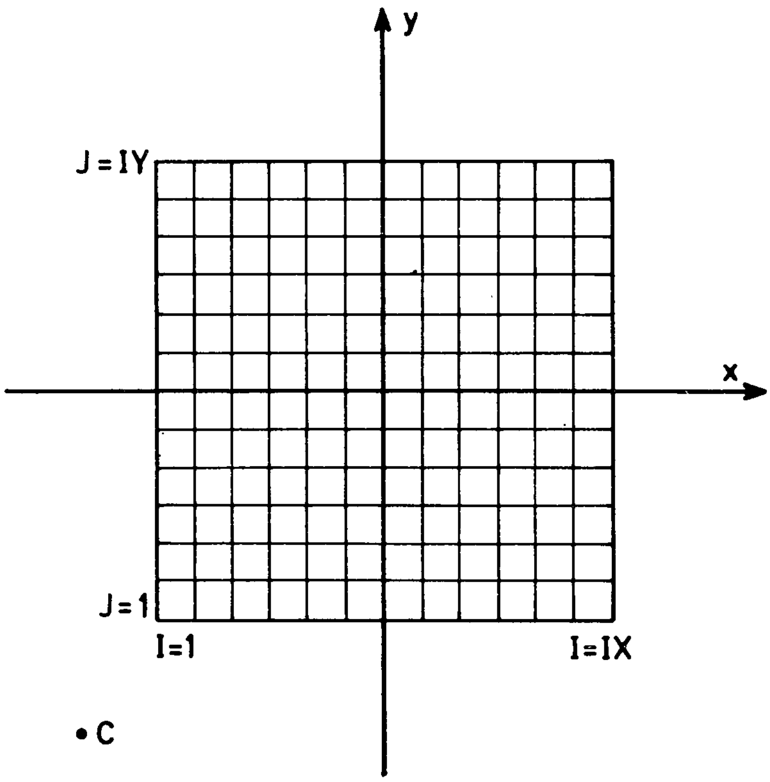
$$JVIS(I, J) = 1 \qquad (3.4.)$$

If the sequence δ_k of eq. (3.1.) contains only one value of δ_k there is no possibility of modifying the condition (3.4.).
For making the best use of TRIVJ the coordinates CX and CY of the observation point C and the coordinates of the grid should satisfy the conditions

$$CX < X(1) < X(2) < \ldots X(IX) \qquad (3.5.)$$

$$CY < Y(1) < Y(2) < \ldots Y(IY)$$

(as shown in fig. 1).



THE GRID

Fig. 1

As already outlined the first test (Sect. 3. 1. a)-based on evaluating δ_k at different values of the abscisses given by eq. (3. 2.)- is always performed for ISEV > 0.
When the conditions (3. 5.) hold, the curve belonging to the surface σ and defined by

$$x = X(1)$$

is always visible, while the curve of σ for

$$x = X(2)$$

gives rise to only one value δ_k, which is not sufficient for modifying the condition (3. 4.). In this case, the only way for improving the visibility check is to perform the second test (Sect. 3. 1. b).
In the case sketched by fig. 2 there is the possibility of creating

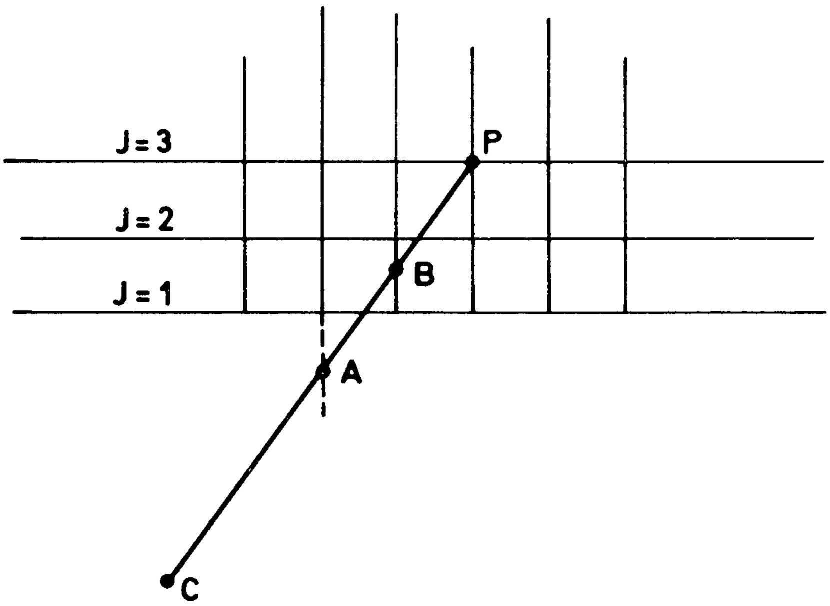


Fig. 2

besides the value of δ_k corresponding to the point B another δ_k for the point A by extrapolating the surface σ out of the definition area. TRIVJ performs this task if the option ISEV = 2 is expressed.

3. 3. The Second Visibility Test

Assuming as points for evaluating δ_k the set P_k defined according to the eq. (3. 3.), the second test (3. 1. b) takes place if ISEV > 2. The extrapolation concerning only the points P having abscissa x = X(2) is performed under the option ISEV = 4.

3. 4. Interpolation Formulas Used in TRIVJ

Let us consider the simple general formulas used for interpolation in the case of the first test (Sect. 3. 1. a).
For the definition of the symbols refer to fig. 3.

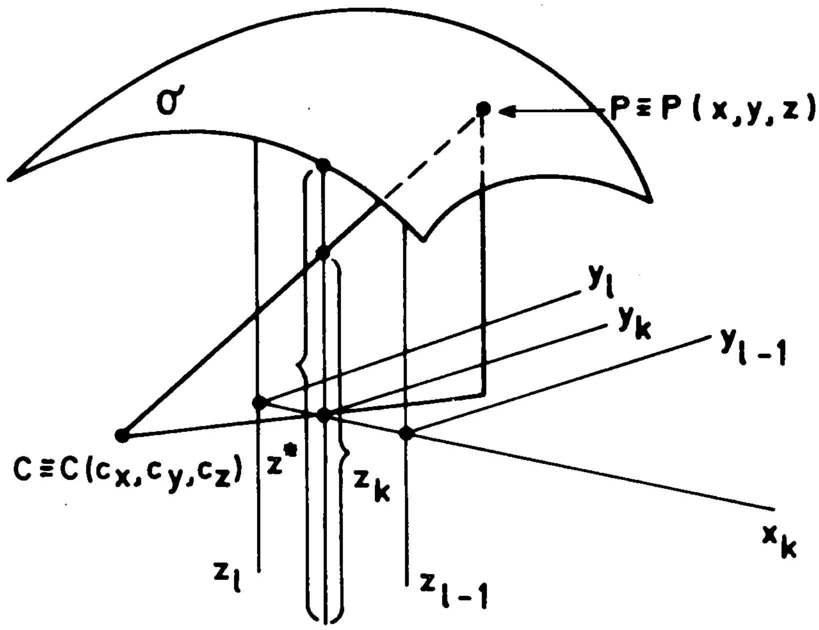


Fig. 3

At the beginning are given:
a) the coordinates of the points C, P;
b) the coordinates x_k, y_l, y_{l-1} belonging to the grid;
c) the coordinates z_l, z_{l-1} which define the surface σ .
Then we determine

$$y_k = \frac{y - C_y}{x - C_x} \cdot (x_k - C_x) + C_y$$

$$z_k = \frac{z - C_z}{x - C_x} (x_k - C_x) + C_z$$

$$z^* = \frac{z_l - z_{l-1}}{y_l - y_{l-1}} (y_k - y_{l-1}) + z_{l-1}$$

4. How to Use TRICE: Time Requirements and Limitations

The part of TRICE that consumes more time is the visibility test: for one test on 4761 points the IBM 1800 computer takes more than one hour and a half. Therefore it is more advisable to use ISEV > 2 only in special cases where no satisfaction was reached with ISEV ≤ 2. Nevertheless when the number of points is reduced the time requested decreases more than proportionally. In case of troubles the user can get better results by arranging the problem according to the scheme of eq. (3. 5.) (scheme of fig. 1).

Peaks and edges defined by few points may still result "transparent" due to the adopted method of interpolation. The user will introduce more points in the definition of such features of the surface σ
If the denominator of the expression of K(eq. 1. 1.) becomes zero, K itself loses its signification.
Reasons of such degeneration are:

I) the line $\overline{CP}$ connecting the observation point C with some point P of the surface σ is parallel to the plane π , that is perpendicular to the line of sight;
II) the observation point C coincides with a point P of the surface σ where the surface itself is defined (coordinates x and y in a mesh point of the grid). Therefore the user must be careful to avoid such topics which will prevent the successful completion of the plot. The choice of the coordinates of C and of the direction of sight may affect the results negatively, but not the choice of the distance d between C and the projection plane π;nevertheless d must not be zero.
A cutting value RANG is foreseen for keeping the plot within suitable limits allowing definition of the scales and execution of a plot although curtailed.

TRICE allows also projection on planes π parallel to the plane x, y: in this case is $\gamma = 0$ and therefore instead of formulas (1. 2.), (1. 3.), formulas (1. 4.), (1. 5.) are used. The coordinate z of C must be definitively higher (or lower) than the coordinate z of the points of σ .

5. Input Data

First card FORMAT (5F10. 0,I10,F10. 0,I10)

Column		
1 - 10	CX	coordinate x of the view point C
11 - 20	CY	coordinate y of the view point C
21 - 30	CZ	coordinate z of the view point C
31 - 40	SIZX	length of the x axis for the plot
41 - 50	SIZY	length of the y axis for the plot
51 - 60	ICALL=	0, TRIFN is called for reading a new surface σ 1, TRIFN is not called and the old points are treated again (different views of the same surface σ).
61 - 70	D1	distance between the view point C and the projection plane π .
71 - 80	ISEV=	(-1, assume the last visibility matrix which has been calculated (0, complete plot including hidden points (1, simple test without extrapolation (2, simple test with extrapolation (3, double test without extrapolation (4, double test with extrapolation.

N. B. The larger is the severity degree ISEV, the higher is the probability of deciding that a point is not visible (see sections 3. 1., 3. 2., 3. 3. for explanations).

Second card FORMAT (4F 10. 0)

1 - 10	ALFA	angle (in degrees) formed by the view line with x axis
11 - 20	BETA	angle (in degrees) formed by the view line with y axis
21 - 30	GAMMA	angle (in degrees) formed by the view line with z axis
31 - 40	RANG	maximum absolute value of the coordinates XR(I), YR(J) of the figure projected on the plane π .

N. B. Avoid view lines which could be perpendicular to some $\overline{CP}$ direction: danger of aborting the plot. The value of RANG is intended to cut down excessive values of XR(I), YR(J) araising from a wrong choise of the view point and view line. Avoid values of RANG which cut away useful parts of the plot or the whole plot itself. (See sect. 4).
If GAMMA=0 or RANG $\leq$ 0 no control is done on RANG.

Third card (optional) FORMAT (5I5), only if ICALL = 0

1 - 5	IREAD =	(0, vectors X(I), Y(I) are built by the program; the matrix Z(I, J) is read from cards (1, whole input from cards (2, input from tape (3, input from disk files
6 - 10	IX	number of points on x axis (IX $\leq$ 69)
11 - 15	IY	number of points on y axis (IX $\leq$ 69)
16 - 20	NTAPE	logic reference of the tape unit to be read if input from tape (NTAPE = 7, 8, 9, 10)

21 - 25	IPRI =	(-1, visibility matrix is printed; vectors X(I), Y(J) and matrix Z(I, J) are printed with FORMAT (35F4. 0) (0, visibility matrix is printed; vectors X(I), Y(J) and matrix Z(I, J) are printed with FORMAT (14E10. 3) (1, vectors X(I), Y(J) and matrix Z(I, J) are printed with FORMAT (35F4. 0); no visibility matrix printed.

N. B. Vectors X(I), Y(J) and matrix Z(I, J) are printed only if TRIFN is called. The visibility matrix is printed only if TRIVJ is called.

Fourth card (optional) FORMAT(4F10. 0), only if ICALL=0 and IREAD=0

1 - 10	XMIN	minimum value of X(I)
11 - 20	XMAX	maximum value of X(I)
21 - 30	YMIN	minimum value of Y(J)
31 - 40	YMAX	maximum value of Y(J)

If ICALL = 0 and IREAD = 1, enter instead of the fourth card, data X(I), Y(J) (separate sets of cards) with FORMAT (8E10. 3).

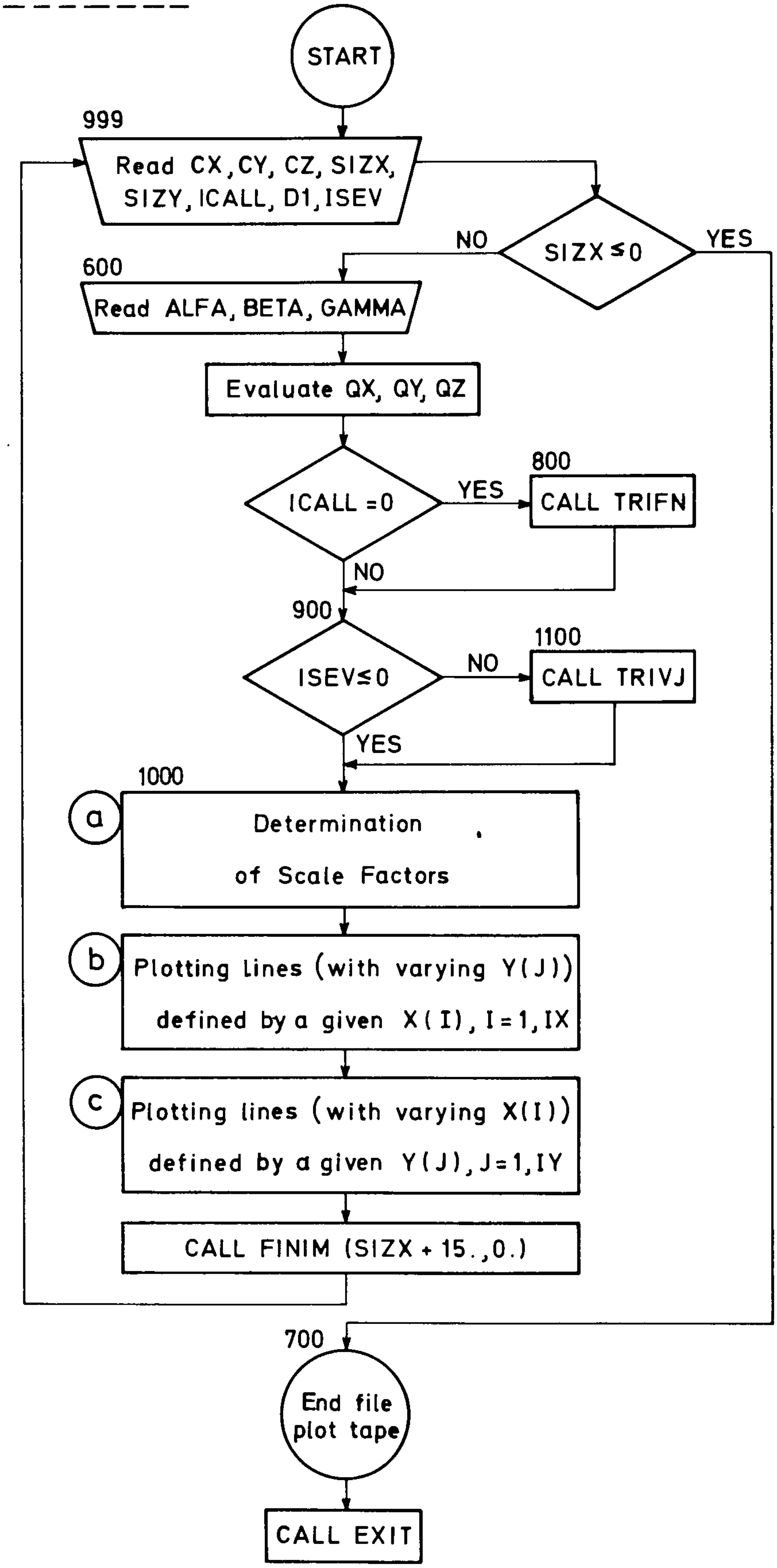
If ICALL = 0 and IREAD = 0, 1, enter data Z(I, J) (separate sets of cards for each value of I) with FORMAT (8E10. 3).

Last card (to be introduced only for the first plot)
The last card indicates on which unit I the plot is expected to be. Columns 1-3 contain **I (I = 0, 1, 2, 3).

N. B. Repeat the complete scheme of input data (except the last card) for each plot requested. Add a blank card at the end of the whole input.

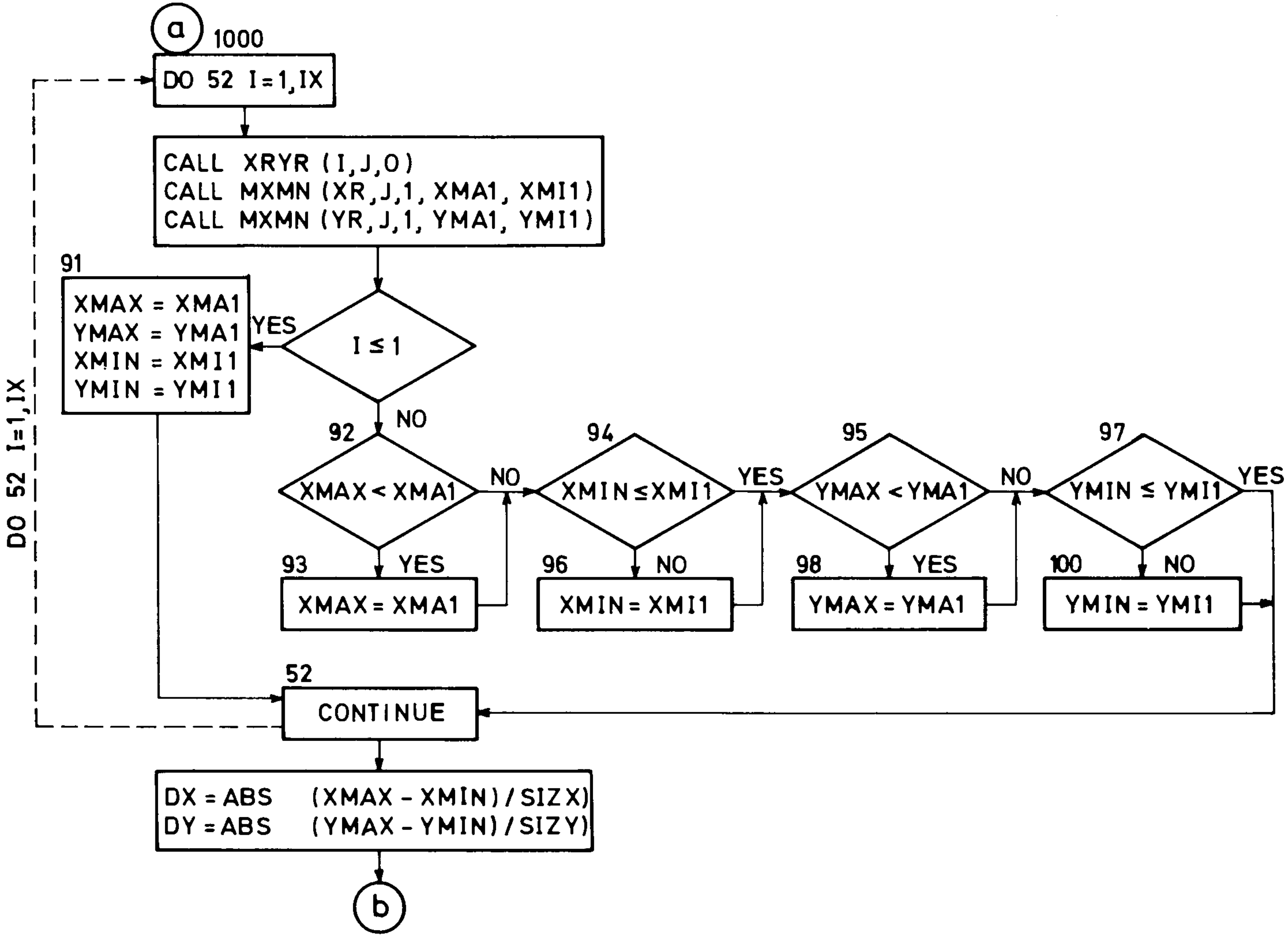
6. BLOCK DIAGRAMS

6.1 Main Program TRICE



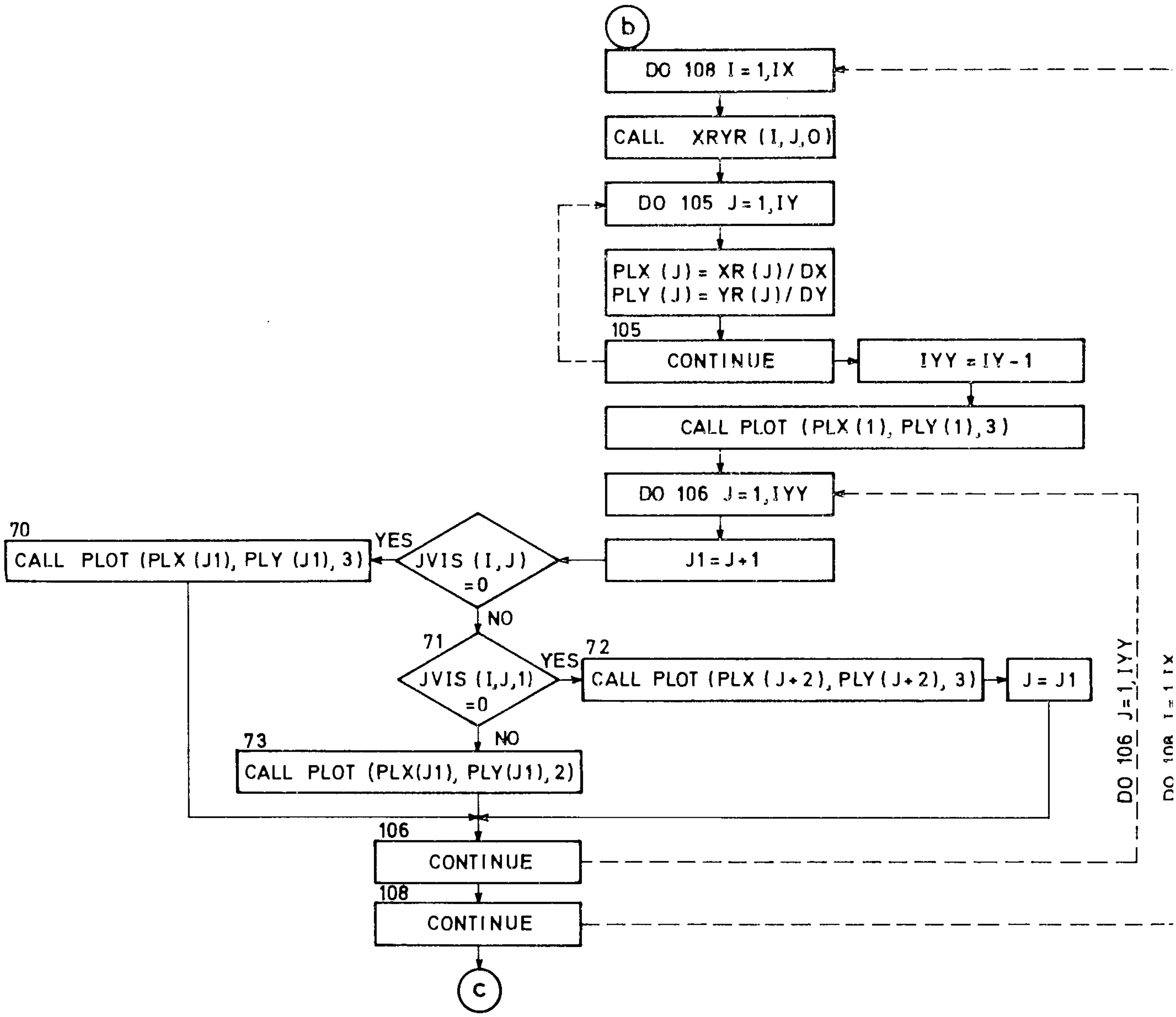
Program TRICE (Continuation 1)

Determination of Scale Factors



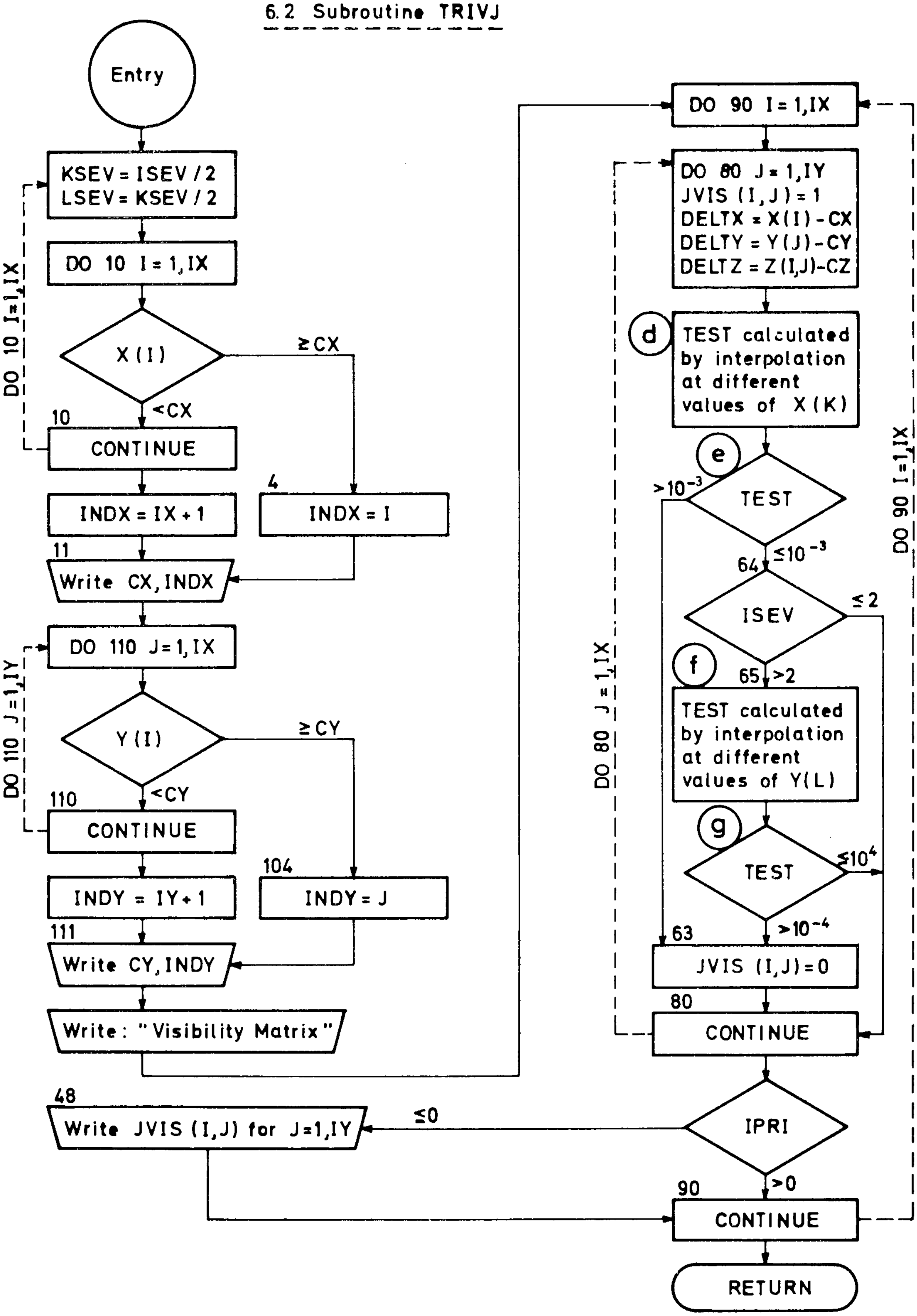
Program TRICE (Continuation 2)

Plotting lines defined by a given X (I)



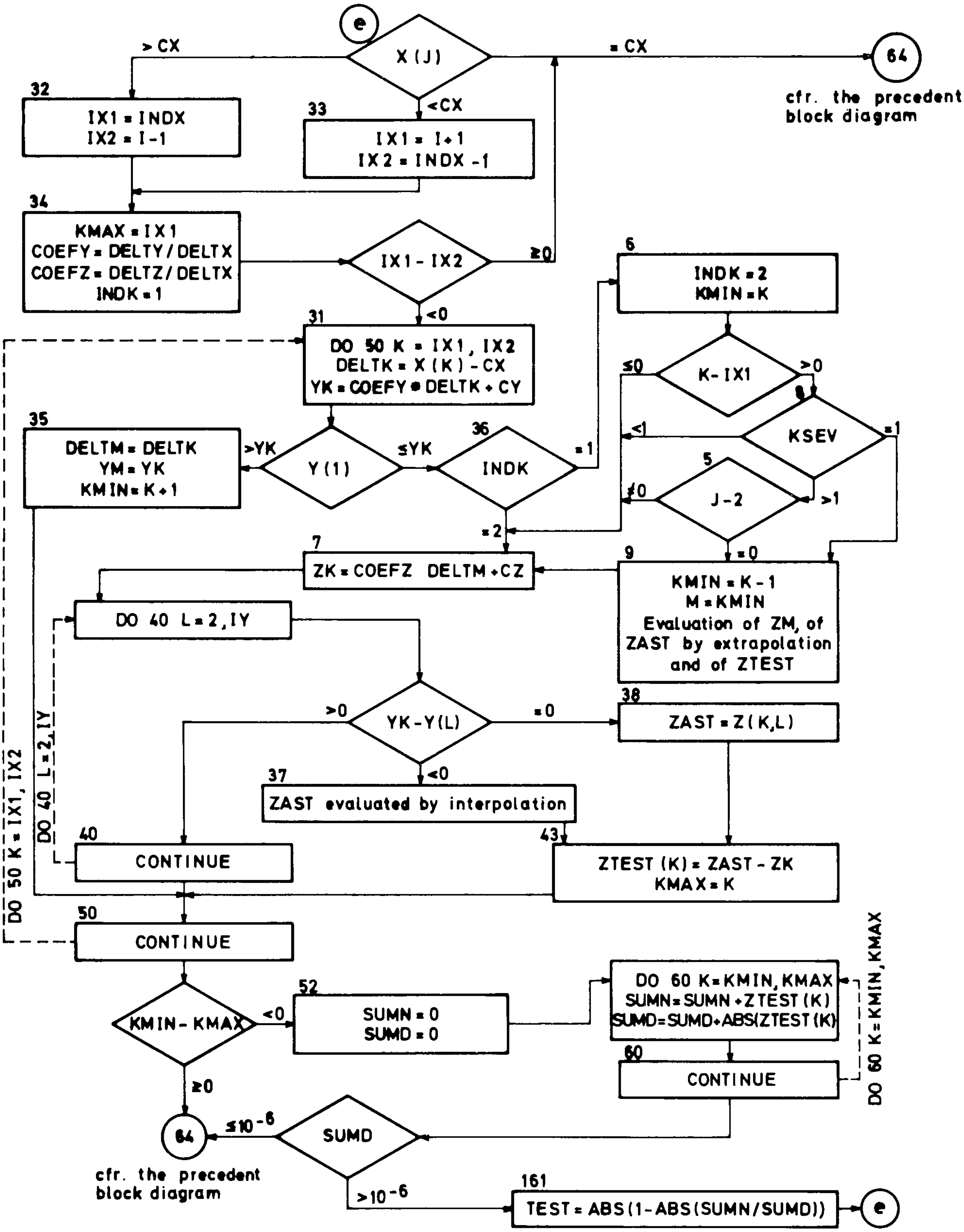
N.B. The same method is used to join with a full line the points with the same coordinate Y (J).

6.2 Subroutine TRIVJ



Subroutine TRIVJ (Contination)

TEST calculated by interpolation at different values of X (K)



N.B. The same method is used - if requested - to calculate TEST by interpolation at different values of Y (L). The only difference is that, contrary to KSVEV = 1, the case LSEV = 1 is not considered apart.

```
// FOR TRICE
*LIST SOURCE PROGRAM
*ONE WORD INTEGERS
*NONPROCESS PROGRAM
*IOCS(CARD,1443 PRINTER)
      DEFINE FILE 12(4800,2,U,IM)
      DEFINE FILE 11( 160,2,U,IM)
      DIMENSION PLX(99) ,PLY(99)
      COMMON XR(69),YR(69),ANG1,ANG2,ANG3,SANG1,QX,QY,QZ,D1,GAMMA,SANG2
      COMMON Y(69),X(69),Z(69,69),JVIS(69,69),IX,IY,CX,CY,CZ,IPRI,ISEV
      COMMON RANG
      EQUIVALENCE(XR(1),PLX(1)) ,(YR(1),PLY(1))
  999 READ(5,1) CX,CY,CZ ,SIZX,SIZY,ICALL,D1,ISEV
    1 FORMAT(5F10.0,I10,F10.0,I10)
      IF(SIZX)600,700,600
  600 READ(5,1) ALFA,BETA,GAMMA,RANG
      WRITE(6,  2)CX,CY,CZ,ALFA,BETA,GAMMA,D1
    2 FORMAT( 1H ,8X,'CX',8X,'CY',8X,'CZ', 6X,'ALFA',6X,'BETA',5X,'GAMMA
     1',8X,'D1' /1H0,7F10.0///)
      FACT=3.14159/180.
      RADIA=ALFA*FACT
      RADIB=BETA*FACT
      RADIC=GAMMA*FACT
      ANG1=COS(RADIA)
      ANG2=        COS(RADIB)
      ANG3=COS(RADIC)
      SANG1=SIN(RADIC)
      SANG2=SIN(RADIB)
      QX=CX+D1*ANG1
      QY=CY+D1*ANG2
      QZ=CZ+ANG3*D1

      IF(ICALL)900,800,900
  800 CALL TRIFN

  900 DO 52 I=1,IX
      CALL XRYR(I,J,0)
      CALL MXMN(XR,IY,1,XMA1,XMI1)
      CALL MXMN(YR,IY,1,YMA1,YMI1)
      IF(I-1) 91,91,92
   91 XMAX=XMA1
      YMAX=YMA1
      XMIN=XMI1
      YMIN=YMI1
      GO TO 52
   92 IF(XMAX-XMA1) 93,94,94
   93 XMAX=XMA1
   94 IF(XMIN-XMI1) 95,95,96
   96 XMIN=XMI1
   95 IF(YMAX-YMA1)97,97,98
   97  YMAX=YMA1
   98 IF(YMIN-YMI1) 52,52,100
  100  YMIN=YMI1
   52 CONTINUE
      DX=ABS((XMAX-XMIN)/SIZX)
      DY=ABS((YMAX-YMIN)/SIZY)
      WRITE(6,6)XMIN,XMAX,YMIN,YMAX
    6 FORMAT(1H0,4E12.5//)

      IF(ISEV)1000,940,1100
  940 DO 960 I=1,IX
      DO 950 J=1,IY
```

```
      JVIS(I,J)=1
  950 CONTINUE
  960 CONTINUE
      GO  TO  1000
 1100 CALL TRIVJ
 1000 DO 108 I=1,IX
      CALL XRYR(I,J,0)
      DO 105 J=1,IY
      PLX(J)=XR(J)/DX
      PLY(J)=YR(J)/DY
  105 CONTINUE
      IYY=IY-1
      CALL PLOT(PLX(1),PLY(1),3)
      DO 106 J=1,IYY
      J1=J+1
      IF(JVIS(I,J))70,70,71
   71 IF(JVIS(I,J1  ))72,72,73
   73 CALL PLOT(PLX(J1),PLY(J1),2)
      GO TO 106
   70 CALL PLOT(PLX(J1 ),PLY(J1 ),3)
      GO TO 106
   72 CALL PLOT(PLX(J+2),PLY(J+2),3)
      J=J1
  106 CONTINUE
  108 CONTINUE
      DO 60 J=1,IY
      CALL XRYR(I,J,1)
      DO 61 I=1,IX
      PLX(I)=XR(I)/DX
      PLY(I)=YR(I)/DY
   61 CONTINUE
      CALL PLOT(PLX(1),PLY(1),3)
      IXX=IX-1
      DO 107 I=1,IXX
      I1=I+1
      IF(JVIS(I,J))80,80,81
   81 IF(JVIS(I1,J))82,82,83
   83 CALL PLOT(PLX(I1),PLY(I1),2)
      GO TO 107
   80 CALL PLOT(PLX(I1),PLY(I1),3)
      GO TO 107
   82 CALL PLOT(PLX(I1+1),PLY(I1+1),3)
      I=I1
  107 CONTINUE
   60 CONTINUE
      CALL FINIM(SIZX+15.,0.)
      GO TO 999
  700 CALL FINTR
      CALL EXIT
      END

FEATURES SUPPORTED
 NONPROCESS
 ONE WORD INTEGERS
 IOCS

CORE REQUIREMENTS FOR TRICE
 COMMON   14868   INSKEL COMMON       0   VARIABLES      62   PROGRAM     820
```

```
*LIST SOURCE PROGRAM
*NONPROCESS PROGRAM
*ONE WORD INTEGERS
      SUBROUTINE XRYR(I,J,K)
      COMMON XR(69),YR(69),ANG1,ANG2,ANG3,SANG1,QX,QY,QZ,D1,GAMMA,SANG2
      COMMON Y(69),X(69),Z(69,69),JVIS(69,69),IX,IY,CX,CY,CZ,IPRI,ISEV
      COMMON RANG
       IF(K)2,49,2
   49 DO 53J=1,IY
      FK=D1/((X(I)-CX)*ANG1+(Y(J)-CY)*ANG2+(Z(I,J)-CZ)*ANG3)
      PX= CX+FK *(X(I)-CX)
      PY= CY+FK *(Y(J)-CY)
      PZ= CZ+FK *(Z(I,J)-CZ)
      IF(GAMMA) 299,300,299
  300 XR(J)=((-QX+PX)*ANG3+(PZ-QZ)*ANG1)/SANG2
      YR(J)=(PY-QY)/SANG2
      GO TO 53
  299 XR(   J)=((PX  -QX  )*ANG2-(PY-QY)*ANG1)/SANG1
      IF(RANG)54,54,51
   51 AB =ABS(XR(J))
      IF(AB-RANG)54,54,55
   55 XR(J)=RANG*XR(J)/AB
   54 YR(   J)=(PZ    -QZ)/SANG1
      IF(RANG)53,53,57
   57 AB=ABS(YR(J))
      IF(AB-RANG)53,53,56
   56 YR(J)=    RANG*YR(J)/AB
   53 CONTINUE
      RETURN
    2 DO 52 I=1,IX
      FK=D1/((X(I)-CX)*ANG1+(Y(J)-CY)*ANG2+(Z(I,J)-CZ)*ANG3)
      PX= CX+FK *(X(I)-CX)
      PY= CY+FK *(Y(J)-CY)
      PZ= CZ+FK *(Z(I,J)-CZ)
      IF(GAMMA)288,287,288
  287 XR(I)=((-QX+PX)*ANG3+(PZ-QZ)*ANG1)/SANG2
      YR(I)=(PY-QY)/SANG2
      GO TO 52
  288 XR(I   )=((PX  -QX  )*ANG2-(PY-QY)*ANG1)/SANG1
      IF(RANG)64,64,61
   61 AB =ABS(XR(I))
      IF(AB-RANG)64,64,65
   65 XR(I)=RANG*XR(I)/AB
   64 YR(I   )=(PZ    -QZ)/SANG1
      IF(RANG)52,52,67
   67 AB=ABS(YR(I))
      IF(AB-RANG)52,52,66
   66 YR(I)=RANG*YR(I)/AB
   52 CONTINUE
      RETURN
      END

FEATURES SUPPORTED
 NONPROCESS
 ONE WORD INTEGERS

CORE REQUIREMENTS FOR XRYR
 COMMON  14868  INSKEL COMMON      0  VARIABLES     18  PROGRAM    558

 END OF COMPILATION
```

```
// FOR TRIFN
*LIST SOURCE PROGRAM
*NONPROCESS PROGRAM
*ONE WORD INTEGERS
      SUBROUTINE TRIFN
      COMMON XR(69),YR(69),ANG1,ANG2,ANG3,SANG1,QX,QY,QZ,D1,GAMMA,SANG2
      COMMON Y(69),X(69),Z(69,69),JVIS(69,69),IX,IY,CX,CY,CZ,IPRI,ISEV
      READ(5,1) IREAD,IX,IY,NTAPE,IPRI
    1 FORMAT(5I5)
      IF(IREAD)20,20,24
   24 GO TO(2,3,5),IREAD
    2 N=5
      GO TO 4
    3 N=NTAPE
      REWIND NTAPE
    4 READ (N,10)(X(I),I = 1,IX)
      READ(N,10)(Y(J),J= 1,IY)
   10 FORMAT(8E10.3)
      GO TO 25
   20 N=5
      READ(5,21)  XMIN,XMAX,YMIN,YMAX
   21 FORMAT(8F10.0)
      XIX=IX-1
      XSTEP=(XMAX-XMIN)/XIX
      X(1)=XMIN
      DO 22 I=2,IX
   22 X(I)=X(I-1)+XSTEP
      YIY=IY-1
      YSTEP=(YMAX-YMIN)/YIY
      Y(1)=YMIN
      DO 23 J=2,IY
   23 Y(J)=Y(J-1)+YSTEP
   25 DO 50  I=1,IX
   50 READ(N,10) (Z(I,J) ,J=1,IY)
      GO TO 6
    5 IM=1
      READ (11'IM)(X(I),I=1,IX),(Y(J),J=1,IY)
      IM=1
      READ (12'IM)((Z(I,J),I=1,IX),J=1,IY)
C  * * * *  O U T P U T  * * * * *
    6 IF(IPRI)82,99,82
   99 WRITE(6,221)(X(I),I=1,IX)
  221 FORMAT(1H0,'TABLE OF X'//(4X,14E10.3))
      WRITE(6,222)(Y(J),J=1,IY)
  222 FORMAT(1H0,'TABLE OF Y'//(4X,14E10.3))
      WRITE(6,91)
   91 FORMAT(1H0 'MATRIX Z'/1H0,'I  * * I IS THE INDEX OF X(I) * * FOR E
     1ACH I,VALUES OF Z(I,J) ARE GIVEN WITH J RUNNING FROM 1 TO IY * *')
      DO 62 I=1,IX
      L=I
      WRITE(6,122) L,(Z(I,J),J=1,IY)
  122 FORMAT(1H0,I2,1X  ,14E10.3/(4X,14E10.3))
   62 CONTINUE
      GO TO 84
   82 WRITE(6,321)(X(I),I=1,IX)
  321 FORMAT(1H0,'TABLE OF X'//(4X,35F4.0) )
      WRITE(6,322)(Y(J),J=1,IY)
  322 FORMAT(1H0,'TABLE OF Y'//(4X,35F4.0) )
      WRITE(6,91)
      DO 83  I=1,IX
      L=I
```

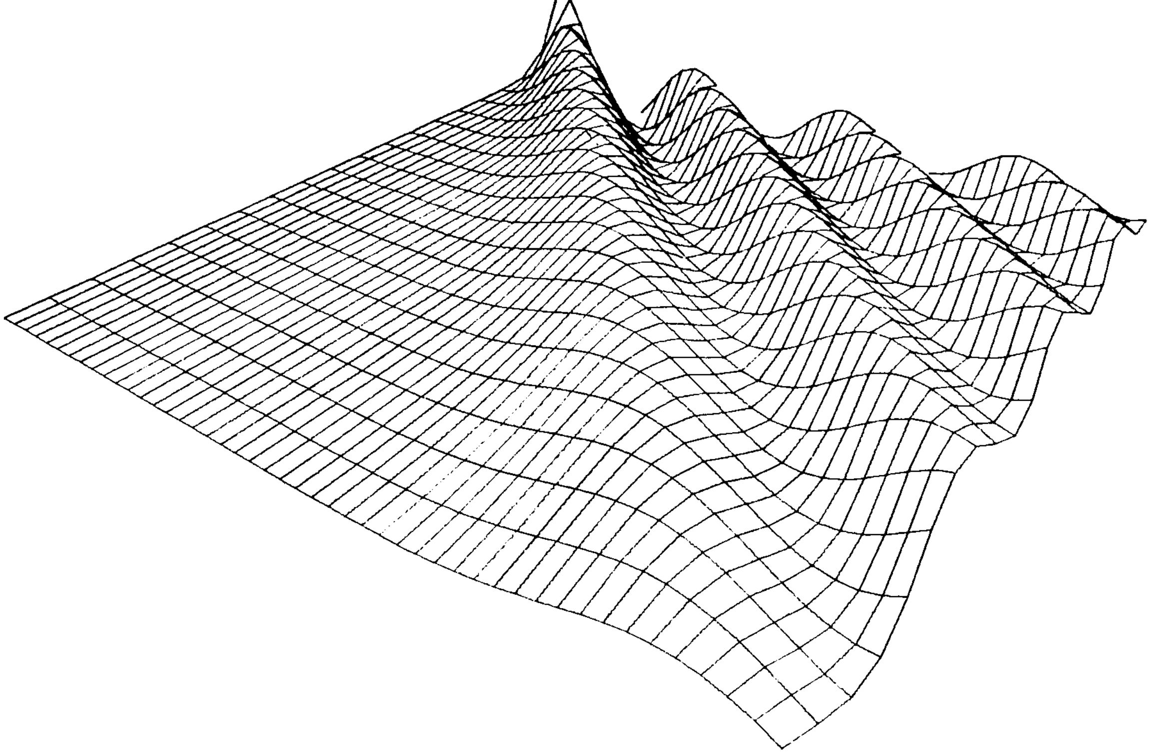
```
123 FORMAT(1H0,I2,1X,35F4.0/4X,35F4.0)
 83 CONTINUE
 84 RETURN
    END

FEATURES SUPPORTED
 NONPROCESS
 ONE WORD INTEGERS

CORE REQUIREMENTS FOR TRIFN
 COMMON  14866  INSKEL COMMON       0  VARIABLES     26  PROGRAM    686

 END OF COMPILATION
```

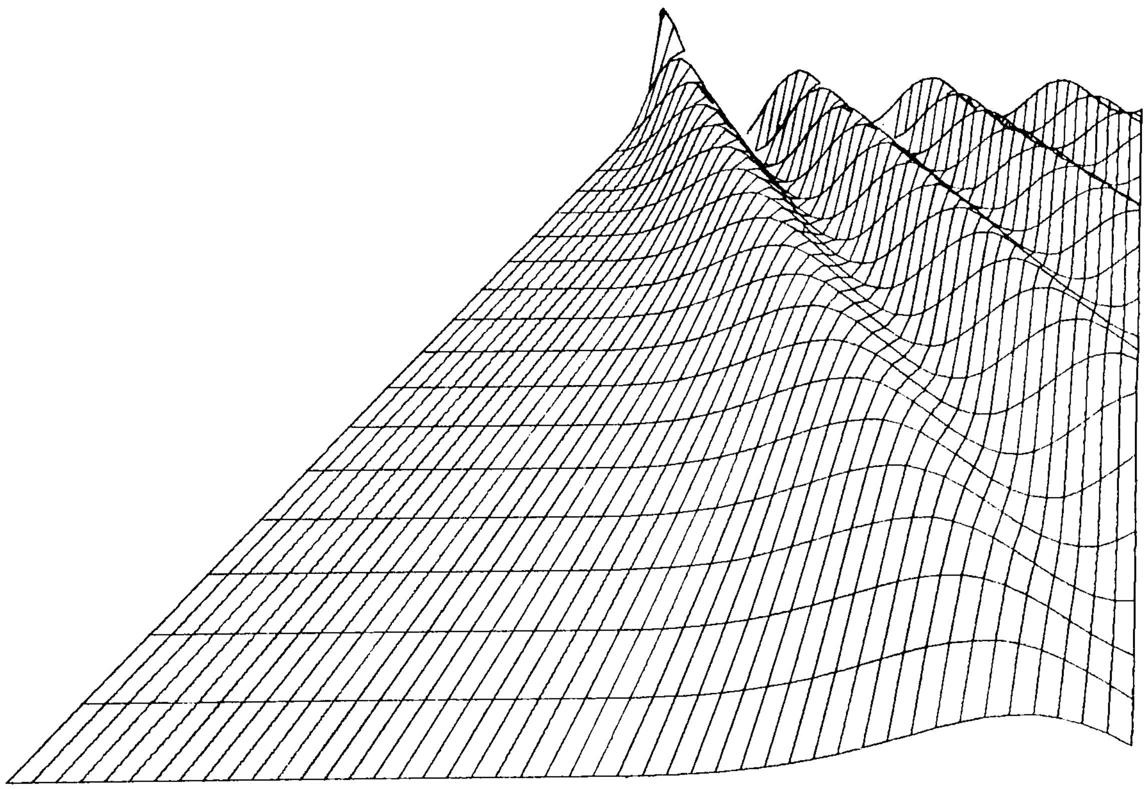
```
// FOR TRIVJ
*LIST SOURCE PROGRAM
*ONE WORD INTEGERS
*NON PROCESS PROGRAM
      SUBROUTINE TRIVJ
      DIMENSION ZTEST(90)
      COMMON XR(69),YR(69),ANG1,ANG2,ANG3,SANG1,QX,QY,QZ,D1,GAMMA,SANG2
      COMMON Y(69),X(69),Z(69,69),JVIS(69,69),IX,IY,CX,CY,CZ,IPRI,ISEV
      KSEV=ISEV/2
      LSEV=KSEV/2

C TRIVJ DECIDES ABOUT THE VISIBILITY OF POINT P(X,Y,Z) FROM C(CX,CY,CZ)
      DO 10 I=1,IX
      IF    (CX-X(I))4,4,10
    4 INDX=I
      GO TO 11
   10 CONTINUE
      INDX=IX+1
   11 WRITE(6,12)CX,INDX
   12 FORMAT(1H0,'TO CX =',F10.2,' CORRESPONDS INDX =',I6)
      DO 110 J=1,IY
      IF(CY-Y(J))104,104,110
  104 INDY=J
      GO TO 111
  110 CONTINUE
      INDY=IY+1
  111 WRITE(6,112)CY,INDY
  112 FORMAT(1H0,'TO CY =',F10.2,' CORRESPONDS INDY =',I6/)
      WRITE(6,72)
   72 FORMAT(1H0,'VISIBILITY MATRIX '//4H   I/)

      DO 90 I=1,IX

      DO 80 J=1,IY
C * * * THE INDICES I,J IDENTIFY THE POINT P(X,Y,Z) TO BE  EXAMINATED
      JVIS(I,J)=1
      DELTX=X(I)-CX
      DELTY=Y(J)-CY
      DELTZ=Z(I,J)-CZ

      IF(CX-X(I))32,64,33
   32 IX1=INDX
      IX2=I-1
      GO TO 34
   33 IX1=I+1
      IX2=INDX-1
   34 KMAX=IX1
      COEFY=DELTY/DELTX
      COEFZ=DELTZ/DELTX
      INDK=1
      IF(IX1-IX2)31,64,64
   31 CONTINUE

      DO 50 K=IX1,IX2
      DELTK=X(K)-CX
      YK= COEFY*DELTK+CY
      IF(YK-Y(1))35,36,36
   35 DELTM=DELTK
      YM=YK
      KMIN=K+1
```

```
 36 GO TO (6,7),INDK
  6 INDK=2
    KMIN=K
    IF(K-IX1)7,7,8
  8 IF(KSEV-1)7,9,5
  5 IF(J-2)7,9,7
  9 KMIN=K-1
    M=KMIN
    ZM=COEFZ*DELTM+CZ
    ZAST=(YM-Y(2))*(Z(M,2)-Z(M,1))/(Y(2)-Y(1))+Z(M,2)
    ZTEST(M)=ZAST-ZM
C ***** C.P AND THE POINT OF COORDINATES (X(K),YK,ZK) ARE ON THE SAME
C         STRAIGTH LINE
  7 ZK=COEFZ*DELTK+CZ
    DO 40 L=2,IY
    IF(YK-Y(L))37,38,40
 37 ZAST=(YK-Y(L-1))*(Z(K,L)-Z(K,L-1))/(Y(L)-Y(L-1))+Z(K,L-1)
C     * YK IS BETWEEN Y(L-1) AND Y(L)
    GO TO 43
 38 ZAST=Z(K,L)
    GO TO 43
 40 CONTINUE
    GO TO 50
 43 ZTEST(K)=ZAST-ZK
    KMAX=K
 50 CONTINUE

    IF(KMIN-KMAX) 52,64,64
 52 SUMN=0
    SUMD=0
    DO 60 K=KMIN,KMAX
    SUMN=SUMN+ZTEST(K)
    SUMD=SUMD+ABS(ZTEST(K))
 60 CONTINUE
    IF(SUMD-0.000001)64,64,161
161 TEST=ABS(1.- ABS(SUMN /SUMD))

    IF(TEST-0.001)64,64,63


 64 IF(ISEV-2)80,80,65


 65 IF(CY-Y(J))132,80,133
132 IY1=INDY
    IY2=J-1
    GO TO 134
133 IY1=J+1
    IY2=INDY-1
134 LMAX=IY1
    COEFX=DELTX/DELTY
    COEFZ=DELTZ/DELTY
    INDL=1
    IF(IY1-IY2)131,80,80
131 CONTINUE

    DO 150 L=IY1,IY2
    DELTL=Y(L)-CY
    XL=COEFX*DELTL+CX
    IF(XL-X(1))135,136,136
135 DELTM=DELTL
```

```
      XM=XL
      LMIN=L+1
      GO TO 150
  136 GO TO (106,107),INDL
  106 INDL=2
      LMIN=L
      IF(L-IY1)107,107,108
  108 IF(LSEV-1) 107,105,105
  105 IF(I-2)107,109,107
  109 LMIN=L-1
      M=LMIN
      ZM=COEFZ*DELTM+CZ
      ZAST=(XM-X(2))*(Z(2,M)-Z(1,M))/(X(2)-X(1))+Z(2,M)
      ZTEST(M)=ZAST-ZM
C ***** C,P AND THE POINT OF COORDINATES (XL,Y(L),ZL) ARE ON THE SAME
C        STRAIGTH LINE
  107 ZL=COEFZ*DELTL+CZ
      DO 140 K=2,IX
      IF(XL-X(K))137,138,140
  137 ZAST=(XL-X(K-1))*(Z(K,L)-Z(K-1,L))/(X(K)-X(K-1))+Z(K-1,L)
C      * XL IS BETWEEN X(K-1) AND X(K)
      GO TO 143
  138 ZAST=Z(K,L)
      GO TO 143
  140 CONTINUE
      GO TO 150
  143 ZTEST(L)=ZAST-ZL
      LMAX=L
  150 CONTINUE

      IF(LMIN- LMAX)152,80,80
  152 SUMN=0
      SUMD=0
      DO 160 K=LMIN,LMAX
      SUMN=SUMN+ZTEST(K)
      SUMD=SUMD+ABS(ZTEST(K))
  160 CONTINUE
      IF(SUMD-0.000001)80,80,61
   61 TEST=ABS(1.- ABS(SUMN /SUMD))

      IF(TEST-0.0001)80,80,63
   63 JVIS(I,J)=0
   80 CONTINUE

      IF(IPRI)  48,48,90
   48 CONTINUE
      WRITE(6,81)I,(JVIS(I,J),J=1,IY)
   81 FORMAT(35I4/(4X,34I4))
   90 CONTINUE


      RETURN
      END

FEATURES SUPPORTED
 NONPROCESS
 ONE WORD INTEGERS

CORE REQUIREMENTS FOR TRIVJ
 COMMON   14866   INSKEL COMMON        0  VARIABLES      248  PROGRAM     1188
```

8. Example of Plots Done by TRICE





Figs. 4, 5: The Bessel function J (cf. Sect. 9. 1.)

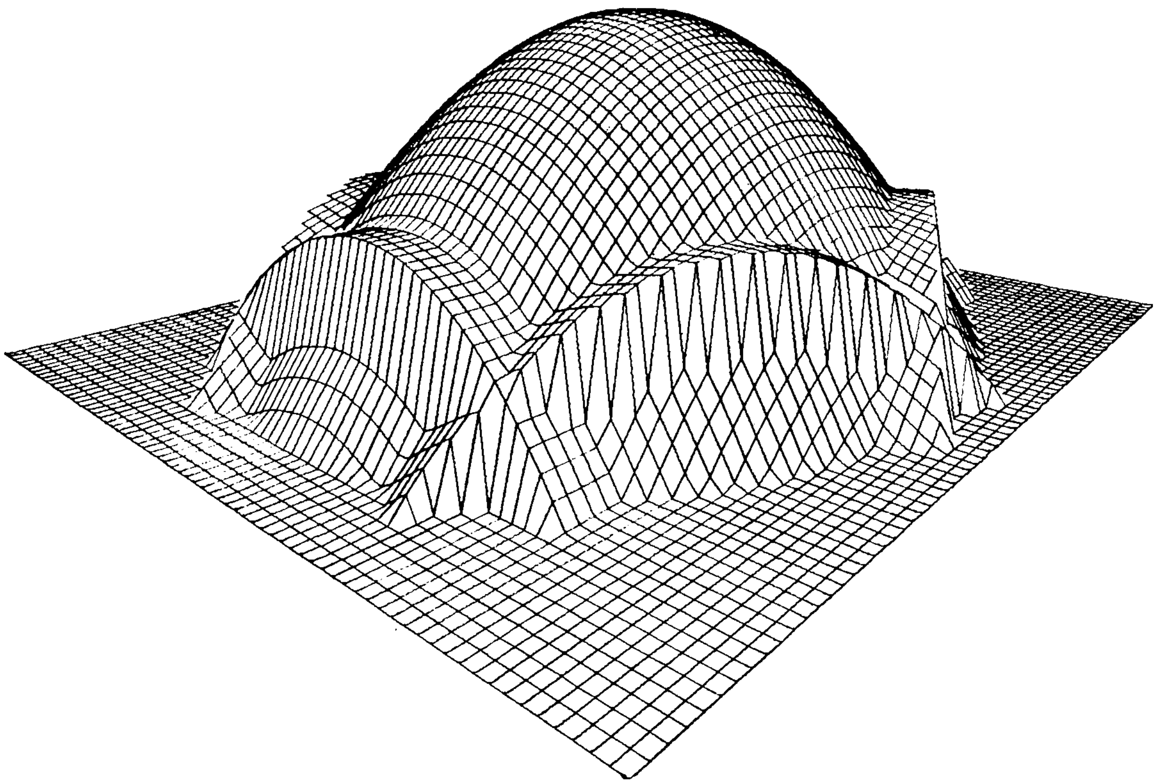


Fig. 6: Surface calculated by TRI (cf. Sect. 9.2.)

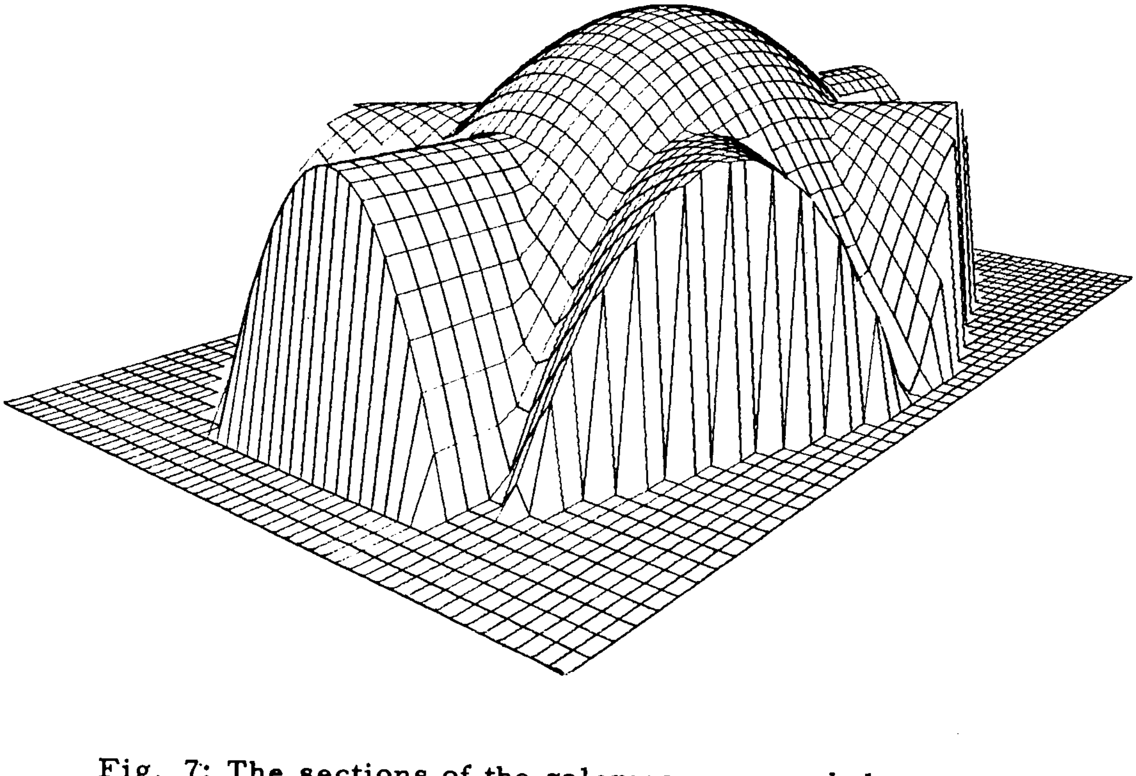


Fig. 7: The sections of the galeries are parabolas.

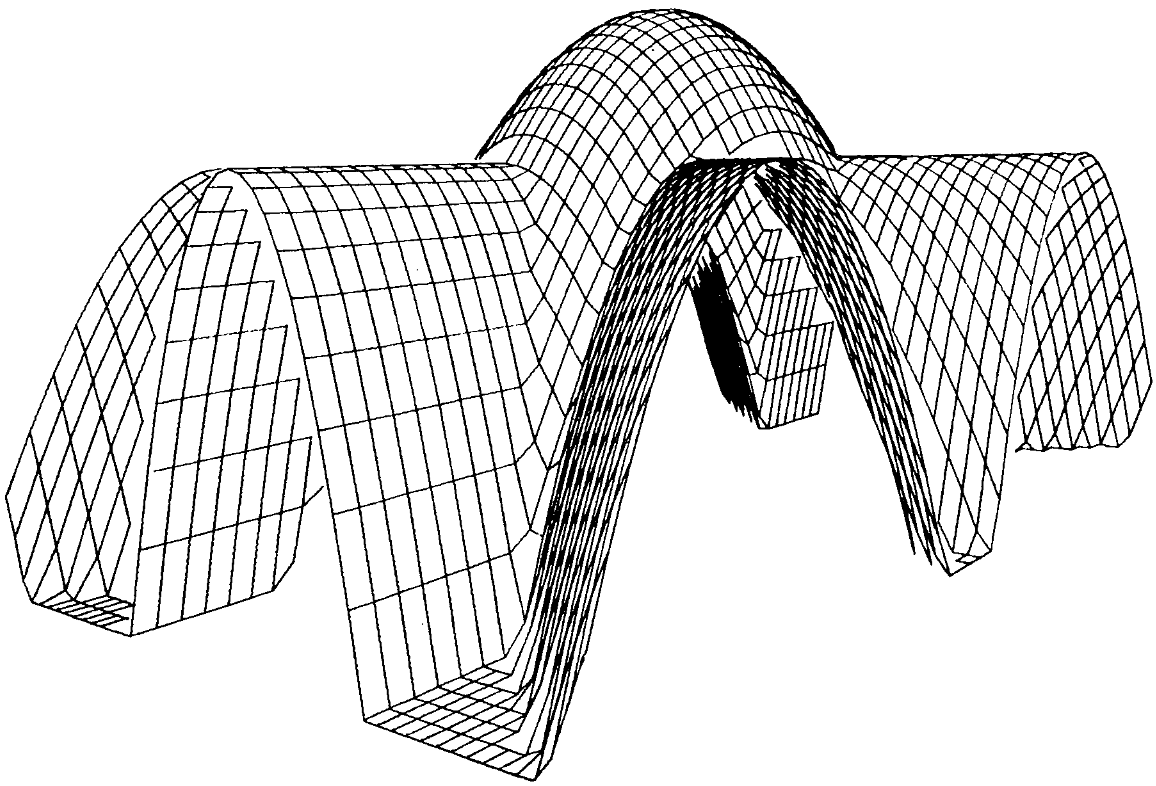


Fig. 8: Parabolic galeries viewed inside.

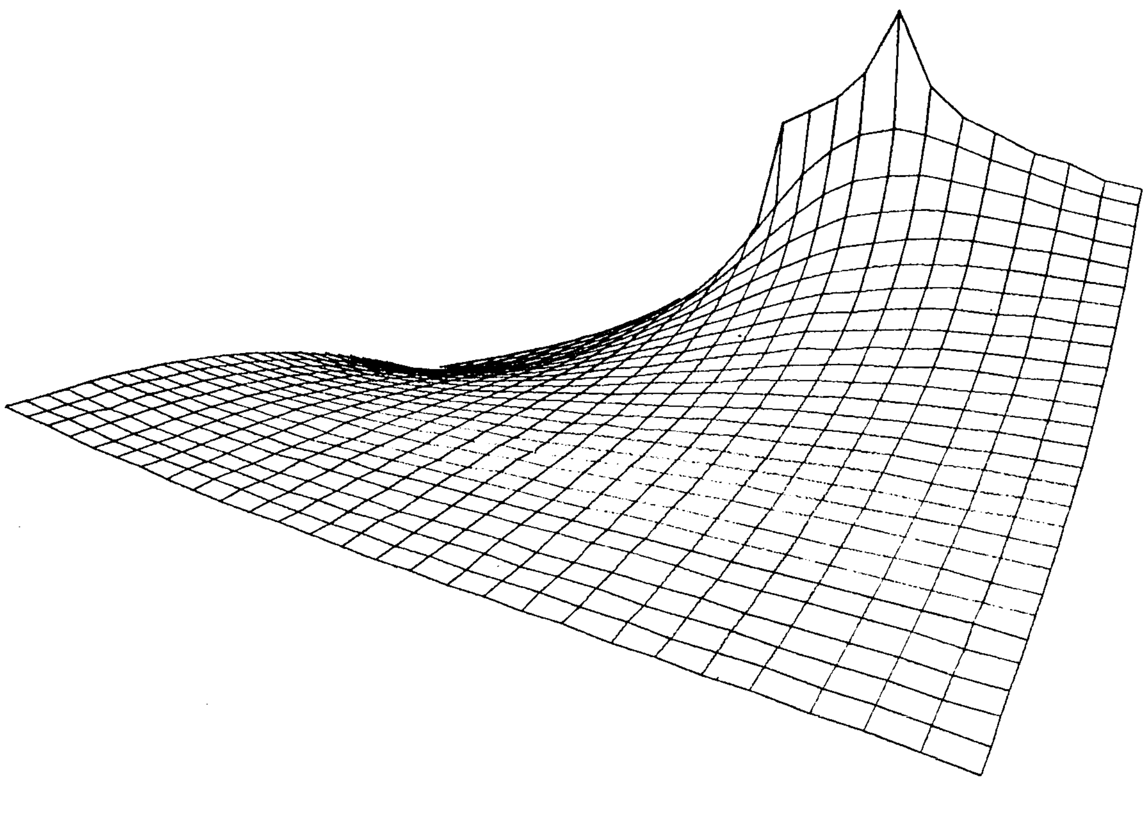


Fig. 9: The elliptic-integral of the first kind (cf. Sect. 9.3.)

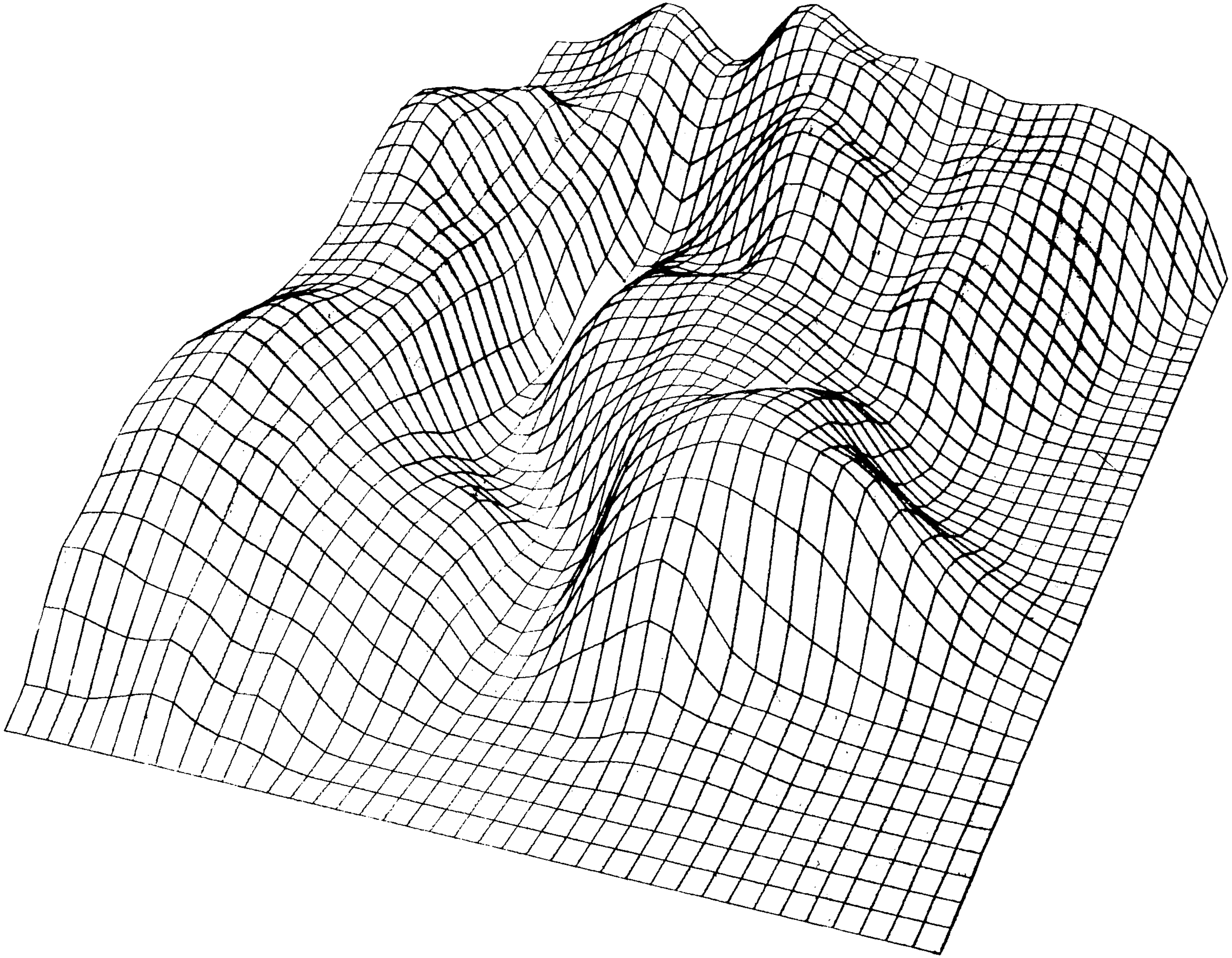


Fig. 10: Plotting data from cards.

9.1. Example of Program Writing the Disk Files TRIXY, TRIZ (Program USBES)

USBES (calling the standard IBM subroutine BESJ, [3]) evaluates as z(x, y) the J Bessel function for given values of x and orders y. The tables of x, y, z are then optionally saved on disk for later use in TRICE.

Input of USBES

One card only FORMAT (7F10.0,I5)

1 - 10	STEPX	interval between two consecutive values of x
11 - 20	STEPY	difference between the order y of two consecutive Bessel functions calculated (STEPY integer but introduced as floating)
21 - 30	YMX	maximum order y of Bessel functions
31 - 40	XMX	maximum value of x
41 - 50	YMI	minimum order y of Bessel functions
51 - 60	XMI	minimum value of x
61 - 70	ACCRY	the requested accuracy (e.g. 0.01)
71 - 75	IPRI =	-1, print the values of x, y, z 0, save on disk x, y, z 1, print and save on disk x, y, z

Limitations for the Choice of y

y integer

$y > 0$, always

$y < 20+10\cdot x-x^{2/3}$, for $x \leq 15$

$y < 90+\frac{x}{2}$, for $x > 15$.

```
// JOB    X     X     X
// FOR USBES
*ONE WORD INTEGERS
*NONPROCESS PROGRAM
*IOCS(CARD,1443 PRINTER,DISK)
*LIST SOURCE PROGRAM
      DEFINE FILE 11( 160,2,U,IM)
      DEFINE FILE 12(4800,2,U,IM)
      COMMON    X(69), Y(69),Z(69,69)
      READ(5,1) STEPX,STEPY,YMX,XMX,YMI,XMI,ACCRY,IPRI
    1 FORMAT(7F10.0,2I5)
      IX=ABS(XMX-XMI)/STEPX
      IY=ABS(YMX-YMI)/STEPY
      N=YMI
      DO 21  J=1,IY
      XX=XMI
      DO 20  I=1,IX
      IER=0
      CALL BESJ(XX,N,BJ,ACCRY,IER)
      XX=XX+STEPX
      Z(I,J)=BJ
   20 CONTINUE
      Y(J)=N
      N=N+STEPY
   21 CONTINUE
      X(1)=XMI
      DO 22 I=2,IX
      X(I)=X(I-1) +STEPX
   22 CONTINUE
      IF(IPRI)24,23,24
   24 WRITE(6,71)(X(I),I=1,IX)
   71 FORMAT(1H0,'TABLE OF X'//(4X,28F5.1))
      WRITE(6,72)(Y(J),J=1,IY)
   72 FORMAT(1H0,'TABLE OF Y'//(4X,28F5.1))
      WRITE(6,91)
   91 FORMAT(1H0 'MATRIX Z'/1H0,'I   * * I IS THE INDEX OF X(I) * * FOR E
     1ACH I,VALUES OF Z(I,J) ARE GIVEN WITH J RUNNING FROM 1 TO IY * *')
      DO 140 I=1,IX
      WRITE(6,122) I,(Z(I,J),J=1,IY)
  122 FORMAT(1H0,I3,14E10.3/(4X,14E10.3))
  140 CONTINUE
   23 IF(IPRI)25,26,26
   26 IM=1
      WRITE(11'IM)(X(I),I=1,IX),(Y(J),J=1,IY)
      IM=1
      WRITE(12'IM)((Z(I,J),I=1,IX),J=1,IY)
   25 CALL EXIT
      END

FEATURES SUPPORTED
 NONPROCESS
 ONE WORD INTEGERS
 IOCS

CORE REQUIREMENTS FOR USBES
 COMMON    9798  INSKEL COMMON      0  VARIABLES      44  PROGRAM    468


 END OF COMPILATION
```

```
// FOR
*ONE WORD INTEGERS
*NONPROCESS PROGRAM
*LIST SOURCE PROGRAM
      SUBROUTINE BESJ(X,N,BJ,D,IER)
      BJ=.0
      IF(N)10,20,20
   10 IER=1
      RETURN
   20 IF(X) 30,30,31
   30 IER=2
      RETURN
   31 IF(X-15.)32,32,34
   32  NTEST=20.+10.*X-X**2/3
      GO TO 36
   34 NTEST =90.+X/2.
   36 IF(N-NTEST) 40,38,38
   38 IER=4
      RETURN
   40 IER=0
      N1=N+1
      BPREV=.0
      IF(X-5.)50,60,60
   50 MA=X+6.
      GO TO 70
   60 MA=1.4*X+60./X
   70 MB=N+IFIX(X)/4+2
      MZERO=MA
      IF(MA-MB)80,90,90
   80 MZERO=MB
   90 MMAX=NTEST
      DO 190 M=MZERO,MMAX,3
      FM1=1.0E-28
      FM=.0
      ALPHA=.0
      IF(M-(M/2)*2)120,110,120
  110 JT=-1
      GO TO 130
  120 JT=1
  130 M2=M-2
      DO 160 K=1,M2
      MK=M-K
      BMK=2.*FLOAT(MK)*FM1/X-FM
      FM=FM1
      FM1=BMK
      IF(MK-N-1)150, 140,150
  140 BJ=BMK
  150 JT=-JT
      S=1+JT
  160 ALPHA=ALPHA+BMK*S
      BMK=2.*FM1/X-FM
      IF(N)180,170,180
  170 BJ=BMK
  180 ALPHA=ALPHA+BMK
      BJ=BJ/ALPHA
      IF(ABS(BJ-BPREV)-ABS(D*BJ))200,200,190
  190 BPREV=BJ
      IER=3
  200 RETURN
      END
```

9.2. Example of Program Writing the Disk Files TRIXY, TRIZ (Program TRI)

TRI (FORTRAN 4, IBM 1800) builds two sets of coordinates x and y defining points on a grid (cf. fig. 1) having as symmetry axes the axes x and y.
A function z(x, y) describing a surface σ is then calculated in such points under the same conditions of symmetry (cf. fig. 1).
The surface σ consists of a dome obtained by rotation of a parabola around the axis z: non cylindrical galeries enter into the dome. Each galery is symmetrical with respect to a different vertical plane τ containing the axis z. The sections of a single galery normal to its symmetry plane τ are hyperbolic cosinus. Let us call axis of a galery the intersection of the plane τ with the plane x, y.

Input of TRI

First card FORMAT(5F10.0, 5I5)

1 - 10	RMAX	the maximum radius of the dome (for z=0)
11 - 20	HCUP	the height of the dome
21 - 30	STEPX	the interval between two values of x
31 - 40	STEPY	the interval between two values of y
41 - 50	DENS	= 1
51 - 55	NX	number of x values in the first quadrant (that is the number of positive values of x; NX≤34)
56 - 60	NY	number of y values in the first quadrant (NY≤34)
61 - 65	NB	number of galeries to be calculated in the first quadrant
66 - 70	IPRI	not used
71 - 75	IESP	the exponent of 0.1 for checking zero values (e.g. IESP=4)

Second card FORMAT(3F10.0)

1 - 10	ANG(L)	the angle between the axis of the L^{th} galery and the axis x
11 - 20	AMPL(L)	half the angle (on the plane x, y and with the vertex in the origin of the coordinates) occupied by the L^{th} galery
21 - 30	LUNG(L)	the length of this galery from the origin of the coordinates

N.B. L runs from 1 to NB, i.e. concerns only the galeries occupying (though partially) the first quadrant. Then this card must be entered NB times.

```
// JOB    X    X
// FOR TRI
*LIST SOURCE PROGRAM
*ONE WORD INTEGERS
*NONPROCESS PROGRAM
*IOCS(CARD,1443 PRINTER,DISK)
      DEFINE FILE 11( 160,2,U,IM)
      DEFINE FILE 12(4800,2,U,IM)
      REAL MARG,LUNG(6)
      DIMENSION M(7),ANG(6),AMPL(6),TANG(6),ZM(7),P(6),Q(6)
      COMMON    C1(69),COEF1(69),RCUP(69),RAXIS(69)
      COMMON    X(69), Y(69),Z(69,69)
      EQUIVALENCE(TANG(1),AMPL(1))
      READ(5,1)RMAX,HCUP,STEPX,STEPY,DENS,NX,NY,NB,IPRI,IESP
    1 FORMAT(5F10.3,6I5)
      A=HCUP/(RMAX*RMAX)
      AM=A+A
      COEFF=DENS/(3.*AM)
      EPSI=1./(10.**IESP)
      WRITE(6,16)RMAX
   16 FORMAT(1H0,'RMAX =',F8.2)

C OCCORRE UNA TAVOLA CHE DIA I DUE PARAMETRI COEF1 E C1 IN FUNZIONE DI RAXIS
      NR= NX+NY
      DELTR=RMAX/NR
      RCUP(1)=0
      DO 120 I=1,NR
      RCUP(I+1)=RCUP(I)+DELTR
  120 CONTINUE
      NR=NR+1
      DO  68  I=1,NR
      R=RCUP(I)
      SQRR=R*R
      ZCUP=HCUP-A*SQRR
      RAM =R*AM
      FACT=1.+RAM*RAM
      ROOT=SQRT(FACT)
      XX=ABS(RAM)-EPSI
      IF(XX)54,54,55
   54 CONTINUE
      TMUZ= 0
      TMUR=-DENS/(AM+AM)
      COIVA=1.618033988
      SIVA=1.272019650
      GO TO 56
   55 TMUZ   =COEFF/RAM*( FACT*ROOT- 1. )
      TMUR  =-TMUZ    /RAM
   56 DTMZX= 3*COEFF*ROOT+TMUR
      DTMZR=DTMZX*AM
      TPAR=DTMZX/ROOT
      GAMMA=1.-DENS*ROOT/DTMZR
      GAMMA=-GAMMA
      DELTA=SQRT(GAMMA*GAMMA+4.)
      COEFY=GAMMA*SQRR/2.
      ID=1
      YY=-COEFY*(DELTA+GAMMA)
      IF(YY)63,64,64
   63 YY=COEFY*(-GAMMA+DELTA)
      ID=2
      IF(YY)68, 64,64
   64 DEQ1=SQRT(YY)
      IF(XX)74,74,75
   75 CONTINUE
```

```
      COIVA=R     /SQRT(SQRR-YY)
      SIVA=R/DEQ1
   74 CONTINUE
      PYTAN  =-TPAR*ROOT*COIVA
      PZVER    =TMUZ*SIVA
C     ALPHA=ATAN(COIVA/SIVA)
      B=PZVER/PYTAN
      B=B+B
      DISCR=SQRT(B*B+4.)
      IF(DISCR+B)66,66,65
   66 WRITE(6,67)B,DISCR
   67 FORMAT(2E12.2)
      GO TO 68
   65 CONTINUE
      VAV=PYTAN/DEQ1
      DENS1    =VAV*ALOG((B+ DISCR)/2.)
      ARGE1=DENS1    /VAV
      E1=EXP(ARGE1)
      COEF1(I)=PYTAN/DENS1
      C1(I)=ZCUP-(E1+1./E1)/2.*COEF1(I)
        RAXIS(I)=R/COIVA
   68 CONTINUE

      READ(5,2)(ANG(L),AMPL(L),LUNG(L),L=1,NB)
    2 FORMAT(3F10.0)
      DO 5  L=1,NB
      ANGOL=ANG(L)
      ANGOL=ANGOL*3.1415927/180.
      P(L)=SIN(ANGOL)
      Q(L)=COS(ANGOL)
      AMP=AMPL(L)
      AMP=AMP      *3.1415927/180.
      TANG(L)=SIN(AMP)/COS(AMP)
      WRITE(6,1) ANG(L),ANGOL,P(L),Q(L),TANG(L)
    5 CONTINUE

      IX1=NX+1
      IX0=NY+2
      IX2=NX+NX+1
      IY1=NY+1
      IY0=NY+2
      IY2=NY+NY+1
      X(IX1)=0
      Y(IY1)=0
      DO 10 I=IX0,IX2
      X(I)=X(I-1)+STEPX
   10 CONTINUE
      DO 20 J=IY0,IY2
      Y(J)=Y(J-1)+STEPY
   20  CONTINUE

      DO 80 I=IX1,IX2

      DO 70 J=IY1,IY2

      M(NB+1)=0
      ZM(NB+1)=0
      Z(I,J)=0
      SQRR=  X(I)*X(I)+Y(J)*Y(J)
      R=SQRT(SQRR)
```

```
   32 M(NB+1)=1
      ZCUP=HCUP-A*SQRR
      ZM(NB+1)=ZCUP
   36 DO 40 L=1,NB
      M(L)=0
      ZM(L)=0
      T=Q(L)*X(I)+P(L)*Y(J)
      IF(T-LUNG(L))37,37,40
   37 D=P(L)*X(I)-Q(L)*Y(J)
      D=ABS(D)
      MARG=T*TANG(L)
      IF(D-MARG-EPSI)39,40,40
   39 M(L)=1
C RICERCA NELLA TAVOLA DI RAXIS E  DEI CORRISPONDENTI PARAMETRI C1 E COEF1
      DO 130 II=2,NR
      IF(T-RAXIS(II))125,124,130
  124 I1=II
      GO TO 131
  125 I1=II
      GO TO 135
  130 CONTINUE
      I1=NR
  131 COEFF=COEF1(I1)
      CC=C1(I1)
      GO TO 136
  135 DELTT=(T-RAXIS(I1-1))/(RAXIS(I1)-RAXIS(I1-1))
      COEFF=COEF1(I1-1)+(COEF1(I1)-COEF1(I1-1))*DELTT
      CC   =C1   (I1-1)+(C1   (I1)-C1   (I1-1))*DELTT
  136 E1=EXP(D/COEFF)
      ZM(L)=COEFF*(E1+1./E1)/2.+CC
   40 CONTINUE
C     CONFRONTO TRA LA CALOTTA E LE GALLERIE
      NBR=NB+1
      DO 60 L=1,NBR
      IF(M(L))60,60,7575
 7575 IF(ZM(L)-Z(I,J))60,60,76
   76 Z(I,J)=ZM(L)
   60 CONTINUE

   70 CONTINUE

   80 CONTINUE

C     COSTRUZIONE DELLA PARTE SIMMETRICA RISPETTO ALL'ASSE Y
      DO 110 I=1,NX
      II=IX2+1-I
      X(I)=-X(II)
      DO 100 J=IY1,IY2
      Z(I,J)=Z(II,J)
  100 CONTINUE
  110 CONTINUE
C     COSTRUZIONE DELLA PARTE SIMMETRICA RISPETTO ALL'ASSE X
      DO 121 J=1,NY
      JJ=IY2+1-J
      Y(J)=-Y(JJ)
      DO 133 I=1,IX2
      Z(I,J)=Z(I,JJ)
  133 CONTINUE
  121 CONTINUE
      IX=IX2
      IY=IY2
```

```
      WRITE(6,21)(X(I),I=1,IX)
   21 FORMAT(1H0,'TABLE OF X'//(4X,28F5.1))
      WRITE(6,22)(Y(J),J=1,IY)
   22 FORMAT(1H0,'TABLE OF Y'//(4X,28F5.1))
      WRITE(6,91)
   91 FORMAT(1H0 'MATRIX Z'/1H0,'I   * * I IS THE INDEX OF X(I) * * FOR E
     1ACH I,VALUES OF Z(I,J) ARE GIVEN WITH J RUNNING FROM 1 TO IY * *')
      DO 140 I=1,IX
      WRITE(6,122) I,(Z(I,J),J=1,IY)
  122 FORMAT(1H0,I3,14E10.3/(4X,14E10.3))
  140 CONTINUE
      IM=1
      WRITE(11'IM)(X(I),I=1,IX),(Y(J),J=1,IY)
      IM=1
      WRITE(12'IM)((Z(I,J),I=1,IX),J=1,IY)
      CALL EXIT
      END

FEATURES SUPPORTED
 NONPROCESS
 ONE WORD INTEGERS
 IOCS

CORE REQUIREMENTS FOR TRI
 COMMON  10350  INSKEL COMMON      0  VARIABLES    212  PROGRAM   1674

 END OF COMPILATION
```

9.3. Example of Program Writing a Tape to be Read by TRICE (Program ELLIN)

ELLIN (FORTRAN 4, IBM 1800) evaluates in the first quadrant of the complex plane defined by t = (x(I), y(J)) the normal elliptic integral of the first kind

$$u(t) = \int_0^t \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}}$$

The real value of k built in is

$$k = 0.8.$$

Therefore both the singularities belong to the real axis in the points x=1, x=1.25. Only one branch of the function u(t) is considered, corresponding to the operation

$$(1-t^2)(1-k^2t^2) = \rho e^{i\varphi}$$

$$(1-t^2)(1-k^2t^2) = \rho^{1/2} e^{i\varphi/2} \quad .$$

The subroutines required for the complex operations are CXSUB, CXMPY, CXDIV, CXRPW, CXABS, CXPOL, CXCRT [4].

Input of ELLIN

Only one card		FORMAT(6I10)
1 - 10	IO = 1	
11 - 20	JO = 1	
21 - 30	IX	total number of points along the real axis (IX≤28)
31 - 40	IY	total number of points along the immaginary axis (IY≤28)
41 - 50	IND =	1, the step between two values of x and y is STEP=0.125
	=	2, the step between two values of x and y is STEP=0.0625
	=	3, the step between two values of x and y is STEP=0.03125
51 - 60	NTAPE	logic tape unit for the output (NTAPE = 7, 8, 9, 10)

```
// JOB    X      X      X
// FOR ELLIN
*IOCS(CARD,1443 PRINTER,MAGNETIC TAPE)
*LIST SOURCE PROGRAM
*ONE WORD INTEGERS
*NONPROCESS PROGRAM
      DIMENSION   Z2(2),ONE(2),FIRST(2),SECON(2),STO(2),F(28),G(28),S(2)
     1,U(28,28),V(28,28),Z(55,55),REAL(55),AIMM (55),X(55),Y(55),AK(2),
     2 T(2),A2(2),DEN(2)
      READ(5,1) IO,JO,IX,IY,IND,NTAPE
    1 FORMAT(8I10)
      IO=1
      JO=1
      M=IX-IO+1
      N=IY-JO+1
      AK(1)=0.64
      AK(2)=0.
      ONE(1)=1.
      ONE(2)=0.
      HALF=0.5
      GO TO(6,7,8),IND
    6 STEP =0.125
      GO TO 9
    7 STEP =0.0625
      GO TO 9
    8 STEP =0.03125
    9 X(1)=STEP/2.
      X(2)=STEP
      DO 10 K=3,M
   10 X(K)=X(K-1)+ STEP
      Y(1)=STEP/2.
      Y(2)=STEP
      DO 11 L=3,N
   11 Y(L)=Y(L-1)+STEP
C     EVALUATE THE FUNCTION TO BE INTEGRATED ALONG THE IMMAGINARY AXIS
      T(1)=0.
      DO 12 L=1,N
      T(2)=Y(L)
      CALL CXMPY(T,T,Z2)
      CALL CXSUB(ONE,Z2,FIRST)
      CALL CXMPY(AK,Z2,A2)
      CALL CXSUB(ONE,A2,SECON)
      CALL CXMPY(FIRST,SECON,S)
      CALL CXRPW(S,DEN,HALF)
      CALLCXDIV(ONE,DEN,STO)
      F(L)=STO(1)
      G(L)=STO(2)
   12 CONTINUE
C     EVALUATION OF THE FUNCTION TO BE INTEGRATED OUT OF THE AXES
      DO14 K=1,M
      DO 15 L=1,N
      T(1)=X(K)
      T(2)=Y(L)
      CALL CXMPY(T,T,Z2)
      CALL CXSUB(ONE,Z2,FIRST)
      CALL CXMPY(AK,Z2,A2)
      CALL CXSUB(ONE,A2,SECON)
      CALL CXMPY(FIRST,SECON,S)
      CALL CXRPW(S,DEN,HALF)
      CALLCXDIV(ONE,DEN,STO)
      U (K,L)=-STO(1)
      V  (K,L)=-STO(2)
```

```
      WRITE(6,19)T,Z2,FIRST,A2,SECON,S,DEN
   14 CONTINUE

      DX1=STEP/6.
      DY1=DX1
      DX=DX1+DX1
      DY=DX
C     THE FIRST INTEGRATION TAKES PLACE ALONG THE IMMAGINARY AXIS
      REAL(J0)=0
      AIMM (J0)=0
C     INTEGRATION FOR THE FIRST POINT OF IMMAGINARY AXIS (J1=1) * HALF STEP
      J1=J0+1
      J=J1
      L=1
      REAL(J)=(0.+4*G(L)+G(L+1))*(-DY1)
      AIMM(J)=(1.+4*F(L)+F(L+1))*DY1
C     INTEGRATION CONTINUES WITH NORMAL STEP * DEFINE AGAIN F(1),G(1),Y(1)
      WRITE(6,19)F(1),G(1)
   19 FORMAT(4X,14E10.3)
      Y(L)=0.
      F(L)=1.
      G(L)=0.
      WRITE(6,21)(F(L),L=1,N)
   21 FORMAT(1H0,'VECTOR F * REAL PART OF THE INTEGRAND * AXIS  Y'/
     1(4X,14E10.3))
      WRITE(6,22)(G(L),L=1,N)
   22 FORMAT(1H0,'VECTOR G * IMMAGINARY PART OF THE INTEGRAND * AXIS Y'/
     1(4X,14E10.3))
C     TWO INTERVALS DY ARE COVERED BY INTEGRATION EACH TIME
      J2=J1+1
      L=1
      DO 40 J=J2,IY
      REAL(J)=REAL(J-2)-(G(L)+4.*G(L+1)+G(L+2))*DY
      AIMM(J)=AIMM(J-2)+(F(L)+4.*F(L+1)+F(L+2))*DY
      L=L+1
   40 CONTINUE
C     INTERPOLATION FOR SMOOTHING VALUES OF REAL,AIMM
      IY1=IY-1
      DO 50 J=J1,IY1
      REAL(J)=(REAL(J-1)+2.*REAL(J)+REAL(J+1))/4.
      AIMM(J)=(AIMM (J-1)+2.*AIMM (J)+AIMM(J+1))/4.
      Z(I0,J)=SQRT( REAL(J)*REAL(J)+AIMM(J)*AIMM(J))
   50 CONTINUE
      J=IY
      Z(I0,J)=SQRT(REAL(J)*REAL(J)+AIMM(J)*AIMM(J))
      WRITE(6,41)(REAL(J),J=1,IY)
   41 FORMAT(1H0,'VECTOR REAL * INTEGRAL ALONG AXIS Y'/(4X ,14E10.3))
      WRITE(6,42)(AIMM(J),J=1,IY)
   42 FORMAT(1H0,'VECTOR AIMM * INTEGRAL ALONG AXIS Y'/(4X ,14E10.3))
      GO TO(51,52,53),IND
   51 ISIN1=I0+8
      ISIN2=I0+10
      GO TO 54
   52 ISIN1=I0+16
      ISIN2=I0+20
      GO TO 54
   53 ISIN1=I0+32
      ISIN2=I0+40
   54 Z(I0,J0)=0
      WRITE(6,56)
```

```
   56 FORMAT(1H0,20X,'THE REAL AND THE IMMAGINARY PART OF THE VALUES INT
     1ERPOSED BETWEEN THE AXIS Y AND THE FIRST PARALLEL'//)
      K=1
      WRITE(6,93)K,(U(K,L),L=1,N)
      WRITE(6,93)K,(V(K,L),L=1,N)

C     EVALUATE THE ELLIPTIC INTEGRAL IN THE FIRST QUADRANT
      I1=I0+1
      I2=I1+1
      DO 70 J=J1,IY
C     INTEGRATION FOR THE FIRST VALUE OF X(I) IN THE QUADRANT UNDER CONSIDERAT.
      K=1
      L=J-J1+2
      R=REAL(J)+(F(L)+4.*U(K,L)+U(K+1,L))*DX1
      AIM=AIMM(J)+(G(L)+4.*V(K,L)+V(K+1,L))*DX1
C     NO Z CALCULATED HERE * IT WILL BE DONE WHEN BEGINNING LOOP 60
      ROLD=REAL(J)
      AIOLD=AIMM(J)
      RNEW=R
      AINEW=AIM
C     DEFINE AGAIN U(1,L),V(1,L) TO BE USED AS FUNCTION TO BE INTEGRATED
C     DEFINED IN THE POINTS OF AXIS Y (I=I0)
      U(K,L)=F(J)
      V(K,L)=G(J)
      IF(J-J1)99,59,68
   59 L=2
      DO 67 I=I1,IX
      FACT=1.
      IF(I-I1)99,62,61
C     INTEGRATION ALONG THE AXIS DEFINED BY  J=J1
   61 K=I-I1
      R=ROLD
      AIM=AIOLD
      R=R  +(U(K,L)+4.*U(K+1,L)+U(K+2,L))*DX
      AIM=AIM  +(V(K,L)+4.*V(K+1,L)+V(K+2,L))*DX
C     INTERPOLATION FOR SMOOTHING THE INTEGRAL ALONG THE AXIS DEFINED BY J=J1
      RR=(ROLD+RNEW+RNEW+R)/4.
      AA=(AIOLD+AINEW+AINEW+AIM)/4.
      Z(I-1,J)=SQRT(RR*RR+AA*AA)
      ROLD=RNEW
      AIOLD=AINEW
      RNEW=R
      AINEW=AIM
   62 K=I-I0+1
      IF(I-ISIN1)66,63,64
   63 Z(I,J0)=2.
      U(K,L-1)=0
      V(K,L-1)=0
      GO TO 67
   64 IF(I-ISIN2)66,65,655
   65 Z(I,J0)=2.66
      U(K,L-1)=0
      V(K,L-1)=0
      GO TO 67
C     INTEGRATION FOR THE VALUES OF X AXIS
  655 FACT=-1.
   66 T(1)=X(K)
      T(1)=X(K)
      T(2)=Y(L-1)
```

```
      CALL CXSUB(ONE,Z2,FIRST)
      CALL CXMPY(AK,Z2,A2)
      CALL CXSUB(ONE,A2,SECON)
      CALL CXMPY(FIRST,SECON,S)
      CALL CXRPW(S,DEN,HALF)
      CALLCXDIV(ONE,DEN,STO)
      TERM=STO(1)*FACT
      VERM=-STO(2)
      XR=R+(V(K,L)+4.*V(K,L-1)+VERM) *DY1
      XAIM=AIM-(U(K,L)+4.*U(K,L-1)+TERM)*DY1
      V(K,L-1)=VERM
      U(K,L-1)=TERM
      Z(I,J0)=SQRT(XR*XR+XAIM*XAIM)
   67 CONTINUE
      I=IX
      Z(I,J)=SQRT( R*R +AIM*AIM )
      GO TO 70
   68 L=J-J0+1
C     INTEGRATION FOR THE OTHER POINTS OF THE FIRST QUADRANT
C     THE INTEGRATION PROCEEDS ALONG AXES DEFINED BY J=J2,IY
      K=1
      DO 69 I=I2,IX
      R=ROLD
      AIM=AIOLD
      R=R+(U(K,L)+4.*U(K+1,L)+U(K+2,L))*DX
      AIM=AIM+(V(K,L)+4.*V(K+1,L)+V(K+2,L))*DX
      RR=(ROLD+RNEW+RNEW+R)/4.
      AA=(AIOLD+AINEW+AINEW+AIM)/4.
      Z(I-1,J)=SQRT(RR*RR+AA*AA)
      ROLD=RNEW
      AIOLD=AINEW
      RNEW=R
      AINEW=AIM
      K=K+1
   69 CONTINUE
      I=IX
      Z(I,J)=SQRT( R*R +AIM*AIM )
   70 CONTINUE
      K=1
      X(K)=0.
      U(1,1)=1.
      V(1,1)=0.


      WRITE(6,73)
   73 FORMAT(1H0,20X,'* * * ARRAYS U AND V CONTAIN THE REAL AND THE IMMA
     1GINARY PART OF  THE  FUNCTION TO BE INTEGRATED * * *'//)
      REWIND NTAPE
      K=M
      II=IX
      DO 75 I=I1,IX
      X(II)= X(K)
      K=K-1
   75 II=II-1
      L=N
      JJ=IY
      DO 76 J=J1,IY
      Y(JJ)=Y(L)
      L=L-1
   76 JJ=JJ-1
      WRITE(6,77)
```

```
   77 FORMAT(1H0,'MATRIX U')
      DO 78 K=1,M
      WRITE(6,93) K,(U(K,L),L=1,N)
   78 CONTINUE
      WRITE(6,79)
   79 FORMAT(1H0,'MATRIX V')
      DO 80 K=1,M
      WRITE(6,93) K,(V(K,L),L=1,N)
   80 CONTINUE
            WRITE(NTAPE,81)(X(I),I=1,IX )
      WRITE(NTAPE,81)                      (Y(J),J=1,IY )
   81 FORMAT(8E10.3)
      WRITE(6,82)(X(I),I=1,IX)
   82 FORMAT(1H0,'VECTOR X'/(4X,14E10.3))
      WRITE(6,83)(Y(I),I=1,IY)
   83 FORMAT(1H0,'VECTOR Y'/(4X,14E10.3))
      WRITE(6,91)
   91 FORMAT(1H0,'MATRIX Z'   )
      DO 95 I=1,IX
      WRITE(NTAPE,81)     (Z(I,J),J=1,IY)
      WRITE(6,93)I,(Z(I,J),J=1,IY)
   93 FORMAT(I4/(4X,14E10.3))
   95 CONTINUE
      END FILE NTAPE
      REWIND NTAPE
   99 CALL EXIT
      END

FEATURES SUPPORTED
 NONPROCESS
 ONE WORD INTEGERS
 IOCS

CORE REQUIREMENTS FOR ELLIN
 COMMON      0  INSKEL COMMON      0  VARIABLES    9850  PROGRAM   2552

 END OF COMPILATION
```

```
// JOB    X     X     X
// DUP
*STOREDATAD    2 FX2 TRIZ         30      9600
*STOREDATAD    2 FX2 TRIXY         1       320
*DUMPLET
// JOB    X     X     X
// XEQ USBES
*FILES(12,TRIZ,2),(11,TRIXY,2)
*CCEND
  0.5         1.        20.        24.       0.        0.5        0.01
// JOB    X     X     X
// XEQ TRICE
*FILES(12,TRIZ,2),(11,TRIXY,2)
*LOCALTRIFN,TRIVJ,XRYR
*CCEND
33.       33.              5.     15.      10.              80.        4
60.      30.        90.       100.
   3   47   20        -1
**0
24.       33.              5.     15.      10.         1    80.        2
90.                90.
   3   47   20        -1

// JOB    X     X     X
// XEQ TRI
*FILES(12,TRIZ,2),(11,TRIXY,2)
*CCEND
10.       7.5              .345     .345       1.   34   34   3   1   4
      0.         52.       8.7
     60.         52.       8.7
 120.            52.       8.7
// JOB    X     X     X
// XEQ TRICE
*FILES(12,TRIZ,2),(11,TRIXY,2)
*LOCALTRIFN,TRIVJ,XRYR
*CCEND
-20.      -20.      7.5      15.      10.                20.         4
35.       55.          90.
   3   69   69
**1

// JOB    X     X     X
// XEQ ELLIN
*CCEND
                          28        28        2        9

// JOB    X     X     X
// XEQ TRICE
*FILES(12,TRIZ,2),(11,TRIXY,2)
*LOCALTRIFN,TRIVJ,XRYR
*CCEND
2.        3.5       4.      15.     10.               2.          4
45.       45.         90.       1000.
   2   28   28    9   -1
**3
```

Acknowledgements

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Alfred Nobel

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