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SORA DYNAMICS AND CONTROL SYSTEM STUDIES USING MEAN-VALUE NEUTRON KINETICS EQUATIONS

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ABSTRACT

The study described in this report deals with dynamics and control of the pulsed fast reactor SORA. It is based on a set of equations for mean-value neutron kinetics. A simulation of the complete set of equations, including thermal reactivity feed-back, is performed. As results, the reactor responses to perturbations of reactivity, inlet coolant temperature and coolant velocity are shown. Control rod malfunctions are investigated; a start-up procedure is proposed. A fast control system is synthesized.

KEYWORDS

TABLE OF CONTENTS

SORA DYNAMICS AND CONTROL SYSTEM STUDIES USING MEAN-VALUE NEUTRON KINETICS EQUATIONS*)

I. INTRODUCTION

The SORA reactor is a fast reactor periodically pulsed by reactivity variations designed as a neutron source for research in neutron physics. The reactor design and experimental use have been described in several meetings, notably at Karlsruhe in 1965 (Ref. 6), at Santa Fe in 1967 (Ref. 7) and at Albuquerque in 1969 (Ref. 4).

The study described in the report was performed in the year 1968 at the request of the SORA Project to answer questions which the project designers had about the reactor dynamics and control.

Mean-value neutron kinetics equations are used for this study. A set of equations for the reactor mean temperatures is used for the calculation of thermal reactivity feedback.

A simulation of the complete set of equations is performed using the digital computer IBM 360/65. As results, the reactor responses to perturbations of reactivity, inlet coolant temperature, and coolant velocity are shown. Also a start-up procedure is proposed. Control rod malfunctions occurring in the uncontrolled reactor are also investigated.

In a classical manner, the set of equations is linearized for small deviations from pulsed steady state to obtain a reactor transfer function which is used for the synthesis of the fast control system. This fast control system is then introduced as a control loop around the reactor. The whole system is then simulated and checked against typical perturbations of reactivity, inlet coolant temperature and coolant velocity.

I wish to acknowledge the help and the interesting suggestions of Mr. Larrimore throughout this study.

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2. Basic Equations for Description of the Reactor

A block-diagram for dynamics of the uncontrolled reactor is
shown in figure 2. See also the list of symbols at the end
of this report. Three parts may be seen in this block diagram:

- core kinetics and conversion to power
- thermal description
- reactivity feedback.

Three inputs are considered for this system:

- external reactivity (ϵ_d)
- inlet coolant temperature ($T_{c,in}$)
- coolant velocity (V_{cool}).

The three most important outputs (from dynamics point of view)
are shown:

- power ($\bar{P}$)
- fuel temperature ($\bar{T}_f$)
- structure temperature ($\bar{T}_g$).

Also available from the set of equations that we will use are
a number of other parameters such as fission rate, outlet coolant
temperature, internal reactivities from fuel and structure. For reasons
of clarity all of these   parameters were not shown in Fig. 2 .

We will analyze now this block-diagram and present the relative
sets of equations.
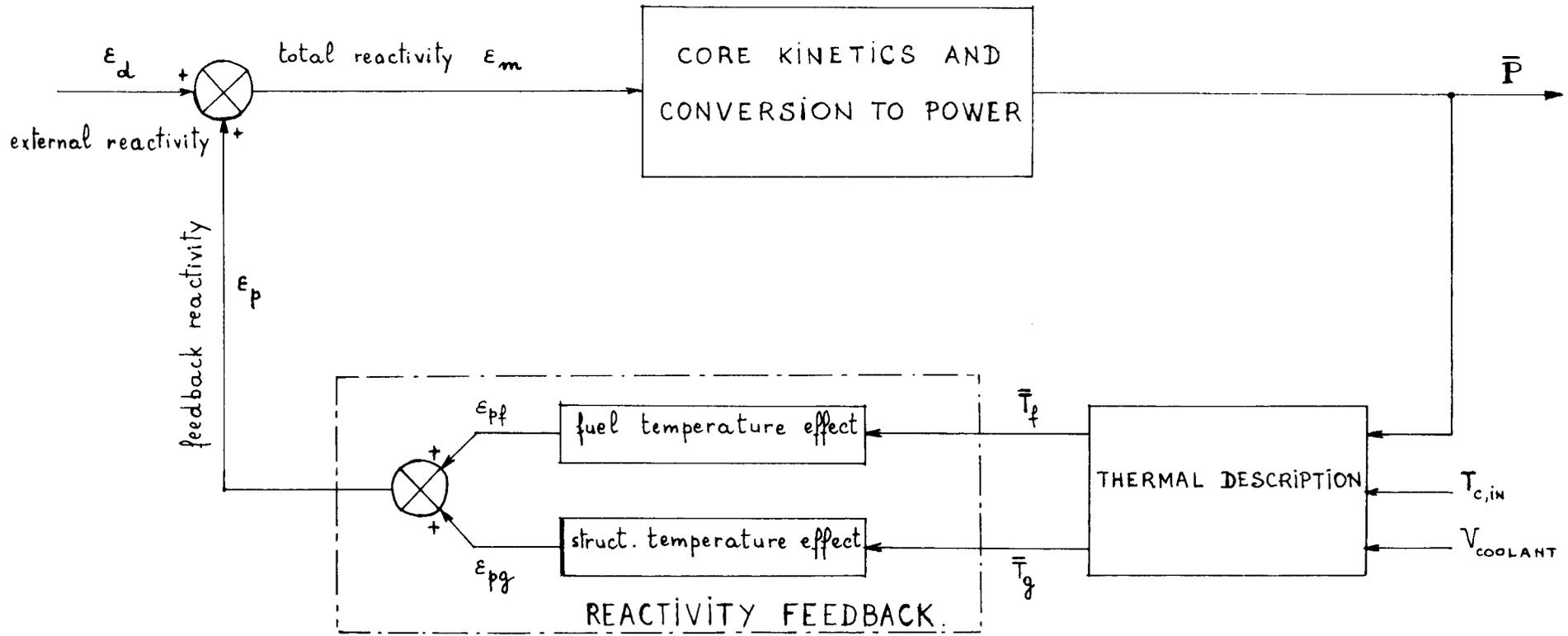
Fig.2. General block-diagram for dynamics of the uncontrolled reactor.

2.1 Core kinetics - Conversion to power

The kinetic theory of a periodically pulsed reactor was established by Bondarenko and Staviskii (Ref.1). Later on, extensive kinetic studies were performed at Ispra, namely by Blässer, Misenta and Raievski (Ref.2). The necessary elements for a clear understanding of the present report will be found in the survey paper of Larrimore (Ref. 3).

Since our main purpose was to get a good approximation for the reactor transients and a reference design for control systems, we decided to use "mean power kinetics" in a point reactor model for such a work. By "mean power kinetics" we mean the time behaviour of the average values over a period of the power and precursor concentrations for times long compared to the pulse period.

In analogy with the multiplication factor for a stationary (non pulsed) reactor, a multiplication factor K is here defined, based (Ref. 2) on production and destruction of delayed neutron precursors:

$$K = \frac{\text{Precursor Production During Period}}{\text{Precursor Decay During Period}} \qquad (2\text{-}1\text{-}1)$$

with K = 1 for pulsed steady-state operation. This multiplication factor, also called "pulsed multiplication coefficient" may be expressed as:

$$K \doteq \frac{M}{T} + \frac{\beta}{\epsilon_o} \qquad (2\text{-}1\text{-}2)$$

where M and ϵ_o are functions of the reactivity level in the reactor. In Figure 2-1 a plot (Ref. 4) of K versus peak prompt reactivity ϵ_m, calculated for the SORA reactor, is shown. The represented curve was fitted as:

$$K = 0,229 + 8,80 \; \epsilon_m + 0,0176 \; e^{4140 \; \epsilon_m} \qquad (2\text{-}1\text{-}3)$$

Now, since the precursor production during a period is $\beta v \bar{w} T$ and the precursor decay during the same time interval is $\sum_i \lambda_i \bar{c}_i T$ it results directly from (2-1-1) that the mean fission rate is:

$$\bar{w} = \frac{K}{\beta v} \left(\sum_i \lambda_i \bar{c}_i + S_o \right) \qquad (2\text{-}1\text{-}4)$$

while the mean delayed neutron precursor concentrations are described by:

MULTIPLICATION FACTOR K
VERSUS
PEAK PROMPT REACTIVITY, $\mathcal{E}_m$

FIG. 2.1

- 9 -
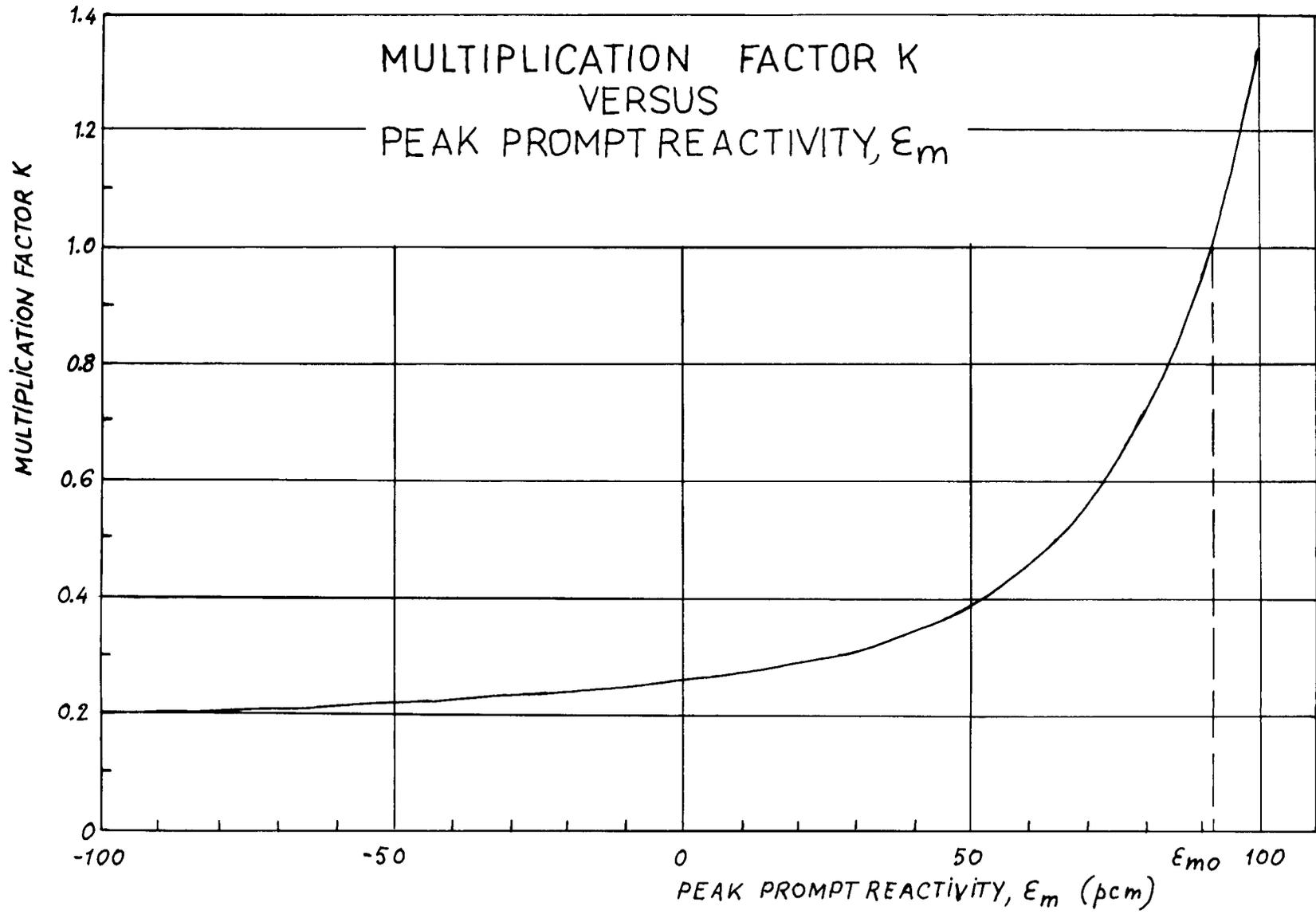
$$\frac{d\overline{C}_i}{dt} = \beta_i \, v \, \overline{w} - \lambda_i \overline{C}_i. \qquad (2\text{-}1\text{-}5)$$

Equations (2-1-2) to (2-1-5) describe completely the reactor kinetics. The corresponding reactor power is straight forwardly obtainable from the mean fission rate $\overline{w}$ as:

$$\overline{P} = \frac{\overline{w}}{C_f} \qquad (2\text{-}1\text{-}6).$$

We will write finally the complete and practical set of equations for kinetics:

$$K = 0,229 + 8,80 \, \epsilon_m + 0,0176 \, e^{4140 \, \epsilon_m}$$

$$\overline{w} = \frac{K}{\beta v} \left(\sum_i \lambda_i \overline{C}_i + S_o \right)$$

$$\frac{d\overline{C}_i}{dt} = \beta_i \, v \, \overline{w} - \lambda_i \overline{C}_i \qquad (2\text{-}1)$$

$$\overline{P} = \frac{\overline{w}}{C_f}$$

2.2 Reactor thermal description

A simple mathematical model (Ref. 5) for the reactor thermal description has been developped in collaboration with the SORA Project, which expresses the mean fuel and structure temperatures in terms of the power, the inlet coolant temperature and the coolant velocity. These mean temperatures are used for the calculation of thermal reactivity feedback.

The SORA reactor core consists of a bundle of approximately 116 fuel rods (Ref. 6 and 7). Assuming a uniform radial power generation and neglecting the heat flow in the vertical direction, a temperature distribution such as shown in Fig. 2-2 may be defined for any element, which enables us to calculate mean temperatures for fuel, bond-clad, and coolant. The heat exchange coefficients, calculated from these mean temperatures, are assumed to hold in in transients around the previous steady state.
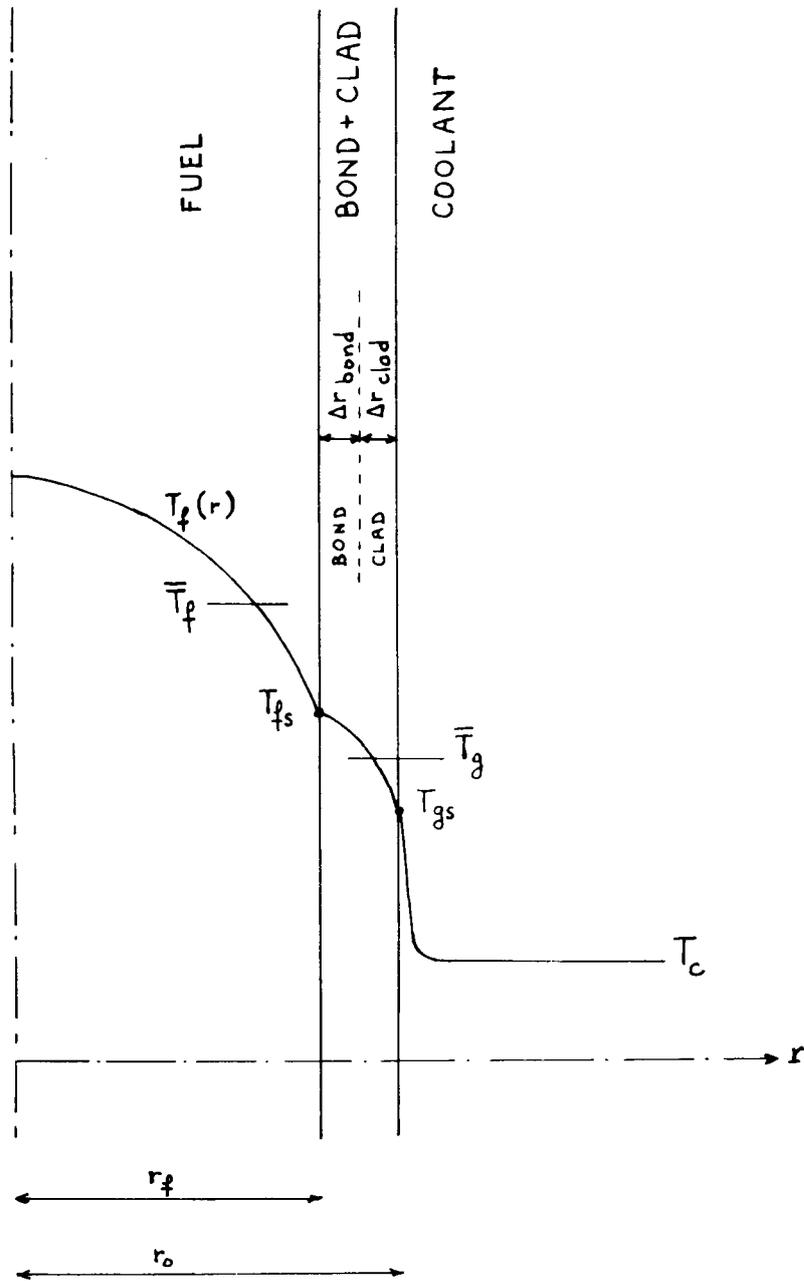
Figure 2.2. Temperature distribution.

2-2-1. Steady-state temperature distributions. Mean temperatures (see figure 2-2).

2-2-1-1. Temperature distribution in fuel. Mean fuel temperature $\overline{T}_f$.

Assuming a uniform volume heat source q''', and k_f independent of the temperature, the temperature distribution in a fuel element is a solution of:

$$q''' + k_f \nabla^2 T_f = 0.$$

Neglecting heat flow in the axial direction, this may be written as:

$$\frac{1}{r} \cdot \frac{d}{dr} \left(r \frac{dT_f}{dr} \right) = - \frac{q'''}{k_f}.$$

Integrating once:

$$\int_0^r d \left(\frac{r dT_f}{dr} \right) = - \frac{q'''}{k_f} \int_0^r r \, dr \qquad\qquad (\frac{dT_f}{dr} = 0 \text{ at } r = 0)$$

$$\frac{dT_f}{dr} = - \frac{q'''}{2k_f} r$$

Integrating again:

$$\int_{T_f(r)}^{T_{fs}} dT_f = - \frac{q'''}{2k_f} \int_r^{r_f} r \, dr$$

$$T_f(r) - T_{fs} = - \frac{q''' \, r_f^2}{4 \, k_f} \left[1 - \left(\frac{r}{r_f} \right)^2 \right].$$

The mean fuel temperature is defined as:

$$\overline{T}_f - T_{fs} = \frac{\displaystyle\int_0^{r_f} \left[T_f(r) - T_{fs} \right] 2\pi \, r dr}{\displaystyle r_f \int_0^r 2\pi \, r dr}$$

$$\overline{T}_f - T_{fs} = \frac{q''}{8k_f} \ r_f^2 \ .$$

Introducing finally the average power generation in fuel per unit of length $q = \pi r_f^2 \ q''$, we obtain:

$$\overline{T}_f = \frac{q}{8 \pi k_f} + T_{fs} \ .$$

2-2-1-2. Temperature distribution in bond-clad. Mean structure temperature $\overline{T}_g$.

For the purpose of this study the bond and clad are combined into a single region. Due to the small thickness, one may write directly (see figure 2-2):

$$T_{fs} - T_{gs} = \frac{q''}{k_g} \ (r_o - r_f)$$

where q'' is the average power per unit area. Introducing the average power generation per unit of length $q = 2\pi r_f q''$, we get:

$$T_{fs} - T_{gs} = \frac{q}{2\pi r_f k_g} \ (r_o - r_f)$$

The mean bond-clad or structure temperature is then defined as:

$$\overline{T}_g = T_{gs} + \frac{1}{2} \ (T_{fs} - T_{gs}) = T_{gs} + \frac{q}{4\pi k_g} \ (\frac{r_o - r_f}{r_f})$$

or:

$$\overline{T}_g = T_{fs} - \frac{1}{2} \ (T_{fs} - T_{gs}) = T_{fs} - \frac{q}{4\pi k_g} \ (\frac{r_o - r_f}{r_f}).$$

2-2-1-3. Temperature distribution in the coolant. Mean coolant temperature(or coolant temperature at half height of core) T_c.

We assume the coolant temperature increases linearly from the inlet to the outlet so that the mean coolant temperature is the coolant temperature at half height of core. Following this assumption:

$$T_{gs} - T_c = \frac{q''}{h_f}$$

where h_f is the heat transfer coefficient between clad and coolant and q'' the average power per unit area. Introducing once again the average power generation per unit of length we obtain:

$$T_{gs} - T_c = \frac{q}{2\pi\, r_f h_f} \quad .$$

2-2-2. Steady state heat transfer coefficients

From the previous equations we derive heat exchange coefficients related to the characteristic temperatures $\overline{T}_f$, $\overline{T}_g$, and T_c.

2-2-2-1. Fuel to clad heat transfer coefficient h_{Fs}

We have found:

$$\overline{T}_f - T_{fs} = \frac{q}{8\pi k_f}$$

$$T_{fs} - \overline{T}_g = \frac{q}{4\pi k_g} \left(\frac{r_o - r_f}{r_f} \right).$$

Defining the heat transfer coefficient h_{Fs} as:

$$h_{Fs} = \frac{q}{\overline{T}_f - \overline{T}_g}$$

we obtain:

$$h_{Fs} = \frac{1}{\frac{1}{8\pi k_f} + \frac{1}{4\pi k_g}\left(\frac{r_o - r_f}{r_f}\right)}$$

2-2-2-2. Clad to coolant heat transfer coefficient

$$\overline{T}_g - T_{gs} = \frac{q}{4\pi kg}\left(\frac{r_o - r_f}{r_f}\right)$$

$$T_{gs} - T_c = \frac{q}{2\pi\, r_f h_f} \quad .$$

Defining the heat transfer coefficient h_{sc} as:

$$h_{sc} = \frac{q}{\overline{T}_g - T_c}$$

it is found:

$$h_{sc} = \frac{1}{\frac{1}{4\pi k_g} \left(\frac{r_o - r_f}{r_f}\right) + \frac{1}{2\pi r_f h_f}} \cdot$$

2-2-3. Heat balance equations

Fuel heat balance equation:

$$C_f \frac{d\overline{T}_f}{dt} = q(t) - h_{Fs}(\overline{T}_f - \overline{T}_g) \cdot \qquad (2-2-3-1)$$

Bond-clad or structure heat balance equation:

$$C_g \frac{d\overline{T}_g}{dt} = h_{Fs}(\overline{T}_f - \overline{T}_g) - h_{sc}(\overline{T}_g - T_c) \qquad (2-2-3-2)$$

Coolant heat balance equation:

$$LC_c \frac{dT_c}{dt} = Lh_{sc}(\overline{T}_g - T_c) - VC_c(T_{c,out} - T_{c,in}) \qquad (2-2-3-3)$$

$$T_c = \frac{1}{2}(T_{c,in} + T_{c,out}) \cdot \qquad (2-2-3-4)$$

Assuming no time lag exists between $\overline{T}_g$ and T_c due to the small heat capacity of the coolant, equation (2-2-3-3) may be turned into:

$$Lh_{sc}(\overline{T}_g - T_c) = VC_c(T_{c,out} - T_{c,in}) \cdot \qquad (2-2-3-5)$$

Equations (2-2-3-4) and (2-2-3-5) are then used to express T_c and $T_{c,out}$ in terms of $\overline{T}_g$ and $T_{c,in}$:

$$T_c = \frac{1}{1 + \frac{2VC_c}{Lh_{sc}}} \cdot \overline{T}_g + \frac{1}{1 + \frac{Lh_{sc}}{2VC_c}} T_{c,in} \qquad (2-2-3-6)$$

$$T_{c,out} = \frac{2}{1 + \dfrac{2VC_c}{Lh_{sc}}} \; \overline{T}_g - \frac{1 - \dfrac{2VC_c}{Lh_{sc}}}{1 + \dfrac{2VC_c}{Lh_{sc}}} \; T_{c,in} \; . \qquad (2-2-3-7)$$

Eliminating finally T_c between (2-2-3-6) and (2-2-3-2), we get for the structure heat balance equation:

$$C_g \frac{d\overline{T}_g}{dt} = h_{Fs} \, (\overline{T}_f - \overline{T}_g) - \frac{h_{sc}}{1 + \dfrac{Lh_{sc}}{2VC_c}} \, (\overline{T}_g - T_{c,in}) \qquad (2-2-3-8)$$

2-2-4. Summary of equations for the reactor thermal description

Results (2-2-3-1), (2-2-3-8), (2-2-3-7) and (2-2-3-4) describe the reactor from a thermal point of view. For clarity, we rewrite here these results:

$$C_f \frac{d\overline{T}_f}{dt} = \frac{p}{nL} \cdot \overline{P}(t) - h_{Fs} \, (\overline{T}_f - \overline{T}_g)$$

$$C_g \frac{d\overline{T}_g}{dt} = h_{Fs}(\overline{T}_f - \overline{T}_g) - \frac{h_{sc}}{1 + \dfrac{Lh_{sc}}{2VC_c}} \, (T_g - T_{c,in})$$

$$T_{c,out} = T_{c,in} + \frac{2}{1 + \dfrac{2VC_c}{Lh_{sc}}} \, (\overline{T}_g - T_{c,\,in})$$

$$T_c = \frac{1}{2} \, (T_{c,in} + T_{c,out})$$

$$(2-2)$$

2-3. Reactivity feedback

A fundamental characteristic of the periodically pulsed reactor is that the reactivity is introduced as a "pulse", that is to say introduced rapidly and removed in the same manner. It has been shown (Ref. 8) that the reactivity feedback during the pulse has a negligible effect on the pulse characteristics due to the short duration of the reactivity pulse. However, the reactivity feedback is important when more than one pulse is considered.

The temperature coefficients of reactivity in SORA have
been calculated to be negative, which simplifies considerably
all problems of control. The temperature effect is split
into two parts: the fuel temperature coefficient ϵ_{pf} (in-
cluding fuel axial expansion and Doppler effect) and the
structure temperature coefficient ϵ_{pg} (including radial struc-
tural expansion) so that:

$$\epsilon_{pf} = \alpha_f(\overline{T}_f - T^*_{REF})$$

$$\epsilon_{pg} = \alpha_g(\overline{T}_g - T^*_{REF})$$

$$\epsilon_p = \epsilon_{pf} + \epsilon_{pg}$$

(2-3)

with α_f and α_g negative.

2-4 Summary of equations

Equations for kinetics, thermal description, and feedback
reactivity are rewritten below. (See Fig. 2-4 for the relative
block diagram.)

ϵ_d = input reactivity signal

ϵ_p = total feedback reactivity signal

ϵ_m = total reactivity = $\epsilon_d + \epsilon_p$.

$$K = 0.229 + 8.80\ \epsilon_m + 0.0176\ e^{4140\ \epsilon_m}$$

$$\overline{w} = \frac{K}{\beta v} \left(\sum_i \lambda_i \overline{C}_i + S_o \right)$$

$$\frac{d\overline{C}_i}{dt} = \beta v\ \overline{w} - \lambda_i \overline{C}_i$$

$$\overline{P} = \frac{\overline{w}}{C_F}$$

$$C_f \frac{d\overline{T}_f}{dt} = \rho \overline{P}(t) - h_{Fs}(\overline{T}_f - \overline{T}_g)$$

$$C_g \frac{d\overline{T}_g}{dt} = h_{Fs}(\overline{T}_f - \overline{T}_g) - \frac{h_{sc}}{1 + \frac{Lh_{sc}}{2VC_c}}(\overline{T}_g - T_{c,in})$$

$$T_{c,out} = T_{c,in} + \frac{\frac{2}{2VC_c}}{1 + \frac{2VC_c}{Lh_{sc}}}(\overline{T}_g - T_{c,in})$$

$$\tag{2-4}$$

$$T_c = \frac{1}{2}(T_{c,in} + T_{c,out})$$

$$\epsilon_{pf} = \alpha_f(\overline{T}_f - T^*_{REF})$$

$$\epsilon_{pg} = \alpha_g(\overline{T}_f - T^*_{REF})$$

$$\epsilon_p = \epsilon_{pf} + \epsilon_{pg}.$$
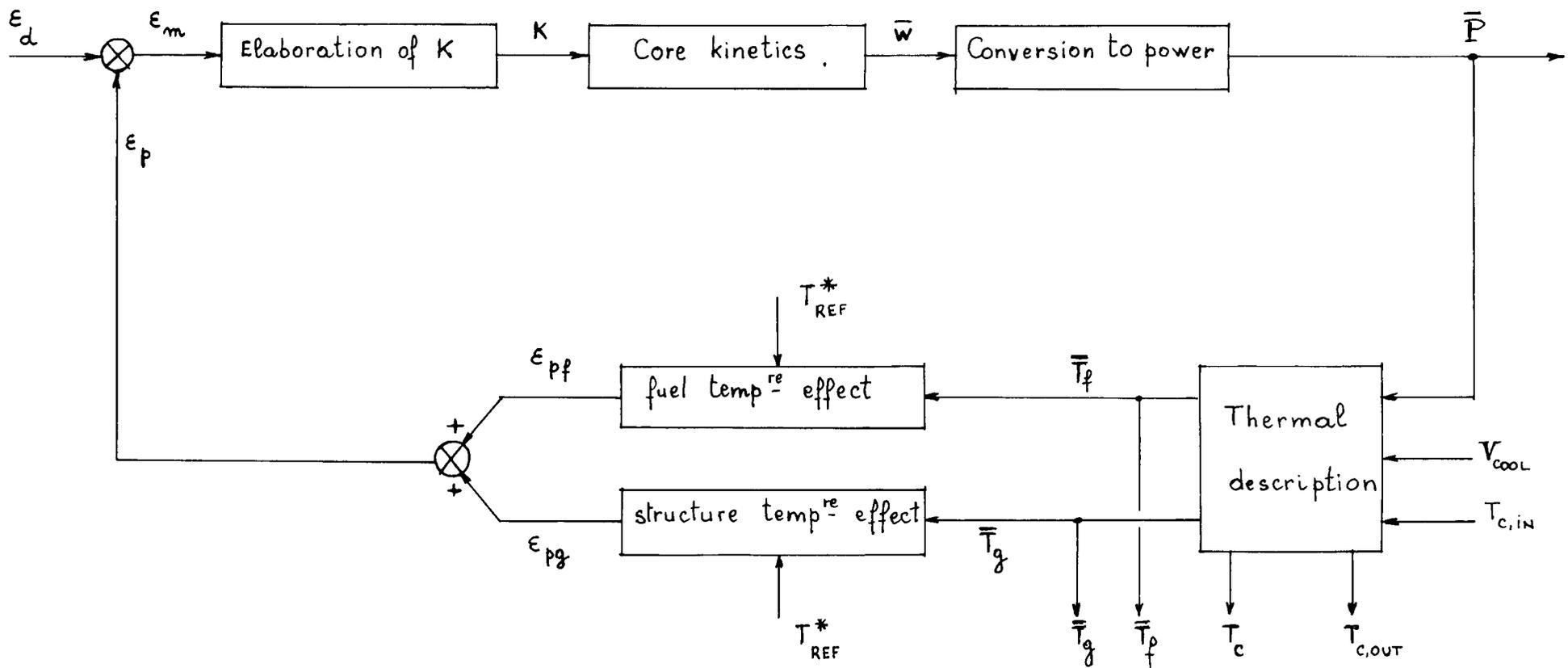
FIG 2.4. Basic block-diagram for SORA dynamics.

— see the summary of equations (2.4) —

3. Simulation of the Uncontrolled Reactor

The simulation of the uncontrolled reactor was based on the set of equations (2-4). The use of a digital computer rather than an analog one was chosen mainly for the following reasons: noise and thermal drift eliminated, easy calculation of $K(\epsilon_m)$, amplitude scale unlimited. Therefore, the simulation consists of an iterative integration of the set of equations (2-4). We used the program SAHYB-2, developped at CETIS: this program was specially developped to meet the requirements of simulation on digital computers (Ref. 9).

It must be noted, however, that an iterative integration by means of a digital computer implies a cumulative error (as time goes on) on the integrated values. But this error may be kept as low as desired by introducing the requested specifications into the program.

We first simulated the effects of:

- a change in reactivity,
- a change in coolant velocity, and
- a change in inlet coolant temperature

on the uncontrolled reactor. These changes were introduced as step functions with the reactor assumed to be stabilized at power 600 kW. The relative initial conditions were calculated from the set of equations (2-4) for the steady-state condition* 600 kW.

In Fig. 3-1 the response to a step of reactivity of 5 pcm is shown: the power jumps immediately by about 100 kW from the initial level 600 kW, which is characteristic of the kinetics of such a reactor (see equation 2-1-4); the power then stabilizes at about 30 kW above the initial level with a time constant of approximately 1 second.

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* See the appendix: Derivation of numerical values for a steady-state condition at power.
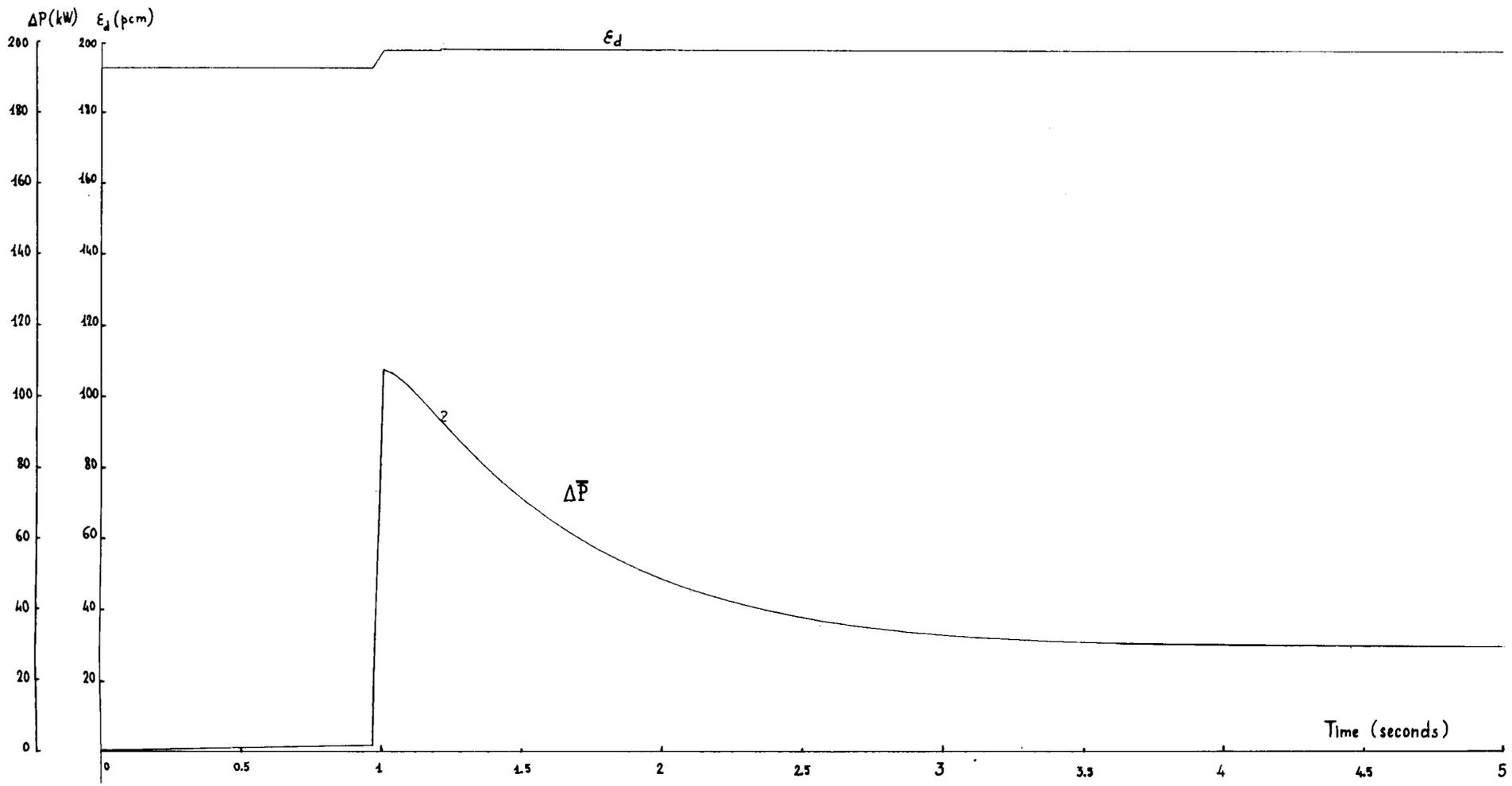
RESPONSE TO A CHANGE ON REACTIVITY

(UNCONTROLLED REACTOR)

FIG. 3-1
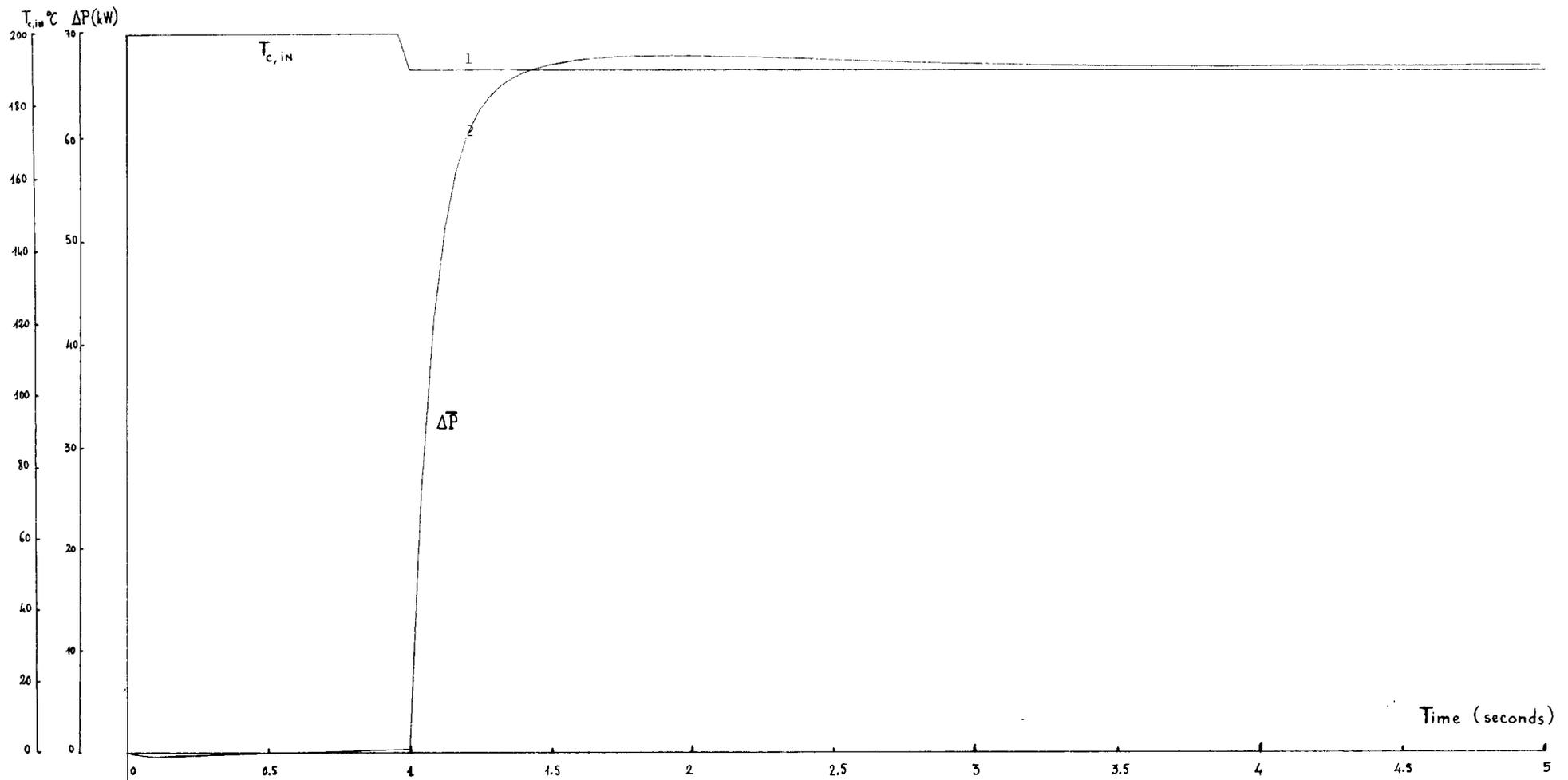
RESPONSE TO A CHANGE ON INLET COOLANT TEMPERATURE

(UNCONTROLLED REACTOR)

FIG. 3-2

In Fig. 3-2, is plotted the power response to a negative step of $10^{\circ}C$ on the inlet coolant temperature. The power stabilizes at about 70 kW from the initial level.

Finally, in Fig. 3-3 the power response to a positive step of 0,25 m/s in the coolant velocity is plotted. This results in a power increase of about 10 kW.

In the two last cases, the power level increases due to the fact that the considered perturbations reduce the reactor temperature.

4. Startup Procedure

Simulating the set of equations (2-4) permits to define:

- a procedure for approach to criticality (K = 1)
- a procedure for bringing the reactor up to power 600 kW.

These procedures are intended as typical insertion rate curves for the uncontrolled reactor (no control, no scram system) and may be considered as the optimum insertion rates meeting the following specifications:

- start from subcritical steady state[*]
- approach to criticality as quick as possible
- period T_R always $\geqslant$ 30 seconds

$$(\; T_R = \frac{\overline{P}}{\frac{d\overline{P}}{dt}} \;)$$

The insertion rates as shown in figure 4 have been found by successively trying numerical values as slopes for ϵ_d. Although these results were derived empirically, they give a good first approximation to an optimal startup procedure based on the previous specifications.

Some conclusions coming from this first approach are:

a) The approach to criticality must be performed <u>very</u> slowly: the reactivity insertion must be held in the order of 0,1 pcm/s if the period has to be kept > 30 s. With this condition,

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[*] See appendix: Initial Conditions for the Subcritical Steady-State.
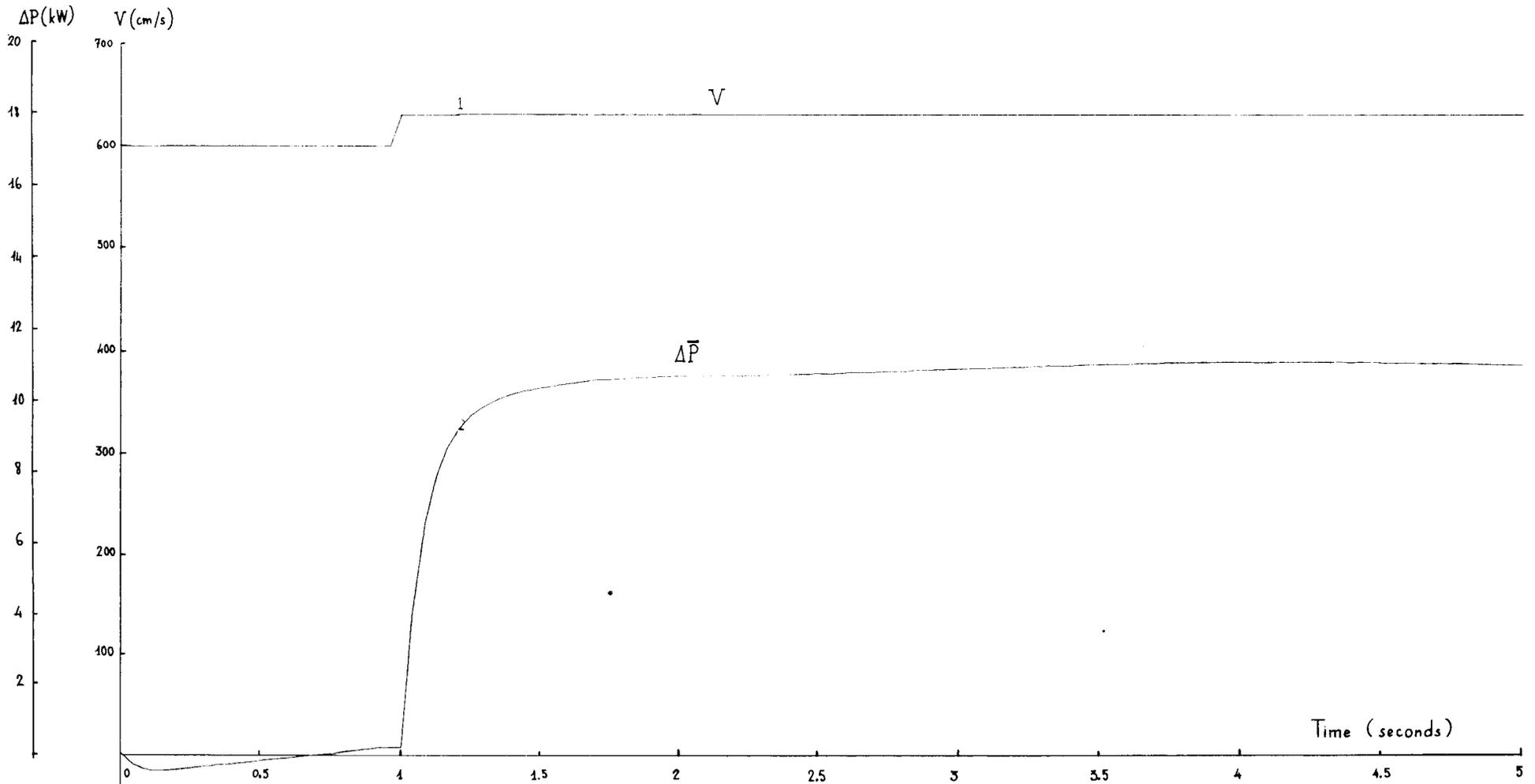
RESPONSE TO A CHANGE ON COOLANT VELOCITY

(UNCONTROLLED REACTOR)

FIG. 3-3

the power level at criticality is reasonably high, that is in the order of 150 mW.

b) The time for bringing the reactor up to power (600 kW) lies in the order of 1000 seconds. This time seems to be the minimum required for starting up to power with the imposed safety condition on the period.

5. Linearized Reactor Description

A linearized reactor description is derived in this section which leads to a transfer function for the whole reactor - including kinetics, thermal description and feedback reactivity. In a successive step comparisons are made, on the basis of the reactor response to perturbations, between the set of non-linear equations (2-4) and the derived transfer functions.

5-1 Derivation of transfer functions

We start with the set of equations (2-4) and consider small deviations from the pulsed steady state.

5-1-1. Derivation of a transfer function for core-kinetics

Assuming the reactor is at steady state at a certain power level, we will denote by ϵ_{mo} the ϵ_m value such that $K(\epsilon_{mo}) = 1$ ($\epsilon_{mo} = 91,1$ pcm) and consider small reactivity derivations $\Delta\epsilon_m$ from this value. Then:

$$K(\epsilon_{mo} + \Delta\epsilon_m) - K(\epsilon_{mo}) = \Delta\epsilon_m \cdot \left(\frac{dK}{d\epsilon_m}\right)_{\epsilon_{mo}}$$

which gives the following transfer function:

$$\Delta K(s) = \left(\frac{dK}{d\epsilon_m}\right)_{\epsilon_{mo}} \cdot \Delta\epsilon_m(s)$$

$$\Delta K(s) = 3,1745 \cdot 10^{-2} \cdot \Delta\epsilon_m(s)$$

with $\Delta\epsilon_m$ expressed in pcm.

In the same manner, considering small deviations from the pulsed steady state, we write the kinetic equations as:

steady state

$K_o = 1$, $\dfrac{d\overline{C}_{io}}{dt} = 0$

$$\overline{w}_o = \dfrac{K_o}{\beta v}\left(\sum_i \lambda_i \overline{C}_i + S_o \right)$$

$$\dfrac{d\overline{C}_{io}}{dt} = \beta v\, \overline{w}_o - \lambda_i \overline{C}_{io}$$

small deviation
from
steady state

$$\overline{w}_o + \Delta\overline{w} = \dfrac{K_o + \Delta K}{\beta v}\left[\sum_i \lambda_i (\overline{C}_{io} + \Delta\overline{C}_i) + S_o \right]$$

$$\dfrac{d}{dt}(\overline{C}_{io} + \Delta\overline{C}_i) = \beta v\, (\overline{w}_o + \Delta\overline{w}) - \lambda_i (\overline{C}_{io} + \Delta\overline{C}_i).$$

The second-order term $\dfrac{\Delta K}{\beta v} \sum_i \lambda_i \Delta\overline{C}_i$ is neglected. Since at steady

state $K_o = 1$ and $\dfrac{d\overline{C}_{io}}{dt} = 0$, one obtains:

$$\Delta\overline{w} = \dfrac{1}{\beta v} \sum_i \lambda_i \Delta\overline{C}_i + \overline{w}_o \Delta K$$

$$\dfrac{d}{dt}(\Delta\overline{C}_i) = \beta_i v\, \Delta\overline{w} - \lambda_i \Delta\overline{C}_i$$

This set of equations is Laplace-transformed on both sides; eliminating $\Delta\overline{C}_i(s)$ the following transfer function is obtained:

$$\dfrac{\Delta\overline{w}(s)}{\Delta K(s)} = \dfrac{\overline{w}_o}{1 - \sum_i \dfrac{\lambda_i}{s + \lambda_i}}$$

In turns, this result is approximated by a one-precursor group model as:

$$\dfrac{\Delta\overline{w}(s)}{\Delta K(s)} = \overline{w}_o \dfrac{s + \lambda}{s}. \qquad \text{(see Ref. 10)}$$

Introducing finally power instead of fission rate, we obtain the following transfer function relating the power variations $\Delta\overline{P}$ to the reactivity variations $\Delta\epsilon_m$:

$$F(s) = \dfrac{\Delta\overline{P}}{\Delta\epsilon_m} = \overline{P}_o \left(\dfrac{dK}{d\epsilon_m} \right)_{\epsilon_{mo}} \cdot \dfrac{s + \lambda}{s}$$

The numerical value chosen for λ is 0.08 sec^{-1}. Then, for the 600 kW power steady state, this gives:

$$\frac{\Delta \bar{P}}{\Delta \epsilon_m} = 1,524 \cdot \frac{12,5\ s + 1}{s}$$

$\Delta \bar{P}$ expressed in kW

$\Delta \epsilon_m$ expressed in pcm

(5-1-1)

5-1-2. Derivation of a transfer function for thermal description

Assuming a constant velocity for the coolant, the set of equations (2-2) consists of linear differential equations and may be directly Laplace-transformed. Since the mean fuel temperature and the mean structure temperature are of main interest, we will establish the results for these two temperatures only,

$$\Delta \bar{T}_f = \frac{\left(sC_g + h_{Fs} + \dfrac{h_{sc}}{1 + \dfrac{Lh_{sc}}{2VC_c}}\right)\ \rho \Delta \bar{P} + \dfrac{h_{Fs}\ h_{sc}}{1 + \dfrac{Lh_{sc}}{2VC_c}}\ \Delta T_{c,\ in}}{s^2 C_f C_g + s\left[h_{Fs}C_g + \left(h_{Fs} + \dfrac{h_{sc}}{1 + \dfrac{Lh_{sc}}{2VC_c}}\right)C_f\right] + \dfrac{h_{Fs}\ h_{sc}}{1 + \dfrac{Lh_{sc}}{2VC_c}}}$$

$$\Delta \bar{T}_g = \frac{\rho h_{Fs}\ \Delta \bar{P} + \dfrac{h_{sc}}{1 + \dfrac{Lh_{sc}}{2VC_c}}\ (sC_f + h_{Fs})\ \Delta T_{c,\ in}}{s^2 C_f C_g + s\left[h_{Fs}C_g + \left(h_{Fs} + \dfrac{h_{sc}}{1 + \dfrac{Lh_{sc}}{2VC_c}}\right)C_f\right] + \dfrac{h_{Fs}h_{sc}}{1 + \dfrac{Lh_{sc}}{2VC_c}}}$$

These results may be synthesized as follows:

$$\Delta \overline{T}_f(s) = \frac{A(s)}{B(s)} \cdot \Delta \overline{P}(s) + \frac{C(s)}{B(s)} \cdot \Delta T_{c,in}(s)$$

$$(5-1-2)$$

$$\Delta \overline{T}_g(s) = \frac{D(s)}{B(s)} \cdot \Delta \overline{P}(s) + \frac{E(s)}{B(s)} \cdot \Delta T_{c,in}(s)$$

where: (expressing power in kW and temperature in $^{\circ}$C)

$$A(s) = (sC_g + h_{Fs} + \frac{h_{sc}}{1 + \frac{Lh_{sc}}{2VC_c}})$$

$$A(s) = 0,22\ s + 2,98$$

$$B(s) = s^2 C_f C_g + s \left[h_{Fs}C_g + (h_{Fs} + \frac{h_{sc}}{1 + \frac{Lh_{sc}}{2VC_c}})\ C_f \right] + \frac{h_{Fs}h_{sc}}{1 + \frac{Lh_{sc}}{2VC_c}}$$

$$B(s) = 3,117\ s^2 + 45,176\ s + 16,174$$

$$C(s) = \frac{h_{Fs}h_{sc}}{1 + \frac{Lh_{sc}}{2VC_c}}$$

$$C(s) = 16,174$$

$$D(s) = \rho h_{Fs}$$

$$D(s) = 18,535$$

$$E(s) = \frac{h_{sc}}{1 + \frac{Lh_{sc}}{2VC_c}}\ (sC_f + h_{Fs})$$

$$E(s) = 15,86\ s + 16,174.$$

5-1-3. Derivation of a transfer function for feedback reactivity

Since α_f and α_g are constant coefficients, we obtain immediately from the set of equations (2-3):

(5-1-3)

$$\Delta\epsilon_{pf}(s) = \alpha_f \, \Delta \overline{T}_f(s)$$

$$\Delta\epsilon_{pf}(s) = -0,42 \, \Delta\overline{T}_f(s)$$

$$\Delta\epsilon_{pg}(s) = \alpha_g \, \Delta \overline{T}_g(s)$$

$$\Delta\epsilon_{pg}(s) = -0,80 \, \Delta \overline{T}_g(s)$$

$$\Delta\epsilon_p = \Delta\epsilon_{pf} + \Delta\epsilon_{pg}$$

with $\Delta\epsilon_p$, $\Delta\epsilon_{pf}$, $\Delta\epsilon_{pg}$ expressed in pcm

$\Delta\overline{T}_f$ and $\Delta\overline{T}_g$ expressed in °C.

From the previous results, (5-1-1), (5-1-2), (5-1-3) one may build the block-diagram represented in Figure 5-1.

5-2. Comparison between the non-linearized and the linearized reactor description

The block diagram shown in Fig. 5-1 and the relative numerical values may be used to get the reactor response to reactivity or inlet coolant temperature perturbations. However, one must remember that the linearized reactor description holds only for small deviations from a particular steady-state; moreover, all of the numerical values from (5-1-1) to (5-1-3) hold only for the 600 kW steady-state. Therefore, it is of importance to check the validity of this linearization and to know how representative the linearized reactor description is with respect to the set of non-linear equations (2-4).

We considered some increasing steps of reactivity (1,2,3,4,5 pcm) and plotted the obtained power responses:

a - using the transfer functions.

b - simulating the complete set of non-linear equations (2-4) by means of the SAHYB programme.
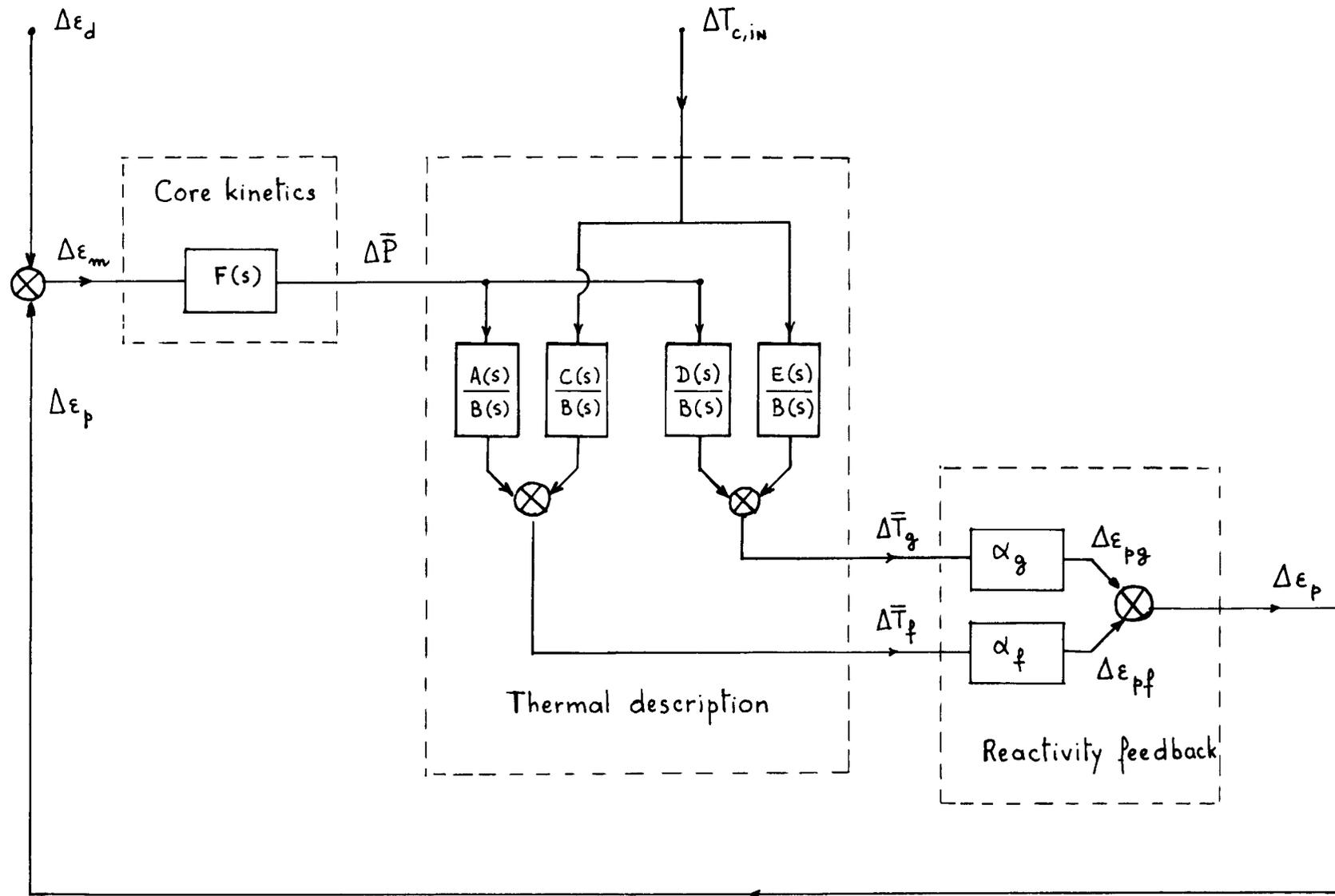
Fig. 5-1.- Linearized reactor description.

(holds for small deviations from pulsed steady state)

In the same manner we considered some increasing steps of inlet coolant temperature (1,2,3,4,5 °C) and plotted the power-response for both the two cases.

5-2-1. Power response to reactivity perturbations

Assuming $\Delta T_{c,in}$ = 0, it is found $\sqrt{\ }$ from the block-diagram represented in figure 5-1, or from the set of results (5-1-1) to $(5-1-3)\_7$:

$$\frac{\Delta \overline{P}(s)}{\Delta \epsilon_d(s)} = 19,05 \ \frac{s^3+14,573 \ s^2+0,348 \ s+0,415}{s^3+15,058 \ s^2+21,946s+1,337}$$

The subroutine POLRT (Ref.11) was used to calculate the roots of these polynominal expressions and gave:

$$\frac{\Delta \overline{P}(s)}{\Delta \epsilon_d(s)} = 19,05 \ \frac{(s+0,08)(s+0,37)(s+14,12)}{(s+0,064)(s+1,56)(s+13,43)}$$

Results are plotted in figure 5-2-1. As is to be expected, the power responses obtained using the transfer function hold good for small perturbations: the more the step height is increased, the less the curves agree. Retaining as good agreement a difference of less than 10 %, it may be seen in figure 5-2-1 that the derived transfer function holds up to about 3 pcm deviation.

It must be pointed out that in these conditions (steps ≤ 3pcm) the transients as well as the final steady states are well contained in the transfer function. If only the final steady states were of interest, the transfer function would hold still higher than 5 pcm - which means that the transfer function might also be used for getting the response to larger slow reactivity deviations.

5-2-2. Power response to inlet-coolant temperature perturbations

Assuming now $\Delta \epsilon_d$ = 0, it is found $\sqrt{\ }$ from the block-diagram of figure (5-1) or from the set of results (5-1-1) to $(5-1-3)\_7$:

Power response ΔP to step reactivity perturbations Δε_d

FIG. 5-2-1

$\dfrac{\Delta P}{(KW)}$

—— using the transfer function $\dfrac{\Delta P}{\Delta \varepsilon_d} = 19.05 \dfrac{(s+0.08)(s+0.37)(s+14.12)}{(s+0.064)(s+1.56)(s+13.43)}$

-·-·- using the complete set of non linear equations for reactor description

ΔP(KW)

extended time scale

5pcm
4pcm
3pcm   Δε_d
2pcm
1pcm

Time (sec.)

5pcm
4pcm
3pcm Δε_d
2pcm
1pcm

Time (sec.)

Power response ΔP to step inlet coolant temperature $\Delta T_{C,IN}$ perturbations (positive steps)

___ using the transfer function $\dfrac{\Delta P}{\Delta T_{C,IN}} = -77,54 \dfrac{(s+0,08)}{(s+0,064)(s+13,43)}$

____ using the complete set of non linear equations for reactor description

$\Delta T_{C,IN} = +1°C$

$\Delta T_{C,IN} = +2°C$

$\Delta T_{C,IN} = +3°C$

$\Delta T_{C,IN} = +4°C$

FIG. 5-2-2

$\Delta T_{C,IN} = +5°C$

$\Delta P (KW)$

$$\frac{\Delta \overline{P}(s)}{\Delta T_{c,in}}(s) = \frac{F(s)\left(\alpha_f C(s) + \alpha_g E(s)\right)}{B(s) - F(s)\left(\alpha_f A(s) + \alpha_g D(s)\right)}$$

$$\frac{\Delta \overline{P}(s)}{\Delta T_{c,in}}(s) = -77,544\ \frac{(s+0.08)(s+1.555)}{(s+0.064)(s+1.56)(s+13,43)}$$

which simplifies as indicated in Figure 5-2-2. Results are shown in this figure.

Considering the set of curves of Figure 5-2-2, it may be seen that the transfer function $\dfrac{\Delta \overline{P}}{\Delta T_{c,in}}$ is quite a good approximation up to 5 $^\circ$C (difference between curves less than about 10%).

Remark 1

So far we considered the power response of the reactor against reactivity or inlet coolant temperature perturbations. However, all of the presented results from (5-1-1) to (5-1-3) may be arranged in such a manner that also the fuel temperature or the structure temperature, or any other parameter of interest might be considered as an output of the system. Such a work has been performed and has shown that the transfer functions always constitute a good approximation for the set of non-linear equations (2-4).

Remark 2

For clarity we have considered responses to reactivity and inlet coolant temperature separately. Due to the linearity of results from (5-1-1) to (5-1-3), mixed transfer functions may be derived where reactivity and inlet coolant temperature appear together; consequently, complex perturbations may be considered where reactivity and inlet coolant temperature are varied simultaneously.

6. Fast Control System

Clearly it seems unreasonable to define a "fast control
system" for the reactor since we deal with mean values for
all of the parameters. Such a fast control system should be
defined using a discrete-time reactor description (Ref.[12])
However, it may be thought that the faster is the control
system based upon mean values, the easier it will work in
the discrete conditions. On the other hand, such a "fast
control system" was intended to keep the reactor at the de-
sired power level against small perturbations of reactivity
(in the order of some pcm) while a "slow control system" was
foreseen against large deviations. In this sense, the set of
equations for mean values might be used for a preliminary de-
finition of the control system. We present here (see also Ref.
13) a possible way to define a fast control system for such a
reactor.

The SORA reactor is equipped with a control rod worth 10 pcm,
vertically mounted as described in Ref. [4] and Ref.[7] . The
so-called fast control rod is a rotating one, directly shafted
to a drive motor.

For our purpose it is of primary interest to use a motor meet-
ing the following requirements:
  - small time constant to achieve a high rapidity response;
  - high rotor inertia with respect to the inertia of the
    driven mechanism: in this manner the load inertia has
    a small effect on the motor time constant;
  - low nominal speed: in this manner the motor may be shafted
    directly on the control rod without any gearbox;
  - power in the range of 100 W.

In order to meet these general requirements, we chose a torque-
motor of the Inland Motor Corporation (Radford, Virginia):

type	T 5135
mechanical time constant	$17 \cdot 10^{-3}$ second
rotor moment of inertia	$3 \cdot 10^{-3}$ lb·ft·sec^2 = $4.05 \cdot 10^{-3}$ kgm^2
power at peak torque	119 Watt
maximum no-load speed	220 rpm.

As may be seen, when such a motor is linked to the load the
overall mechanical time constant changes into:

$$\left(\begin{array}{c}\text{Mechanical time constant}\\ \text{of the motor}\end{array}\right) x \; \frac{\left(\begin{array}{c}\text{Moment of inertia}\\ \text{of the motor}\end{array}\right) + \left(\begin{array}{c}\text{Moment of inertia}\\ \text{of the load}\end{array}\right)}{\left(\begin{array}{c}\text{Moment of inertia}\\ \text{of the motor}\end{array}\right)}$$

$$17 \cdot 10^{-3} \; x \; \frac{4.05 \cdot 10^{-3} + 3.54 \cdot 10^{-4}}{4.05 \cdot 10^{-3}} = 18.5 \cdot 10^{-3} s.$$

Denoting by θ the position of the motor shaft (which is also
the angular position of the fast control rod) and by U the vol-
tage applied to the motor, we get as a transfer function between
these two parameters (see nomenclature and numerical values):

$$\frac{\theta}{U} = \frac{1}{k} \cdot \frac{1}{s(1 + \tau s + \tau \tau' s^2)}$$

or

$$\frac{\theta}{U} = \frac{1.4652 \cdot 10^4}{s(s+260)(s+73.35)} \tag{6-1}$$

θ expressed in radians

U expressed in volts.

Now, to synthesize the fast control loop:

a - we will assume the reactor in pulsed steady state condition
    at 600 kW and use the linearized reactor description holding
    for small deviations from the previous steady state, i.e.:

$$\frac{\Delta \bar{P}}{\Delta \epsilon_d} = 19.05 \frac{(s+0.08)(s+0.37)(s+14.12)}{(s+0.064)(s+1.56)(s+13.43)} \tag{6-2}$$

b - we will close the loop and plot a root-locus. (This plot gives direct information about stability and system performances.) From an analysis of this plot the gain will be chosen.

c - the so-defined fast control system will be added to the set of non-linear equations (2-4) and the obtained overall reactor description will be checked against reactivity perturbations by means of the SAHYB programme.

Steps (a), (b) and (c) will be repeated introducing new elements in the loop to improve the control system performances until satisfactory results are obtained.

6-1. Closing the loop directly

The loop is closed as shown in the block diagram of figure 6-1-1. The fast control system consists of an amplifier of gain G delivering the voltage U to the motor described by 6-1; the angular position of the fast control rod is converted into reactivity (10 pcm for 180 $^\circ$ angular rotation); this control reactivity is applied to the reactor described by 6-2.

As may be seen, small power demands from the considered state 600 kW are considered and denoted by $\Delta \bar{P}_o$; possible small reactivity perturbations $\Delta \epsilon_d$ are also considered.

Since we are mainly interested in the whole system response to reactivity perturbations, we will not impose any specification for the response to power demand and assume $\Delta \bar{P}_o=0$, which simplifies the system synthesis.

Let us consider now the closed loop transfer function $\frac{\Delta P}{\Delta \epsilon_d}$ (s). We obtain directly from the block diagram represented in Figure 6-1-1.
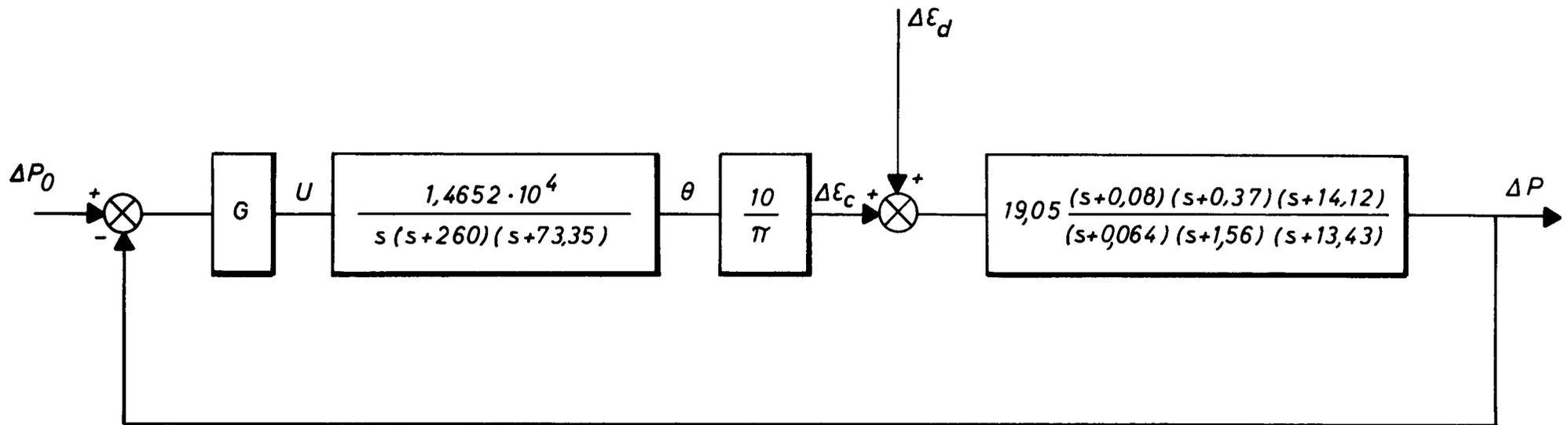
FIG.6-1-1. CLOSING THE LOOP DIRECTLY
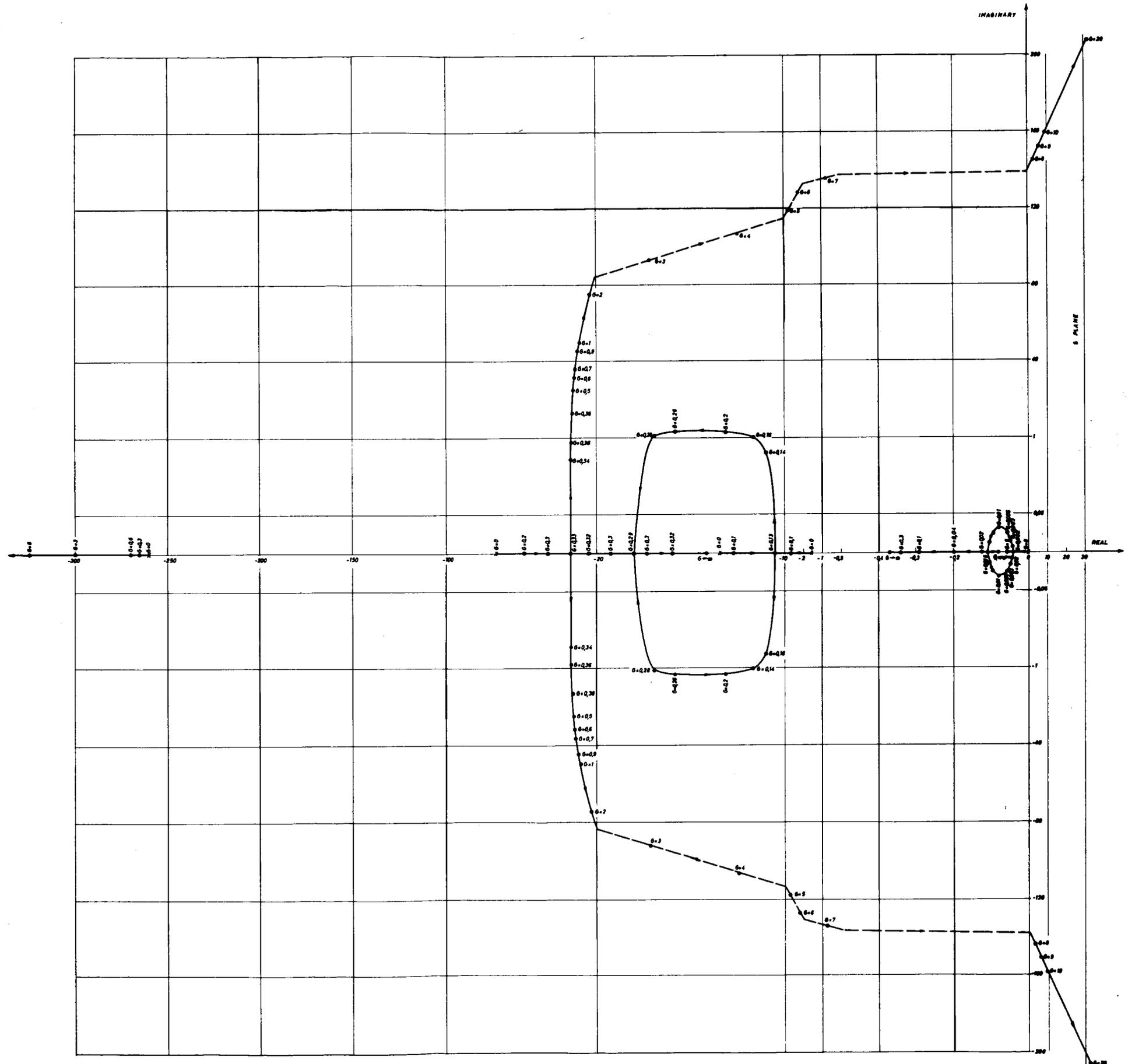
ROOT-LOCUS DIAGRAM — STEP 1   FIG.6.1.2

$$\frac{\Delta \bar{P}}{\Delta \epsilon}\bigg|_d = \frac{19.05\ s(s+0.08)(s+0.37)(s+14.12)(s+73.35)(s+260)}{\sqrt{s(s+260)(s+73.35)(s+0.064)(s+1.56)(s+13.43)}}$$
$$+ 8.8847 \cdot 10^5 \cdot G(s+0.08)(s+0.37)(s+14.12)\_7$$

The denominator of this expression contains G as a parameter. Developping and ordering in the successive powers of s we obtain for this denominator a polynominal expression, the roots of which govern the overall stability and the time domain transient response of the system.

A plot of the values of these roots (root-locus) graduated with the corresponding values of G is shown in Fig. 6-1-2. $\sqrt{}$This root locus was plotted using the subroutine POLRT - Ref.(11).$\_7$ For G = 0, all of the poles start from the open-loop transfer function poles. As G is increased, they tend to the zeros of the open-loop transfer function or to infinity. So we get three asymptotes: one of them is the real axis, the two others are $\pm \frac{2\pi}{3}$ from the real axis.

As may be seen, even for small G the poles (crosses) nearer the origin tend quickly to the two nearer zeros (circles). The two further poles meet together, separate, come again to the real axis and separate: one of them tends to the third zero while the other one meets with the fourth pole; these two last separate and become complex. In such a case, choosing a gain G higher than 0.35 results in two control poles - the farthest one starting from - 260 and tending to infinity may be neglected.

We will choose a gain G = 0.6 corresponding to a small overshoot for the step response (for this value of G the system is quite stable and the two control poles are such that imaginary part $\doteq$ real part = 32) and a settling time of about 1/32 second.
For such a value of G, the exact roots were calculated always using the subroutine POLRT and led to the following result:

$$\frac{\Delta P}{\Delta \epsilon}\bigg|_d = \frac{19.05s(s+260)(s+73.35)(s+14.12)(s+0.37)(s+0.08)}{(s+270)(s+14.5)(s+0.354)(s+0.08)(s+31.7+j31.5)(s+31.7-j31.5)}$$
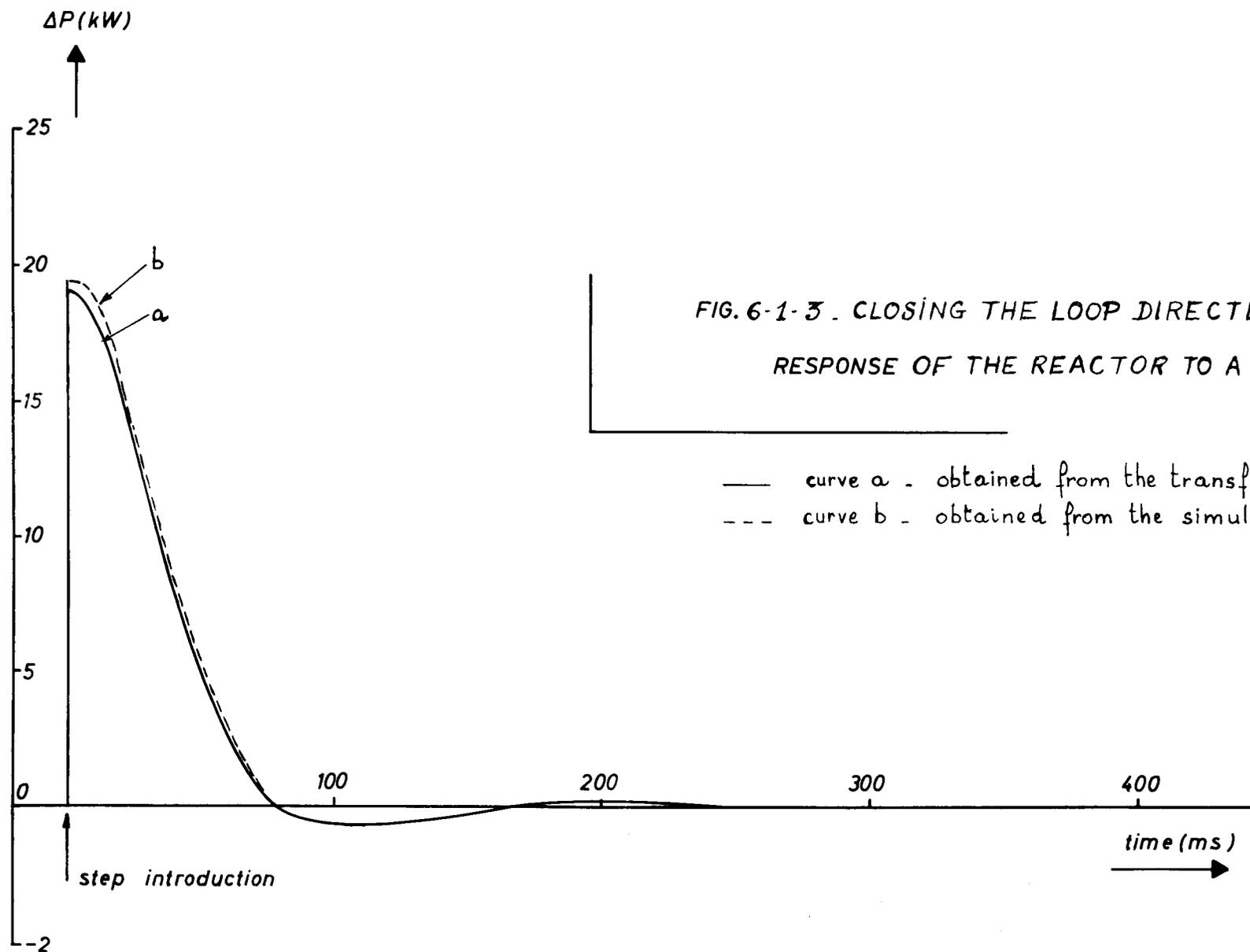
FIG. 6-1-3. CLOSING THE LOOP DIRECTLY

RESPONSE OF THE REACTOR TO A STEP OF 1 pcm

—— curve a — obtained from the transfer function
--- curve b — obtained from the simulation.

which simplified as:

$$\frac{\Delta P}{\Delta \epsilon_d} = 19.05 \frac{s(s+260)(s+73.35)}{(s+270)(s+31.7+j\ 31.5)(s+31.7-j\ 31.5)}$$

This last result, used to plot the power response against a step of reactivity of 1 pcm gave the result shown in Fig. 6-1-3, curve a. From this curve, we see a settling time of about 40 ms and an overshoot less than 5 %, which agrees perfectly with the previsions if one takes into account the simplifications that have been made in obtaining the last transfer function.

At this point we will verify the validity of our treatment by introducing the defined control system into the simulation of the set of non-linear equations (2-4). In other words, we add to the set (2-4) describing the reactor the set of equations describing the control loop, make G = 0.6, and use the SAHYB program for the simulation. We considered a step of reactivity of 1 pcm and obtained the overall power response shown in Fig. 6-1-3 curve b. A comparison between the two responses (a) and (b) leads to the conclusion that the linearized description used to synthesize the fast control system is more than adequate for our purpose: it may be said that the two responses are coincident.

6-2. First improvement of the fast control system performances

A simple means of improving the previous results consists in introducing a tachometer dynamo as a loop around the motor, as shown in the block-diagram of figure 6-2-1. The primary loop is closed exactly as before; a secondary loop is added around the motor. A tachometer dynamo delivering 5 Volt at 100 rpm has been chosen. In terms of transfer function, we introduce the simple block 0.5 s as a feedback around the motor, so that we get now:

$$\frac{\theta}{U} = \frac{1.4652 \cdot 10^4}{s(s+204)(s+130)}$$

With this new (motor and dynamo) description, we will repeat completely the procedure described in § 6-1 and determine the new
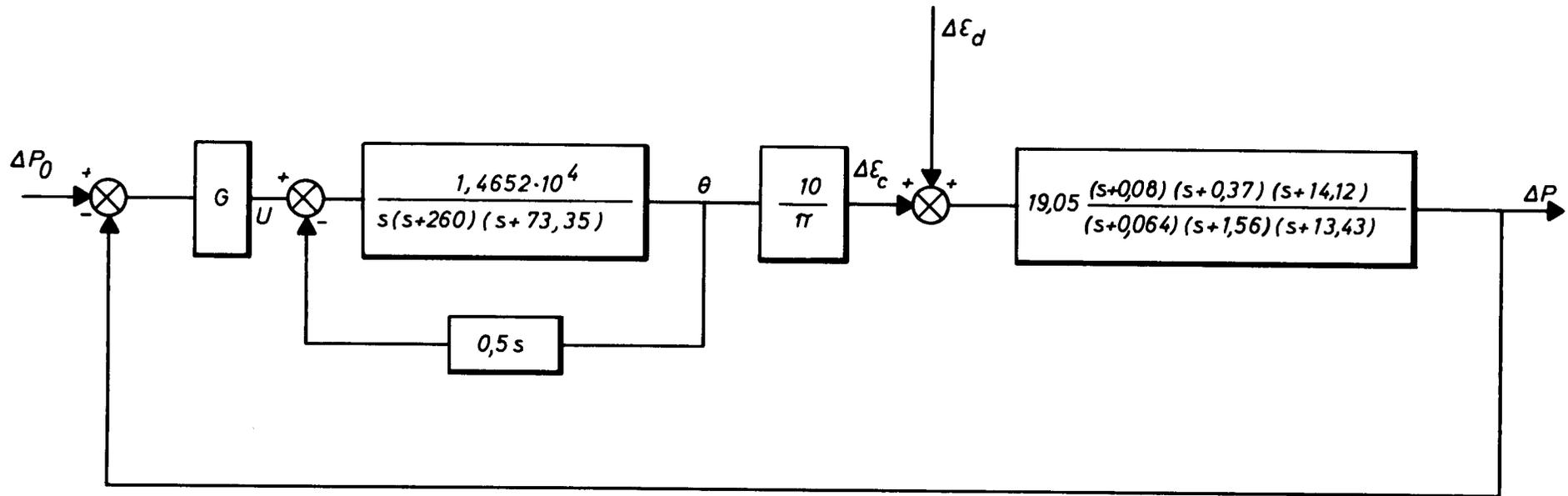
FIG. 6-2-1 _ FIRST IMPROVEMENT

ROOT - LOCUS DIAGRAM - STEP 2   FIG. 6.2.2

FIG. 6-2-3. FIRST IMPROVEMENT
RESPONSE OF THE REACTOR TO A STEP OF 1 pcm.

———— curve a. obtained from the transfer function

– – – – curve b. obtained from the simulation.

value for the gain G.

The overall closed-loop transfer function becomes:

$$\frac{\Delta \overline{P}}{\Delta \epsilon_d} = \frac{19.05\ s(s+0.08)(s+0.37)(s+14.12)(s+130)(s+204)}{\underline{/}\ s(s+204)(s+130)(s+0.064)(s+1.56)(s+13.43)} $$
$$+8.8847 \cdot 10^5 \cdot G(s+0.08)(s+0.37)(s+14.12)\underline{/}$$

Developping the denominator, we obtain a polynominal expression which contains G as a parameter. The root-locus for this expression is plotted in Fig. 6-2-2 with the corresponding values of G.

This root-locus is quite similar to the first one: the main difference lies in the fact that the two (symmetric) asymptotic branches start farther from the origin. Choosing as before two complex poles on these branches will result in a quicker transient.

We chose a gain G = 1, corresponding to a small overshoot in the step-response, a good settling time and a quite safe relative stability.

For G = 1, the exact roots of the polynomial were calculated and the overall transfer function resulted in:

$$\frac{\Delta P}{\Delta \epsilon_d} = \frac{19.05\ s(s+0.08)(s+0.37)(s+14.12)(s+130)(s+204)}{\underline{/}(s+0.0801)(s+0.3572)(s+14.4546)(s+47.8+j38.3)}$$
$$x(s+47.8-j38.3)(s+238.4))$$

which simplified as:

$$\frac{\Delta P}{\Delta \epsilon_d} = \frac{19.05\ s(s+130)(s+204)}{(s+238.4)(s+47.86+j\ 38.35)(s+47.86-j\ 38.35)}$$

This last expression, used to plot the response to a step of reactivity $\Delta \epsilon_d$ of 1 pcm showed a settling time of about 35 ms and a small overshoot as expected, see fig. 6-2-3, curve a.

Finally, introducing the so defined control system in the simulation and setting the gain at the new value G = 1, gave the result as shown in fig.6-2-3 curve b. As may be seen from a comparison between figures 6-1-3 and 6-2-3, the performances of the control system are somewhat improved in the second case.

6-3. Second improvement of the fast control system performances

The performances obtained as in § 6-2 may still be improved by means of a lead-compensation network. This network would change the arrangement of poles and zeros on the real axis: the operation results in a larger real part of the complex control poles, and thus in a faster response to the perturbations. We will introduce a lead compensation network $\frac{s+100}{s+500}$ as shown in the block diagram of figure 6-3-1.
We get then for the transfer function of the closed loop:

$$\frac{\Delta P}{\Delta \epsilon_d} = \frac{19.05 \ s(s+0.08)(s+0.37)(s+14.12)(s+130)(s+204)(s+500)}{\sqrt{\begin{array}{l} s(s+0.064)(s+1.56)(s+13.43)(s+130)(s+204)(s+500) \\ + \ 8.8847 \cdot 10^5 \cdot G(s+0.08)(s+0.37)(s+14.12)(s+100) \end{array}}}$$

The root locus for the denominator of this expression is plotted in Fig. 6-3-2, with G as a parameter. As may be seen this root locus is somewhat different from the previous ones: the two asymptotic branches start much farther from the origin; this time, we have not simply two control poles but another one which tends to - 100 as G is increased.

To get a fast transient response it is of interest to approach this pole - 100 with a high gain. On the other hand, a too high gain would bring the complex poles too much on the right and would slow down the response. The choice G = 10 is a good compromise for fast response, small undershoot and good stability. It may be seen that for G = 10 we get complex poles with real part = imaginary part.
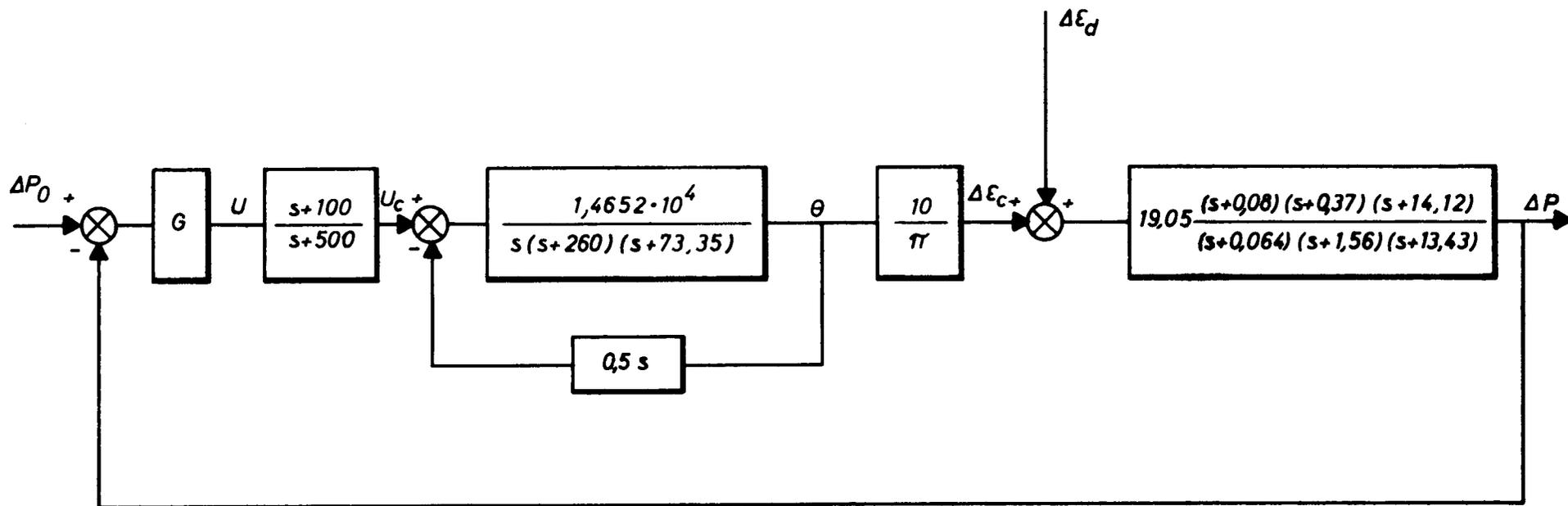
FIG. 6-3-1 _ SECOND IMPROVEMENT
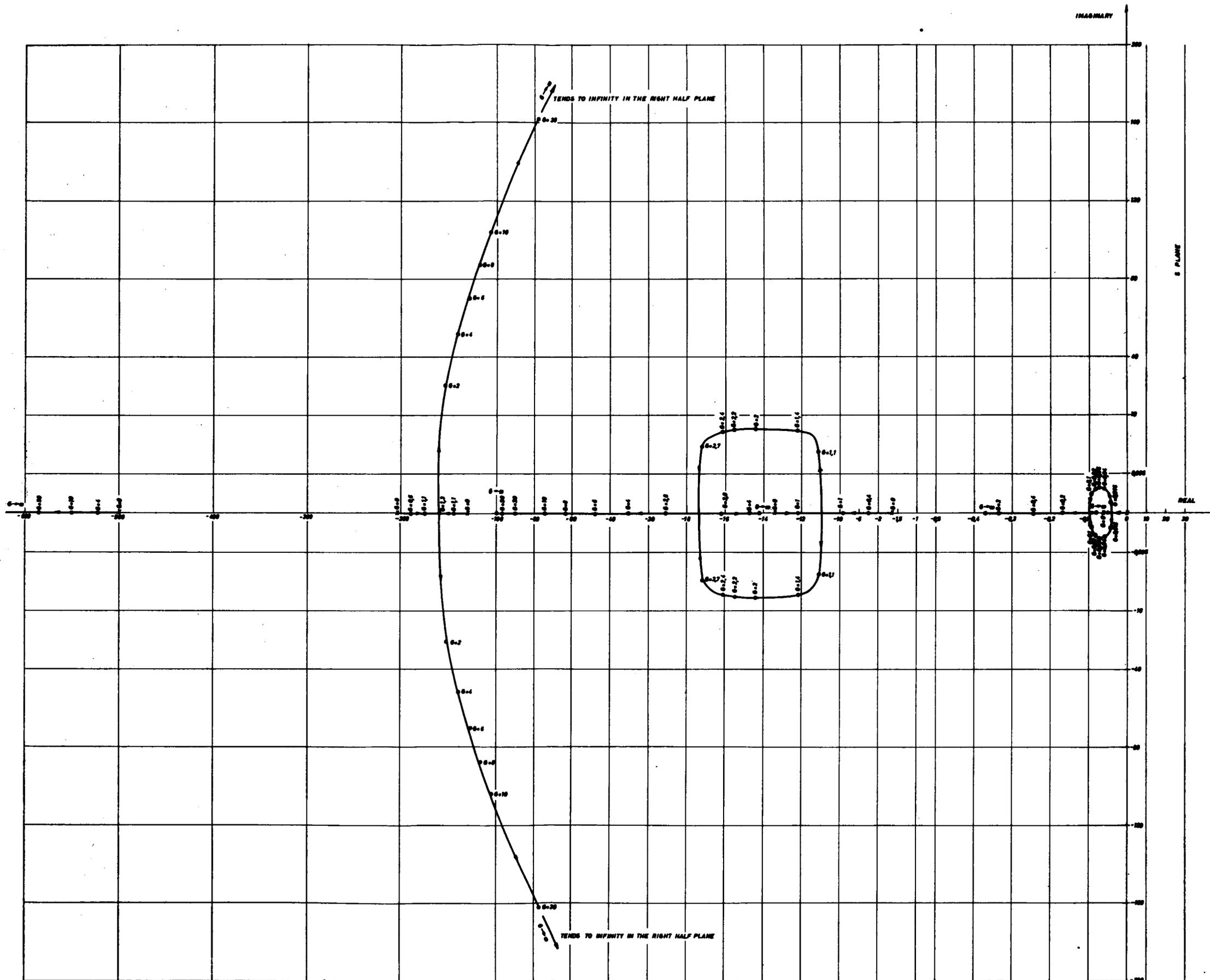
ROOT- LOCUS DIAGRAM - STEP 3   FIG. 6.3.2
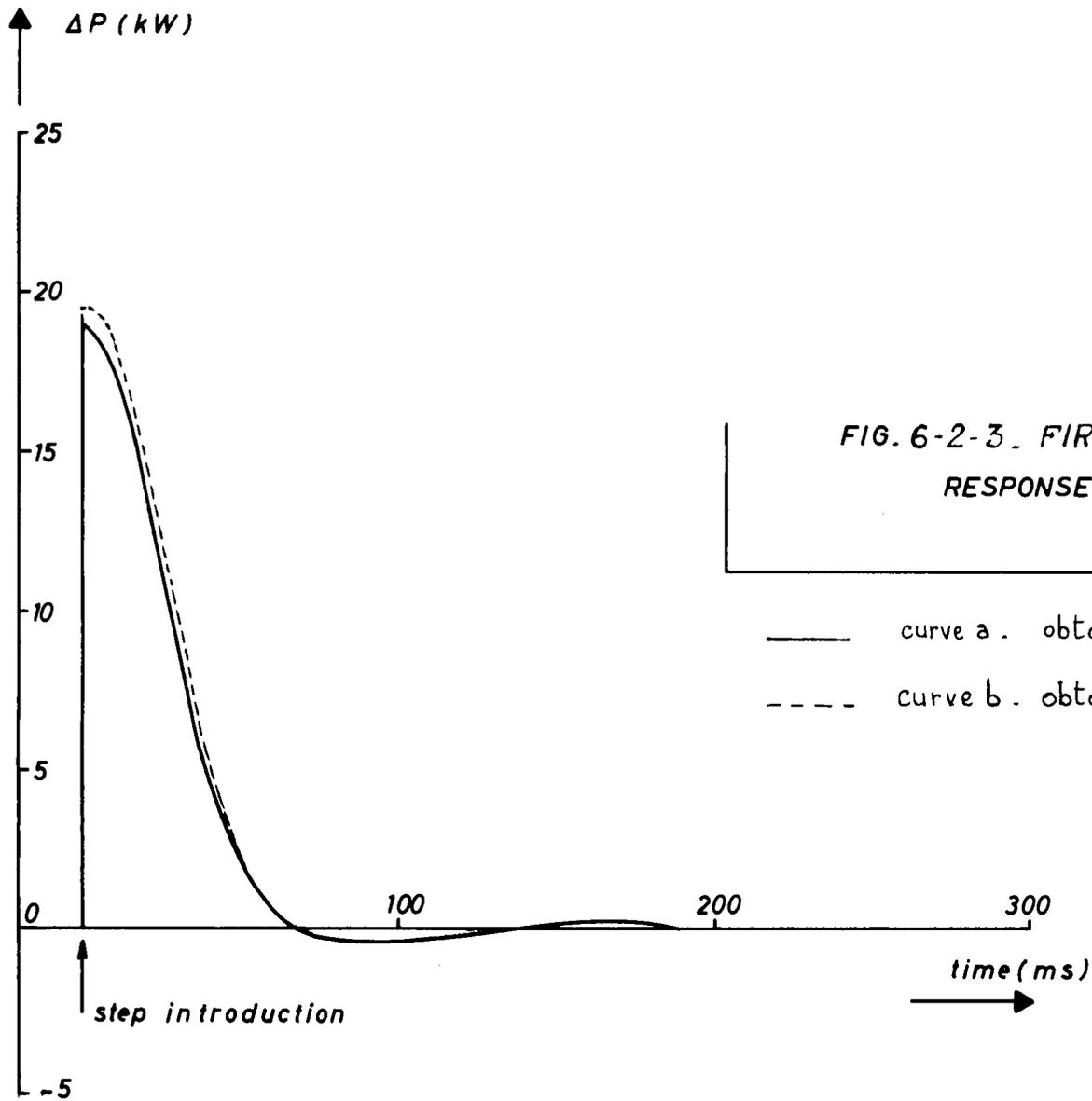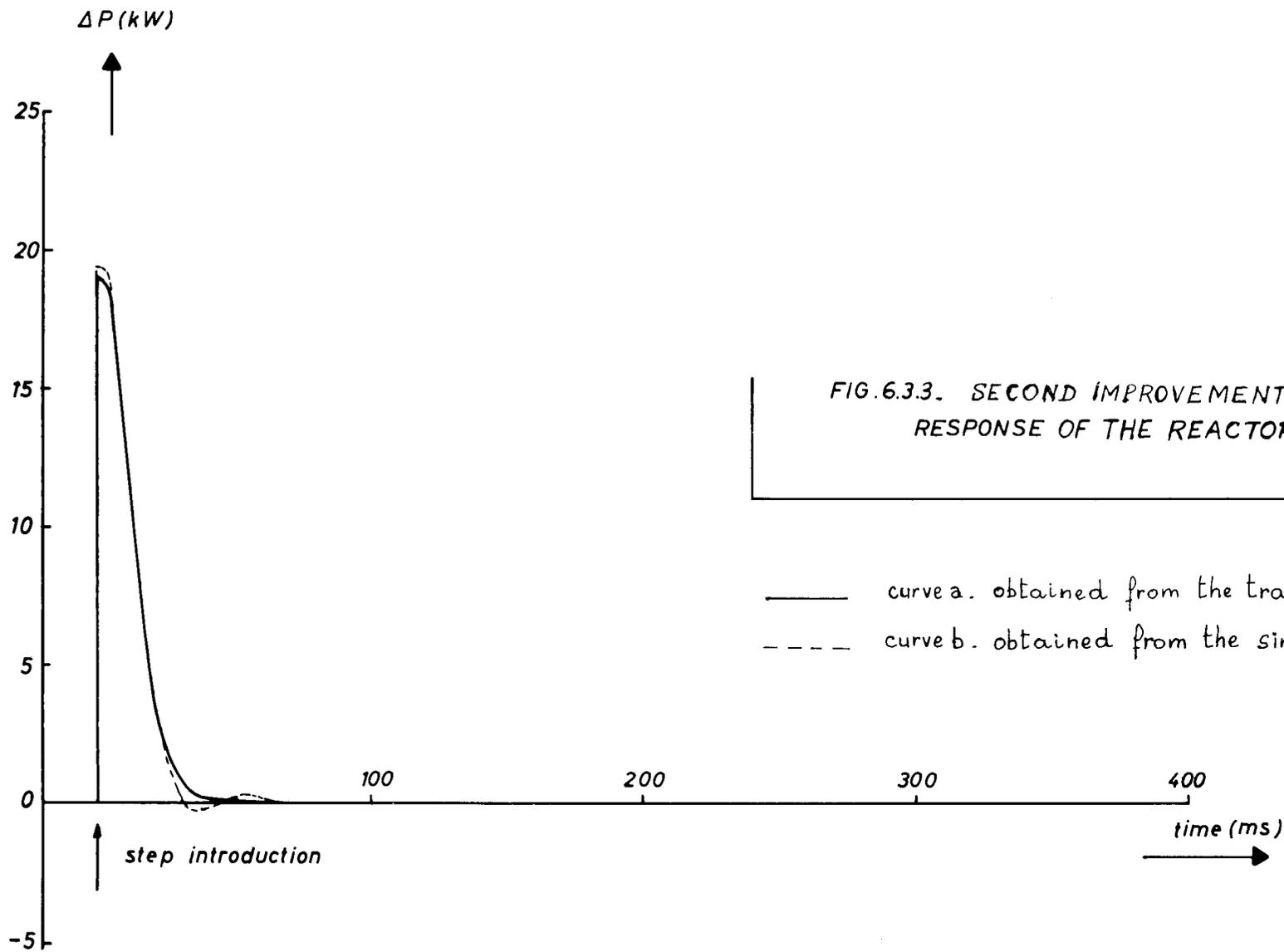
FIG.6.3.3. SECOND IMPROVEMENT
RESPONSE OF THE REACTOR TO A STEP OF 1 pcm

—————— curve a. obtained from the transfer function

— — — — curve b. obtained from the simulation.
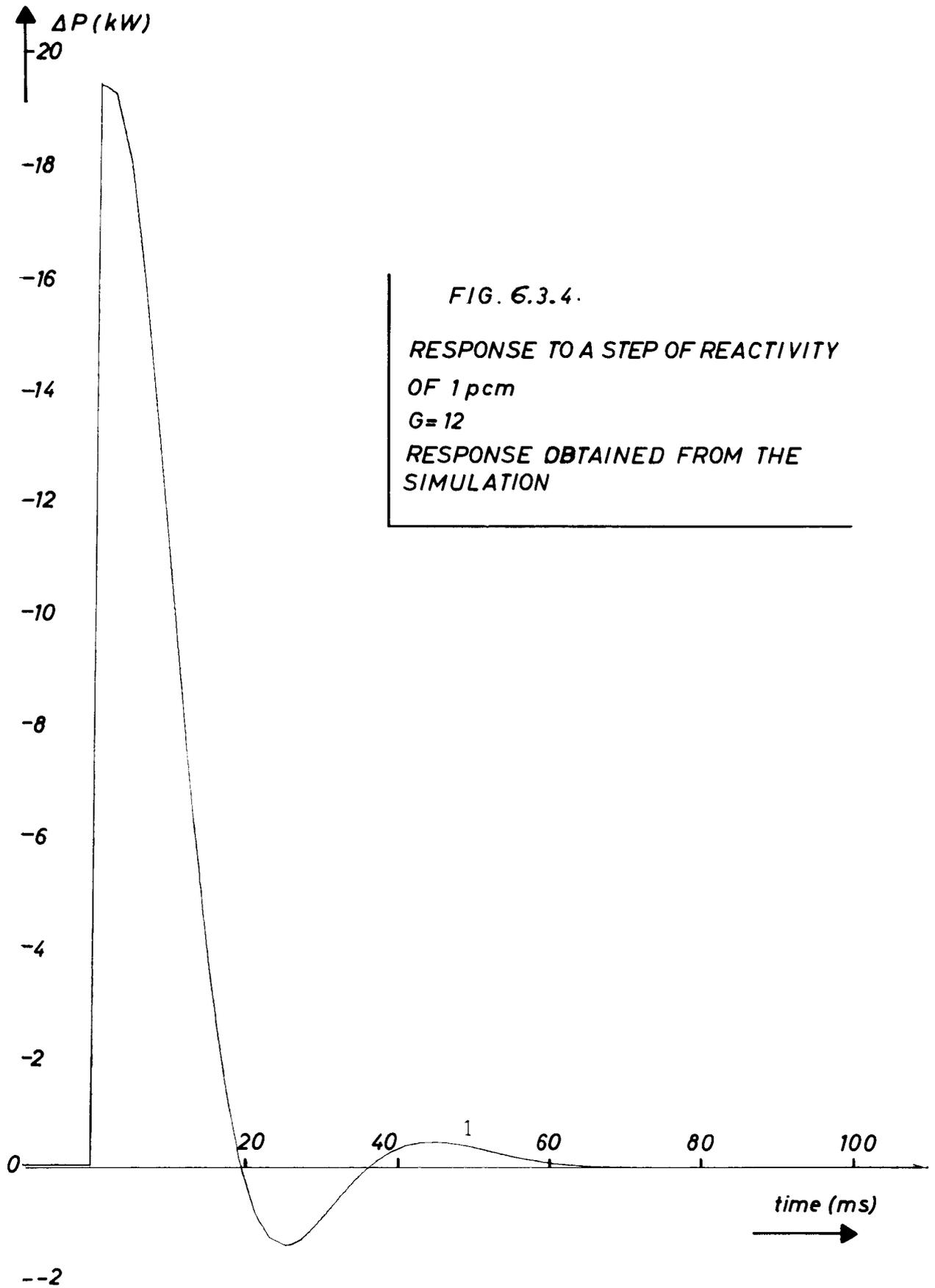
FIG. 6.3.4.

RESPONSE TO A STEP OF REACTIVITY
OF 1 pcm
G = 12
RESPONSE OBTAINED FROM THE
SIMULATION

The settling time results from the combination of these two poles and of the third one which tends to -100. For the chosen gain G = 10, the exact roots of the denominator were found and gave for the transfer function the already simplified expression:

$$\frac{\Delta \overline{P}}{\Delta \epsilon_d} = 19.05 \ \frac{s(s+130)(s+204)(s+500)}{(s+74.46)(s+550)(s^2+210s + 21816)}$$

Let us now see how this final control system works when added to the set of equations (2-4). Once again the SAHYB program was used for the simulation; a step of reactivity of 1 pcm was introduced as a perturbation. The response of the overall system (reactor + fast control system such as synthesized in this paragraph) is shown in figure 6-3-3. As it may be seen, the fast control system performances are considerably improved with respect to the two first results 6-1-3 and 6-2-3.

One might think that increasing still the gain would lead to a better response. However, the response presents some oscillations as shown in Fig. 6-3-4 which was plotted with G = 12.

6-4. Basic set of equations for the controlled reactor - Conclusion

As a first conclusion, an important one, all of the results show that the SORA reactor is controllable, although the fast control system synthesis was based upon a set of equations for "mean values"; only the performances of the control system will change a little bit with respect to the real case, due to the sampling effect on the power - in terms of automatic control, the power will be "sampled and hold" at the frequency of the pulsation device.

Major improvements were not investigated, since the obtained settling time lies in the order of 20 ms, which is the smallest time interval that one might take under consideration, always due to the validity of the set of equations (2-4).

As concerns the feasibility of such a control system, some diffi-
culties arise if perturbations really occur under the form of
step-functions as we assumed until now, since saturation phenomena
(on the amplifier, on the motor) would occur reducing the indicated
performances. However, it seems more reasonable from a practical
point of view to expect perturbations in the form of "ramp-func-
tions". The situation is then quite sure; the simulation showed
that the so-defined control system worked well with ramp-functions
until about 50 pcm/second, which is far above expected ramp rates
$\boxed{/}$a response to this type of perturbation is shown in Fig. 6-4-1$\boxed{/}$.

Basic set of equations for the controlled reactor.

ϵ_c = control reactivity

ϵ_d = input reactivity signal

ϵ_p = feedback reactivity

ϵ_m = total reactivity

core kinetics

$$K = 0.229 + 8.80\,\epsilon_m + 0.0176\,e^{4140\,\epsilon_m}$$

$$\bar{w} = \frac{K}{\beta v}\left(\sum_i \lambda_i \bar{C}_i + S_o\right)$$

$$\frac{d\bar{C}_i}{dt} = \beta_i v\,\bar{w} - \lambda_i \bar{C}_i$$

$$\bar{P} = \frac{\bar{w}}{C_f}$$

Thermal description

$$C_f \frac{d\bar{T}_f}{dt} = \rho\bar{P} - h_{Fs}(\bar{T}_f - \bar{T}_g)$$

$$C_g \frac{d\bar{T}_g}{dt} = h_{Fs}(\bar{T}_f - \bar{T}_g) - \frac{h_{sc}}{1 + \frac{Lh_{sc}}{2VC_c}}(\bar{T}_g - T_{C,IN})$$

$$T_{c,out} = T_{c,in} + \frac{2}{1+\dfrac{2VC_c}{Lh_{sc}}} \; (\overline{T}_g - T_{c,in})$$

$$T_c = \frac{1}{2} \, (T_{c,in} + T_{c,out})$$

Feedback reactivity

$$\epsilon_{pf} = \alpha_f (\overline{T}_f - T_{REF}^*)$$

$$\epsilon_{pg} = \alpha_g (\overline{T}_g - T_{REF}^*)$$

$$\epsilon_p = \epsilon_{pf} + \epsilon_{pg}$$

(6-4)

Fast control-system

$$U = G(\overline{P}_o - \overline{P})$$

$$\frac{dU_c}{dt} = \frac{dU}{dt} + \omega_1 U - \omega_2 U_c$$

$$U_c - \eta \, \frac{d\theta}{dt} = k \, \frac{d\theta}{dt} + \frac{RJ}{k} \, \frac{d^2\theta}{dt^2} + \frac{J}{k} \, \frac{d^3\theta}{dt^3}$$

$$\epsilon_c = \gamma\theta$$

Typical responses of the controlled reactor to perturbations are shown in figures 6-4-1, 6-4-2, 6-4-3, 6-4-4.

FAST CONTROL SYSTEM PERFORMANCES
POWER RESPONSE TO REACTIVITY PERTURBATIONS

CURVE 1 : response to a change of 1 pcm at 45 pcm/second
CURVE 2 :   ..   .. 5 ..   ..

POWER (KW)

TIME (milliseconds)

RAMP INSERTED

FIG.6-4-1

- 60 -

Curve 1. Response to a change (step) of 5°C on inlet coolant temperature

FAST CONTROL SYSTEM PERFORMANCES
POWER RESPONSE TO INLET COOLANT TEMPERATURE PERTURBATION

FIG. 6-4-2

Power (kW)

600,3 —	
600.2 —	
600.1 —	
	step inserted
600 —	1 2 3 4 5 6 7 8 9 10
	Time (seconds)
599.9 —	
	1
599.8 —	
	Curve 1. Power response to a reduction on the coolant velocity
599.7 —	(from 6 m/s to 5.5 m/s as a step function)
599.6 —	
599.5 —	FAST CONTROL SYSTEM PERFORMANCES
	POWER RESPONSE TO COOLANT VELOCITY PERTURBATION
599.4 —	
599.3 —	
599.2 —	
599.1 —	
599 └	FIG. 6-4-3

- 62 -

FAST CONTROL SYSTEM PERFORMANCES
POWER RESPONSE TO A SET POINT CHANGE

Curve 1. Response to a change of 25 kW on the set point

FIG.6-4-4   Time (seconds)

7. Malfunctions

Control rod malfunctions occurring in the uncontrolled reactor have been simulated as a part of reactor safety studies. The set of equations (2-4) was used for this purpose. We investigated the malfunctions of:

- the fast control rod (by inserting 10 pcm at 20 pcm/s)
- the slow control rod (by inserting 40 pcm at 0.83 pcm/s)
- the regulation rod (by inserting 300 pcm at 3 pcm/s).

Now, in order to have a first assessment of the importance of a reduction in the values of the reactivity feedback coefficients, each malfunction was repeated changing these coefficients. Four cases were considered:

	α_f	α_g
Case 1	Reference value $(-0.42\text{pcm}/^\circ C)$	Reference value $(-0.80\text{pcm}/^\circ C)$
Case 2	Half reference value $(-0.21\text{pcm}/^\circ C)$	Half reference value $(0.40\text{pcm}/^\circ C)$
Case 3	Reference value $(-0.42\text{pcm}/^\circ C)$	0
Case 4	0	Reference value $(-0.80\text{pcm}/^\circ C)$

For each malfunction, and in each case, the following parameters have been plotted: power, period, fuel temperature, structure temperature. Results are presented in the figures from 7-1 to 7-6. Also the effect of a total loss of coolant flow occurring at 600 kW power has been investigated. The corresponding result is shown in figure 7-7.

UNCONTROLLED REACTOR - 600 kw POWER - FAST CONTROL ROD MALFUNCTION - (10 pcm at 20 pcm/s)

FIG.2-1

UNCONTROLLED REACTOR - 600 kw POWER - SLOW CONTROL ROD MALFUNCTION - (40 pcm at 0B3 pcm/s)

FIG.7-2

UNCONTROLLED REACTOR - 600 kw POWER - REGULATION ROD MALFUNCTION - (300 pcm at 3 pcm/s)

FIG. 7-3

UNCONTROLLED REACTOR - POWER 100 kw - FAST CONTROL ROD MALFUNCTION - (10 pcm at 20 pcm/s)

FIG. 7-4

UNCONTROLLED REACTOR-POWER 100 kw-SLOW CONTROL ROD MALFUNCTION-(40 pcm at 0.83 pcm/s)

FIG.7-5

UNCONTROLLED REACTOR - POWER 100 kw - REGULATION ROD MALFUNCTION - (300 pcm at 3 pcm/s)

FIG 7-6

UNCONTROLLED REACTOR = 600 KW POWER. COOLING MALFUNCTION
(coolant velocity reduced to ~0)

FIG 7-7

Time (seconds)

Power (kW)

Fuel temperature (°C)

Fuel temperature
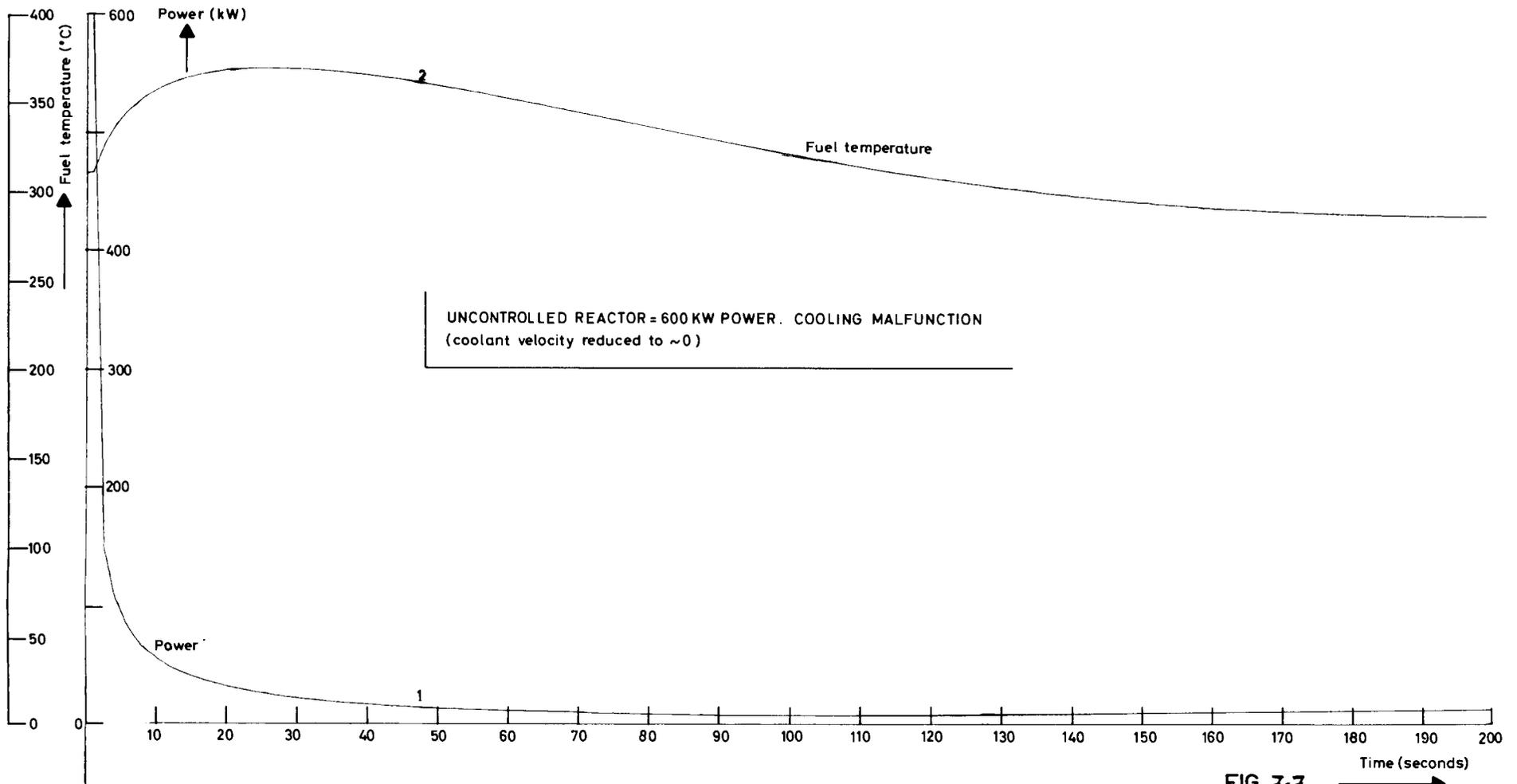
Power

Conclusion

As may be noted, this study has been performed using extensively only two programmes: the SAHYB-programme for simulation and the subroutine POLRT for finding the roots of polynominals. This method has proved to be powerful since it permitted - in a short time and without any difficulty - to answer most questions about the SORA dynamics and control, such as: reactor response to perturbations, start-up procedure, design of the fast control system, malfunctions, etc. Therefore, the initial choice of a digital computer for such a study seems retrospectively to be a good one.

Of interest also is the possibility of <u>extending the simulation to the whole plant</u>: the set of equations (6-4) may be completed with a heat exchanger description and safety circuits. In this manner, a complete logical start-up procedure should be defined.

However, the obtained results lie on the assumption of mean values equations as a reactor description. Although most important questions may be treated upon this basis, another study was started, based on a more accurate reactor kinetics description: the discrete time reactor description which consists of a set of recurrent equations. A comparison between the results obtained from this ultimate representation and the results presented here shows a posteriori the high degree of accuracy of the reactor description that we have used. This coincidence might be expected from the theoretical studies (Ref. 2 and 3).

Appendix 1 - Derivation of numerical values for a steady-state condition at power

Let us assume a pulsed steady state condition at power $\overline{P}_o$. We may neglect the contribution of the source-term S_o; then we get the following set of equations for kinetics:

$$K = 0,229 + 8,80 \, \epsilon_m + 0,0176 \, e^{4140 \, \epsilon_m} = 1$$

$$\overline{w}_o = \frac{K}{\beta v} \left(\sum_i \lambda_i \overline{C}_i + S_o \right) \quad \text{with } K = 1$$

$$\frac{d\overline{C}_{io}}{dt} = \beta_i \, v \, \overline{w}_o - \lambda_i \overline{C}_{io} = 0$$

$$\overline{P}_o = \frac{\overline{w}_o}{C_F}$$

from which we derive $\overline{w}_o = C_F \overline{P}_o$ and successively:

$$\overline{C}_{10} = v \frac{\beta_1}{\lambda_1} \overline{w}_o$$

$$\overline{C}_{20} = v \frac{\beta_2}{\lambda_2} \overline{w}_o$$

$$\vdots$$

etc.

$$\vdots$$

$$\overline{C}_{60} = \frac{\beta_6}{\lambda_6} \overline{w}_o$$

for the set of equations "thermal description", we get:

$$0 = \rho \overline{P}_o - h_{Fs} \left(\overline{T}_{fo} - \overline{T}_{go} \right)$$

$$0 = h_{Fs} \left(\overline{T}_{fo} - \overline{T}_{go} \right) - \frac{h_{sc}}{1 + \frac{Lh_{sc}}{2VC_c}} \left(\overline{T}_{go} - T_{c,in} \right)$$

from which $\overline{T}_{fo}$ and $\overline{T}_{go}$ may be derived, while $T_{c,out,o}$ and

T_{co} are calculated successively. Results are given here below:

$$\overline{C}_{10} = v \frac{\beta_1}{\lambda_1} C_F \overline{P}_o$$

$$\overline{C}_{20} = v \frac{\beta_2}{\lambda_2} C_F \overline{P}_o$$

$$\overline{C}_{30} = v \frac{\beta_3}{\lambda_3} C_F \overline{P}_o$$

$$\overline{C}_{40} = v \frac{\beta_4}{\lambda_4} C_F \overline{P}_o$$

$$\overline{C}_{50} = v \frac{\beta_5}{\lambda_5} C_F \overline{P}_o$$

$$\overline{C}_{60} = v \frac{\beta_6}{\lambda_6} C_F \overline{P}_o$$

$$\overline{T}_{fo} = T_{c,in} + \rho \overline{P}_o \left(\frac{1}{h_{Fs}} + \frac{1}{h_{sc}} + \frac{L}{2VC_c} \right)$$

$$\overline{T}_{go} = T_{c,in} + \frac{\rho \overline{P}_o}{h_{sc}} \left(1 + \frac{L h_{sc}}{2VC_c} \right)$$

$$T_{c,out,o} = T_{c,in} + \rho \overline{P}_o \frac{L}{2VC_c}$$

$$T_{co} = T_{c,in} + \rho \overline{P}_o \frac{L}{2VC_c}$$

Numerical values of these parameters at different power levels are given in the following table.

$\overline{P}_0$ (kW)	$\overline{c}_{10}$	$\overline{c}_{20}$	$\overline{c}_{30}$	$\overline{c}_{40}$	$\overline{c}_{50}$
100	$0,33383232 \times 10^{12}$	$0,45385316 \times 10^{13}$	$0,64990908 \times 10^{14}$	$0,80666532 \times 10^{14}$	$0,33245973 \times 10^{15}$
200	$0,66766471 \times 10^{12}$	$0,90770642 \times 10^{13}$	$0,12998183 \times 19^{15}$	$0,16133306 \times 10^{15}$	$0,66491972 \times 10^{15}$
300	$0,10014971 \times 10^{13}$	$0,13615596 \times 10^{14}$	$0,19497276 \times 10^{15}$	$0,24199960 \times 10^{15}$	$0,99737972 \times 10^{15}$
400	$0,13353290 \times 10^{13}$	$0,18154122 \times 10^{14}$	$0,25996368 \times 10^{15}$	$0,32266613 \times 10^{15}$	$0,13298397 \times 10^{16}$
500	$0,16691610 \times 10^{13}$	$0,22692661 \times 10^{14}$	$0,32495454 \times 10^{15}$	$0,40333259 \times 10^{15}$	$0,16622994 \times 10^{16}$
600	$0,20029941 \times 10^{13}$	$0,27231183 \times 10^{14}$	$0,38994545 \times 10^{15}$	$0,48399906 \times 10^{15}$	$0,19947594 \times 10^{16}$

Numerical values of parameters at different power levels.

c_{60}	$\overline{T}_{fo}(^{o}C)$	$\overline{T}_{go}(^{o}C)$	$T_{c,out,o}(^{o}C)$	$T_{co}(^{o}C)$
$0,14895400 \times 10^{15}$	218,42065	211,45950	210,72224	205,36111
$0,29790779 \times 10^{15}$	236,84132	222,91902	221,44450	210,72224
$0,44686182 \times 10^{15}$	255,26199	234,37852	232,16675	216,08337
$0,59581585 \times 10^{15}$	273,68262	245,83806	242,88901	221,44450
$0,74476988 \times 10^{15}$	292,10327	257,29736	253,61125	226,80562
$0,89372391 \times 10^{15}$	310,52393	268,75708	264,33325	232,16673

Numerical values of parameters at different power levels (cont.)

Appendix 2 - Derivation of numerical values for a subcritical steady-state condition

We will calculate the numerical values for precursor concentrations and mean temperatures assuming that all of the control rods are removed. (ϵ_d = 0). The feedback reactivity may then be neglected (ϵ_p = 0), therefore ϵ_m = 0, which gives the following value for the pulsed multiplication coefficient:

$$K = 0,2466$$

In such a steady state:

$$\frac{d\bar{C}_{10}}{dt} = \ldots\ldots = \frac{d\bar{C}_{60}}{dt} = 0$$

which gives:

$$\bar{C}_{10} = \nu \frac{\beta_1}{\lambda_1} \bar{w}_o$$

$$\cdot$$
$$\cdot$$
$$\cdot$$
$$\bar{C}_{60} = \nu \frac{\beta_6}{\lambda_6} \bar{w}_o$$

In turns, these values are introduced into the equation:

$$\bar{w}_o = \frac{K}{\beta\nu} (\sum_i \lambda_i \bar{C}_i + S_o)$$

so that

$$\bar{w}_o = \frac{K}{\beta\nu} (\nu\beta_1 \bar{w}_o + \ldots + \nu\beta_6 \bar{w}_o + S_o) = \frac{K}{\beta\nu}(\beta\nu \bar{w}_o + S_o).$$

The value for $\bar{w}_o$ is derived from this last result:

$$\bar{w}_o = \frac{K}{1-K} \cdot \frac{S_o}{\beta\nu}$$

and permits to calculate the precursor concentrations for the considered steady-state (K = 0,2466). It is found:

$$\bar{C}_{io} = \frac{1}{\lambda_i} \cdot \frac{\beta_i}{\beta} \cdot \frac{K}{1-K} \cdot S_o.$$

For such a steady-state, numerical values are:

$$\overline{C}_{10} = 2,202997 \cdot 10^4$$

$$\overline{C}_{20} = 2,995030 \cdot 10^5$$

$$\overline{C}_{30} = 4,2888294 \cdot 10^6$$

$$\overline{C}_{40} = 5,3232759 \cdot 10^6$$

$$\overline{C}_{50} = 2,19394090 \cdot 10^7$$

$$\overline{C}_{60} = 9,82964943 \cdot 10^6$$

and
$$\overline{P}_{o} = 6,599096 \cdot 10^{-6} \text{ kW.}$$

Appendix 3 - Reactor period

The reactor period is defined as $\dfrac{\bar{P}}{\dfrac{d\bar{P}}{dt}}$ or $\dfrac{\bar{w}}{\dfrac{d\bar{w}}{dt}}$; this parameter

is of main interest during a start-up. We will outline the
method which has been used to calculate this parameter. It
consists of expressing the period in terms of the derivatives
and functions which already appear in the set of equations
(2-4) for the uncontrolled reactor. (Of course the derivation
does not hold for the reactor submitted to control since
other terms would appear in the final expression due to the
control reactivity.)

$$\bar{w} = \frac{K}{\beta v} \quad \sum_i \lambda_i \bar{C}_i$$

$$\frac{d\bar{w}}{dt} = \frac{1}{\beta v} \cdot \frac{dK}{dt} \cdot \sum_i \lambda_i \bar{C}_i + \frac{K}{\beta v} \sum_i \lambda_i \frac{d\bar{C}_i}{dt}$$

with

$$\frac{dK}{dt} = 8,80 \left(\frac{d\epsilon_d}{dt} + \frac{d\epsilon_{pf}}{dt} + \frac{d\epsilon_{pg}}{dt} \right) + 72,864 \left(\frac{d\epsilon_d}{dt} + \frac{d\epsilon_{pf}}{dt} + \frac{d\epsilon_{pg}}{dt} \right) e^{4140\epsilon_m}$$

In our simulation the external reactivity ϵ_d is always introduced under the
form of a ramp function, so that the derivative $(d\epsilon_d/dt)$ is the slope of
this ramp and remains constant as long as this slope is not varied.
On the other hand ϵ_{pf} and ϵ_{pg} are related to the temperatures $\bar{T}_f$
and $\bar{T}_g$ so that we get for the derivative of K:

$$\frac{dK}{dt} = 8,80 \left(\frac{d\epsilon_d}{dt} + \alpha_f \frac{d\bar{T}_f}{dt} + \alpha_g \frac{d\bar{T}_g}{dt} \right) + 72,864 \left(\frac{d\epsilon_d}{dt} + \alpha_f \frac{d\bar{T}_f}{dt} + \alpha_g \frac{d\bar{T}_g}{dt} \right) e^{4140\epsilon_m}$$

The reciprocal of the period is then expressed as:

$$\frac{1}{K} \cdot \frac{dK}{dt} + \frac{\sum_i \lambda_i \frac{dC_i}{dt}}{\sum_i \lambda_i \cdot C_i}$$

which is finally written under the form:

$$\frac{1}{K} \cdot \frac{dK}{dt} + \frac{K}{\beta v \, \overline{P} C_F} \cdot \sum_i \lambda_i \frac{d\overline{C}_i}{dt} \quad .$$

We have used this last expression to calculate the reciprocal of the period, changing the $\frac{d\epsilon_d}{dt}$ term as requested on account of the time interval under calculation into the program.

Appendix 4 - Note on the simulation

Below is an example of how the simulation was performed.
The subroutine DER describing the reactor $\sqrt{}$i.e. the complete
set of equations (6-4)$\underline{}$ is introduced into the main programme
SAHYB. The case given simulates the response of the controlled
reactor to a reactivity perturbation of 1 pcm, applied as
a ramp function of 45 pcm/second slope. As may be noted, the
subroutine DER calls in turn the subroutine DRAW, which enables
the designer to get directly drawn the functions of interest.
(In the present case, the power deviation was plotted as re-
presented in Fig. 6-4-1 - curve 1).

```
C
C
```

```
ISN 0022      UNIT=1.L+ 5
ISN 0023      ELT 1=115
ISN 0024      HEIGHT=24
ISN 0025      CF UEL=5  6
ISN 0026      CCLAD= 616
ISN 0027      CC UEL= 154
ISN 0028      VC UEL=6
ISN 0029      HFS=5 16
ISN 0030      HSS=50 39
ISN 0031      TCI I=27
ISN 0032      TF =3.1 52E+ 2
ISN 0033      TG =2  8873E+ 2
```

```
C ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
C          REFERENCE TEMPERATURE FOR THE WHOLE REACTOR
C ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
```

```
ISN 0034      TREF=2
```

```
C ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
C          FEEDBACK REACTIVITY COEFFICIENTS
C ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
```

```
ISN 0035      FUTC=-  42E-
ISN 0036      SCTC=- 0 87E-
```

```
C ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
C          FAST CONTROL  SYSTEM PARAMETERS
C ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
```

```
ISN 0037      C=12
ISN 0038      Q=1 3
ISN 0039      S=2 E-
ISN 0040      W=4 4E- 3
ISN 0041      R=5 7
```

```
C ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
C          REACTIVITY EPSD0 FOR STEADY STATE
C ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
```

```
ISN 0042      EPSD0 =016  6E-  5-FUTC*(TF -TREF)-SCTC*(TG -TREF)
```

```
C
C
C
C     CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C              PERTURBATION DESCRIPTION                                 C
C     CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

ISN 0043   T0=10.E-2
ISN 0044   T1=50.E-12
ISN 0045   . RR=450.E-15

```
C     CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C              WRITING AND DRAWING                                      C
C     CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

ISN 0046   CALL WRITE (4HEPSD,EPSD,4HEPSC,EPSC,4HDPKW,DPKW,3HPKW,PKW,1HU,U,4H
                 1UCOR,UCOR,5HVTACH,VTACH,4HVDIR,VDIR,5HDTETA,DTETA,3HACC,ACC)
ISN 0047   CALL DRAW(4HDPKW,DPKW)

ISN 0048   100 CONTINUE

```
C     CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C         EPSD: EXTERNALLY INTRODUCED REACTIVITY                        C
C     CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

ISN 0049   EPSD=EPSD +RAMP(T0,T1)*RR

```
C     CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C         FUEL AND STRUCTURE TEMPERATURES TF AND TG                     C
C     CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

ISN 0050   TF=X(7)
ISN 0051   TG=X(8)

```
C     CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C         FUEL AND STRUCTURE TEMPERATURES DEVIATIONS                    C
C     CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

ISN 0052   DTF=TF-TREF

```
ISN 0053          DTS=TS-TREF


C
C                  TOTAL FEEDBACK REACTIVITY EPSP
C

ISN 0054          EPSPF=RUTS*DTF
ISN 0055          EPSPS=SETS*DTS
ISN 0056          EPSP=EPSPF+EPSPS


C
C            FAST CONTROL ROD ANGLE , POSITION TETA
C            RESULTING CONTROL REACTIVITY EPSC
C

ISN 0057          TETA=X(7)
ISN 0058          EPSC=ELC(1-2/0.145)*TETA


C
C                  TOTAL REACTIVITY EPST
C

IS  0059          EPST=EPSD+EPSP+EPSC


C
C            PULSED MULTIPLICATION COEFFICIENT PMCK         C
C            FISSION RATE FRFS                              C
C            REACTOR POWER IN KW PKW                        C
C            POWER DEVIATION FROM STEADY STATE DPKW         C
C

ISN 0060          PMCK=0.329+0.3*EPST+0.176*EXP(414.6*EPSM)
ISN 0061          FRFS=PMCK/(BETA*GAM)*(ALA1*X(1)+ALA2*X(2)+ALA3*X(3)+ALA4*X(4)+ALA5
                 1*X(5)+ALA6*X(6))
ISN 0062          PKW=FRFS/CF
ISN 0063          DPKW=PKW-600.


C
C            FAST CONTROL LOOP EVOLUTION                    o
C            ANGULAR SPEED OF THE FAST CONTROL ROD DTETA    o
C            ACCELERATION OF THE FAST CONTROL ROD ACC       o
```

```
C     .........  VOLTAGE VT=24
C     ........ ... VOLTAGE
C     ...........  OUTPUT VOLTAGE ...
C     VOLTAGE APPLIED TO THE MOTOR VT=
C
      VT=X(7)
      VF=X(6)
      VA=X(5)
      VF=VFST+...
      J=X(3)-...
      J=X(2)+...(          ,      ,  ...,J)
      VT=X(7)-VT/24
C
C
C  STATEMENT OF  INITIAL DESCRIPTION
C
      D(1)=...-...-...X(1)
      D(2)=...-...-...X(2)
      D(3)=...-...-...X(3)
      D(4)=...-...-...X(4)
C
C  STATEMENT OF EQUATIONS FOR THERMAL DESCRIPTION
C
      DX(1)=1./C...(...+...)...C/...-(X(2)-X(1))
      DX(2)=1./C...(X(3)-X(1))-1./C...)...C/(1.+...C/(...C...
     1C...L))*(X(3)-TCT))
C
C  STATEMENT OF EQUATIONS FOR THE FAST CONTROL SYSTEM
C
      DX(1)=X(1)
      DX(2)=X(3)
      DX(1)=1/(S...)*VD...-)*2./(S...)*X(..)-1/S*X(11)
      DX(1)=1..S...+(..-...)/...-S-...*X(12)
```

```
IS.1 ---34            RETURN
IS.1 ---35            END
```

C
C

NOMENCLATURE

Symbol	Signification	Value	Units
A_c	coolant channel area per fuel element $$A_c = 2(\sqrt{3} - \frac{\pi}{2})\, r_o^{\,2}$$	0.1814	cm^2
c_{bond}	specific heat of bond	0.91	$Wxs/g/^\circ C$
C_{bond}	heat capacity of bond per unit length $C_{bond}\ {}^{2\pi r_o \Delta r_{bond}}\ \rho_{bond}\ c_{bond}$	0.066	$Wxs/cm/^\circ C$
c_c	specific heat of coolant	0.91	$Wxs/g/^\circ C$
C_c	heat capacity of coolant per unit length $C_c = A_c\, \rho_c c_c$	0.134	$Wxs/cm/^\circ C$
c_{clad}	specific heat of clad	0.50	$Wxs/g/^\circ C$
C_{clad}	heat capacity of clad per unit length $C_{clad}\quad 2\pi\ r_o\,\Delta r_{clad}\ \rho_{clad} c_{clad}$	0.550	$Wxs/cm/^\circ C$
c_f	specific heat of fuel	0.19	$Wxs/g/^\circ C$
C_f	heat capacity of fuel per unit length $C_f = \pi r^2_f\, \rho_f c_f$	5.06	$Wxs/cm/^\circ C$
C_F	conversion factor from fission rate to kW power $C_F = \dfrac{\overline{w}}{\overline{P}}$	3.1×10^{13}	fission/ (sec.kW)
$\overline{C}_i$	mean value of the i'th delayed neutron precursor concentration	function	without
G	amplifier gain (see fast control system)	12	
h_f	heat transfer coefficient from clad to coolant $(Nu = 0{,}15.Pr^{0,34})$	1.6	$W/cm^2/^\circ C$
h_{Fs}	heat transfer coefficient from fuel to mixed bond-clad	5.16	$W/cm/^\circ C$
h_{sc}	heat transfer coefficient from mixed bond-clad to coolant	5.89	$W/cm/^\circ C$
I	current through the motor	function	A

NOMENCLATURE cont.

Symbol	Signification	Value	Units
J	total inertia of the fast control device =	4.4×10^{-3}	kgm^2
	(rotor moment of inertia	4.05×10^{-3}	kgm^2
	$+$ load of inertia)	3.54×10^{-4}	kgm^2
k	back EMF coefficient for the motor	1.3	Volt/rad/s
k_{bond}	heat conductivity of bond	0.24	$W/cm/^{\circ}C$
k_c	heat conductivity of coolant	0.24	$W/cm/^{\circ}C$
k_{clad}	heat conductivity of clad	0.18	$W/cm/^{\circ}C$
k_f	heat conductivity of fuel	0.24	$W/cm/^{\circ}C$
k_g	heat conductivity of mixed bond-clad $$kg = \frac{\Delta r_{bond} + \Delta r_{clad}}{\frac{\Delta r_{bond}}{k_{bond}} + \frac{\Delta r_{clad}}{k_{clad}}}$$	0.20	$W/cm/^{\circ}C$
K	pulsed multiplication coefficient $$K = \frac{\text{precursor production during period}}{\text{precursor decay during period}}$$	function	
l	self-inductance of the motor actuating the fast control rod	2×10^{-2}	Henry
L	fuel rod height	24	cm
M	ratio of fissions in a pulse to the mean fission rate	function	second
n	number of fuel rods	116	-
p	conversion factor from kW into w	10^3	-
$\overline{P}$	mean power of the reactor (over one period)	function	kW
$\overline{P}_o$	nominal mean power	600	kW
q	average power generated in fuel per unit length $= \dfrac{p\overline{P}}{nL}$	function	W/cm
q"	average power generated in fuel per unit of area $= \dfrac{q}{2\pi r_f}$	function	W/cm^2

NOMENCLATURE cont.

Symbol	Signification	Value	Units
q''	average power generated in fuel per volume unit $= \dfrac{q}{\pi r_f^2}$	function	W/cm^3
r_o	outer fuel element radius $r_o = r_f + \Delta r_{bond} + \Delta r_{clad}$	0.75	cm
r_f	fuel slug radius	0.70	cm
R	rotor resistence of the motor	6.7	ohm
S_o	neutron source	10^7	neutron/sec
T	period length of the pulsed reactor	2×10^{-2}	second
T_c	mean coolant temperature (=temperature at half height core)	function	^oC
$T_{c,in}$	inlet coolant temperature	function	^oC
	nominal value	200	^oC
$T_{c,out}$	outlet coolant temperature	function	^oC
$\overline{T}_f$	mean fuel temperature	function	^oC
$\overline{T}_g$	mean bond-clad temperature	function	^oC
T^*_{REF}	reference temperature for reactivity measurements	200	^oC
T_R	reactor period $(\overline{P}/d\overline{P}/dt)$	function	second
V	voltage applied to the motor or at the input of the lead network or at the input of the whole system motor + tachometer	function	Volt
U_c	output of the lead network	function	Volt
U	coolant velocity in the channel	function	
	nominal value	600	cm/s
$\overline{w}$	mean fission rate over a period	function	neutron/s
$\overline{w}_o$	nominal mean fission rate (corresponding to nominal mean power)	1.86×10^{16}	neutron/s

NOMENCLATURE cont.

Symbol	Signification	Value	Units
α_f	fuel temperature coefficient	-0.42	pcm/$^{\circ}$C
α_g	structure temperature coefficient	-0.80	pcm/$^{\circ}$C
β	total delayed-neutron precursor production per fission neutron	6.4×10^{-3}	–
β_i	i'th delayed neutron precursor production per fission neutron		
	β_1	0.1668×10^{-3}	–
	β_2	0.8184×10^{-3}	–
	β_3	2.6066×10^{-3}	–
	β_4	1.2024×10^{-3}	–
	β_5	1.3615×10^{-3}	–
	β_6	0.2444×10^{-3}	–
γ	conversion factor from the angular position of the fast control rod into pcm – $\gamma = \dfrac{10}{\pi}$	3.18	pcm/rad
ΔP	small power deviation from steady state resulting of a perturbation	function	kW
ΔP_o	small power demand from the nominal steady state 600 kW	function	kW
Δr_{bond}	bond thickness	0.02	cm
Δr_{clad}	clad thickness	0.03	cm
$\Delta\epsilon_d$	small external reactivity perturbation	function	pcm
$\Delta\epsilon_m$	small reactivity deviation from ϵ_{mo} $\underline{/}K(\epsilon_{mo}) = 1\underline{/}$	function	pcm

NOMENCLATURE cont.

Symbol	Signification	Value	Units
$\Delta\epsilon_p$	small feedback reactivity deviation from steady-state	function	pcm
	$\Delta\epsilon_p = \begin{cases} \Delta\epsilon_{pf} \ldots \text{from fuel} \\ \Delta\epsilon_{pg} \ldots \text{from structure} \end{cases}$	function function	pcm pcm
ϵ_o	average prompt reactivity between pulses (absolute value)	0.038	–
ϵ_c	control reactivity from the fast control rod ($\epsilon_c = \gamma\theta$))	function	pcm
ϵ_d	externally introduced reactivity	function	pcm
ϵ_m	maximum prompt reactivity	function	pcm
ϵ_{mo}	total reactivity at steady state = max prompt reactivity at steady state such as $K(\epsilon_{mo}) = 1$	91.1	pcm
ϵ_p	feedback reactivity	function	pcm
ϵ_{pf}	feedback reactivity from fuel	function	pcm
ϵ_{pg}	feedback reactivity from structure	function	pcm
θ	angular position of the fast control rod (starting from steady-state)	function	radian
λ	mean decay constant of delayed neutron precursor	0.08	$(\text{sec})^{-1}$
λ_i	decay constant of the i'th delayed neutron precursor		$(\text{sec})^{-1}$
	λ_1	3.8723	$(\text{sec})^{-1}$
	λ_2	1.3975	$(\text{sec})^{-1}$
	λ_3	0.31083	$(\text{sec})^{-1}$
	λ_4	0.11552	$(\text{sec})^{-1}$
	λ_5	0.031738	$(\text{sec})^{-1}$
	λ_6	0.012716	$(\text{sec})^{-1}$

NOMENCLATURE cont.

Symbol	Signification	Value	Units
η	gain of the tachometer dynamo	0.5	Volt/ (rad/sec)
ν	total number of neutrons (prompt and delayed) produced per fission	2.5	–
ρ	conversion factor from mean power $\bar{P}$ generated in the reactor into average power q generated in fuel per unit length $= \frac{p}{nL}$	0.359	$(cm)^{-1}$
ρ_{bond}	specific mass of bond	0.81	g/cm^3
ρ_c	specific mass of coolant	0.81	g/cm^3
ρ_{clad}	specific mass of clad	7.92	g/cm^3
ρ_f	specific mass of fuel	17.3	g/cm^3
τ	electromechanical time constant of the fast control system (motor + control rod)	18.5×10^{-3}	second
τ'	electrical time constant of the motor $(\tau' = \frac{1}{R})$	3×10^{-3}	second
ω_1	zero of the compensation network introduced in the fast control system	10^2	rad/sec
ω_2	pole of the compensation network introduced in the fast control system	5×10^2	rad/sec

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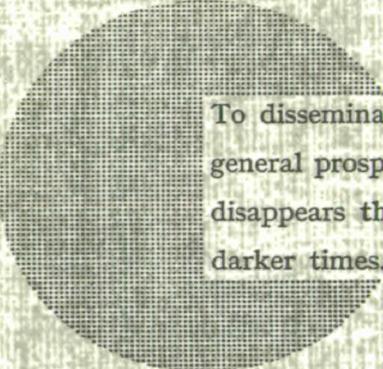
To disseminate knowledge is to disseminate prosperity — I mean general prosperity and not individual riches — and with prosperity disappears the greater part of the evil which is our heritage from darker times.

Alfred Nobel

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