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EUROPEAN ATOMIC ENERGY COMMUNITY — EURATOM

**A DYNAMIC MODEL FOR THE COOLING CHANNELS
OF A BOILING NUCLEAR REACTOR WITH
FORCED CIRCULATION AND HIGH PRESSURE LEVEL**

by

G. FORTI

1968



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Joint Nuclear Research Center
Ispra Establishment - Italy

Reactor Physics Department
Reactor Theory and Analysis

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European Atomic Energy Community — EURATOM
Joint Nuclear Research Center — Ispra Establishment (Italy)
Reactor Physics Department — Reactor Theory and Analysis
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The model is one-dimensional, and includes treatment of subcooled boiling and liquid superheating. It includes a new form of boiling heat transfer correlations and a correction of Bankoff's slip correlation for diabatic conditions.

In the present formulation the model is limited to constant pressure level of the whole system, but an extension to slowly variable pressure is easy in principle.



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Summary

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The model is one-dimensional, and includes treatment of subcooled boiling and liquid superheating. It includes a new form of boiling heat transfer correlations and a correction of Bankoff's slip correlation for diabatic conditions.

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KEYWORDS

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COOLANT LOOPS

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SUBCOOLED BOILING
SUPERHEATING
TIME
SLIP
PRESSURE

A DYNAMIC MODEL FOR THE COOLING CHANNELS
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CIRCULATION AND HIGH PRESSURE LEVEL (+)

1) Scope of the model

The purpose of this work is to build a physical model for the dynamic behaviour of the cooling channels of boiling nuclear reactors or experimental loops.

The model will be used for numerical calculations of transients in the time domain, and should possibly be coupled to spatial neutron kinetics calculations in one or two dimensions.

For this purpose we tried to simplify the representation of phenomena as much as possible without losing the significance of the results. Therefore we introduce the following fundamental assumptions:

a) Every channel is treated in one spatial dimension.

Equations will be established for average quantities over the cross section of the channel. This implies, in case of heating elements arranged in bundles, that phenomena are well represented by means of an average channel, or alternatively of some parallel channels without mixing coupling.

b) We admit that the liquid is incompressible, as well as the vapour, which is presumed to be saturated everywhere, and we neglect the hydrodynamic work in the energy balance in comparison with the heat transmission and internal energy terms.

The assumption of incompressibility brings to neglect all the phenomena of sound and shock waves propagation. The model is therefore not conceived for treatment of very fast transients in which pressure waves may have an essential importance. Neglecting of hydrodynamic work is generally well justified in practical applications.

These assumptions allow the simplification essential to the model, i.e. decoupling of the momentum equation from the energy and mass continuity equations. The momentum equation can thereby be integrated in space over the channel giving at any time, through the inlet velocity, the boundary condition for the system of the remaining equations.

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- c) All the physical constants of the coolant are assumed to be space and time independent.

The model will therefore permit the analysis of transients in which the inlet temperature of the coolant is not very far from saturation and the pressure drop along the channel may be considered small in comparison with the pressurization level. This is generally pretty well verified in forced circulation systems, and may sometimes be admitted also for natural circulation systems.

The time independence assumption prevents the treatment of pressure loss accidents. This limitation is not essential and could be removed pretty easily. We admit it here for the sake of simplifying the model as much as possible.

We thought it important to include in the model a detailed representation of subcooled boiling as it is known that the void fraction in the subcooled zone may reach high values in practical cases.

2) Fundamental variables and equations of the model.

With the assumptions stated, and for simplicity's sake we will take as zero temperature the saturation temperature and zero enthalpy the saturated liquid enthalpy.

We will assume further that the channel have constant cross section A and heated perimeter p . Indicating by ϕ the heat flux through the heated perimeter, the power density (averaged over cross section) reaching the coolant is given by

$$Q = \frac{p}{A} \phi + Q_{dir}$$

where Q_{direct} is the power density directly added to the coolant by neutron moderation and radiation absorption.

We will indicate by Ψ the source of vapour volume (averaged over the cross section) i.e. the vapour volume generated per unit volume ^{and time} over the whole cross section of the channel.

Leaving aside for the instant the momentum equation, which will be decoupled from the others because of our assumptions, we shall refer all the balance equation to the unit cross section.

Our fundamental variables will be:

w = total surface velocity or flow rate. It is the total volumetric flow divided by the area of the channel cross section.

q_v = vapour surface velocity or vapour flow rate. It is the vapour volumetric flow divided by the cross section area

α = void fraction.

Ratio of the vapour volume to the total volume at a given axial location.

H = enthalpy of the liquid.

In terms of the more commonly employed variables v_v = average velocity of vapour and v_l = average velocity of liquid, our variables are given by

$$w = \alpha v_v + (1-\alpha)v_l$$

$$q_v = \alpha v_v$$

The present choice of the variables have the advantage of giving the simplest expression to the conservation equations and permits an unambiguous formulation excluding any approximation. For a thorough discussion of the matter we refer the reader to the work of Zuber (Ref.1).

Let us further indicate by $q_l = w - q_v$ the surface velocity of the liquid, by ρ the liquid density, by γ the ratio between vapour and liquid densities and by λ the latent heat of vaporization; the z coordinate axis will be oriented in the direction of the flow.

In the assumption of incompressibility, the total volume continuity equation is written

$$(1) \quad \frac{\partial w}{\partial z} = (1-\gamma)\nabla$$

as $(1-\gamma)\nabla$ is the volume variation due to vaporization.

The vapour volume continuity equation reads

$$(2) \quad \frac{\partial \alpha}{\partial t} = \Psi - \frac{\partial q_V}{\partial z}$$

Finally the last equation which may be derived from first principles is the energy conservation equation: indicating by E the total energy contained in the unit volume and by L the energy flow through unit section, it may be written

$$\frac{\partial E}{\partial t} = Q - \frac{\partial L}{\partial z}$$

with the assumptions already mentioned we may substitute

$$E = \rho H(1-\alpha) + \rho \gamma \lambda \alpha$$

$$L = \rho H q_1 + \rho \gamma \lambda q_V$$

dividing by ρ and replacing $q_1 = w - q_V$ and $\frac{\partial \alpha}{\partial t}$ and $\frac{\partial w}{\partial z}$ from equations (1) and (2), we will finally get:

$$(3) \quad (1-\alpha) \frac{\partial H}{\partial t} = \frac{Q}{\rho} - \gamma (\lambda - H) \Psi - (w - q_V) \frac{\partial H}{\partial z}$$

3) Nature of the empirical correlations of the model

The equations (1) (2) (3) are the fundamental equations of the model directly deductible from first principles in the assumptions stated.

We have 3 equations for the 4 unknown variables α , w , q_V , H , and therefore the dynamic system is not fully determined, although we consider Ψ and Q as known quantities. The total momentum equation cannot be used to complete the set of equations as it will introduce the pressure as a new variable. In our assumptions the momentum equation will be decoupled from the others and may be separately integrated in space to give inlet acceleration of coolant.

What is really needed to determine completely the system is the motion equation of the single vapour (or liquid) phase which will give us a connection between the two velocities w and q_V . But as nothing is known about the momentum exchange between the phases,

the only way out is to assume an empirical correlation which will tie together the two velocities with the void fraction α as a parameter. Such a correlation is commonly called slip correlation as it determines the Slippage between the two phases. In the literature many forms of this correlation may be found; it is however a priori clear that no correlation may have general validity, as its form should depend on the type of flow regime in the system.

A satisfactory solution of the problem will be got only when the character of every type of flow regime will be specified together with the transition conditions from one to the other type.

This is not even foreseen presently, and all what can be done is to specify a recipe which will have a limited range of validity and should be selected in relation with the practical problems that have to be treated.

Another empirical correlation is needed to specify the source Ψ of vapour volume. If it could be assumed that everywhere the liquid and vapour are at equilibrium, the problem will be solved easily, but actually it is necessary to take into account the lack of equilibrium at least in the subcooled zone, in which it is known that the void fraction may reach significant values.

We shall therefore consider the source term Ψ formed by the two addends Ψ_s , source at the heating surface, and Ψ_b which is the bulk source term and may be negative if recondensation take place.

If we want to force thermal equilibrium in the bulk boiling zone ($H = 0$) we will take $\Psi = \frac{0}{\rho\gamma\lambda}$ eliminating thus the energy equation (3).

The simplest choice for Ψ_b in the subcooled zone will be $\Psi_b = 0$, assuming that the bubbles formed at the surface have no time to collapse significantly in the bulk of the coolant.

A more general form for Ψ_b will correlate it to the enthalpy (or temperature) difference from saturation H and the void frac-

tion α (α is of course related to the boundary surface between phases). The Ψ_s term will be correlated to the heat flux which is due to the boiling transfer mechanism.

Further correlations are needed to evaluate the heat flux ϕ . In stationary problems the heat flux is a datum, and correlations are only needed to evaluate the heating surface temperature in terms of the heat flux and bulk coolant temperature. This is of course no more true for dynamics where heat flux is an unknown variable. As in practical cases the heat flux may vary in time much more abruptly, due to change of surface resistance, than the surface temperature, which has a time constant related to its heat capacity, it will be convenient to take surface temperature as the independent variable for the correlations, as well as for expressing the transition conditions from convective to boiling heat transfer regime.

4) Correlations of the model

We give here the correlations adopted in our model that are discussed in detail in the appendixes.

a) Slip correlation (see appendix A)

$$q_v = \frac{\alpha}{K} w - \frac{z_e}{K} \Psi_s$$

K is Bankoff (Ref.2) constant and depends on the pressure. For the water it can be taken following Jones (Ref.3)

$$K = 0.71 + 0.29 \frac{P}{P_{cr}}$$

z_e is a new parameter, which has the dimensions of a length and should be fixed experimentally. Its order of magnitude is the same as the hydraulic diameter of the channel and it represents physically a relaxation length for the establishment of void asymptotic profile for bubbles of vapour generated at the heating surface.

b) Heat transmission to coolant (see appendix B)

$$\phi = h(T_s - T) \quad T_s^2 \ll \theta (T_s - T)$$

$$\left\{ \begin{array}{l} \phi = \phi_b + \phi_c \\ \phi_b = h' T_s^n \quad \text{for } T_s^2 > \theta (T_s - T) \\ \phi_c = (h(T_c - T) - h' T_c^n) \left(1 - \frac{T_s - T_c}{\theta_f - \theta}\right) \geq 0 \\ T_c = \frac{\theta}{2} \left(1 + \sqrt{1 - \frac{4T}{\theta}}\right) \quad \theta_f = 1.4^{1/n} \left(\frac{h}{h'}\right)^{1/n-1} \end{array} \right.$$

h is the convective heat transfer coefficient which for turbulent flow is given by the well known formula

$$\frac{hD}{K} = \text{Nusselt} = \text{constant} \times \text{Reynolds}^{0.8} \times \text{Prandtl}^{0.4}$$

Many expressions are given in the literature for h'. The inversion of the given formulae is generally needed for deriving h' from the values given in the references (see for instance Ref.4.5) while the exponent n has a value between 3 and 4.

The parameter θ which has the dimensions of a temperature and physically is interpreted as the temperature difference between heating surface and saturation at the inception of nucleation in saturated liquid. The value of θ may be fixed by experience or derived from theory (Ref.6) as
$$\theta = \frac{8\sigma}{K \frac{\partial p_{\text{sat}}}{\partial T}} h$$

As explained in Appendix B, we prefer the expression

$$\theta = \left(\frac{h}{nh'}\right)^{\frac{1}{n-1}}$$

c) Vapour source correlations (see appendix c)

$$\Psi = \Psi_s + \Psi_b$$

$$\Psi_s = \tau \frac{P}{A} \frac{\phi b}{\rho \gamma \lambda} \quad \tau = \frac{1}{1 + \epsilon}$$

$$\Psi_b = \begin{cases} R_0 \alpha T & T < 0 \\ R_1 \alpha T & T > 0 \end{cases}$$

According to Bowring (Ref. 7) ϵ is a positive constant which depends only on the pressure.

Values of the parameters R_0 and R_1 are not available at present even as a guess. According to Bowring, in practical application R_0 can be taken as 0 (no recondensation in subcooled zone). A first guess for R_1 could be infinite, i.e. equilibrium in the bulk boiling zone with no superheating of liquid. In this case equation (3) should be replaced by

$$H = 0$$

$$\Psi = \frac{Q}{\rho \gamma \lambda}$$

d) Finally, as first approximation the relation between T and H may be simply taken as $H = C_p T$ with a C_p kept constant at saturation value as long as the inlet subcooling is not too large.

5) Momentum equation

Let us write the motion equation for the fluid in the form

$$A \frac{dG}{dt} = -A \frac{dp}{dz} - A \bar{\rho} g - f$$

where $\bar{\rho}$ is the average density of the fluid, g is the gravity acceleration in the direction opposite to the flow and f the friction force.

G is here the momentum of fluid per unit cross section and is expressed in terms of our variables as

$$G = \rho q_\rho + \rho \gamma q_V = \rho (w - (1 - \gamma) q_V)$$

Passing from Lagrange to Euler representation we have

$$\frac{dG}{dt} = \frac{\partial G}{\partial t} + \frac{\partial}{\partial z} L_G$$

with L_G we indicate the momentum flow per unit cross section.

Integrating the motion equation over the whole channel (and dividing by the cross section A)

$$P_{in} - P_{out} - \Delta p_f - \Delta p_g = \int_{channel} \frac{\partial G}{\partial t} dz + L_{G_{out}} - L_{G_{in}}$$

where Δp_f is the pressure loss due to friction, Δp_g is the gravity head $\bar{\rho}gL$ and $\Delta p_a = L_{G_{out}} - L_{G_{in}}$ is the so called acceleration pressure loss. We shall rewrite the equation in the form

$$\frac{\partial}{\partial t} \int_{channel} G dz = \Delta p - \Delta p_g - \Delta p_f - \Delta p_a$$

The actual pressure drop Δp over the whole channel will be imposed by the external conditions of the loop. The gravity head is easily calculated at each instant of time. The friction pressure drop must be evaluated as function of local flow rate. It is normally expressed by

$$\Delta p_f = \int_{channel} R_f K_f \rho \left(\frac{G}{\rho}\right)^2 dz$$

K_f is here the friction factor for the liquid alone, while R_f is the so called two phase friction factor and may be evaluated by the well known Lockhart and Martinelli correlation (Ref. 8) or any other of the many existing correlations (see Ref. 4,5).

The acceleration pressure drop is impossible to evaluate exactly, because the flow of momentum at a give section depends on the velocity profiles in the section itself.

For pure liquid we have:

$$L_G = \epsilon \rho w^2 = \epsilon \frac{G^2}{\rho}$$

with $\epsilon = \frac{\langle v_z^2 \rangle}{\langle v_z \rangle^2}$ a weighing factor not far from 1 for turbulent

flow, because the profile is markedly flat.

For two phase flow,

$$L_G = \rho \left(\epsilon_l \frac{q_l^2}{(1-\alpha)} + \epsilon_v \gamma \frac{q_v^2}{\alpha} \right)$$

with ϵ_l and ϵ_v weighing factors for liquid and vapour that, assuming no local slip, may be expressed as:

$$\epsilon_v = \frac{\langle \alpha v_z^2 \rangle \langle \alpha \rangle}{\langle v_z \rangle^2} \quad \epsilon_l = \frac{\langle (1-\alpha) v_z^2 \rangle \langle 1-\alpha \rangle}{\langle (1-\alpha) v_z \rangle^2}$$

If the flow is highly turbulent for both phases (or for the mixture), so that the velocity profile is nearly flat, the weighing factors are not far from 1.

We have, now

$$G = \rho (w - (1-\gamma)q_v)$$

and therefore

$$\frac{\partial G}{\partial z} = \rho \left(\frac{\partial w}{\partial z} - (1-\gamma) \frac{\partial q_v}{\partial z} \right)$$

and, replacing $\frac{\partial w}{\partial z}$ from equation (1)

$$\frac{\partial G}{\partial z} = \rho (1-\gamma) \left(\Psi - \frac{\partial q_v}{\partial z} \right)$$

or, by equation (2)

$$\frac{\partial G}{\partial z} = \rho (1-\gamma) \frac{\partial \alpha}{\partial t}$$

integrating over z

$$G = G_{inlet} + \rho (1-\gamma) \left(\int_0^z \Psi dz - q_v(z) \right)$$

$$\text{or } G = G_{inlet} + \rho (1-\gamma) \int_0^z \frac{\partial \alpha}{\partial t} dz$$

replacing G in the left hand side of momentum equation

$$\frac{\partial}{\partial t} \int_{\text{channel}} G dz = L \frac{\partial}{\partial t} G_{\text{inlet}} + \rho(1-\gamma) \frac{\partial}{\partial t} \int_0^L dz \int_0^z \alpha dz - \int_0^L q_v dz$$

or

$$\frac{\partial}{\partial t} \int G dz = L \frac{\partial}{\partial t} G_{\text{inlet}} + \rho(1-\gamma) \frac{\partial^2}{\partial t^2} \int_0^L dz \int_0^z \alpha dz$$

Assuming that the coolant enters in the channel as a pure liquid, we get the final form of the momentum equation:

$$(4) \quad \rho L \frac{\partial v_{\text{inlet}}}{\partial t} = \Delta p - \Delta p_g - \Delta p_f - \Delta p_a - \Delta p_d$$

where L is the total length of the channel and Δp_d is a dynamic term containing the second derivative of the void fraction

$$\Delta p_d = \rho(1-\gamma) \frac{\partial^2}{\partial t^2} \left(\int_0^L dz \int_0^z \alpha dz' \right)$$

It is worth of noting that for slowly varying phenomena (if the void fraction is linear in time) the apparent inertia of the channel will be equal to that of a purely liquid channel, whatever the void fraction.

6) Scheme for numerical calculations

The model described will be employed for numerical calculations. A FORTRAN code is now being developed for this purpose.

The differential equations (1) (2) (3) are reduced to finite form by the backwards difference method in space and time.

Starting from known values of inlet velocity and enthalpy at the time t, a new value of enthalpy is evaluated for the next point along the Z axis by energy equation (3) coupled with convective heat transmission equations in the heating element (the heating power is given as function of time) and so on mesh after mesh until a text will show that subcooled boiling occurs. Then an iterative procedure is employed for the next points.

A guess is taken for the heat flux ϕ , from which, by means of heat equations in the element the surface temperature T_s is derived. From T_s we get $\phi_b = h' T_s^n$ and then the vapour source terms Ψ_s, Ψ_b, Ψ (correlations C). From the continuity equation (1) the flow rate w is found, and from equation (2), in which q_v is replaced by slip correlation A1, α is calculated. Then q_v , then H from energy equation (3) and the liquid temperature T . From the values of T_s and T , a new value for the heat flux ϕ is derived and the process is iterated to convergence on ϕ . No explicit calculation of bulk boiling boundary is needed, as the equations are the same across the boundary. Only if equilibrium is assumed in the bulk boiling region, a test on the enthalpy is necessary, so that if H is greater than 0 the enthalpy is set to zero and equation (3) is employed to get the value of the vapour source Ψ . The rest of the procedure is unchanged.

When the calculation is completed for all the channel, the $\Delta p_g, \Delta p_f, \Delta p_a$ and Δp_d are evaluated. The momentum equation (4) is then employed, with externally given Δp , to get a new value for v_{inlet} for the next time step (Δp constant ^{for} parallel channel behaviour analysis). Alternatively the inlet flow may be assumed constant and the static pressure drop Δp derived (flat pump characteristics). It should be noted that the use of momentum equation illustrated corresponds to an explicit calculation of the coolant flow; a more safe method should employ a complete recycling of the whole calculation until the momentum equation is satisfied. This would however multiply the calculation time, and it is not felt to be necessary for many practical cases.

APPENDIX A

Slip correlation

Among the different types of correlation available, we have chosen the correlation introduced by Bankoff (Ref. 2) and we have modified it to take into account the effect of void production which may be of some importance in many cases, especially for dynamics.

Bankoff's model has been proposed by the author for bubble flow; it is therefore somewhat dared to extend its validity to very high fraction conditions in which this regime is no more possible.

This is what have done Jones (Ref. 3), by modifying Bankoff's correlation to make it tend to the limit 1 when the void fraction goes to 1.

We think that this procedure is not very well founded on physical grounds, as the limit 1 has no real meaning; actually, in many practical cases, at high void fractions corresponds an annular flow, and therefore the velocity ratio increases without limits when the liquid annulus thickness decreases.

Still it may be objected that Bankoff's original formula gives a divergent value for slip ratio s when $\alpha = K$ ($K \gg 0.71$). This objection has no value for two reasons. The first, already mentioned, is that Bankoff's model is however out of the range of validity for a void fraction as high as K . The second reason is that, if we want to extend, for practical computing reasons, the model beyond its validity range, there is nothing physically absurd in assuming that the slip ratio should diverge for a certain value of α and become negative beyond it. This may be simply interpreted, in case annular flow, assuming that the liquid on the walls become stopped, and afterwards falls back counterflow to the vapour. For vertical channels with upwards flow this may well happen, and has actually been observed in dynamic cases.

The sole justification of Jones' correction should therefore be a better agreement with experimental data, but as there are no great differences between the two in the range of actual validity of the formulae, we prefer in this work to maintain Bankoff formulation, which has a simpler expression. In terms of our fundamental varia-

bles Bankoff's correlation is written.

$$q_v = \frac{\alpha w}{K}$$

With K a constant depending on pressure - Bankoff gives for water $K = 0.71 + 0.0001 p$ - with pressure in psia: we can here accept Jones' correction to bring but the right limit at critical pressure $K = 0.71 + 0.29 p/p_c$ (On physical grounds it should perhaps be preferred a dependence on the ratio $= \rho v / \rho$ which is more directly linked to the void profiles).

We remember now that Bankoff formula is derived assuming that the void and velocity profiles depend only on the hydrodynamics characteristics of the channel (and that no local slip occurs this error should be small for forced circulation channels). A through discussion of the matter is given in the paper by Zuber and Findley. (Ref. 9)

Bankoff correlation is therefore strictly applicable to adiabatic conduits of sufficient length in which no phase changes occur and velocity and void profiles are asymptotic with zero void at the channel surface and maximum in the central zone of the conduit. If we want to extend it to the dynamic case for diabatic channels with change of phase, it may be noted that the void profile shall depend in this case also on the vapour source distribution.

We may fraction the vapour source into the bulk term Ψ_b and the surface term Ψ_s ; if we suppose that Ψ_b is everywhere proportional to the void fraction α ; it will not alter the void profile, while this will happen with Ψ_s , which is localized at the surface of the channel.

As a first approximation, to take into account this effect, we shall suppose that the voids profile in a given section of the channel may be considered intermediate between Bankoff's profile and the surface source profile, which, if we consider that the fluid near the walls is still, gives zero velocity for the vapour: we shall then have

$$q_v = \frac{\alpha}{K} w (1 - \epsilon)$$

where ϵ is the relative weight of the surface distribution of bubbles to the total void profile.

For ϵ we take the expression $\epsilon = \frac{\Psi_s \tau}{a}$ where τ is the time constant which measures the relaxation time of the bubbles, i.e. the time after which the bubbles born at the surface reach the asymptotic distribution.

A tentative expression for τ is $\tau = \frac{z_e}{w}$ where z_e is a kind of relaxation length related to the distance after which the bubbles born at the surface reach the asymptotic profile. As it is reasonable to suppose that the transverse migration velocity of the bubbles is proportional to the axial velocity of the fluid, the value of z_e should be significantly independent of w .

It may be estimated as order of magnitude observing visually the distance after which the bubble generated at a fixed point in the surface take the asymptotic profile.

The modified Bankoff formula will finally read

$$q_v = \frac{a}{K} w \left(1 - \frac{z_e \Psi_s}{a w}\right) = \frac{a}{K} w - \frac{z_e}{K} \Psi_s$$

It will be noted that the deduction of the formula given implies that τ_s should be constant along the z axis. Practically it may be accepted for the general case, provided Ψ_s vary little over a distance of the order of z_e .

It should however be remembered that ϵ will reach at most the maximum value 1, and this will happen when all voids are concentrated at the walls. This is exactly what happens at the far end of subcooled boiling zone: with this care we may hope that our final formula

$$\begin{cases} q_v & = & \frac{a}{K} w - \frac{z_e}{K} \Psi_s & q_v > 0 \\ q_v & = & 0 \end{cases}$$

will represent also the subcooled boiling zone with a careful choice of z_e .

The z_e parameter should be determined by experience, by fitting the axial void profiles in static condition. It is however expected that it should be of the same order of magnitude as the hydraulic diameter of the channel, for if this should not be true, the com-

plete mixing hypothesis that supports one dimensional model shall also be wrong.

It is stressed that the model proposed does not claim to have general validity and is a priori limited to forced circulation systems, or however fast circulating turbulent systems with inlet speed of the order of 1 m/s or more.

APPENDIX B

Heat transmission at the heating surface.

The heat transmission in the purely convective region is well represented, for turbulent flow, by the well known correlation for heat transmission coefficient h :

$$\frac{hD}{K} = A \text{ Reynolds}^{0.8} \text{ Prandtl}^{0.4}$$

where D is the hydraulic diameter of the channel and K the liquid inlet conductivity. A is given as 0.023 for water (see p.ex.Ref. 4). In the nucleate boiling zone we shall assume a formula of the type (Ref. 4)

$$\phi_D = h' T_s^n \quad \text{with} \quad 3 < n \leq 4$$

To evaluate the coefficient h' different formulae have been proposed, but in any case they include some experimentally determined coefficients depending on type of coolant, geometry, and heating surface.

The nucleate boiling region is sometimes subdivided in two regions: one in which the nucleation centers are still not very frequent, so that part of the heat is still transmitted by the turbulent convective mechanism, and the second in which all of the heat is practically transmitted by nucleate boiling mechanism (fully developed nucleate boiling).

The transition between the two regions is continuous. (It has been stated (Foster and Grief) that the transition point is found for a heat flux equal to 1.4 times the heat flux ϕ_0 of fig. 3).

The general situation for the heat transmission is illustrated in fig. B1, in which the heat flux is represented as function of the heating surface temperature T_s , and where the fluid temperature T is taken as a parameter.

(We are not interested here to what happens beyond the DNB point or boiling crisis point):

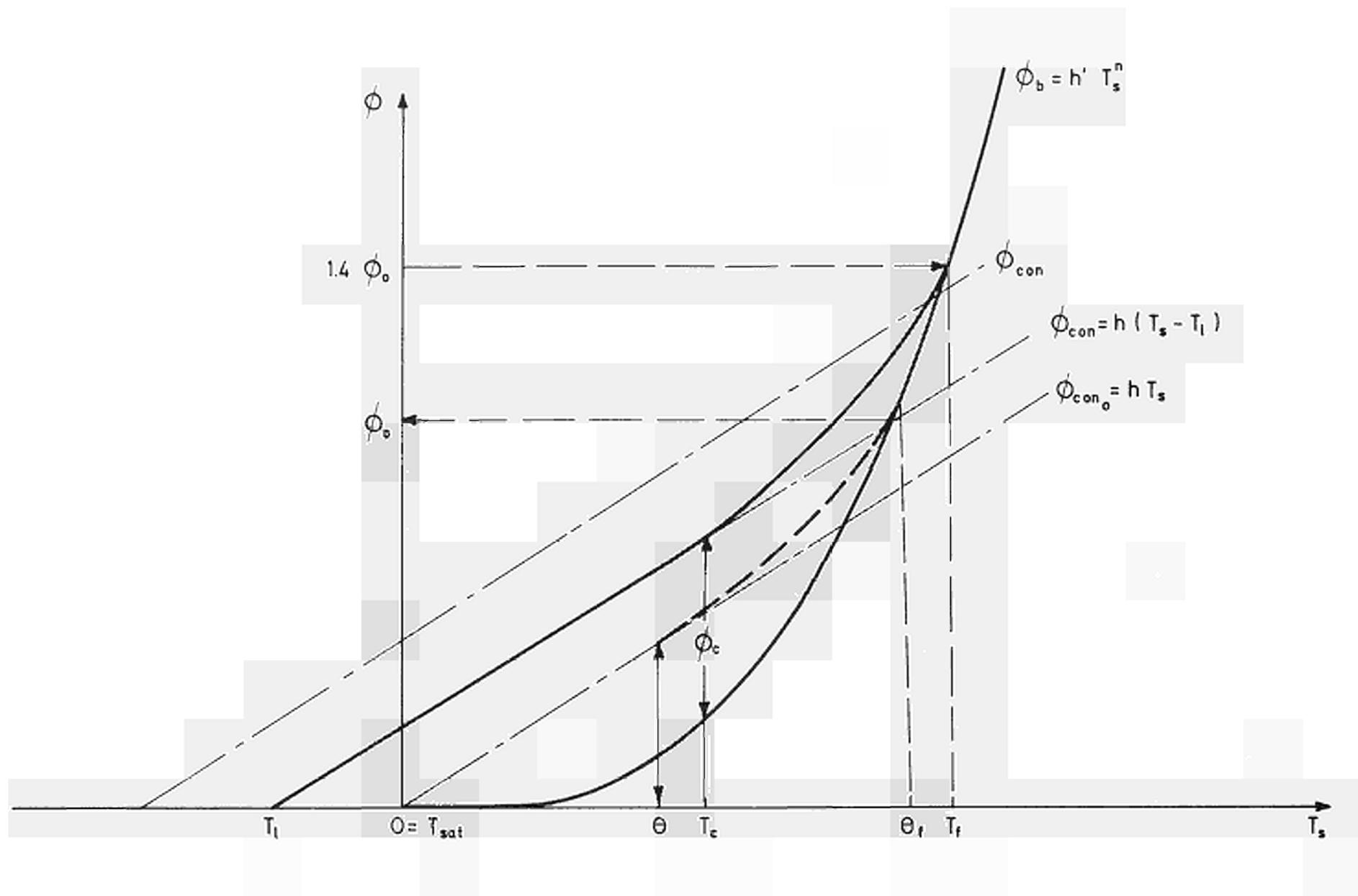


Fig.B1 HEAT FLUX AS FUNCTION OF HEATING SURFACE TEMPERATURE

With reference to the figure 1B, we assume for simplicity and continuity that for wall temperatures greater than T_C (threshold temperature for nucleate boiling) the heat flux ϕ will be equal to the nucleate boiling flux ϕ_b plus a convective term ϕ_c which decreases linearly to zero when the temperature T_s increases over T_C to a value corresponding to $1.4 \phi_0$. This formulation brings the advantage of representing in a simple way and without discontinuity the relationship between heat flux and wall temperature T of the coolant. From the figure we see actually that

$$(B1) \quad \phi_c = (h(T_C - T) - h' T_C^n) \left(1 - \frac{T_s - T_C}{T_f - T_C}\right) \geq 0$$

$$\phi = h' T_s^n + \phi_c$$

For determining T_C we will use the model proposed by Bergles and Rosen how (ref. 6).

Let us suppose that nucleate boiling starts when the temperature in the liquid layer next to the heating wall reaches a value permitting the existence of a bubble of finite radius in thermal equilibrium with the surrounding liquid. The temperature of the vapour bubble will be saturation temperature corresponding to the bubble pressure; namely, if the bubble is spherical:

$$T_v = T_{sat}(p_0) + \frac{2\sigma}{r} \frac{\partial T_{sat}}{\partial p}; \text{ and taking according}$$

to our convection $T_{sat} = 0$

$$T_v = \frac{2\sigma}{r} \frac{\partial T_{sat}}{\partial p}$$

The temperature of the superheated liquid at a point corresponding to the center of the bubble; if the bubble is in contact with the wall will be:

$$T_1 = T_s - \frac{r}{K} \phi \text{ if } \phi \text{ is the convective heat flux}$$

$\phi = p(T_s - T)$. Equating T_v and T_e we obtain a second order equation in r :

$$\frac{h}{K} (T_s - T)r^2 - T_s r + 2\sigma \frac{\partial T_{sat}}{\partial p} = 0$$

The threshold condition for nucleate boiling will therefore be given by putting to zero the discriminant

$$T_c^2 = \frac{8\sigma}{K \frac{\partial p}{\partial T_{sat}}} \quad h(T_c - T) = \theta(T_c - T)$$

with
$$\theta = \frac{8\sigma}{K \frac{\partial p}{\partial T_{sat}}} h$$

The given expression for θ is a consequence of the assumption somewhat oversimplified in the model about the spherical shape of the bubble at threshold and about heat transmission in the liquid layer next to the wall.

We will keep therefore only its form

$$T_c^2 = \theta (T_c - T)$$

with θ function of pressure h which will be found experimentally. In practical the form

$$\theta / h = \frac{8\sigma}{K \frac{\partial p}{\partial T_{sat}}}$$

may be used as guess in absence of better information.

Another way to evaluate θ perhaps to be preferred is the following:

when $T = 0$ the criterium for boiling transmission becomes

$$T_s^2 = \theta T_s \quad \text{or} \quad T_s = \theta$$

But in this case we may say that nucleate boiling will begin when boiling heat transfer becomes more efficient than convection. Referr-

ing to the fig.1B, we can say that this will happen when

$$\frac{d\phi_b}{dT_s} \geq \frac{d\phi_{\text{convective}}}{dT_s} \quad \text{or} \quad nh'T_s^{n-1} \geq h$$

we can than write

$$\theta = \left(\frac{h}{nh'}\right)^{1/n-1}$$

It is worth noting that assuming, as somebody does for simplicity, that nucleate boiling transmission begins in the junction point of the $\phi_{\text{convective}}$ and ϕ_b curves leads not only to a sharp discontinuity in the slope of $\phi(T_s)$ that may be harmful for a good convergence of numerical calculations, but also to a systematic underestimation of the heat flux in the partial nucleate boiling region.

Finally, for simplicity's sake, we will replace in formula B1 the denominator of the decreasing factor $T_f - T_c$, by the saturation temperature value $\theta_f - \theta$, which can be determined once for all for a given channel condition. It can be easily shown that

$$\theta_f = 1.4^{1/n} \left(\frac{h}{h'}\right)^{1/n-1}$$

APPENDIX C

Vapour source correlations

We divide the volumetric source of vapour at a given level Ψ into two parts

$$\Psi = \Psi_s + \Psi_b$$

Ψ_s represents the source at the heating surface and Ψ_b the source in the bulk of the coolant:

Ψ_s will be of course proportional to the heat transferred by the boiling mechanism. Should all this heat go into bubbles, we should have

$$\Psi_s = \frac{p}{A} \frac{\phi_b}{\gamma \rho \lambda}$$

Actually in the boiling not all of the heat is transferred by the bubbles, but a relevant part of it is transferred by the supplementary turbulent agitation of liquid value to bubble motion. In effect each bubble detaching causes a volume of superheated liquid in the layer next to the heating surface to be injected into the bulk of the liquid and be replaced by new liquid. In spite of some attempts, no sound theory exists about the fraction of heat transferred by this mechanism, as generally no information is available on the volume ratio between vapour and liquid transferred from the source to the bulk; some measurements of bubble volume against heat flux exist for water at atmospheric pressure. The matter is examined by Bowring (Ref. 7), whose conclusions are that the ratio ϵ between the heat transferred by liquid agitation and vapour transfer is a function of the sole pressure, and is not sensitive to the bulk fluid subcooling. An attempt to introduce the dependence on temperatures is done by Solberg and Bakstad (Ref. 10). Unfortunately the volume relation between liquid and vapour displaced is merely assumed in their treatment, as well as the relevant temperature for the superheated liquid of the source layer. As no experimental validation of their formulation is available, we will here keep the Bowring formulation which is simpler. Defining $\tau = \frac{1}{1+\epsilon}$ we have thus

$$\Psi_s = \tau \frac{p}{A} \frac{\phi_b}{\gamma \rho \lambda}$$

The bulk source term \dot{V}_b will depend on the amount of non equilibrium of the system. That means that the quantity of coolant evaporated or condensed will depend on the subcooling or overheating of the liquid, assuming as we do that the vapour is always at saturation temperature. Moreover, as the evaporation or condensation process is mass exchange process between physically separated phases, it is reasonable that the rate of exchange should depend on compact surface between phases. For small void contents, this is in turn proportional to the void fraction α ; we will then write:

$$\dot{V}_b = R_c \alpha T \quad T < 0$$
$$R_e \alpha T \quad T > 0$$

where the coefficients R_c and R_e for recondensation and evaporation respectively are not necessarily equal since the two process are physically different.

It should be mentioned that for high void contents the surface between phases is no more proportional to α , as the bubbles begin to coalesce together. To take into account this phenomenon Solberg and Bakstad (Ref. 10) assume a dependence on $\alpha(1-\alpha)$, but this is, by their own admission, quite arbitrary. In practice, at high void contents the flow regime will become annular, and the exchange surface will again be increasing with α . We keep therefore the simpler linear form, as all our model will however break down for too high void content. Furthermore we do not include a term independent from α , as done by Solberg and Bakstad, as this is obviously absent at low void content in the subcooled region (no condensation can occur if no vapour is present) which is the most important for the practical application, and the determination from experience of a further independent parameter in the superheated region is unpractical.

No information is available at present to give values for the R_c and R_e coefficients. From a practical point of view, Bowring (Ref: 7) showed that recondensation is negligible in most cases in the subcooled region, and therefore, lacking further information we will put $R_c = 0$; on the other hand, in the bulk boiling region, also if some superheating occurs, we may think that this will not be very relevant, at least for not too fast transients. So when better information is missing we may take $R_e = \infty$ i.e. thermal equilibrium in the bulk boiling region.

References

- 1) Zuber. Symposium on two phase flow dynamics.
Session 8. Eindhoven, September 1967 - to be published
- 2) Bankoff. A variable density single-fluid model for two-phase flow with particular reference to steam water flow.
Trans. of the ASME Series C 82,4 (1960)
- 3) Jones. Hydrodynamic stability of a boiling channel.
KAPL 2170 (1961)
- 4) Lottes et al. Boiling water reactor technology status of the art report.
Vol.1 Heat transfer and hydraulics - ANL 6561 (1962)
- 5) Tong. Boiling heat transfer and two phase flow.
John Wiley & Sons. NewYork (1965)
- 6) Bergles and Rosenhow. The determination of forced convection surface boiling heat transfer.
Trans. of the ASME 63 HT 22
- 7) Bowring. Physical model, based on bubble detachment, and calculation of steam voidage in the subcooled region of a heated channel.
HPR 29 Inst.Atomenergi - Kieller (1962)
- 8) Lokhart and Martinelli. Proposed correlation of data for isothermal two phase component flow in pipes.
Chemical Engineering Progress 45 Pag.39 (1949)
- 9) Zuber and Findlay. Average volumetric concentration in two phase flow systems.
Trans. of the ASME 87c Pag.453 (1965)
- 10) Solberg and Backstad. Symposium on two phase flow dynamic.
Session 7.4 - Eindhoven, September 1967 - to be published

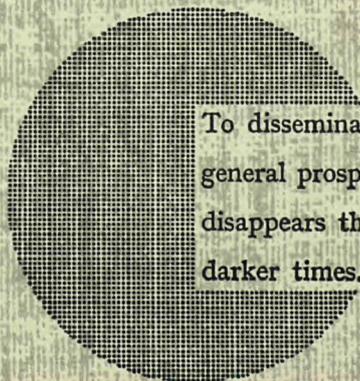
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