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EUROPEAN ATOMIC ENERGY COMMUNITY — EURATOM

CRAYON G1
A CODE FOR THE CALCULATION OF THE IRRADIATION BEHAVIOUR OF FUEL RODS

by

M. FANTOZZI and B. HUBER

1969

ORGEL Program

Joint Nuclear Research Center
Ispra Establishment - Italy

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ABSTRACT

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KEYWORDS

C-CODES
RADIATION EFFECTS
FUEL RODS
TEMPERATURE
SWELLING
FISSION PRODUCTS
GASES
PRESSURE
STRESSES
STRAIN
ORGEL REACTOR
FUEL ELEMENTS
URANIUM CARBIDES

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CRAYON G 1

A CODE FOR THE CALCULATION OF THE
IRRADIATION BEHAVIOUR OF FUEL RODS(*)

1. Introduction

The code CRAYON G1 (1) calculates the irradiation behavior of a fuel rod. It gives in the main the evolution of the following characteristics during fuel element service:

- fuel and sheath temperatures;
- irradiation swelling of fuel;
- fission gas release and gas pressure;
- sheath stresses and deformations.

The objectives of the code CRAYON are:

- to yield a better insight of what happens in a fuel element;
- to predict the failure of fuel elements;
- to clarify the influence of design characteristics and working conditions in order to contribute to a better optimization of the reactor.

The code CRAYON was made for application to the fuel element of the ORGEL reactor. This fuel element consists of UC rods sheathed with SAP. However, general relations are utilized to describe physical properties and irradiation behavior; the material constants to be filled in these relations are input data. Consequently, the code may be employed also for other materials than UC and SAP.

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(1) G stands for gaseous bonding; 1 means first version

(*) Manuscript received on 28 January, 1969

Generally, the code applies to the following conception of a fuel rod:

- full cylindrical fuel;
- gaseous bonding(1);
- plain or finned, virtually free-standing(2) sheath.

The operating conditions of the fuel rod can be changed step-wise during the calculation procedure, in order to simulate reactor power variation, fuel element shuffling or fuel degradation.

The following principal idealizations are employed for temperature calculation:

- temperatures are taken symmetrical with respect to rod axis(3);
- axial heat transfer in the rod is neglected;
- steady-state equations are employed.

The utilization of steady-state equations for temperature calculation means that the code does not apply to very rapid transients. However, for a 2 cm-diameter UC/SAP rod, the error in fuel temperature will be less than 10°C if the coolant temperature transient does not exceed 1°C/s. The calculable transient is the more rapid as the rod is smaller.

A parametrical study by means of the code CRAYON is given in [21].

---

(1) A similar code has been made for liquid metal bonding: Code CRAYON L 1.

(2) "Virtually free-standing" is to say that sheath is not collapsing under coolant pressure; this does not exclude forced contact with fuel.

(3) This supposes that neutron flux, fuel-to-sheath clearance and sheath-to-coolant heat transfer coefficient are symmetrical.

2. Calculation procedure

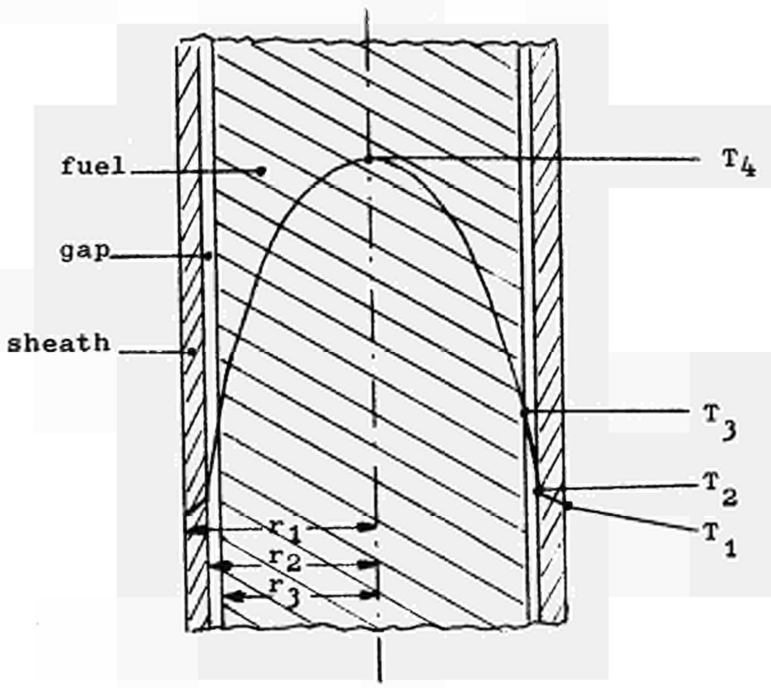
The fuel length is divided into n equal sections. The operation time is divided into a series of "positions" of given duration. Along each section and during each position, the operating conditions (input data α, q, T_c, Pe, Q) are taken constant.

The duration of each position is divided into small time increments Δt. During each time increment, all values are taken constant.

The code has been programmed on the 360-IBM computer.

3. Temperature

Starting from the given coolant temperature, the calculation proceeds from the outside to the center of the rod. Except for gap width, the initial cold dimensions (r_1, r_2, r_3) can be employed for temperature calculation.



3.1. Sheath temperature

Outer surface temperature:

$$T_1 = T_c + \frac{q}{2\pi r_1 \mu \eta \alpha} \tag{1}$$

Inner surface temperature:

$$T_2 = T_1 + \frac{q}{\pi k_s} \cdot \frac{r_1 - r_2}{r_1 + r_2} \tag{2}$$

Average sheath temperature:

$$T_s = \frac{T_1 + T_2}{2} \tag{3}$$

3.2. Heat transfer in the gap

Radiation heat flux is negligible compared to total heat flux, even for large temperature differences between fuel and sheath. For instance, if fuel surface is at 1000°C and sheath at 450°C, radiation heat flux is about 0.7 W/cm^2.

The calculation of the code is based on conventional heat conduction in the bonding gas. In the case of contact between fuel and sheath, an hypothetical gas layer of uniform thickness ε is assumed. This means, theoretically, that heat transfer by direct solid contact(1) is neglected and that gap width is considered large compared to the mean free path of gas molecules. In practice, the assumption of hypothetical gas layer is to be considered as a mathematical model with the thickness to be adjusted according to irradiation results.

---

(1) In the case of the ORGEL fuel element, the contact pressure between fuel and sheath will never be very strong because of the low coolant pressure and the low mechanical resistance of the sheath.

A more sophisticated and physically truer model - as given in the literature [1] [2] [3] [4] [5] - may be employed when the constants utilized in these models will be known more exactly than now.

As the temperature difference between fuel and sheath in the case of contact is not very large, its present rather rough calculation does not degrade the validity of the final results of the code.

The temperature T at any point of the gap is given by the equation:

$$\int_{T_2}^{T} k_g \, dT = \frac{q}{\pi (r_2+r_3)} \; j_r \qquad (4)$$

with:

$$0 \leq j_r \leq j^*$$

where j_r is the distance from the inner surface of sheath.

Utilizing equation (59) for thermal conductivity k_g, one obtains from (4):

$$T^{C_2+1} - T_2^{C_2+1} = \frac{C_2+1}{C_1} \cdot \frac{q}{\pi (r_2+r_3)} \cdot j_r \qquad (5)$$

3.3. Fuel temperature

From (5) one obtains with $j_r = j^*$ for the fuel surface temperature T_3:

$$T_3^{C_2+1} - T_2^{C_2+1} = \frac{C_2+1}{C_1} \cdot \frac{q}{\pi (r_2+r_3)} \cdot j^* \qquad (6)$$

This equation cannot be resolved independently, since the gap width j* depends on T_3, because of the thermal expansion of the fuel. For calculation of the gap width, see section 5.3.

The distribution of heat generation within the fuel [6] [7] may be expressed by:

$$q_v = q_{vo} \, I_o \, (Kr) \tag{7}$$

Eliminating q_{vo} by the condition of heat balance

$$q = \int_o^{r_3} 2\pi r \, q_v \, dr = 2\pi q_{vo} \frac{r_3}{K} \, I_1 \, (K \, r_3) \tag{8}$$

one obtains:

$$q_v = \frac{q}{2\pi} \cdot \frac{K}{r_3} \cdot \frac{I_o \, (Kr)}{I_1 \, (Kr_3)} \tag{9}$$

The temperature distribution in the fuel follows the Fourier equation:

$$\frac{1}{r} \, \frac{d}{dr} \, (k_f \, r \, \frac{dT}{dr}) = - \, q_v \tag{10}$$

From (9) and (10), one obtains by integration:

$$\int_{T_3}^{T_r} k_f \, dT = \frac{q}{4\pi} \cdot \xi \tag{11}$$

with

$$\xi = 2 \, \frac{I_o \, (Kr_3) - I_o \, (Kr)}{Kr_3 \, \cdot \, I_1 \, (Kr_3)} \tag{12}$$

Taking thermal conductivity of fuel independent from temperature, one obtains the following temperature distribution:

$$T = T_3 + \frac{1}{k_f} \cdot \frac{q}{4\pi} \xi \qquad (13)$$

The fuel center temperature T_4 is:

$$T_4 = T_3 + \frac{1}{k_f} \cdot \frac{q}{4\pi} \quad \xi' \qquad (14)$$

with

$$\xi' = 2 \frac{Io\ (Kr_3) - 1}{Kr_3\ I_1\ (Kr_3)} \qquad (15)$$

The medium temperature, averaged on the fuel section, is:

$$T_f = T_3 + \frac{1}{k_f} \cdot \frac{q}{4\pi} \cdot \xi'' \qquad (16)$$

with

$$\xi'' = \frac{2}{Kr_3} \left[\frac{I_o\ (Kr_3)}{I_1\ (Kr_3)} - \frac{2}{Kr_3} \right] \qquad (17)$$

The equations (14) and (16) are employed in the code; ξ' and ξ'' are input data which may be obtained from Fig. 1, knowing either F_o or Kr_3.

Generally, the self-shielding coefficient F_o is given from neutron calculation. F_o is defined as the ratio of average heat generation density in the fuel to heat generation density at the fuel surface and is expressed by:

$$F_o = \frac{2}{Kr_3} \cdot \frac{I_1\ (Kr_3)}{I_o\ (Kr_3)} \qquad (18)$$

If the neutron flux depression is small, K may be calculated with sufficient accuracy from

$$K = \sqrt{3\ \Sigma_a\ (\Sigma_t - \frac{4}{5}\ \Sigma_a)} \qquad (19)$$

4. Sheath stress (1)

4.1. Maximum mechanical stress

As a criterion for the failure of the end plug weld, the axial stress σ_a at the ends of the rod is calculated. Since the fins are often removed at the ends of the rod, the calculation is based on the tube section without fin section. It is supposed that there is no forced contact between fuel and end-plug.

$$\sigma_a = \frac{p_i\,\pi\,r_2^{\,2} - Q}{\pi\,(r_1^{\,2} - r_2^{\,2})} \cdot 10^{-2} \qquad (20)$$

As a criterion for sheath failure, the tangential stress σ_t between fins is calculated. Only the difference between internal gas pressure and external coolant pressure is taken into account. If there is forced contact between fuel and sheath, the plastic deformation of sheath will be considered as an additional rupture criterion.

$$\sigma_t = \frac{r_2}{r_1 - r_2}\ (p_i - p_e) \cdot 10^{-2} \qquad (21)$$

---

(1) Tensile stress is taken positive, compressive stress negative.

4.2. Average mechanical stress

These stresses are used in the later calculation to evaluate the overall elastic deformation of sheath.

For the calculation of axial stress $\bar{\sigma}_a$, it is again supposed that there is no forced contact between fuel and end - plug. No ratcheting effect is taken into account.

The tangential stress $\bar{\sigma}_t$ will only be used in the later calculation if there is no forced contact between fuel and sheath.

General equations

$$\bar{\sigma}_a = \frac{p_i \pi r_2^2 - Q}{S} \cdot 10^{-2} \qquad (22)$$

where S [cm^2] is the cross-section of sheath

$$\bar{\sigma}_t = \frac{1}{2\pi} \oint \sigma_{t\theta} \cdot d\theta \qquad (23)$$

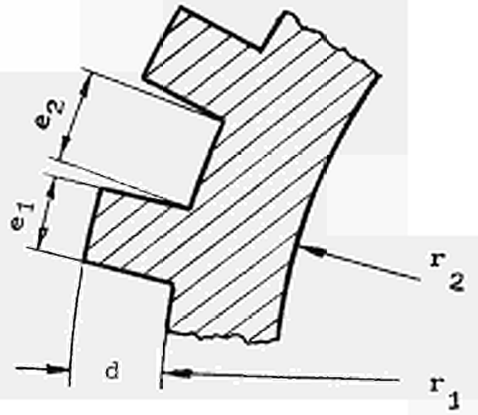
with:

$$\sigma_{t\theta} = \frac{r_2}{r_\theta - r_2} \cdot (p_i - p_e) \times 10^{-2} \qquad (24)$$

where r_θ [cm] is the radius of outer profile of sheath, dependent on angle θ [-] .

Though this calculation of $\bar{\sigma}_t$ is a rather rough approximation, it is considered satisfactory, since the influence of $\bar{\sigma}_t$ on the later calculation is small.

Equations for rectangular fins:



By application of the given general equations to fins of rectangular cross-section, the following equations are obtained, which are employed in the code:

$$\bar{\sigma}_a = \frac{p_i \pi r_2^2 - Q}{(r_1^2 - r_2^2 + 2 r_1 . d \frac{e_1}{e_1 + e_2}) \pi} \cdot 10^{-2} \tag{25}$$

$$\bar{\sigma}_t = \frac{\sigma_{t1} \cdot e_1 + \sigma_t \cdot e_2}{e_1 + e_2} \tag{26}$$

with:

$$\sigma_{t1} = \frac{r_2}{r_1 - r_2 + d} \cdot (p_i - p_e) \cdot 10^{-2} \tag{27}$$

4.3. Thermal stress

The stress due to the radial temperature gradient is calculated. For simplification, the formula for plain sheath and elastic deformation is employed. The stress occurs in axial and tangential direction and is positive (tensile) at the outer and negative (compressive) at the inner surface of sheath.

$$\sigma_{th} = (\pm) \frac{(T_2 - T_1) \lambda^*_s E}{2 (1-\nu)} \tag{28}$$

5. Dimensional variations

5.1. Thermal expansion

Thermal expansion of sheath:

$$\varepsilon_s = (T_s - T_a) \cdot \lambda_s \tag{29}$$

The thermal expansion of fuel depends on cracking of fuel:

Case A

There are no open cracks in the fuel. In this case, local thermal expansion has to be averaged over the fuel section, in order to get overall expansion:

$$\varepsilon_f = \frac{1}{\pi r_3^2} \int_o^{r_3} \lambda_f \, (T-T_a) \; 2\pi r \, dr \tag{30}$$

Case B

Fuel is completely cracked, so that all thermal stresses in the fuel are relaxed. In this case, thermal expansion has to be averaged over the fuel radius:

$$\varepsilon_f = \frac{1}{r_3} \int_o^{r_3} \lambda_f \, (T-T_a) \, dr \tag{31}$$

If the expansion coefficient λ_f is taken independent of radius, one sees from equations (30) and (31) that fuel expands as if being at uniform temperature T_f:

$$\varepsilon_f = \lambda_f \, (T_f - T_a) \tag{32}$$

with

$$T_f = \frac{2}{r_3^2} \int_o^{r_3} T \, r \, dr \tag{33}$$

in the case A (T_f = fuel temperature averaged on fuel section), and with

$$T_f = \frac{1}{r} \int_o^{r_3} T \, dr \qquad (34)$$

in the case B (T_f = fuel temperature averaged on fuel radius)(1).

In the code, case A is assumed. T_f is calculated from equation (16) which is obtained from equation (33) by integration.

5.2. Irradiation swelling of fuel

The swelling behavior of UC is generally described by the ratio R of diameter increase to burn-up, this ratio depending on the fuel center temperature and on the nature of UC (porosity, stoichiometry) [8] [9] [10]

$$S = R_{(T_4)} \cdot \tau \qquad (36)$$

This implies the assumptions that:

- swelling at constant fuel temperature increases linearly with burn-up;
- swelling is independent of fission rate;
- swelling depends only on fuel center temperature, but not on the radial temperature profile.

The last assumption is evidently a rather rough approximation. If, however, for simplification, only one parameter is used to describe temperature dependence of swelling, the center temperature is likely the most significant parameter.

---

(1) For uniform heat generation, one would obtain:

$$T_f = \frac{T_3 + T_4}{2} \qquad \text{for case A}$$

$$T_f = \frac{T_3 + 2\,T_4}{3} \qquad \text{for case B}$$

Swelling is assumed to be isotropic, i.e. percentage diameter and length increases are equal.

The temperature dependence of given swelling data can be described by a formula of the type:

$$R_{(T_4)} = C_1 + C_2 \exp\left(-\frac{C_3}{T_4}\right) \tag{37}$$

where the first term on the right side may be interpreted as to stand for solid fission product accumulation and the second term for the action of fission gases.

Equation (36) applies only to constant fuel temperature; no correlation is known for variable temperature. The assumption is made here, that equation (36) may be extended to variable fuel temperature by the formula:

$$S = \int_0^{\tau} R_{(T_4)} \cdot d\tau \tag{38}$$

with equation (37), one obtains the formula used in the code:

$$S = \sum_{j=1}^{J} \left[C_1 + C_2 \exp\left(-\frac{C_3}{T_4}\right)\right]_j \cdot \Delta\tau_j \tag{39}$$

5.3. Fuel - Sheath clearance and sheath deformation

It is supposed that no plastic deformation of sheath occurs in axial direction. The axial clearance between fuel and end-plug is then:

$$j_a^* = j_a + \frac{1}{n} \cdot \sum_{i=1}^{n} \left(\varepsilon_s + \frac{\sigma_a - \nu \sigma_t}{E} - \varepsilon_f - S\right)_i \tag{40}$$

Plastic deformation of sheath in tangential direction is supposed to occur if the deformation imposed by forced contact with fuel exceeds the elasticity limit ζ . In order to decide if there is contact between fuel and sheath, the radial clearance is calculated, which would exist without mechanical interaction of fuel and sheath:

$$j_o = j + \varepsilon_s \, r_2 + \Delta r_m - (\varepsilon_f + S) \cdot r_3 \qquad (41)$$

with:

$$\Delta r_m = \Delta r_e + (\Delta r_p)_{j-1} \qquad (42)$$

$$\Delta r_e = \frac{r_1 + r_2}{2} \cdot \frac{\bar{\sigma}t - \nu \bar{\sigma}a}{E} \qquad (43)$$

where $(\Delta r_p)_{j-1}$ is the plastic deformation of sheath, occurred during the previous fuel operation.

The following three cases must be distinguished:

No contact	Contact, but no new plastic deformation	Contact and new plastic deformation	
$j_0 - \varepsilon > 0$	$0 \geq j_0 - \varepsilon \geq \Delta r_e - \zeta \frac{r_1+r_2}{2}$	$\Delta r_e - \zeta \frac{r_1+r_2}{2} > j_o - \varepsilon$	(44)
$j^* = j_o$	$j^* = \varepsilon$	$j^* = \varepsilon$	(45)
$\Delta r_p = (\Delta r_p)_{j-1}$	$\Delta r_p = (\Delta r_p)_{j-1}$	$\Delta r_p = \varepsilon - \zeta \frac{r_1+r_2}{2} - j_o + \Delta r_m$	(46)

6. Bonding gas

6.1. Fission gas release

The fission gas production at unit fission energy production is:

$$A = \frac{V}{N\ E'} \sum y_s \qquad (47)$$

with:

$E' = 200$ MeV/fission $= 3{,}2 \times 10^{-11}$ Wsec/fission

$\sum y_s = 0.254$ [11]

one obtains:

$$A = 3.24 \times 10^{-10} \ \frac{cm^3\ NTP}{W.sec} = 28 \ \frac{cm^3\ NTP}{MWd}$$

The fission gas production in one fuel section is then:

$$G = \Psi \cdot A \circ \frac{1}{n} \sum_{j=i}^{J} q_j \ \Delta t_j \qquad (48)$$

The calculation of fission gas release is based on Booth's diffusion model [8] [12] [13]. If fission production rate and diffusion coefficient are invariable with time, the fraction of released to produced gas is:

$$f = 4 \sqrt{\frac{D'\ t}{\pi}} - \frac{3}{2} D'\ t \qquad (49)$$

Generally, fission gas production rate and diffusion constant are variable with time. As an approximation, equation (39) will be employed with a time-averaged diffusion coefficient:

$$D' = \frac{1}{t_j} \sum_{j=i}^{J} D'_j \ \Delta t_j \qquad (50)$$

From (39) and (40), one obtains:

$$f = 4 \sqrt{\frac{1}{\pi} \sum_{j=i}^{J} D'_j \, \Delta t_j} - \frac{3}{2} \sum_{j=i}^{J} D'_j \, \Delta t_j \qquad (51)$$

In order to take into account the radial variation of fuel temperature, a three-point Simpson's rule is applied [12]:

$$\bar{f} = \frac{1}{6} \left[f(T_3) + 4f(T_f) + f(T_4) \right] \qquad (52)$$

The fission gas release from the whole fuel rod is:

$$F = \sum_{i=i}^{n} G_i \, \bar{f}_i \qquad (53)$$

6.2. Conductivity of bonding gas

Experimental results show that, within the range of interest, the thermal conductivity may be considered as independent of pressure [14]. The conductivity of an inert monoatomic gas is (see [15] p.8):

$$k_g = \frac{1.9801 \times 10^{-4}}{\sigma^2 \, \Omega^{(2,2)*}} \sqrt{\frac{T}{M}} \qquad [\text{cal/cm sec °K}] \qquad (54)$$

One sees from Fig. 2 that, for the gases and temperatures of interest, the reduced collision integral can be approximated by the relation:

$$\Omega^{(2,2)*} = 1.1575 \, (T^*)^{-0.1475} \qquad (55)$$

where [16]:

$$T^* = \frac{K_B}{\varepsilon^*} \, T \qquad (56)$$

Gas	Ar	Xe	Kr	He
ε^*/K_B =	119.9	10.22	164.7	206.9

Utilizing these relations, one obtains from equation (54):

$$k_{g_i} = c_{1i} \cdot T^{c2} \qquad \text{W/cm °K} \tag{57}$$

Gas	Ar	Xe	Kr	He
$c_{1i} \times 10^6$	4.8682	2.7192	1.7068	37.634

c_2 = 0.6475 (independent of the nature of gas)

The conductivity of a gas mixture is [17]:

$$k_g = 0.5 \left(\sum k_{g_i} \beta_i + \frac{1}{\sum \beta_i / k_{g_i}} \right) \tag{58}$$

From equation (57) and (58), one obtains:

$$k_g = c_1 \, T^{c2} \tag{59}$$

with

$$c_1 = 0.5 \left(\sum c_{1i} \beta_i + \frac{1}{\sum \beta_i / c_{1i}} \right) \tag{60}$$

It is assumed that the fission gas is 84.5% Xe and 15.5% Kr.

6.3. Pressure of bonding gas

The quantity of bonding gas is:

$$H = H_o + F \tag{61}$$

The state of the gases being far from critical point, the Boyle-Marioette's law can be applied. For non-uniform temperature, it is:

$$p_i \int \frac{dV}{T} = 3.78.10^{-3}\ H \tag{62}$$

where dV is a volume element at temperature T and the integral has to be taken over the whole disposable space in the rod.

For the radial gap of one axial section, one obtains with

$$dV = \frac{1}{n}\ \pi\ (r_2 + r_3)\ dj_r$$

and (from equation (5) by differentiation)

$$T^{c_2}\ dT = \frac{1}{c_1} \cdot \frac{q}{\pi\ (r_2+r_3)}\ dj_r$$

the equation:

$$\frac{dV}{T} = \frac{1}{n} \cdot \frac{\pi^2}{q}\ (r_2 + r_3)^2\ c_1\ T^{c_2-1}\ dT$$

and, integrating from T_2 to T_3:

$$\int_{gap} \frac{dV}{T} = \frac{1}{n} \frac{\pi^2}{q}\ (r_2+r_3)^2\ \frac{C_1}{C_2}\ (T_3^{\ c_2} - T_2^{\ c_2}) \tag{63}$$

The gas plenum at the top of the rod is:

$$V_t = V_p + j_a^*\ \pi\ r_2^{\ 2} \tag{64}$$

Its temperature is supposed to be:

$$T_t = \left(\frac{T_f + T_c}{2} \right)_{i\ =\ n} \tag{65}$$

Furthermore, a chamber beyond the rod with the volume V_c and the temperature equal to the coolant temperature at the upper rod section may be taken into account.

With these volumes, one obtains from equation (62) the pressure of the bonding gas:

$$p_i = \frac{3.78 \cdot 10^{-3} \cdot H}{\frac{V_t}{T_t} + \left(\frac{V_c}{T_c}\right)_{i=n} + \sum_{i=1}^{n} \int_{gap} \frac{dV}{T}} \qquad (66)$$

7. Properties of fuel and sheath

7.1. Relations

Thermal conductivity

The thermal conductivities of fuel and sheath are considered to be independent of temperature.

Thermal expansion coefficients

It is supposed that the average expansion coefficients between 0 °C and T °C are:

fuel: $$\lambda_f \Big|_o^T = g_1 + g_2 T$$

sheath: $$\lambda_s \Big|_o^T = a_1 + a_2 T$$

The average expansion coefficients between ambient and operation temperature are then:

$$\lambda_f = g_1 + g_2 (T_f^* + T_a^*) \qquad (67)$$

$$\lambda_s = a_1 + a_2 (T_s^* + T_a^*) \qquad (68)$$

The true expansion coefficient of sheath is:

$$\lambda_s^* = \frac{d\,(g_1 T_s^* + g_2 T_s^{*2})}{d\,T_s^*} = g_1 + 2\, g_2\, T_s^* \tag{69}$$

Mechanical properties of sheath

Young's modulus:

$$E = b_1 - b_2\, T_s \tag{70}$$

Mechanical resistance:

$$R^* = h_1 + h_2\, T_s + h_3\, T_s^2 \tag{71}$$

R^* is employed for the calculation of the security coefficient against sheath rupture.

Apparent diffusion coefficient of fission gases in the fuel:

$$D' = D'_o \ \exp\ \left(- \frac{Q^*}{RT} \right) \tag{72}$$

7.2. Constants for UC and SAP

The constants given here, being input data, can be changed without changing the code.

The constants concerning the irradiation behavior of UC have been adjusted to irradiation data of SAP-clad UC rods (see section 8). By that means, the error due to simplified calculation models and inexact constants is minimized, because the different inaccuracies largely compensate each other.

Consequently, a set of so adjusted constants is to be considered as an assembly of which no constant may be changed independently. If, for instance, the thermal conductivity would be changed, swelling and diffusion constants would have to be re-adjusted.

The following list gives the values used for cast near stoichiometric UC and for SAP 7%:

Thermal conductivities

$k_f = 0.173$ W/cm °C (see section 8)

$k_s = 2.03$ W/cm °C [19]

Thermal expansion coefficients:

$g_1 = 9.8 \cdot 10^{-6}$ $g_2 = 1.3 \cdot 10^{-9}$ [18]

$a_1 = 20.38 \cdot 10^{-6}$ $a_2 = 9 \cdot 10^{-9}$ [19]

Young's modulus of SAP [19] :

$b_1 = 8950$ $b_2 = 5.925$

Mechanical resistance of sheath

The creep rupture resistance is utilized [19]

$h_1 = 50.15$ $h_2 = - 0.107$ $h_3 = 6.2 \cdot 10^{-5}$

Apparent diffusion coefficient (see section 8)

$D_o' = 0.0045$ $\sec^{-1}$

$Q^* = 60.000$ cal/mole

Irradiation swelling of UC (see section 8)

$C_1 = 7 \times 10^{-7}$ tU/MWd $\quad C_2 = 1.8 \times 10^{-3}$ tU/MWd $\quad C_3 = 11000$ °K

8. Adjustment of constants

There was considerable uncertainty about some constants concerning the irradiation behavior of UC. These constants were adjusted by application of the code to the data of the NRX-721 experiment.

8.1. The NRX-721 experiment [20]

This experiment was chosen because it was the most representative one for the conception and the operating conditions of ORGEL fuel elements.

The NRX-721 rods consist of cast uranium carbide fuel (14 mm diameter, 180 mm length), argon bonding and finned SAP sheath.

In phase I, eight rods were irradiated up to a maximum burn-up of 8,000 MWd/t. After non-destructive examination, half of these rods were irradiated in phase II, together with two new rods containing central thermocouples. This second phase was terminated after indication of a sheath rupture at a maximum burn-up of 11,000 MWd/t.

8.2. Application of X-721 data to the code

From chemical analysis, lower carbon content was given for the rods FMA, FMF and FMR ($C = 4.66 \div 4.67$ wt%) than for the other rods ($C = 4.73 \div 4.83$ wt%). As the ORGEL fuel will be hyper-stoichiometric ($C + N_2 + O_2 = 4.8 \div 4.95$ wt%) and as it is believed that free uranium metal enhances irradiation swelling, the rods FMA, FMF and FMR were not considered for the adjustment calculation.

The variation of irradiation conditions - as coolant temperature, heat rating, etc. - being small along each rod, the calculation was done with length-averaged values.

Time-averaged values were employed for each of the irradiation phases I and II. After each irradiation phase, a cold phase - with zero-power and 45°C temperature - was included into the calculation, in order to take into account the fuel/sheath interaction, due to different thermal expansion, which may occur between irradiation and examination.

8.3. The adjustment calculation

The criteria for the adjustment of code constants were that experimental values and calculated values of the following parameters were as near as possible:

- Linear irradiation swelling of fuel ($\Delta D/D$). 1/3 of volumetric swelling from post-irradiation data was taken as reference. This value showed less scatter than dimensional measurements and was therefore thought to be more reliable.
- Diameter change of sheath.
- Fission gas release.

Question was of the following constants:

- in-pile thermal conductivity k_f;
- constants concerning the irradiation swelling: C_1, C_2 and C_3 in equation (39);
- constants concerning the fission gas release D_o' and Q^* in equation (72).

First, a calculation was done with estimated values. Then, comparing the results of this calculation with the adjustment criteria from post-irradiation data, new values were guessed and tried, and so on, until no further improvement of accordance seemed possible.

In this manner, only few constants could be modified for each new trial. At the beginning, the most uncertain values - i.e. C_3, C_4 and D_o' - were adjusted. Then it was tried to adjust also the other constants, but no better accordance was obtained by the few trials being done. So, the initially estimated values were conserved for these constants.

The following values were retained:

k_f = 0.19 W/cm °C (estimated from literature)

C_2 = 7×10^{-7} tU/MWd (estimated from volume of solid fission products)

C_3 = 1.8×10^{-3} tU/MWd (adjusted)

C_4 = 11000 °K (adjusted)

D_o' = 4.5×10^{-3} $\sec^{-1}$ (adjusted)

Q = 60,000 cal/mole (estimated from literature)

The relation for swelling which is obtained with the retained constants is represented in Fig. 4.

Table 1 gives the measured and the calculated values of the adjustment criteria (swelling, sheath deformation, fission gas release). Fig. 5 gives the measured and the calculated fuel center temperature of the rod FMS.

It is thought that the following reasons contribute significantly to the differences:

- Inexact input data (see section 8.4.)
- Measurement error of post-irradiation data
- Variation of fuel composition

The calculated plastic sheath deformation at the sheath rupture of rod FMB is 0.55% (averaged on the sheath circumference). This value is recommended as sheath rupture criterion.

8.4. New data

At the time when the adjustment was done, some data were not given and had to be estimated by us. Meanwhile, further information is available. Several new data differ from the data utilized in the adjustment calculation. The values for which this difference seems to be significant are discussed in the following. It is felt that an adjustment to the new data would somewhat improve the agreement of code and experience, but would not change much the adjusted constants.

Flux depression coefficient ξ'

Value utilized in the adjustment calculation: $\xi' = 0.91$
New value [20] : $\xi' = 0.83$

From equation (14), it can be seen that the same results T_4 (and, in consequence, same fuel swelling) are obtained with:

$$\xi' = 0.91 \; ; \; k_f = 0.19 \text{ W/cm °C}$$

(values utilized in the adjustment calculation), and with:

$$\xi' = 0.83 \; ; \; k_f = 0.173 \text{ W/cm °C}$$

It is therefore proposed to utilize the value k_f = 0.173 W/cm °C together with the adjusted constants given in this report. This value is also within the range of measured data given in the literature.

Heat-rating and burn-up of the lower rods (FMB, FMK, FMT)

The following table compares the utilized data (A) with the new values (B) from [20].

Rod	Heat-rating (W/cm) Phase I A	Phase I B	Phase II A	Phase II B	Total burn-up (MWd/t) A	B(1)
FMB	1042	1024	795	966	11200	11800
FMK	1004	948	654	895	10400	10850
FMT	769	649	289	612	7060	7360

Fuel-sheath clearance of rod FMS

Value utilized in the adjustment calculation : $\varepsilon = 68\,\mu$

New value : $\varepsilon = 52\,\mu$

The utilization of the new value would decrease the calculated fuel center temperature. This may indicate that the thermal conductivity of the fuel was lower than 0.173 W/cm°C.

(1) The burn-up values given in [20] are thermal MWd/t. The values given here are values from [20] multiplicated by 1.06, in order to give fission MWd/t.

9. List of symbols

A	ratio of fission gas production to fission energy production [cm^3 NTP/W sec]
a_1, a_2	constants for λ_s
b_1, b_2	constants for E
C_1, C_2, C_3	constants for S
c_1, c_2	constants for k_g
D'	apparent fission gas diffusion coefficient [sec^{-1}]
D'_o	constant for D' [sec^{-1}]
d	height of fins [cm]
E	Young's modulus of sheath [kg/mm^2]
E'	average energy released per fission [W sec/fission]
e_1	width of fins [cm]
e_2	distance between fins [cm]
F	quantity of released fission gas [cm^3 NTP]
F_o	self-shielding coefficient [-]
f	fraction of fission gas released [-]
G	quantity of fission gas produced in a section [cm^3 NTP]
g_1, g_2	constants for λ_f
h_1, h_2, h_3	constants for R^*
H	quantity of bonding gas [cm^3 NTP]
H_o	quantity of initial gas charge [cm^3 NTP]
I_o, I_1	Bessel functions of imaginary argument
j	initial radial clearance at ambient temperature [cm]
j^*	radial clearance during operation [cm]
j_a	initial axial clearance at ambient temperature [cm]
j_a^*	axial clearance during operation [cm]

j_r	distance from inner sheath surface in the gap [cm]
j_o	criterion for contact between fuel and sheath [cm]
K_B	Boltzmann's constant [1.38×10^{-23} W sec/ °K]
k_f	thermal conductivity of fuel [W/cm °K]
k_g	thermal conductivity of bonding gas [W/cm °K]
k_s	thermal conductivity of sheath [W/cm °K]
l	fuel length (1) [cm]
M	molecular weight [g/mole]
N	Avogadro's number [$6.03 \cdot 10^{23}$ atoms/mole]
n	number of axial sections [-]
P_e	pressure of the coolant [kg/cm^2]
P_i	pressure of the bonding gas [kg/cm^2]
Q	externally applied axial compressive load [kg]
Q^*	activation energy for fission gas diffusion [cal/mole]
q	linear heat rating [W/cm]
q_v	heat generation density [W/cm^3]
q_{vo}	heat generation density at the fuel center [W/cm^3]
R	gas constant [1.98726 cal/mole °K]
R^*	mechanical resistance of sheath [kg/mm^2]
r	distance from rod axis [cm]
r_1	outer radius of sheath (1) [cm]
r_2	inner radius of sheath (1) [cm]
r_3	fuel radius (1) [cm]
S	irradiation swelling of fuel [-]

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(1) initial values at ambient temperature

T	temperature [°K]
T_1	outer surface sheath temperature [°K]
T_2	inner surface sheath temperature [°K]
T_3	fuel surface temperature [°K]
T_4	fuel center temperature [°K]
T_a	ambient temperature [°K]
T_c	coolant temperature [°K]
T_f	average fuel temperature in a section [°K]
T_s	average sheath temperature in a section [°K]
T_t	average temperature of gas plenum at the top of the rod [°K]
T^*	reduced temperature [-]
T_a^*	ambient temperature [°C]
T_f^*	average fuel temperature in a section [°C]
T_s^*	average sheath temperature in a section [°C]
t	time [sec]
V	volume disposable to bonding gas [cm^3]
V_c	volume of gas chamber beyond the rod [cm^3]
V_p	gas volume in the upper end-plug [cm^3]
V_t	gas volume at the top of the rod [cm^3]
y_s	yield of a fission gas per fission [atoms/fission]
α	sheath coolant heat transfer coefficient [W/cm^2 °K]
β_i	atomic fraction of a gas on total bonding gas [-]
ε	equivalent radial clearance allowing for surface roughness [cm]
ε_f	overall thermal expansion ratio of fuel [-]
ε_s	overall thermal expansion ratio of sheath [-]
ε^*	maximum attraction energy between colliding gas a'

ζ	elasticity limit of sheath material [-]
η	fin efficiency [-]
K	reciprocal of effective neutron diffusion length in the fuel [cm^{-1}]
λ_f	average expansion coefficient of fuel from T_a^* to T_f^* [$°K^{-1}$]
λ_s	average expansion coefficient of sheath from T_a^* to T_s^* [$°K^{-1}$]
λ_s^*	real expansion coefficient of sheath at T_s^* [$°K^{-1}$]
μ	fin coefficient (surface ratio)[-]
ν	Poisson's ratio [-]
ξ, ξ', ξ''	coefficients concerning the temperature distribution in the fuel [-]
σ	low velocity collision diameter [Å]
σ_a	axial sheath stress at the ends of the rod [kg/mm^2]
σ_t	tangential sheath stress between fins [kg/mm^2]
σ_{t1}	tangential sheath stress on fins [kg/mm^2]
σ_{th}	sheath stress due to radial temperature gradient [kg/mm^2]
$\bar{\sigma}_a$	average axial sheath stress [kg/mm^2]
$\bar{\sigma}_t$	average tangential sheath stress [kg/mm^2]
τ	burn-up (fission energy per ton Uranium) [MWd/t]
Υ	ratio of total fission energy to thermal energy released in the fuel [-]
$\Omega^{(2,2)*}$	reduced collision integral [-]

Indices:

i	index for axial section
j	index for time interval

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TABLE 1

Experiment NRX-721

Comparison of measured and calculated values

	Rod	Heat rating w/cm	Final burn-up MWd/tU	Swelling ΔV/3V % m	Swelling ΔV/3V % c	Sheath deformation* μ m	Sheath deformation* μ c	Fission gas release cm3 NTP m**	Fission gas release cm3 NTP c
Phase I	FMH	911	7,000	0.95	1.17	23.5	15.7	0.84	1.38
Phase I	FMD	1,004	7,780	1.08	1.10	33.0	29.4	0.92	0.80
Phase I	FMJ	1,042	8,014	0.90	1.16	33.0	34.8	1.23	0.84
Phase I	FMB	1,042	8,014	n.m.	1.12	20.3	33.0	n.m.	0.76
Phase I	FMK	1,004	7,780	n.m.	1.11	12.1	27.5	n.m.	0.86
Phase I	FMT	769	5,900	n.m.	0.68	0	0	n.m.	0.34
Phase II	FMB	795	11,200	1.81	1.40	45.7	48.9	n.m.	1.06
Phase II	FMK	654	10,400	1.31	1.29	33.0	40.0	0.87	1.16
Phase II	FMT	289	7,060	0.82	0.75	16.5	3.7	0.64	0.41
Phase II	FMS	982	3,700	0.90	0.89	20.3	0	n.m.	0.79

m = from post-irradiation measurements

c = calculated by the code

n.m. = not measured

* radial; elastic + plastic deformation; after cooling of the rod to 45°C

** total gas (measured) minus filling gas (calculated from initial dimensions)

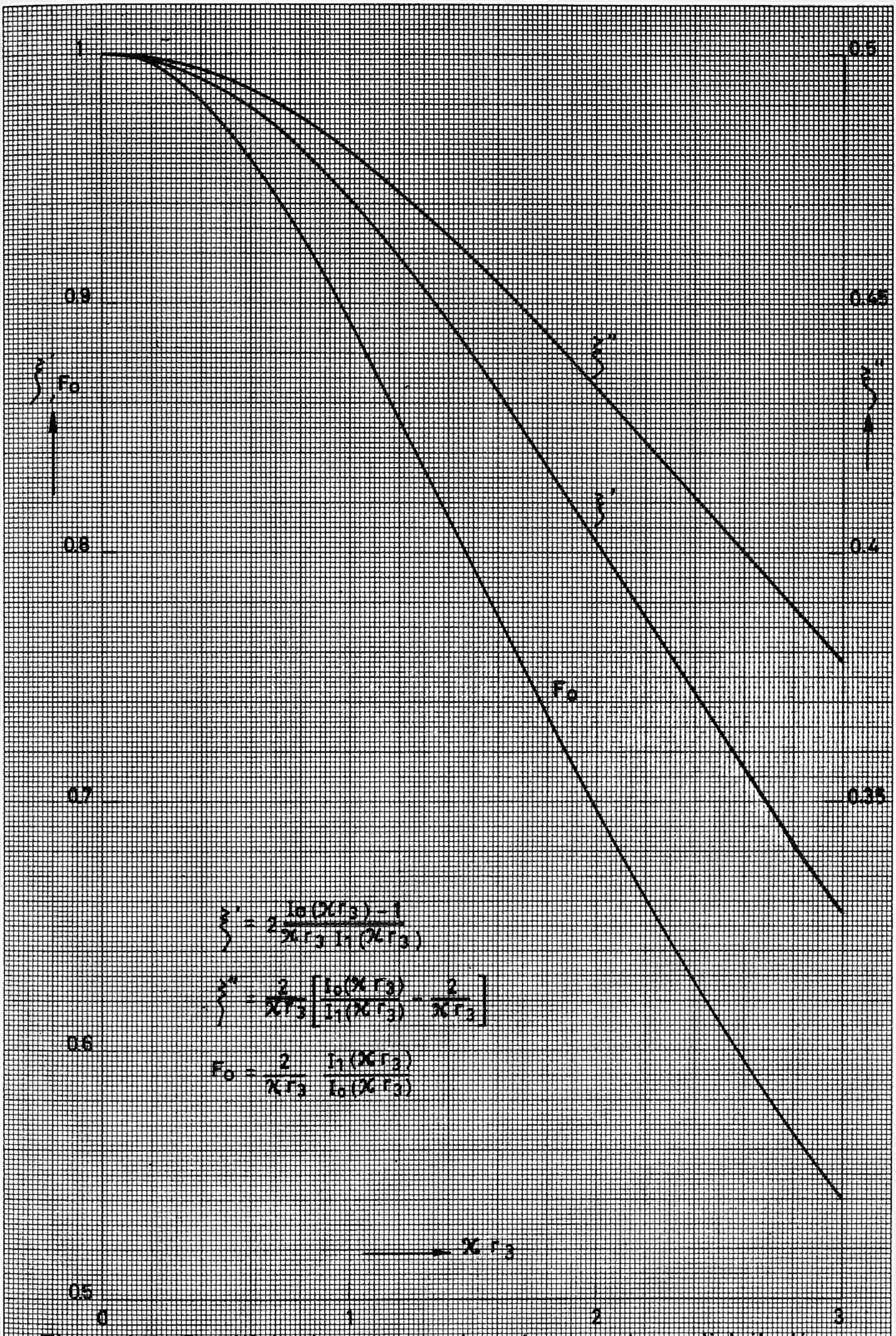

Figure 1 Coefficients concerning temperature distribution in the fuel

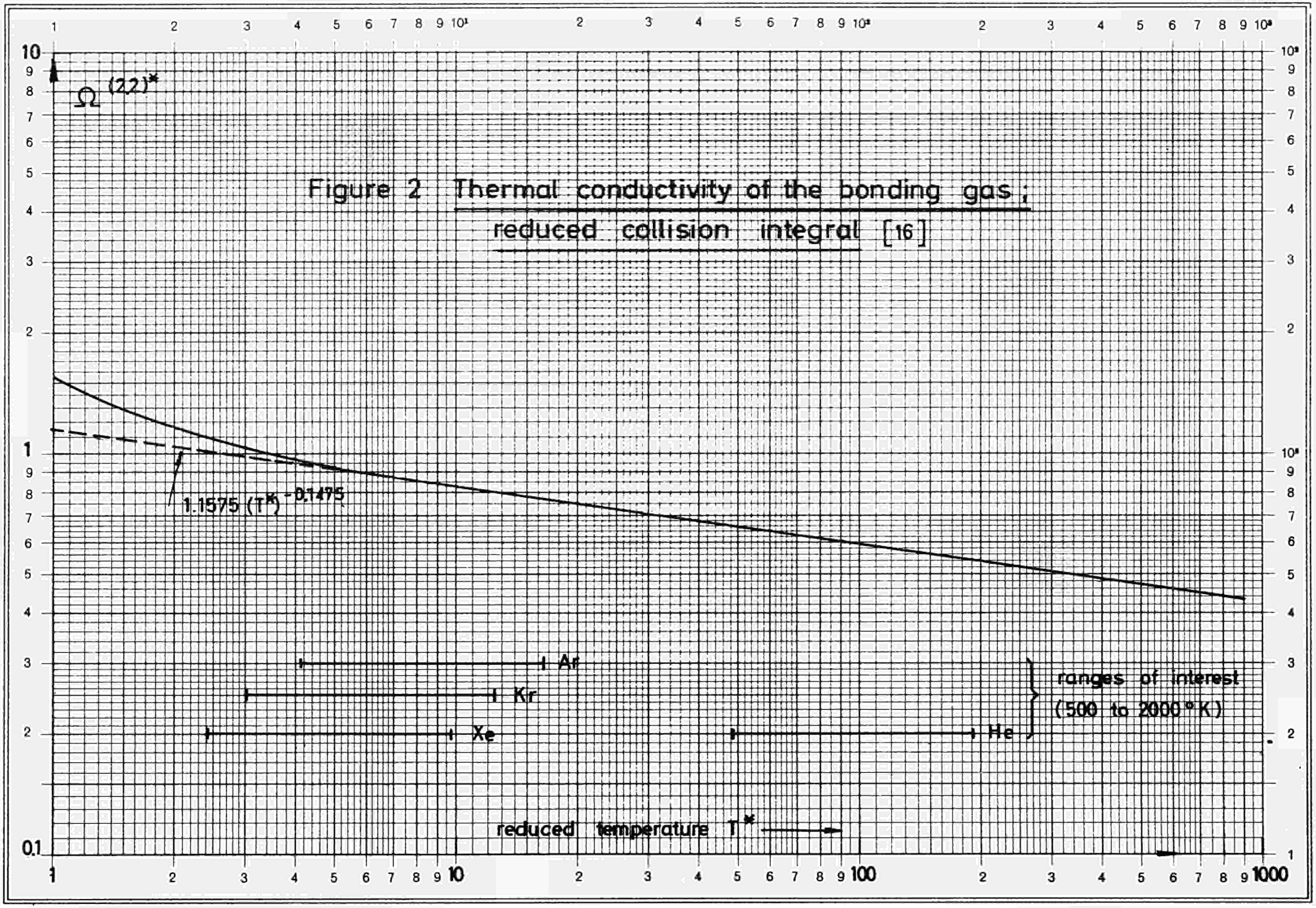

Figure 2 Thermal conductivity of the bonding gas; reduced collision integral [16]

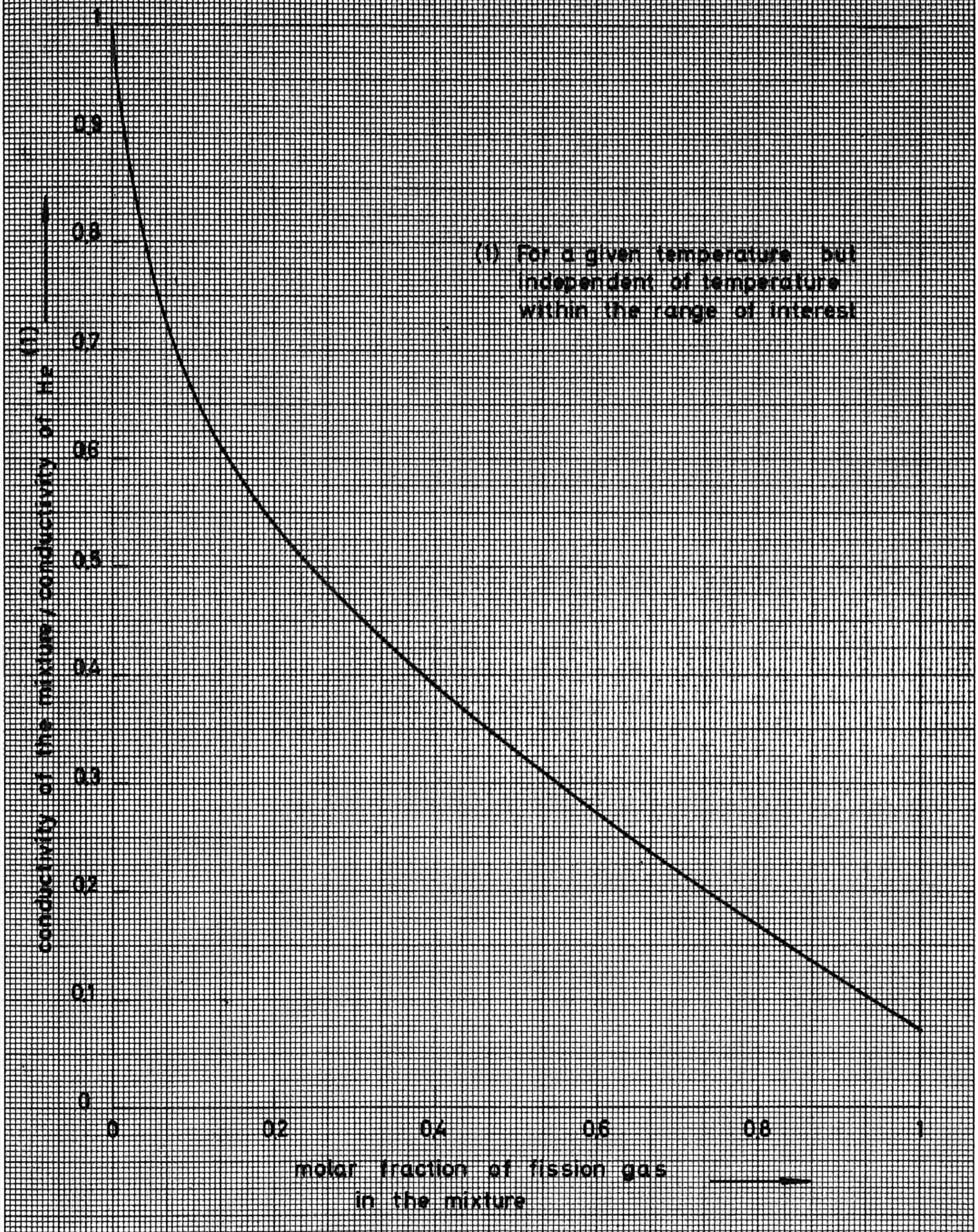

Figure 3 Thermal conductivity of a mixture of He and fission gas

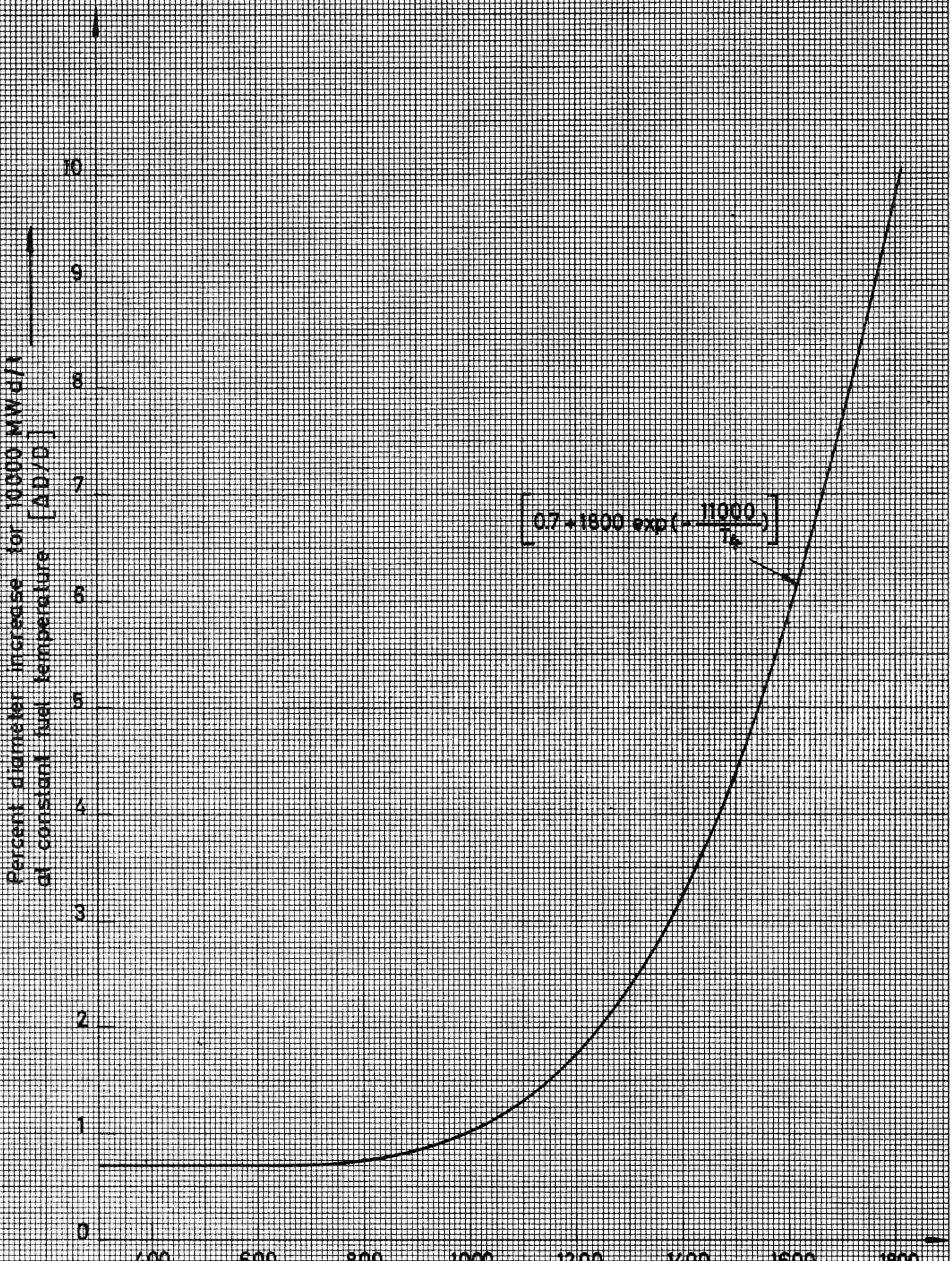

Figure 4 Irradiation swelling of UC

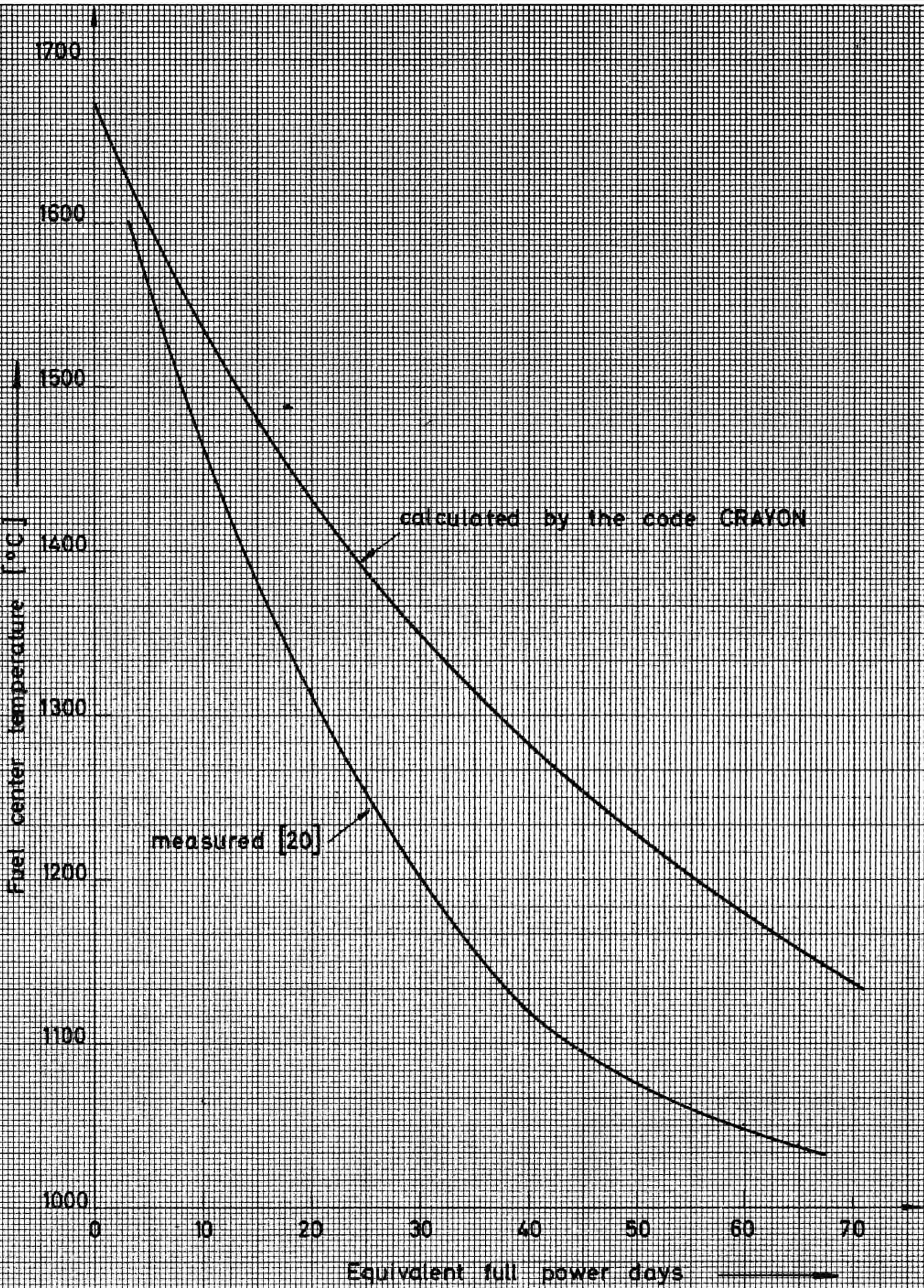

Figure 5 Comparison of measured and calculated fuel center temperature of rod FMS

To disseminate knowledge is to disseminate prosperity — I mean general prosperity and not individual riches — and with prosperity disappears the greater part of the evil which is our heritage from darker times.

Alfred Nobel

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