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FINITE-ELEMENT ANALYSIS OF RADIATION-DAMAGE STRESSES IN THE GRAPHITE OF MATRIX FUEL ELEMENTS

by

J. Donea

1970

Joint Nuclear Research Center
Ispra Establishment - Italy
Physical Chemistry Department

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A computer program based on the developments presented in this paper is being prepared.

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ABSTRACT

A finite-element procedure is presented for the determination of the radiation damage stresses in the graphite of matrix fuel elements. The assumption of a generalized plane strain situation is made. The graphite is assumed to be transversely isotropic. Irradiation induced creep as well as radiation damage and thermal strains are taken into account by using an incremental type of analysis.

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KEYWORDS

POWER
REACTOR CORE
GAS COOLED REACTORS
PRESSURE
COOLANT LOOPS
HELIUM
FUELS
TEMPERATURE
GRAPHITE

1. Introduction *)

It has been shown that the impregnation of graphite with suitable molten metals leads to an impervious material with excellent neutronic characteristics (ref. [1]).

A new concept of matrix fuel element for thermal reactors has been developed based on this material. The cross-section of such an element is shown for illustrative purposes in figure 1 with fuel pellets and cooling channels arranged in a square pattern.

The fabrication of the fuel element takes place during the impregnation process of the graphite [2]. The fuel and the graphite canning are thus brought into contact across a thin layer of impregnating metal.

The purpose of the present paper is to describe a method of analysis for the determination of the radiation-damage stresses in the graphite of such matrix fuel elements.

Steady and transient loads are considered including the interaction between the fuel and graphite matrix (swelling and fission gas pressure), the coolant pressure, the axial load as well as radiation-damage and thermal strains.

The graphite is assumed to be transversely-isotropic and irradiation induced creep is accounted for. The assumption of a generalized plane strain is made, i.e. the total axial strain is a constant which in general is non-zero.

The finite-element method is used to solve numerically the variational problems associated with the determination of temperatures and stresses in the graphite. The inclusion of non-linear strains in the finite-element method is handled by using an incremental type of procedure, i.e.

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the life of the fuel element is divided into suitable intervals.

The method presented in this paper is quite general and is independent of the type of creep law used.

2. Assumptions.

A certain number of assumptions are made in order to bring the problem of predicting the stresses in the graphite matrix into a tractable range.

2.1 Repetitive stress field.

The fuel element is regarded as the assembly of basic cells and it is assumed that the stress field is repetitive from cell to cell, except near the edge of the element. Hence, for the determination of the repetitive stress field, the problem reduces to the analysis of a single cell.

2.2 Generalized plane strain.

The fuel element can be considered as being very long with respect to its transverse dimensions. A condition of generalized plane strain can thus be assumed, i.e. the total axial strain is a constant which in general is non zero. The solutions obtained are thus valid over the central region of the element.

2.3 Transverse isotropy of graphite.

The axial direction of the element is supposed to correspond with the extrusion direction of graphite. It is assumed that a rotational symmetry of graphite properties exists within the plane perpendicular to the element axis. This assumption combined with that of plane strain reduces to four the independent elastic constants that are necessary to define the stress-strain relations.

2.4 Interaction between fuel and graphite matrix.

As already mentioned, the fuel and the graphite canning are brought into contact across a thin layer of impregnating metal.

Since Young's modulus of both possible fuels (UC and UO_2) is very large as compared to the graphite modulus, it will be assumed that the fuel expands freely.

At the start of the life of the element, radial displacements due to differential thermal expansion are thus imposed along the fuel-graphite interface. During the life of the element, these boundary conditions have to be adjusted to take into account the irradiation effects on the fuel (swelling or fission gas release, or both).

3. Method of analysis.

The life of the fuel element is divided into suitable intervals. At the start of the first interval the temperature distribution in the element is calculated. To simplify the analysis, the thermal conductivity of graphite is taken as independent of temperature.

The thermal strains are then evaluated and since the radiation-damage and creep strains are zero, the stresses in the graphite can be calculated. Initial boundary conditions consist of imposed thermal displacements at the fuel-graphite interface and of given pressures along the edge of the element in contact with the coolant.

During the interval, damage to the graphite causes a decrease of its thermal conductivity and changes of dimensions. The next step in the calculation is thus the derivation of the thermal and radiation-damage strains at the end of the interval. The boundary conditions between fuel and graphite matrix are then adjusted to take into account the irradiation effects on the fuel.

The creep strain increments during the interval are evaluated by an iterative process. A first estimate of the stresses at the end of the interval is obtained using the creep strain increments of the preceeding interval (zero for the first step). A first approximation of the creep strain increments is then obtained by assuming that the stresses remain constant, having a mean value between the known stresses at the start of the interval and the first estimate of the final stresses. A second estimate of the final stresses can then be obtained and the process is continued until adequate convergence is attained.

The same steps in the calculation are repeated until either the final neutron dose is reached or until the stress distribution does not change anymore, i.e., a steady-state condition is established.

4. Expressions for strains and stresses.

4.1 Total strains.

Suppose the z-coordinate corresponds to the axial direction of the fuel element and let u,v,w be the displacement components along O_x, O_y and O_z. The assumption of generalized plane strain makes w independent of x and y and the in-plane components u and v independent of the axial coordinate. There are thus only four non-zero components in the total strain matrix and they are defined in terms of displacements by well-known relations:

$$\varepsilon_x^t = \frac{\partial u}{\partial x} \quad ; \quad \varepsilon_y^t = \frac{\partial v}{\partial y} \quad ; \quad \gamma_{xy}^t = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad ; \quad \varepsilon_z^t = \text{constant} \qquad (1)$$

It is assumed that during a time interval the total change in strain can be separated into elastic and non-linear parts

$$\{\Delta \overset{t}{\varepsilon}\} = \{\Delta \overset{e}{\varepsilon}\} + \{\Delta \overset{n}{\varepsilon}\} \qquad (2)$$

where $$\{\Delta\varepsilon\}^T = (\Delta\varepsilon_x , \Delta\varepsilon_y , \Delta\gamma_{xy} , \Delta\varepsilon_z) \qquad (3)$$

4.2 Stress-strain relations.

In view of the assumption of transverse isotropy the increments of the in-plane stresses are related to the changes of the elastic strains by

$$\{\Delta\sigma\} = [D^*] \{\Delta \overset{e}{\varepsilon}\} \qquad (4)$$

where $$\{\Delta\sigma\}^T = (\Delta\sigma_x , \Delta\sigma_y , \Delta\tau_{xy}) \qquad (5)$$

$$[D^*] = \frac{E_2}{(1+\nu_1)(1-\nu_1-2n\nu_2^2)} \begin{bmatrix} n(1-n\nu_2^2) & m(\nu_1+n\nu_2^2) & 0 & m\nu_2(1+\nu_1) \\ m(\nu_1+n\nu_2^2) & m(1-n\nu_2^2) & 0 & m\nu_2(1+\nu_1) \\ 0 & 0 & \dfrac{m(1-\nu_1-2n\nu_2^2)}{2} & 0 \end{bmatrix} \qquad (6)$$

and $$n = E_1 / E_2$$

In the elasticity matrix (6) the constants E_1, ν_1 are associated with the behaviour perpendicular to the extrusion direction and E_2, ν_2 with the behaviour parallel to that direction.

Substituting eq. (2) into eq. (4) yields the relation between the incremental stress, total strain and non-linear strain.

$$\{\Delta\sigma\} = [D^*]\left\{\{\Delta\overset{t}{\varepsilon}\} - \{\Delta\overset{n}{\varepsilon}\}\right\} \tag{7}$$

The expression for the axial stress increment is given by Hooke's law

$$\Delta\sigma_z = E_z\left(\Delta\varepsilon_z^t - \Delta\varepsilon_z^n\right) + \nu_z\left(\Delta\sigma_x + \Delta\sigma_y\right) \tag{8}$$

The relations (7) and (8) for the stress increments involve the change of the total axial strain. This last can be eliminated by equating the total load increment on a transverse section of the element to the change of the applied axial load ΔP_z during the interval

$$\Delta P_z + \iint_{\text{Section}} \Delta\sigma_z \, dx \, dy = 0 \tag{9}$$

4.3 Non-linear strains.

The non-linear part of the strain increment is due to thermal dilatation, radiation damage and creep

$$\{\Delta\overset{n}{\varepsilon}\} = \{\alpha\,\Delta T\} + \{\Delta\overset{w}{\varepsilon}\} + \{\Delta\overset{c}{\varepsilon}\} \tag{10}$$

4.3.1 Thermal strains.

A finite-element procedure is used to derive the two-dimensional temperature distribution in the fuel and in the graphite matrix. The continuous body is replaced by a system of discrete triangular elements into which the temperature field is approximated by a linear polynominal with the corner point temperatures as parameters [3]. Boundary values of temperature and heat flux, internal heat generation and heat-transfer coefficients may be a function of position.

The nodal values of the temperature are obtained by minimizing the functional associated with Poissons's equation. The thermal strain vector is then constructed for each finite element by multiplying the appropriate coefficient of thermal expansion by the arithmetic mean of the three nodal temperatures.

4.3.2 Radiation-damage strains.

Irradiation induced dimensional changes in graphite are functions of both the neutron dose and the irradiation temperature. Since the thickness of graphite canning is small, the assumption of a uniform Wigner strains distribution can be made. Nevertheless, any expression giving the Wigner strains as a function of dose and temperature could easily be incorporated in the present analysis.

4.3.3 Creep strains.

The transient creep of graphite is usually neglected and the steady creep rate assumed to be independent of the neutron dose and directly proportional to the applied stress.

The multiaxial creep laws for this particular case are readily obtained by an analogy to linear elasticity.

Since the method presented in this paper holds for any type of creep law a more general way of expressing the multiaxial strain rates in term of stresses is given.

It has been shown [4] to be fruitful to introduce the speed of energy dissipation during the creep process and to consider this speed as a single valued function of the state of stress.

This can be written as

$$\{\sigma\}^T \{\dot{\varepsilon}^c\} = f(\sigma) \geq 0 \qquad (11)$$

where $\{\dot{\varepsilon}^c\}$ is the creep strain rate vector, $\{\sigma\}$ the stress vector and $f(\sigma)$ the speed of energy dissipation function.

The scalar function $f(\sigma)$ can only define a vector by means of its partial derivatives with respect to the components of the stress vector, i.e.

$$\{\dot{\varepsilon}^c\} = \left\{ \frac{\partial f(\sigma)}{\partial \{\sigma\}} \right\} \frac{f(\sigma)}{\{\sigma\}^T \left\{ \frac{\partial f(\sigma)}{\partial \{\sigma\}} \right\}} \qquad (12)$$

Energy dissipation functions can be obtained from monoaxial tensile experiments by defining the creep rate for a given stress level. For the steady creep of graphite one has

$$\dot{\varepsilon}_1^c = B\,\sigma_1 \qquad (\sigma_2 = \sigma_3 = 0) \qquad (13.a)$$

$$f(\sigma) = B\,\sigma_1^2 \qquad (13.b)$$

To transform this experimental result into a multiaxial state of stress, an equivalent stress is introduced, according to von Mises criterion

$$\sigma_e^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x\sigma_y - \sigma_y\sigma_z - \sigma_z\sigma_x + 3\,\tau_{xy}^2 \qquad (13.c)$$

For the monoaxial experiment

$$\sigma_e = \sigma_1 \qquad (13.d)$$

hence

$$f(\sigma) = B\,(\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x\sigma_y - \sigma_y\sigma_z - \sigma_z\sigma_x + 3\tau_{xy}^2) \qquad (13.e)$$

The individual components of the creep rate can then be obtained from eq. (12). The result for the steady creep of graphite is

$$\begin{aligned}
\dot\varepsilon_x^c &= B\left[\sigma_x - \tfrac{1}{2}(\sigma_y + \sigma_z)\right] \\
\dot\varepsilon_y^c &= B\left[\sigma_y - \tfrac{1}{2}(\sigma_x + \sigma_z)\right] \\
\dot\gamma_{xy}^c &= 3B\tau_{xy} \\
\dot\varepsilon_z^c &= B\left[\sigma_z - \tfrac{1}{2}(\sigma_x + \sigma_y)\right]
\end{aligned} \qquad (14)$$

5. Finite-element formulation of the stress problem.

In the present analysis the strain and stress distributions are derived from the calculus of variations by minimizing the potential energy of the loaded structure.

The finite-element technique known as the displacement method is used to solve numerically the variational problem associated with the principle of minimum potential energy. For a complete description of the method the reader is referred to Zienkiewicz's basic text [3].

The continuous body is divided into triangular elements interconnected at their corner points. These elements have the advantage of being able to fit any boundaries and, also, permit the use of a graded mesh.

5.1 Characteristics of a triangular finite element.

Within a typical triangular element (fig. 2) with nodes i,j,m numbered in anti-clockwise order the in-plane components u and v of the displacement vector are expressed by linear polynominals with the displacement values at the corner points as parameters

$$\begin{Bmatrix} u(x,y) \\ v(x,y) \end{Bmatrix} = \begin{bmatrix} N_i & 0 & N_j & 0 & N_m & 0 \\ 0 & N_i & 0 & N_j & 0 & N_m \end{bmatrix} \{U^e\} \tag{15.a}$$

where

$$\{U^e\}^T = (u_i, v_i, u_j, v_j, u_m, v_m) \tag{15.b}$$

$$N_i = \frac{1}{2A^e}(a_i + b_i x + c_i y)$$

$$a_i = x_j y_m - x_m y_j$$

$$b_i = y_j - y_m \qquad \text{(... cyclic order i,j,m)}$$

$$c_i = x_m - x_j$$

A^e = area of triangle ijm.

The continuity requirements on the displacements along the edges of adjacent elements are thus automatically satisfied.

The in-plane components of the total strain increment are defined in terms of the change in displacements by eq. (1). In view of eq. (15.a) this can be written as

$$(\Delta\varepsilon_x^t, \Delta\varepsilon_y^t, \Delta\gamma_{xy}^t)^T = [B]\{\Delta U^e\} \tag{16}$$

with

$$[B] = \frac{1}{2A^e}\begin{bmatrix} b_i & 0 & b_j & 0 & b_m & 0 \\ 0 & c_i & 0 & c_j & 0 & c_m \\ c_i & b_i & c_j & b_j & c_m & b_m \end{bmatrix}$$

The total axial strain increment is given by eq. (9), i.e.

$$\Delta\varepsilon_z^t = \frac{1}{E_z A_{tot}}\left[\sum_{elements}\left\{E_z \Delta\varepsilon_z^* - \nu_z(\Delta\sigma_x + \Delta\sigma_y)\right\}A^e - \Delta P_z\right] \tag{17}$$

where

$$A_{tot} = \sum A^e$$

Combining eqs. (7) and (16) the increments of the in-plane stresses can be expressed as

$$(\Delta\sigma_x, \Delta\sigma_y, \Delta\tau_{xy})^T = [D][B]\{\Delta U^e\} + D^{**}(\Delta\varepsilon_z^t, \Delta\varepsilon_z^t, 0)^T - [D^*]\{\Delta\hat{\varepsilon}\} \qquad (18)$$

where the matrix $[D]$ consists of the first three columns of the matrix $[D^*]$ given in (5) and where

$$D^{**} = \frac{n \nu_2 E_2}{(1-\nu_1-2n\nu_2^2)}$$

The axial stress increment is

$$\Delta\sigma_z = E_2(\Delta\varepsilon_z^t - \Delta\varepsilon_z^n) + \nu_2(\Delta\sigma_x + \Delta\sigma_y) \qquad (19)$$

The next step in the finite-element analysis is to define element nodal forces which are statically equivalent to the in-plane boundary stresses.

Using the theorem of virtual work it is easy to show that these forces are

$$\{\Delta F^e\} = [k^e]\{\Delta U^e\} + [B]^T D^{**} A^e (\Delta\varepsilon_z^t, \Delta\varepsilon_z^t, 0)^T - [B]^T[D^*]A^e\{\Delta\hat{\varepsilon}\} \qquad (20)$$

with

$$\{\Delta F^e\}^T = (\Delta F_{xi}, \Delta F_{yi}, \Delta F_{xj}, \Delta F_{yj}, \Delta F_{xm}, \Delta F_{ym})$$

$[k^e]$ = stiffness matrix of the triangular element.

The first term in eq. (20) represents the forces induced by the displacements of the nodes. The last two terms represent the nodal forces required to balance the total axial strain and the non-linear strains.

5.2 Overall equilibrium equations.

The structure is considered to be loaded incrementally by external forces $\{\Delta R\}$ applied at the nodes.

The equilibrium conditions of a typical node i are established by equating each component of $\{\Delta R_i\}$ to the sum of the component forces of type (20) contributed by the elements meeting at node i.

Provided the non-linear strains and the total axial strain can be considered to be known, the resulting linear equilibrium equations

$$\{\Delta R_i\} = \sum \{\Delta F_i\} \tag{21}$$

contain the nodal displacements as unknowns.

Once the nodal displacements have been obtained, the element strains and stresses are readily computed using equations (16) to (19).

5.3 Solution technique.

The life of the fuel element is divided into suitable intervals and in each interval the total increment of strain is divided into its elastic, thermal, radiation-damage and creep components. If the increment of non-linear strains during an interval can be considered to be known, the change of stress can be found as an ordinary elasticity problem in presence of initial strains. The solution technique is thus as follows:

1) At the start of the life of the fuel element suitable boundary conditions are imposed and the elastic stresses are determined

with the thermal strains considered as initial strains and with the total axial strain constrained to satisfy eq. (17).

2) The increments of the nodal forces required to balance the non-linear strains that developed during a time interval are then computed. For this purpose the creep strain changes are evaluated from eq. (14) by the iterative process described in section 2 and are added to the changes of the thermal and radiation-damage strains.

3) After adjustment of the boundary conditions, the change in nodal point displacements are evaluated from eq. (21) with the increment of the total axial strain computed from eq. (17).
The total change in strain is determined by eq. (16) and the incremental stresses are obtained from eqs. (18) and (19).

4) Steps 2 and 3 are repeated for the subsequent intervals until either the maximum neutron dose has been reached or until the stress distribution does not change anymore, i.e. a steady-state condition is established.

6. Conclusions.

A method has been presented for predicting the radiation-damage stresses in the graphite of matrix fuel elements.

The proposed method of solution is based on the analysis of generalized plane strain situations by a finite-element procedure. The use of an incremental type of approach makes it easy to incorporate in the analysis arbitrary radiation-damage and creep laws.

A computer program based on the developments presented in this paper is being prepared.

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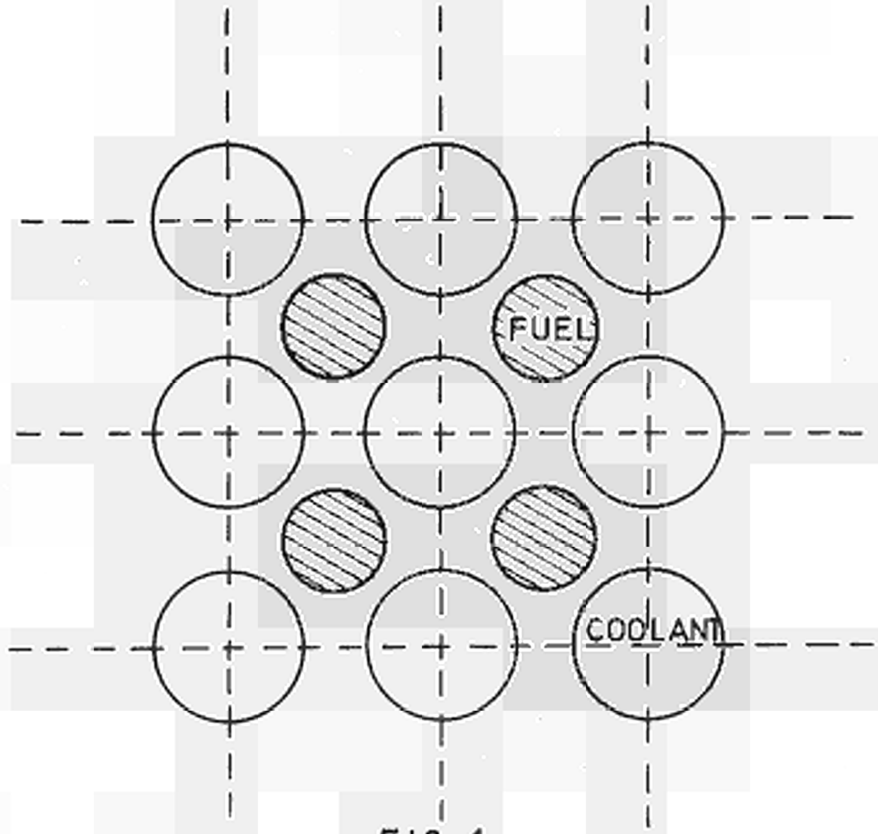

FIG. 1

Idealized cross-section of a matrix fuel element

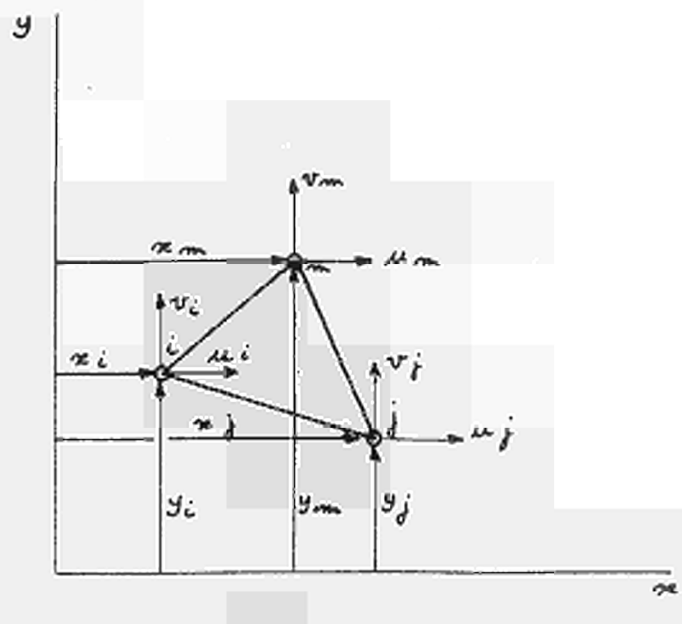

FIG. 2 Triangular finite element

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Alfred Nobel

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