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EUROPEAN ATOMIC ENERGY COMMUNITY - EURATOM

INTERACTING KNOTS IN GEOMETRODYNAMICS

by

K.-H. MÜLLER

1968

Joint Nuclear Research Center
Ispra Establishment - Italy

Reactor Physics Department
Reactor Theory and Analysis

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SUMMARY

Physics can be interpreted as the totality of all physically perceptible aspects of the fluctuating space-age continuum. This geometrization leads, among other facts, to discrete stable and normalizable solutions called space-age knots. Such knots transport perturbation and interact with another. Kinematics and dynamics of systems of a sufficient high knot density are governed by a transport equation.

KEYWORDS

FIELD THEORY
SPACE-TIME
UNIFIED MODEL
SINGULARITY
TENSORS
PARTICLE MODEL

C O N T E N T S

INTERACTING KNOTS IN GEOMETRODYNAMICS[(+)]

I. Introduction

An observer perceives an object. Comparing a sequence of states occupied by the object he recognizes a metamorphosis. For the measurement of this change he relates it to an arbitrarily chosen four-dimensional coordinate-system $\boldsymbol{X}$, marking out the different "cells" constituting the object, by their position both in "space" $\boldsymbol{r}$: (x_1,x_2,x_3) and in "age" x_4.

To characterize the progress of change he procures, with the aid of a metronome, a monotonically increasing scale t and projects the order of the states onto it, uniquely.

The "properties" of an object are nothing but relations between its "conserved quantities", i.e. certain characters which stay unchanged during the evolution. A distinction and classification of the elements of an ensemble are based on those quantities, they also allow a system of objects to be called a structured organism.

The observer interprets his perception as an "aspect" of a continuous deformation [1] of the four-dimensional "space-age" surrounding him.

The correlation of space-age deformation and aspect makes possible the consideration of all the physical world as the totality of the physically noticable aspects of the changing space-age.

For simplicity, a corresponding theory can be based on the hypothesis that all states and properties of an object are functions only of the "position" $\boldsymbol{X}$ and of the "deviation" $d\boldsymbol{x}$ from the anticipated state, or the "velocity", $\boldsymbol{u} = \frac{dx}{dt}$; but they will not possess any immediate t-dependence.

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II. Formalism

The velocity field $\mathfrak{u}$ generally involves two components, $\mathfrak{u}'$ and $\mathfrak{u}''$, for which the following relations hold

$$\mathrm{DIV}\ \mathfrak{u}' = 0 \quad ; \qquad \mathrm{ROT}\ \mathfrak{u}' = S \neq 0 \tag{1}$$

$$\mathrm{ROT}\ \mathfrak{u}'' = 0 \quad ; \qquad \mathrm{DIV}\ \mathfrak{u}'' = \delta \neq 0 \tag{2}$$

where $S = \delta = 0$ at $x \notin G = \sum_k G_k$; G and k are finite and uniformly continuous in the G_k's.

Using§ $\mathrm{DIV}\ S = -\ s'$; $\mathrm{DIV}\,\delta I = s''$; $\mathrm{ROT}^*\ \mathfrak{u}'$ dual to $\mathrm{ROT}\ \mathfrak{u}'$; $\mathrm{DIV}\ (I \circ \mathrm{DIV}\ \mathfrak{u}'') = \mathrm{GRAD\ DIV}\ \mathfrak{u}''$ the systems (1) and (2) can be rewritten as follows

$$\mathrm{DIV\ ROT}\ \mathfrak{u}' = -s' \quad ; \qquad \mathrm{DIV\ ROT}^*\ \mathfrak{u}' = 0 \tag{3}$$

$$\mathrm{GRAD\ DIV}\ \mathfrak{u}'' = s'' \tag{4}$$

or shortened into

$$\Box\ \mathfrak{u} = s \tag{5}$$

where $\Box \equiv \mathrm{GRAD\ DIV} - \mathrm{DIV\ ROT}$ and $s \equiv s' + s''$.

Via the relations

$$X = -\ \mathrm{ROT}\ \mathfrak{u} \quad ; \qquad Y = \frac{1}{2}\, I \cdot \mathrm{DIV}\, \mathfrak{u} \qquad \text{and } Z = X + Y \tag{6}$$

we attach the tensors X, Y and Z to the vectors $\mathfrak{u}'$, $\mathfrak{u}''$ and call X "Maxwell tensor", because it is governed by the "Maxwell equations"

$$\mathrm{DIV}\ X = -s' \qquad \text{and} \qquad \mathrm{DIV}\ X^* = 0 \tag{7}$$

following from (3).

The second part of this chapter will deal with "conserved"quantities.

A quantity φ satisfying

$$\frac{d\varphi}{dt} = 0 \tag{8}$$

is called "absolutely conserved".

An integration of the identity:

$$\frac{d\varphi}{dt} = \frac{d\varphi}{dx}\cdot u + \frac{d\varphi}{du}\cdot\frac{du}{dt} = \left[\frac{d\varphi}{dx} - \frac{d}{dt}\left(\frac{d\varphi}{du}\right)\right]\cdot u + \frac{d}{dt}\left(\frac{d\varphi}{du}\cdot u\right) \equiv 0$$

leads to

$$\int_{t_0}^{t_1}\left[\frac{d\varphi}{dx} - \frac{d}{dt}\left(\frac{d\varphi}{du}\right)\right]\cdot u(t)\,dt = \int_{x(t_0)}^{x(t_1)}\left[\frac{d\varphi}{dx} - \frac{d}{dt}\left(\frac{d\varphi}{du}\right)\right]\cdot dx = -\left.\frac{d\varphi}{du}\cdot u\right|_{t_0}^{t_1}$$

Since $x(t_1) = x(t_0)$ for a closed path of integration, the right hand side of this equation has to be zero for a regular φ, i.e. φ satisfies the relation

$$\frac{d}{dt}\left(\frac{d\varphi}{du}\cdot u\right) = 0$$

Thus we obtain for an absolutely conserved quantity φ the conditions

$$\left.\begin{aligned} \frac{d\varphi}{dx} - \frac{d}{dt}\left(\frac{d\varphi}{du}\right) &= 0 \\ \varphi = \varphi\left(x, \frac{u}{u_4}\right) & \end{aligned}\right\} \tag{9}$$

The first of them could be called "Lagrange equation".

The above t-integration applied to the indentity

$$\frac{du^2}{dt} = \frac{du^2}{dx} - \frac{d}{dt}\left(\frac{du^2}{du}\right)\cdot u + \frac{d}{dt}\left(\frac{du^2}{du}\cdot u\right) \tag{10}$$

leads to

$$u^2 = c^2 = \text{constant} \tag{11}$$

The field $\varphi = u^2$ is, therefore, homogeneous and isotropic; u involves only three free components, i.e.

$$u : \left(v, \sqrt{c^2 - v^2}\right), \quad \text{where} \quad v : (u_1 u_2 u_3) \tag{12}$$

One finally can eliminate the measure dt and relate all properties of the different aspects to the variable

$$dx_4 = \sqrt{c^2 - v^2}\, dt \tag{13}$$

Instead of u, we use now

$$w \equiv \frac{v}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \tag{14}$$

III. Space-age knots

The special case $w \equiv 0$ marks the homogeneous "empty" or "flat" space-age. Convective inhomogeneities, $w \not\equiv 0$, imprint a structure on the space-age continuum. If they are localizable in a coordinate system and if there are conserved quantities characterizing them, they are called "space-age knots". They present themselves to the observer in form of aspects, which will - due to equation (9) - mostly not be related to the u-field itself, but to its "cylindrical" aspect $p : (w, c)$. Since $p_4 = c$, space-age knots involve only three variable p-components.

Via the "vortex atoms" [2], [3] the literature places a lot of different space-age knots at our disposal. They always consist of the same space-age points, are indistructable, impenetrable and impress a multiply connected topology into space-age. We, therefore, can call them "elementary particles". They are able to form couples, chains and clusters, i.e. composed units of action.

We look now for normalizable and steady solutions of (5) for which purpose the above mentioned correlation of ψ and u will be helpful.

The equations $u^2 = c^2$, ROT u = S and $|S| = 0$ reduce the identity

$$\frac{du}{dt} \equiv \left(u\cdot\frac{d}{dx}\right)u = GRAD\,\frac{u^2}{2} - (ROT\,u)\,u$$

to the equation

$$\frac{du}{dt} + Su \equiv 0 \tag{15}$$

Introducing a potential V by

$$Su = \frac{dV}{dx}$$

we obtain from (9) the Euler equation

$$\sqrt{c^2-v^2}\,\frac{\partial v}{\partial x_4} - v\times \mathrm{rot}\,v + \mathrm{grad}\left(\frac{v^2}{2}+V\right) = 0 \tag{16}$$

where $v:(u_1\ u_2\ u_3)$. We restrict our considerations by the requirement

$$\mathrm{div} = 0 \text{ and } \quad \frac{d\phi}{dt} = \sqrt{c^2-v^2}\,\frac{\partial\phi}{\partial x_4} + v_0\,\frac{\partial\phi}{\partial x_3} = 0$$

ϕ = constant being the equation of a rigid axially symmetric surface always containing the same points and moving along its axis with constant velocity v_o.

Due to axial symmetry, Stokes' stream-function ϕ can be introduced by

$$v:\left(-\frac{1}{2\pi\rho}\,\frac{\partial\phi}{\partial x_3}\,;\ f(\phi)\,;\ \frac{1}{2\pi\rho}\,\frac{\partial\phi}{\partial\rho}+v_0\right) \tag{17}$$

where $\rho^2 = x_1^2 + x_2^2$ and f is an arbitrary function of ϕ. Eliminating the unknown potential V from (16) we obtain

$$\left(\frac{\partial\phi}{\partial\rho}\,\frac{\partial}{\partial x_3} - \frac{\partial\phi}{\partial x_3}\,\frac{\partial}{\partial\rho}\right)\left[\frac{1}{\rho}\left(\frac{\partial v_\rho}{\partial x_3} - \frac{\partial v_3}{\partial\rho}\right) - \frac{1}{2\pi\rho^2}\,f\cdot\frac{df}{d\phi}\right] = 0$$

or, finally, after a formal integration

$$\frac{\partial^2\phi}{\partial\rho^2} - \frac{1}{\rho}\,\frac{\partial\phi}{\partial\rho} + \frac{\partial^2\phi}{\partial x_3^2} = -2\pi\rho^2 F - f\cdot\frac{df}{d\phi} \tag{18}$$

where $F(\phi)$ is an arbitrary function of ϕ. This differential equation governs the stream function ϕ of solutions u satisfying the above assumptions.

A stream function related to the spherical coordinates $(r, \vartheta, \varepsilon)$ is better suited to physical problems. It satisfies the corresponding equation

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1-y^2}{r^2}\frac{\partial^2\phi}{\partial y^2} = -2\pi F r^2(1-y^2) - f\cdot\frac{df}{d\phi} \tag{19}$$

where $y = \cos\vartheta$

The simplest case characterized by $F = F_1$ = constant and $f.\frac{df}{d\phi} = F_2$ = constant should be mentioned explicitly. The left hand side of (19) is separated by the ansatz $\phi = g(r)h(y)$. The functions $g = \sum a_n r^n$ and h(y) satisfy the differential equations

$$\left.\begin{aligned} r^2\frac{d^2g}{dr^2} - n(n-1)g = 0 \\ (1-y^2)\frac{d^2h}{dy^2} + n(n-1)h = 0 \end{aligned}\right\} \tag{20}$$

div V = 0 is guaranteed, as can easily be verified.

<u>Example:</u> "Hicks' Dyade" [4]

A spindle-shaped resting solid nucleus (I) is covered by a spining vortex ring (II). The stream function for the different regions (fig. 1) are given by

$$\left.\begin{aligned} \phi_{I} &= 0 \\ \phi_{II} &= \frac{\pi F_1}{5} r^2(a^2-r^2)\sin^2\vartheta - \frac{F_2}{2}(a-r)^2 \\ \phi_{III} &= \frac{2\pi F_1}{15}\frac{a^2}{r}(a^3-r^3)\sin^2\vartheta \end{aligned}\right\} \tag{21}$$

This aggregate can easily be modified to a spherical nucleus.

The special case $F_2 = 0$ is called "Hill's vortex" [5]. Because of completeness will still add the Maxwell tensor

$$X(\xi,\eta) = \begin{pmatrix} 0 & \eta_3 & -\eta_2 & \xi_1 \\ -\eta_3 & 0 & \eta_1 & \xi_2 \\ \eta_2 & -\eta_1 & 0 & \xi_3 \\ -\xi_1 & -\xi_2 & -\xi_3 & 0 \end{pmatrix} = X^*(\eta,\xi)$$

for region (III). The vectors ξ and η are given by

$$\left.\begin{aligned} \xi &= -\frac{1}{2\sqrt{c^2-v^2}}\,\mathrm{grad}\, v^2; \quad \eta = 0 \\ v^2 &= \left(\frac{2F_1 a^2}{15}\right)^2 \left[\left(1-\frac{a^3}{r^3}\right)^2 + \frac{3\cdot a^3}{4\cdot r^3}\left(4-\frac{a^3}{r^3}\right)\sin^2\vartheta\right] \end{aligned}\right\} \qquad (22)$$

IV. Vortex rings

A circular vortex ring characterized by radius r_o of aperture, r_1 of transverse section, by div $\mathbf{v} = 0$ and rot $\mathbf{v} = \begin{cases} \omega & \text{at the file} \\ 0 & \text{outside} \end{cases}$ corresponds to the ϕ-distribution:

$$\phi = -\frac{\omega r_1^2}{\pi}\sqrt{r_o r}\left[\left(\frac{2}{\lambda}-\lambda\right)K(\lambda) - \frac{2}{\lambda}E(\lambda)\right]; \quad \lambda^2 = \frac{4 r_o r}{x_3^2 + (r_o+r)^2} \qquad (23)$$

The symbols E and K denote normal elliptic integrals.

The center of the ring linearly migrates with velocity [3]

$$V_o = \frac{\omega r_1^2}{2 r_o}\left(\log\frac{8 r_o}{r_1} - \frac{1}{4}\right) \qquad (24)$$

A reference to an interesting stability phenomenon should be added here.

A vortex ring with a finite cross section, although an instantaneously possible form, is not steady. This instability can very clearly be elucidated by discussing the behaviour of "Hill's vortex" [5], which is a spinless vortex ring possessing a spherical shaped "stream-surface". During its migration the spheroid alters and becomes a spindle. A spin of the ring around its axis of symmetry, however, obstructs that tendency of alteration.

In the case of a hydrodynamical vortex ring this fact can be understood with the aid of Bernoulli's equation.

At present we shall be content to note that the stability of space-age knots is closely connected with the presence of an additional spin motion.

Two or more vortex rings interact with one another [3] , [6] , i.e. they change their form and velocity.

The deformations dissolve into eigen-vibrations of the rings [3] , [4] . A perturbation calculation establishes, besides some stability criteria, the corresponding spectra of the different eigen-vibrations (The vortex can be hollow!).

The bending vibrations which a single vortex ring executes, when it is slightly disturbed from its circular form, is characterized by the "frequency spectrum"

$$\nu = \frac{n\sqrt{n^2-1}}{2r_0} v_0 \qquad (25)$$

where v_0 denotes the linear velocity of the ring and $2\pi r$ the periphery bending in the n'th harmonic.

Due to the proportionality, $\nu \sim v_0$ the discrete spectral lines (25) of a vortex ring possess lower frequencies than the lines in the corresponding spectrum of a faster migrating ring ("red - shift"). This shift is independent of the direction of motion.

If the excited rings are elements of a particle cloud rotating with velocity $v_0 \sim \frac{1}{r}$ around an axis, r = 0, the relation $\nu \sim \frac{1}{r}$ holds, i.e. the spectrum (25) of an element far from the axis possesses a red-shift in comparison with that of an element moving nearer, since it migrates more slowly.

The "scattering" of a spinless vortex ring at a spherical obstacle will be discussed now. (fig.2)

According to the usual vortex dynamics the scattering angle can be expressed by [3]

$$\left.\begin{aligned} \sin\Theta &= \frac{45}{128}\pi R^3 \frac{a^2}{d^6}\frac{\omega}{V_0} \\ a &= a_0\left(1+\frac{1}{16}R^3\frac{a_0^2}{d^6}\frac{\omega}{V_0}\right) \end{aligned}\right\} \qquad (26)$$

provided that the minimal distance d between vortex and obstacle is large in comparison with the radius a of the ring aperture. Relation (26) shows the deviation Θ as a function of vorticity ω , velocity v_o and the geometrical data of vortex and obstacle; in short, it governs the "small angle scattering" of a vortex ring at a spherical obstacle.

Introducing the relation (24) equation (26) contains nothing but geometrical data.

The central collision of a ring and a spherical obstacle possesses a remarkable result [7] . Fig. 3 presents two sequent phases of the collision. The approaching vortex ring widens and finally covers the obstacle.

V. Coupled rings

Two vortex rings created one after the other by the same source and procedure possess the same initial radius, sense and strength of rotation.

The interaction of such a couple (fig. 4) consists in an expansion of the preceding ring and in a simultaneous contraction of the following one. Due to its higher velocity the latter reaches and slippes through the other. This procedure repeats now with changed order [6] .

The volume V enclosed by the "control surface" C pulses and the vortex density ρ related to V varies periodically between extrema.

The "dynamical coupling" mentioned just now is able to connect more than two vortex rings to chains and clusters.

The elements of a cluster form a common velocity field of complex dynamics. Attraction and repulsion permanently change in space-age. Vortex chains build up and decay continuously. Thus there are, simultaneously, both single rings and chains of different length completely matted. The velocity fields of the different vortices partially annul one another. Only the interaction between neighboured elements will be important.

Fast vortices of the boundary domain are able to leave the cluster, but they will simultaneously be widened, i.e. delayed. This fact corresponds to an "evaporation energy" A depending from the radius R of the (spherical) cluster and tending to an asymptotic value A_{∞}. Only vortices whose kinetic energy $E > A$ leave the cluster. Due to that the corresponding partial density decreases, at least within the boundary domain. The velocity distribution show a cut-off (fig. 5). Single vortex rings tramping about outside will be attracted and catched by the cluster. An equilibrium between cluster and surroundings establishes, i.e. a mean radius R_{*} and a certain density distribution ρ_{*} will be reached (fig. 6).

A high energy knot hitting such a cluster disturbs the equilibrium; the cluster becomes "excited". During the interaction the "outer energy", $\frac{1}{2}\sum_{k}(v_0^2)_k$, of the confounded vortices diminishes in favour of eigen-vibrations.

VI. Interacting clusters

Our interest is focussed now to the system-behaviour of a knot-gas; the internal structure of possible knots will be omitted here.

We start from the non linear "t ansport equation"

$$\left(c\frac{\partial}{\partial x_4} + w\cdot\frac{d}{dr} + a\cdot\frac{d}{dw}\right)\varphi = A(\varphi) \tag{27}$$

of a pure elastically colliding gas of uniform particles. φ denotes the distribution function and A the (in general nonlinear) collision operator acting on φ.

An interaction where φ satisfies

$$\int_W m\,\frac{d\varphi}{dt}\,dW = 0 \tag{28}$$

will be called mass-conserving " and if, in addition,

$$\int_W m\,w\,\frac{d\varphi}{dt}\,dW = 0 \tag{29}$$

"momentum-cons." m is defined by

$$m = \frac{m_o}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = m_0\sqrt{1+\left(\frac{w}{c}\right)^2}\ ; m_o = \text{constant}.$$

m_o is a parameter characterizing the type of particle.

The conditions (28) and (29) require that all fluctuations of $m\varphi$ and the momentum $mw\varphi$, related to the interval dt, vanish in the w-average.

A unique description of the transport process requires the knowledge of the velocity and the acceleration function

$$w = w(x) \qquad \text{and} \qquad a(x) = \sqrt{1+\left(\frac{w}{c}\right)^2}\ \frac{dw}{dt}$$

While the transport of a dense gas is essentially influenced by the structure of the w(x)-function, a theory of diluted gases can be based in a first order approximation, upon the assumption of uncorrelated x and w.

A weighted integration of (27) leads to the following system of differential equations:

$$\left.\begin{aligned} c\frac{\partial \rho}{\partial x_4} + \frac{\partial(\rho v_k)}{\partial x_k} = 0 \\ \left(c\frac{\partial}{\partial x_4} + v_k\frac{\partial}{\partial x_k}\right)v_i + \frac{1}{\rho}\frac{\partial P_{ik}}{\partial x_k} + a = 0 \end{aligned}\right\} \qquad (30)$$

where

$$\rho = \int_W m\varphi\, dW \quad ; \quad v = \frac{1}{\rho}\int_W m w \varphi\, dW ; \quad P_{ik} = \int_W m(w_i - v_i)(w_k - v_k)\varphi\, dW$$

System (17) is independent of the collision term $A(\varphi)$ and therefore valid both for binary and higher order collisions, i.e. for diluted and for dense gases. For simplicity we restrict our studies to the special case

$$a = 0 \quad ; \quad P_{ik} = p\,\delta_{ik} \quad ; \quad \rho = \text{const.} \quad ; \quad \text{rot}\, v = 0 \qquad (31)$$

According to the terminology of hydrodynamics p will be called "pressure" and $F = \int_B p \cdot df$ "force" impressed to surface B by the gas.

A classical calculation leads to "Bjerknes" formula" [8]

$$F_{12} = -\frac{\rho \dot{V}_1 \dot{V}_2}{4\pi r^2} - \frac{6\rho}{(4\pi)^2 r^5}\left[\frac{\rho_1-\rho}{2\rho_1+\rho}V_1\dot{V}_2^2 + \frac{\rho_2-\rho}{2\rho_2+\rho}V_2\dot{V}_1^2\right] \qquad (32)$$

where $\dot{V}_k \equiv c\dfrac{\partial V}{\partial x_4}$ and $F_{12} = F_{21}$. It describes the interaction of two pulsators embedded into ahomogeneous gas of the density ρ. The volumes of the clusters are named V_1 and V_2, their "mean" densities ρ_1 and ρ_2 and their central distance r.

$$F_{12} \begin{matrix} < \\ > \end{matrix} 0 \quad \text{means} \quad \begin{matrix} \text{attraction} \\ \text{repulsion} \end{matrix}$$

"Stable" clusters, i.e. clusters always consisting of a constant number $M_k = \rho_k V_k$ of vortex rings require $V_k = \sigma_k M_k$, where $\sigma_k = \dfrac{1}{\rho_k}$ represents the specific volume covered - in the mean - by a single vortex ring.

Using the abbreviations

$$\left.\begin{aligned} \gamma &= \frac{\varrho}{4\pi}\dot{\sigma}_1\dot{\sigma}_2 \\ \gamma' &= \frac{\varrho_1-\varrho}{2\varrho_1+\varrho}\left(\frac{\sigma_1}{\sigma_2}\right)\left(\frac{\dot{\sigma}_2}{\dot{\sigma}_1}\right)R_2^3 + \frac{\varrho_2-\varrho}{2\varrho_2+\varrho}\left(\frac{\sigma_2}{\sigma_1}\right)\left(\frac{\dot{\sigma}_1}{\dot{\sigma}_2}\right)R_1^3 \end{aligned}\right\} \tag{33}$$

equation (32) can be rewritten in

$$F_{12}(r, x_4) = -\gamma(x_4)\frac{M_1 M_2}{r^2}\left[1 + 2\frac{\gamma'(x_4)}{r^3}\right] \tag{34}$$

This formula governs the interaction of stable clusters.

A cluster consisting of a large number of vortices shows, with respect to its internal dynamics, a stochastic behaviour.

The interaction described by the first term of (34) and stochastically varying between attraction and repulsion will vanish "in the average" The second term, however, guarantees an always attractive contribution.

The interaction of two equal structured clusters, $M_1 = M_2 = M$, averaged over a finite interval τ can be expressed by

$$F(r) = -\frac{M^2}{r^5}\int_\tau \gamma\gamma'\frac{dx_4}{\tau} < 0 \tag{35}$$

and will, therefore, always be attractive.

A quadrupel (fig. 7) of identical spheres centered at the corners of a regular tetrahedron and touching each other represents a stable aggregate of clusters of a high symmetry, as the central distances of neighboured clusters are equal and absolutely minimal.

To interpret the scattering of clusters and cluster-aggregates one has to combine eq. (35) and the results of the classical collision theory. The relation

$$\Theta = \pi - 2d\sqrt{1-U(d)}\int_d^\infty \left\{r^4(1-U(r)) - r^2d^2(1-U(d))\right\}^{-1/2} dr$$

correlates the deviation Θ, the minimum d of central distance and the normalized potential $\mathcal{V}(r)$.

For $F \sim r^{-5}$, i.e. $\mathcal{V}(r) = \frac{\alpha^4}{r^4}$ and $\varkappa = \frac{\alpha^2}{\sqrt{d^4+\alpha^4}}$ we obtain the scattering law

$$\Theta = \pi - 2\sqrt{1-2\varkappa^2}\; K(\varkappa) \tag{36}$$

$K(\varkappa)$ symbolizes the complete elliptic Legendre integral.

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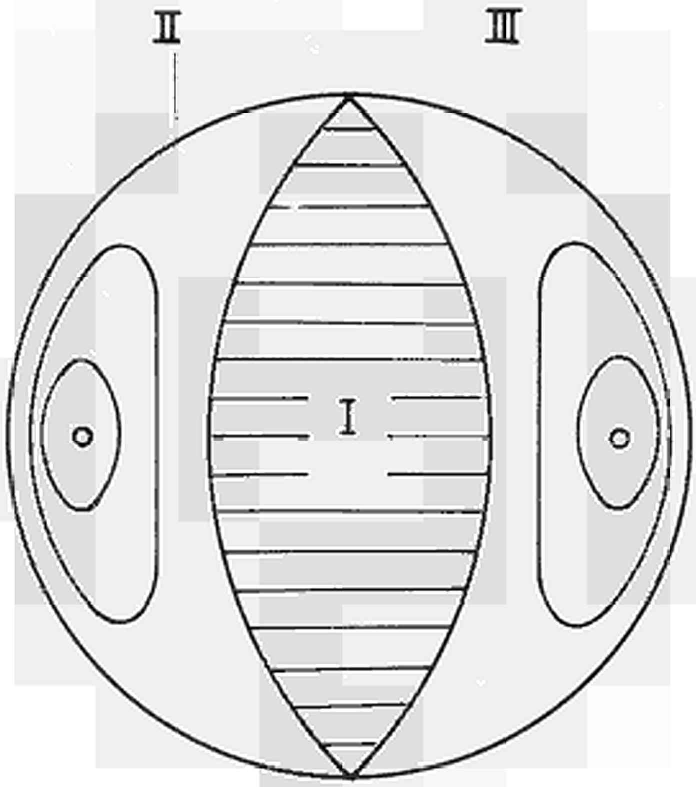

fig. 1

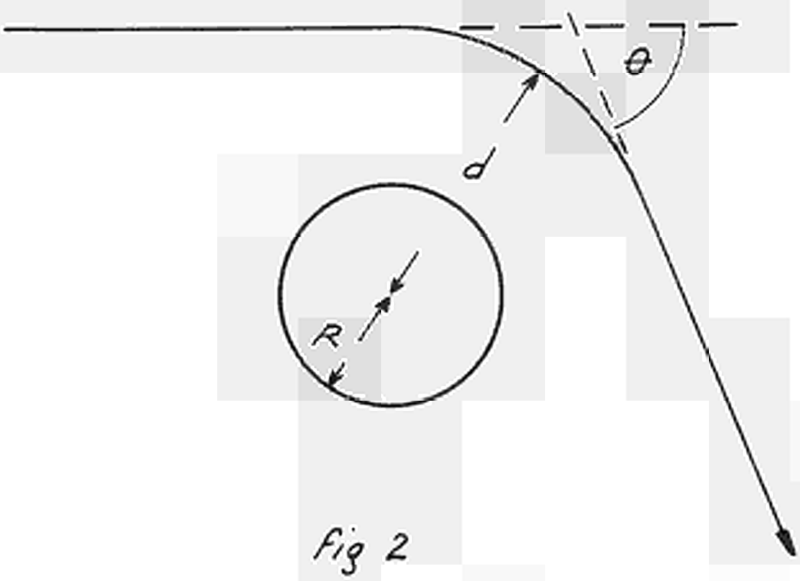

fig 2

fig. 1 Spindle-shaped nucleus (I) covered by a spining vortex ring (II).

fig. 2 Spinless vortex ring scattered by a spherical obstacle.

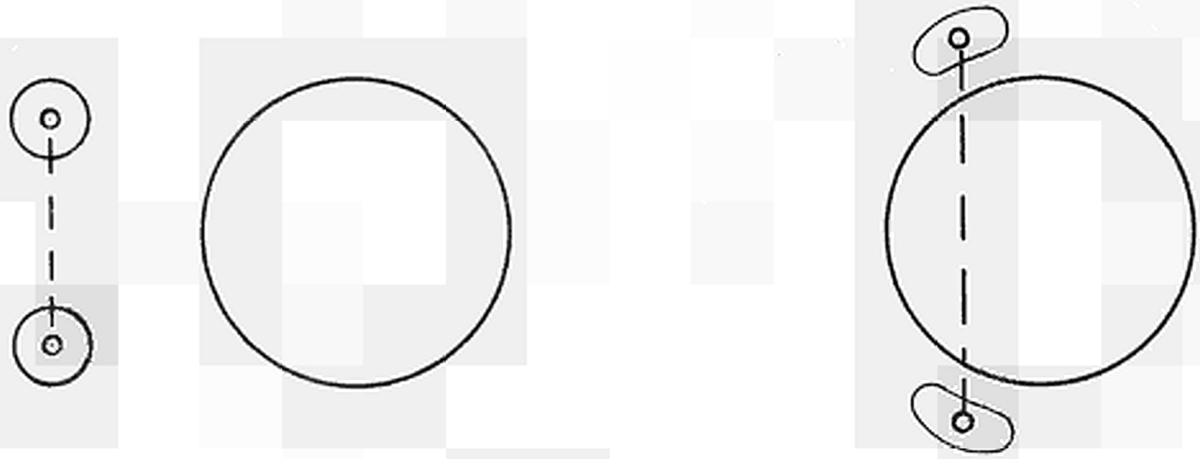

fig. 3

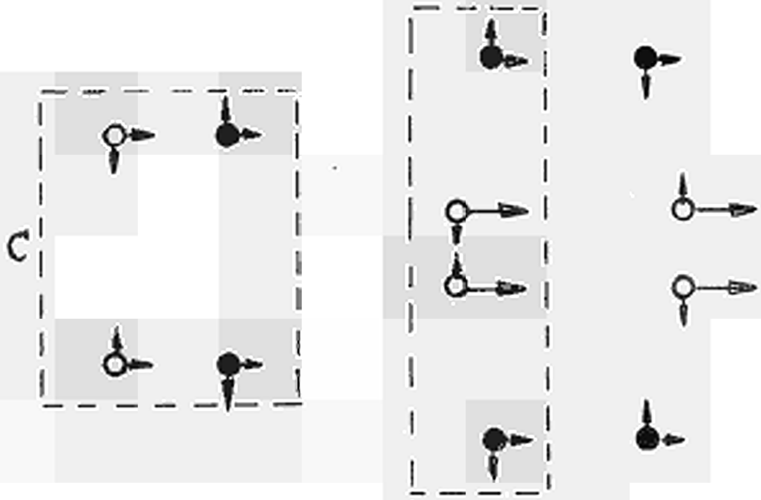

fig. 4

fig. 3 "Stripping" of a vortex ring due to a central collision.

fig. 4 Pulsating vortex density during the interaction of two rings.

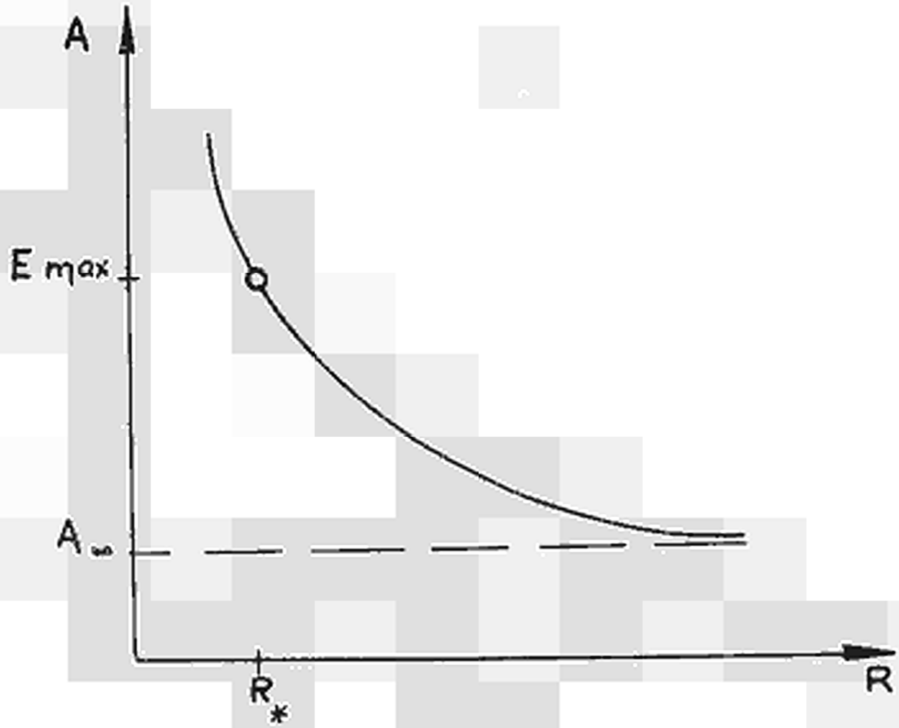

fig. 5

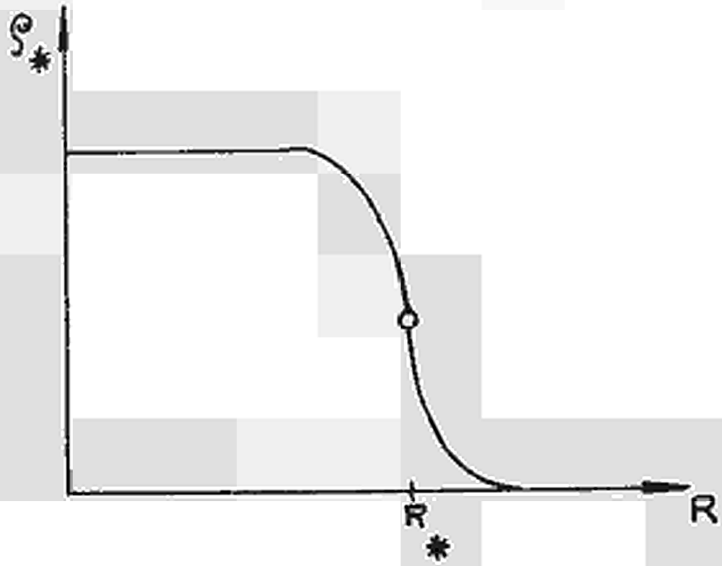

fig. 6

fig. 5 Surface/Volume ratio causes velocity cut-off (E_{max}) and radius R_*.

fig. 6 Distribution of particle density in a cluster.

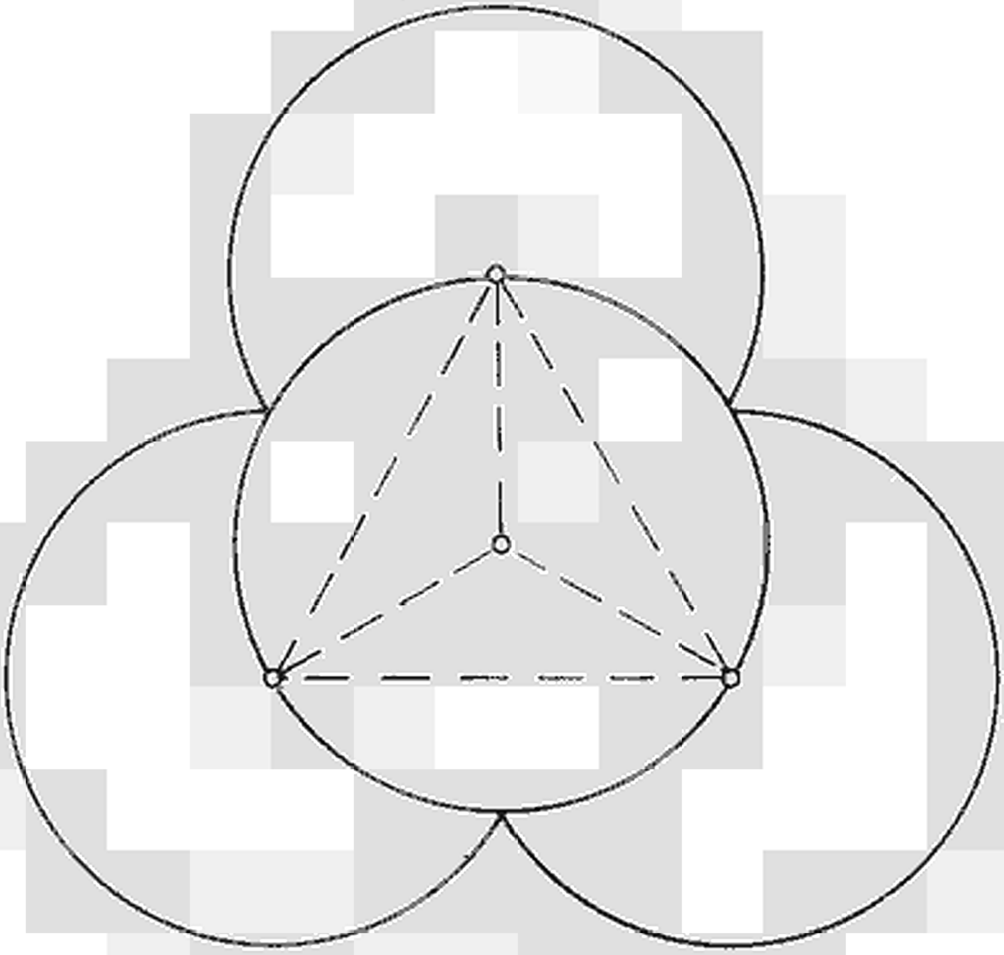

fig. 7

fig. 7 Four equidistanced spheres represent a stable unit of action.

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Alfred Nobel

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