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EUROPEAN ATOMIC ENERGY COMMUNITY - EURATOM

ANISOTROPIC COLLISION PROBABILITIES IN GENERAL CYLINDRICAL GEOMETRY

by

L. AMYOT

1968

Joint Nuclear Research Center
Ispra Establishment - Italy

Reactor Physics Department
Reactor Theory and Analysis

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Printed by Guyot, s.a.
Brussels, June 1968

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SUMMARY

Based on the work of Takahashi for one-dimensional systems, a general formalism is presented for anisotropic collision probabilities in general cylindrical geometry. The method is based on an expansion in spherical harmonics of fluxes, sources, cross-sections and collision probabilities. The resulting expressions for the zeroth and first order collision probabilities are simply related respectively to the classical isotropic collision probabilities and the directed probabilities used by Benoist in his thesis on the diffusion coefficient.

KEYWORDS

TRANSPORT THEORY
SERIES EXPANSION
SPHERICAL ARMONICS
ANISOTROPY
COLLISION INTEGRAL
PROBABILITY
INTEGRALS

ANISOTROPIC COLLISION PROBABILITIES IN GENERAL CYLINDRICAL GEOMETRY

Introduction (*)

In recent years, the method of first collision probabilities has received considerable attention in the development of calculation methods for lattice cell analysis. The success of this technique is attributable to the surprising accuracy of which it is capable at a relatively low cost in machine-time. However, the virtues of the classical method of collision probabilities were somewhat overshadowed by its inability to treat in a correct manner physical situations in which the assumptions of scattering, source and flux isotropy were not strictly valid. The transport approximation, normally resorted to in such cases, is difficult to justify from a theoretical point of view and, at any rate, cannot have a wide range of applicability.

Starting with the work of Takahashi(8) on cylindrical systems, this restriction has been lifted by the introduction of anisotropic collision probabilities. Later, Harper(7) has studied the case of linear anisotropy for general two-dimensional systems. In the present work, the work of Harper is generalized to any order of anisotropy. The formalism and method of derivation are akin to those applied by Takahashi to the one-dimensional case. The resulting expressions for the zeroth and first order collision probabilities are closely related to Benoist's directed probabilities(15) which were used in his thesis on the diffusion coefficient.

In view of the fact that the collision probabilities of any order are always functions of the same geometrical argument, it is not expected that the machine-time will increase very rapidly with the order of anisotropy.

(*) Manuscript received on March 28, 1968.

Boundary conditions are not considered in the present report. The study of Harper does not indicate, however, that their introduction in cell problems would cause any great difficulty. It is also noted that, as in all previous work, the cylindrical systems considered are assumed to be axially infinite.

1. The Integral Transport Equation

Let $G(E,\vec{r}'\to\vec{r},\vec{\Omega}'\to\vec{\Omega})$ be the angular flux of energy E produced at point $\vec{r}$ along $\vec{\Omega}$ by unit emission of neutron of the same energy at point $\vec{r}'$ along $\vec{\Omega}'$. Then the angular flux of energy E at point $\vec{r}$ along $\vec{\Omega}$ will be given by:

$$\phi(\vec{r},E,\vec{\Omega}) = \int_{(V)} d\vec{r}' \int_{(4\pi)} d\vec{\Omega}'\, G(E,\vec{r}'\to\vec{r},\vec{\Omega}'\to\vec{\Omega})\, H(E,\vec{r}',\vec{\Omega}') \qquad (1)$$

where the function $H(E,\vec{r}',\vec{\Omega}')$ represents the emission density at point $\vec{r}'$ along $\vec{\Omega}'$ for neutrons of energy E. The integration is over the whole range of space occupied by the source. The quantity G, which relates the field to its cause, is, by definition, the Green's function for the neutron field[1]. Its explicit mathematical expression will be given later in the development.

The emission density may include contributions independent from the field intensity-in this case the angular flux - and others which are functions of the field intensity. The linear nature of the Boltzmann equation for the neutron transport problem translates the physical fact that neutron-neutron interactions are relatively unimportant and we may write:

$$H(E,\vec{r}',\vec{\Omega}') = S(E,\vec{r}',\vec{\Omega}') + \int_{0}^{\infty} dE' \int_{(4\pi)} d\vec{\Omega}''\, \Sigma(\vec{r}',E'\to E,\vec{\Omega}''\to\vec{\Omega}')\, \phi(\vec{r}',E',\vec{\Omega}'') \qquad (2)$$

The exact form of the factor of proportionality which, in general, will depend on the multiplying properties of the medium needs not be specified at this point. The second term on the right-hand side of equ. (2) conveys only the information that, in general, a collision between a neutron and a nuclide situated at point $\vec{r}'$ will modify the energy and direction - and possibly result in a multiplication or absorption - of the incident neutron.

Equation (1) combined with the definition (2) for the emission density constitutes the general form of the steady state integral equation for neutron transport[2] and forms the basis of all subsequent developments in the present work.

2. Expansion in Spherical Harmonics

It is found, in practice, that the solution of neutron transport problems is simplified considerably when the angular dependence of the various quantities entering the balance equation is removed, hence the popularity of the isotropic approximation which, moreover, has proved to be adequate for the treatment of many physical situations. This suggests that a suitable method for the study of anisotropic problem might be to expand every angular - dependent function in a series of orthogonal polynomials with the aim of obtaining a set of equations for the flux components from which the angular variable has been eliminated.

Other types of orthogonal polynomials - v.g. Tschebyscheff[3] Jacobi[4] have occasionally been used but by far the most widespread technique of the kind is the expansion in spherical harmonics. Essentially, the success of this method stems from the peculiar properties of the transfer cross section $\Sigma(\vec{r}', E' \to E, \vec{\Omega}'' \to \vec{\Omega}')$ [5]. Non-zero contributions to this function are mostly due to fission and scattering events. While the term ascribed to fission and inelastic scattering may usually be considered as isotropic, the kinematics of an elastic scattering collision is such that the angle of scattering is uniquely related to the energy change. To take full advantage of the decomposition in orthogonal polynomials, it will thus be important to choose a set of polynomials for which one is able to express functions of the angle between two vectors ($\vec{\Omega}''$ $\vec{\Omega}'$) in terms of functions of the individual vectors $\vec{\Omega}''$, $\vec{\Omega}'$. The convenience of the spherical harmonics expression technique is closely connected with the fact that such relationships exist and possess a particularly simple form in the case of these polynomials.

A serious drawback of the spherical harmonics method, as applied to the integro-differential Boltzmann equation, is that for highly anisotropic flux distributions the expression converges very slowly. However, it has been shown[6] that for a given order of expansion in spherical harmonics, the integral form of the transport equation is inherently more accurate. Furthermore, at least for the variant based on the use of collision probabilities, the requirements in machine-time are considerably more modest than in the case of the integrodifferential equation approximated to the same order of expansion[7,8].

Various definitions of the spherical harmonics appear in the literature[9]; in the present work, we will asume the following form:

$$Y_n^m(\vec{\Omega}) = H_n^m P_n^m(\cos\theta) e^{im\phi} \tag{3}$$

where the coefficients H_n^m are given by:

$$H_n^m = \left[\frac{(2n+1)(n-m)!}{4\pi (n+m)!} \right]^{1/2} \tag{4}$$

The quantities P_n^m (cos Θ) are the associated Legendre functions of the first kind:

$$P_n^m(\cos\theta) = \sin^m\theta \, \frac{d^m}{d(\cos\theta)^m} P_n(\cos\theta) \tag{5}$$

P_n(cos Θ) represents the Legendre polynomial of order n; Θ is the polar angle and ϕ the azimuthal angle defining the vector $\vec{\Omega}$.

Let us then expand the angular flux and source in terms of the functions (3):

$$\phi(\vec{r}, E, \vec{\Omega}) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \phi^{n,m}(\vec{r}, E) Y_n^m(\vec{\Omega}) \tag{6}$$

$$S(\vec{r}, E, \vec{\Omega}) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} S^{n,m}(\vec{r}, E) Y_n^m(\vec{\Omega}) \tag{7}$$

Since the transfer cross section is defined by means of the single variable $\vec{\Omega}'\cdot\vec{\Omega}''$, a suitable series representation can be given in terms of the Legendre polynomials $P_n(\vec{\Omega}'\cdot\vec{\Omega}'')$. Thus:

$$\Sigma(\vec{r}', E'\to E, \vec{\Omega}''\to\vec{\Omega}') = \sum_{n=0}^{\infty}\left(\frac{2n+1}{4\pi}\right)\Sigma_n(\vec{r}', E'\to E)\, P_n(\vec{\Omega}'\cdot\vec{\Omega}'') \tag{8}$$

The Legendre polynomial $P_n(\vec{\Omega}'\cdot\vec{\Omega}'')$ may be written in terms of the variables $\vec{\Omega}', \vec{\Omega}''$ by the application of the addition theorem:

$$\begin{aligned} P_n(\vec{\Omega}'\cdot\vec{\Omega}'') &= \sum_{m=-n}^{n}\frac{(n-m)!}{(n+m)!}\, P_n^m(\cos\theta')\, P_n^m(\cos\theta'')\, e^{im(\phi'-\phi'')} \\ &= \frac{4\pi}{2n+1}\,(H_n^m)^2\, P_n^m(\cos\theta')\, P_n^m(\cos\theta'')\, e^{im(\phi'-\phi'')} \end{aligned} \tag{9}$$

Combining equs. (8) and (9), we find:

$$\Sigma(\vec{r}', E'\to E, \vec{\Omega}''\to\vec{\Omega}') = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\Sigma_n(\vec{r}', E'\to E)\, Y_n^m(\vec{\Omega}')\, Y_n^{m*}(\vec{\Omega}'') \tag{10}$$

where
$$Y_n^{m*}(\vec{\Omega}'') = H_n^m\, P_n^m(\cos\theta'')\, e^{-im\phi''} \tag{11}$$

is the complex conjugate of the function $Y_n^m(\vec{\Omega}'')$.

The substitution of the expansions(6), (7) and (10) into (2) yields

$$H(E,\vec{r}',\vec{\Omega}') = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} S^{n,m}(\vec{r}',E)\, Y_n^m(\vec{\Omega}')$$

$$+ \int_0^{\infty} dE' \int_{(4\pi)} d\vec{\Omega}'' \left[\sum_{n=0}^{\infty} \sum_{m=-n}^{n} \Sigma_n(\vec{r}', E'\to E)\, Y_n^m(\vec{\Omega}')\, Y_n^{m*}(\vec{\Omega}'') \right] \left[\sum_{n=0}^{\infty} \sum_{m=-n}^{n} \phi^{n,m}(\vec{r}',E')\, Y_n^m(\vec{\Omega}'') \right]$$

The integral in the last equation may be reduced to a simpler form by the use of the orthogonality property of the spherical harmonic

$$\int_{(4\pi)} Y_{n'}^{m'*}(\vec{\Omega})\, Y_n^m(\vec{\Omega})\, d\vec{\Omega} = \delta_{n'n}\, \delta_{m'm} \qquad (12)$$

Thus, we may write

$$H(E,\vec{r}',\vec{\Omega}') = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left[S^{n,m}(\vec{r}',E) + \int_0^{\infty} dE'\, \Sigma_n(\vec{r}', E'\to E)\, \phi^{n,m}(\vec{r}',E') \right] Y_n^m(\vec{\Omega}') \qquad (13)$$

This result suggests that a convenient expansion for the function $G(E, \vec{r}'\to\vec{r}, \vec{\Omega}'\to\vec{\Omega})$ would be

$$G(E,\vec{r}'\to\vec{r},\vec{\Omega}'\to\vec{\Omega}) = \sum_{n,n'=0}^{\infty} \sum_{m=-n}^{n} \sum_{m'=-n'}^{n'} G^{(n',m')\to(n,m)}(E,\vec{r}'\to\vec{r})\, Y_n^m(\vec{\Omega})\, Y_{n'}^{m'*}(\vec{\Omega}') \qquad (14)$$

Substituting (13) and (14) into (1), we find, with the help of (12):

$$\phi(\vec{r},E,\vec{\Omega}) = \sum_{n,n'=0}^{\infty} \sum_{m=-n}^{n} \sum_{m'=-n'}^{n'} \left[\int_{(V)} d\vec{r}'\, G^{(n',m')\to(n,m)}(E,\vec{r}'\to\vec{r})\, H^{n',m'}(E,\vec{r}') \right] Y_n^m(\vec{\Omega}) \qquad (15)$$

or

$$\phi^{n,m}(\vec{r},E) = \int_{(4\pi)} d\vec{\Omega}\, \phi(\vec{r},E,\vec{\Omega})\, Y_n^{m*}(\vec{\Omega})$$

$$= \int_{(V)} d\vec{r}' \sum_{n'=0}^{\infty} \sum_{m'=-n'}^{n'} G^{(n',m')\to(n,m)}(E,\vec{r}'\to\vec{r}) \left[S^{n',m'}(\vec{r}',E) + \int_0^{\infty} dE' \cdot \Sigma_{n'}(\vec{r}',E'\to E)\, \phi^{n',m'}(\vec{r}',E') \right] \quad (16)$$

The angular dependent integral equation (1) has thus been replaced by the infinite set of equations (16) from which the angular dependence has been removed. Definitions (7), (10) and (14), coupled with the orthogonality property (12) lead to the following equation for the parametric functions entering equ. (16):

$$S^{n,m}(\vec{r},E) = \int_{(4\pi)} d\vec{\Omega} \cdot S(\vec{r},E,\vec{\Omega})\, Y_n^{m*}(\vec{\Omega}) \quad (17)$$

$$\Sigma_n(\vec{r}',E'\to E) = \int_{(4\pi)} d\vec{\Omega} \int_{(4\pi)} d\vec{\Omega}'\, \Sigma(\vec{r}',E'\to E,\vec{\Omega}'\to\vec{\Omega})\, Y_n^{m*}(\vec{\Omega})\, Y_n^{m}(\vec{\Omega}') \quad (18)$$

$$G^{(n',m')\to(n,m)}(E,\vec{r}'\to\vec{r}) = \int_{(4\pi)} d\vec{\Omega} \int_{(4\pi)} d\vec{\Omega}'\, G(E,\vec{r}'\to\vec{r},\vec{\Omega}'\to\vec{\Omega})\, Y_n^{m*}(\vec{\Omega})\, Y_{n'}^{m'}(\vec{\Omega}') \quad (19)$$

Thus, in principle, one could evaluate the source, cross section and Green's function components through the use of the set (17)-(19) and then proceed to the solution of equ . (16), including in the expansion for the flux as many terms as required to obtain the desired accuracy. Similar procedures have actually been used by several authors[10, 11, 12] and, for simple, one-dimensional geometries, very reasonable machine-times may result. However, for complicated two-dimensional geometries such as reactor cells with the fuel in the form of rod clusters, the evaluation of the integral over space on the R.H.S. of equ. (16) becomes extremely difficult and a scheme based on the use of collision probabilities is to be preferred.

3. Generalized Collision Probabilities[13]

The medium of propagation may always be considered as made up of a number of homogeneous regions sufficiently small that the spatial distributions of fluxes and sources need not be taken into account in a detailed fashion but, for all practical purposes, may be supposed to remain flat throughout each individual region. We may then define:

$$\phi_j^{n,m}(E) = \frac{1}{V_j} \int_{(V_j)} d\vec{r} \; \phi^{n,m}(\vec{r},E) \tag{20}$$

$$S_j^{n,m}(E) = \frac{1}{V_j} \int_{(V_j)} d\vec{r} \cdot S^{n,m}(\vec{r},E) \tag{21}$$

$$\Sigma_{m,j}(E' \to E) = \frac{1}{V_j} \int_{(V_j)} d\vec{r} \cdot \Sigma_m(\vec{r},E' \to E) \tag{22}$$

$$\Delta_{ij}^{(n',m')\to(n,m)}(E)=\frac{1}{V_j}\cdot\frac{\int_{(V_j)}d\vec{r}\int_{(V_i)}d\vec{r}'\,G^{(n',m')\to(n,m)}(E,\vec{r}'\to\vec{r})\,H^{n',m'}(\vec{r},E)}{\frac{1}{V_i}\int_{(V_i)}d\vec{r}'\,H^{n',m'}(\vec{r}',E)}$$

$$=\frac{1}{V_j}\cdot\int_{(V_j)}d\vec{r}\int_{(V_i)}d\vec{r}'\,G^{(n',m')\to(n,m)}(E,\vec{r}'\to\vec{r}) \qquad (23)$$

The corresponding balance equations become

$$\phi_j^{n,m}(E)=\sum_i\sum_{n'=0}^{\infty}\sum_{m'=-n'}^{n'}\Delta_{ij}^{(n',m')\to(n,m)}(E)\left[S_i^{n',m'}(E)+\int_0^{\infty}\Sigma_{n',i}(E'\to E)\phi_i^{n',m'}(E')dE'\right] \qquad (24)$$

Let us now investigate in more detail the mathematical form of the generalized collision probabilities $\Delta_{ij}^{(n',m')\to(n,m)}(E)$. The Green's function for neutrons which have suffered no collision en route from the emission point $\vec{r}'$ to the field point $\vec{r}$ is specified by:

$$d\vec{r}.G(\vec{r}'\to\vec{r},\vec{\Omega}'\to\vec{\Omega},E)=\frac{e^{-\tau(\vec{r}';\vec{r})}}{|\vec{r}'-\vec{r}|^2}\,\delta(\vec{\Omega}.\vec{\Omega}')\,\delta\left(\vec{\Omega}.\frac{\vec{r}'-\vec{r}}{|\vec{r}'-\vec{r}|}\right)d\vec{r}$$
$$=e^{-\tau(\vec{r}';\vec{r})}\,\delta(\vec{\Omega}.\vec{\Omega}')\,\delta(\vec{\Omega}.\vec{\Omega}^*)\,d|\vec{r}'-\vec{r}|\,d\vec{\Omega}^* \qquad (25)$$

where the angular delta functions $\delta(\vec{\Omega}.\vec{\Omega}')$ has the following properties:

$$\delta(\vec{\Omega}.\vec{\Omega}')=0$$
$$\int g(\vec{\Omega}')\,\delta(\vec{\Omega}.\vec{\Omega}')\,d\vec{\Omega}'=g(\vec{\Omega}) \qquad (26)$$

and we have defined

$$\vec{\Omega}^{*} = \frac{\vec{r}' - \vec{r}}{|\vec{r}' - \vec{r}|} \tag{27}$$

Equ. (25) states that the angular flux of energy E produced along $\vec{\Omega}$ in the volume element

$$d\vec{r} = |\vec{r} - \vec{r}'|^{2}\, d|\vec{r}' - \vec{r}|\, d\vec{\Omega}^{*}$$

around point $\vec{r}$ by unit emission of neutrons of the same energy at point $\vec{r}'$ along $\vec{\Omega}'$ is obtained by applying Lambert's exponential law and introducing the first flight condition through the use of the delta functions. Thus, using equ. (19), as well as the properties (26) of the delta function we may write:

$$\Delta_{ij}^{(n',m')\rightarrow(n,m)}(E) = \frac{1}{V_j} \int_{(j)} d|\vec{r}' - \vec{r}| \int_{(V_i)} d\vec{r}' \int_{(\vec{\Omega}^{*})} d\vec{\Omega}^{*}\, e^{-\tau(\vec{r}';\vec{r})}\, Y_{n}^{m*}(\vec{\Omega}^{*})\, Y_{n'}^{m'}(\vec{\Omega}^{*}) \tag{28}$$

Further developments will be restricted to the case of a system of arbitrary size but effectively infinite in axial extent and the properties of which remain constant along this axial direction . Geometrical periodicity will not be introduced at this stage. The nature of the geometrical system to be considered is such that the dependence on the polar angle can be conveniently integrated out and all further considerations restricted to some plane of reference, normal to the axial direction. Thus the optical path from point $\vec{r}'$ to point $\vec{r}$ may be expressed as:

$$\tau(\vec{r}';\vec{r}) = \frac{\alpha(\vec{r}';\vec{r})}{\sin\theta^*}$$

where $\alpha(\vec{r}';\vec{r})$ is the projection of $\tau(\vec{r}';\vec{r})$ onto the plane z=0. If similarly, s is the projection of $|\vec{r}'-\vec{r}|$ onto the reference plane, we obtain:

$$\Delta_{ij}^{(n',m')\to(n,m)} = \frac{H_n^m H_{n'}^{m'}}{V_j} \int_{(j)} \frac{ds}{\sin\theta^*} \int_{(V_i)} d\vec{r}' \int_{(\phi^*)} d\phi^* \, e^{i(m'-m)\phi^*} \int_0^{\pi} d\theta^* \sin\theta^* \, e^{-\frac{\alpha(\vec{r}';\vec{r})}{\sin\theta^*}} P_n^m(\cos\theta^*) P_{n'}^{m'}(\cos\theta^*) \tag{30}$$

The associated Legendre function $P_\nu^\mu(\cos\theta)$ is given by:

$$P_\nu^\mu(\cos\theta) = \frac{1}{2^\nu} \sin^\mu\theta \sum_{0\leq r\leq \frac{\nu-\mu}{2}} (-1)^r \cdot \frac{1}{r!} \cdot \frac{(2\nu-2r)!}{(\nu-r)!} \cdot \frac{\cos^{(\nu-\mu-2r)}\theta}{(\nu-\mu-2r)!} \tag{31}$$

Since

$$\int_0^{\pi} e^{-\frac{x}{\sin\theta}} \sin^q\theta \cos^{2p+1}\theta \, d\theta = 0 \tag{32}$$

it appears immediately that for $|(n-m)+(n'-m')|$ odd, the probability $\Delta_{ij}^{(n',m')\to(n,m)}$ will be equal to zero. We need therefore consider and distinguish only two cases according to whether (n - m), (n'- m') are both odd or both even. In the first case, we obtain, after substitution of (31) into (30) and some algebric manipulation

$$\Delta_{ij\,(\text{odd-odd})}^{(m',m')\to(m,m)} = \frac{1}{V_i}\int_{(j)} da \int_{(V_i)} d\vec{r}' \int_{(\phi^*)} d\phi^* e^{i(m'-m)\phi^*}\int_0^{\pi} d\theta^* e^{-\frac{\alpha(\vec{r}':\vec{r})}{\sin\theta^*}} \sum_{K=0}^{\frac{1}{2}(m-m-1)} \sum_{K'=0}^{\frac{1}{2}(m'-m'-1)} \sum_{l=0}^{K+K'+1} c_l \sin^{m+m'+2l} \tag{33}$$

where

$$c_l = \frac{H_m^m H_{m'}^{m'}}{2^{m+m'}} \cdot \frac{(-1)^{\frac{1}{2}(m-m+m'-m'-2K-2K'+2l-2)}}{\left[\frac{1}{2}(m-m-2K-1)\right]!\left[\frac{1}{2}(m'-m'-2K'-1)\right]!} \cdot \frac{(m+m+2K+1)!\,(m'+m'+2K'+1)!}{\left[\frac{1}{2}(m+m+2K+1)\right]!\left[\frac{1}{2}(m'+m'+2K'+1)\right]!} \cdot \frac{(K+K'+1)!}{(2K+1)!\,(2K'+1)!\,l!\,(K+K'+1-l)!} \tag{34}$$

Similarly, in the second case, we have:

$$\Delta_{ij\,(\text{even-even})}^{(m',m')\to(m,m)} = \frac{1}{V_i}\int_{(j)} da \int_{(V_i)} d\vec{r}' \int_{(\phi^*)} d\phi^* e^{i(m'-m)\phi^*}\int_0^{\pi} d\theta^* e^{-\frac{\alpha(\vec{r}':\vec{r})}{\sin\theta^*}} \sum_{K=0}^{\frac{1}{2}(m-m)} \sum_{K'=0}^{\frac{1}{2}(m'-m')} \sum_{l=0}^{K+K'} c'_l \sin^{m+m'+2l} \tag{35}$$

where

$$c'_l = \frac{H_m^m H_{m'}^{m'}}{2^{m+m'}} \cdot \frac{(-1)^{\frac{1}{2}(m-m+m'-m'-2K-2K'+2l)}}{\left[\frac{1}{2}(m-m-2K)\right]!\left[\frac{1}{2}(m'-m'-2K')\right]!} \cdot \frac{(m+m+2K)!\,(m'+m'+2K')!}{\left[\frac{1}{2}(m+m+2K)\right]!\left[\frac{1}{2}(m'+m'+2K')\right]!} \cdot \frac{(K+K')!}{(2K)!\,(2K')!\,l!\,(K+K'-l)!} \tag{36}$$

Now, define the volume element $d\vec{r}'$ in V_i as equal to $dx\,dy$ where dx is an elemental increase in length along ϕ^* and dy is the elemental thickness of the volume element. Introduce, further, the Bickley functions $Ki_n(x)$ given by:

$$Ki_n(x) = \int_0^{\frac{\pi}{2}} e^{-\frac{x}{\sin\theta}} \sin^{n-1}\theta \, d\theta \tag{37}$$

Noting that

$$Ki_{m+1}(x) = \int_x^\infty Ki_m(t)\,dt \tag{38}$$

we can perform the integration over ds in V_j and dx in V_i:

$$\Delta_{ij\,(\text{odd-odd})}^{(n',m')\to(n,m)} = \frac{2}{\Sigma_i \Sigma_j V_j} \int_{(\phi^*)} d\phi^* e^{i(m'-m)\phi^*} \int dy \sum_{K=0}^{\frac{1}{2}(n-m-1)} \sum_{K'=0}^{\frac{1}{2}(n'-m'-1)} \sum_{\ell=0}^{K+K'+1} c_\ell Ki^*_{(3+m+m'+2\ell)}(\alpha_{\phi^*,y}) \tag{39}$$

$$\Delta_{ij\,(\text{even-even})}^{(n',m')\to(n,m)} = \frac{2}{\Sigma_i \Sigma_j V_j} \int_{(\phi^*)} d\phi^* e^{i(m'-m)\phi^*} \int dy \cdot \sum_{K=0}^{\frac{1}{2}(n-m)} \sum_{K'=0}^{\frac{1}{2}(n'-m')} \sum_{\ell=0}^{K+K'} c_\ell Ki^*_{(3+m+m'+2\ell)}(\alpha_{\phi^*,y}) \tag{40}$$

where (41)

$$Ki^*_m(\alpha_{\phi^*,y}) = Ki_m(\alpha_{i,j}) - Ki_m(\alpha_{i,j}+\alpha_i) - Ki_m(\alpha_{i,j}+\alpha_j) + Ki_m(\alpha_{i,j}+\alpha_i+\alpha_j)$$

The symbols $\alpha_i, \alpha_j, \alpha_{ij}$ denote respectively the projections onto the reference plane of the optical paths along ϕ^* in V_i, V_j and the intervening medium[14]. The integrations over ϕ^* and y must include all neutrons paths crossing both bodies i and j.

The self-collision probability of a convex body takes a slightly different form:

$$\Delta_{ii\,(\text{even-even})}^{(n',m')\to(n,m)} = \frac{1}{V_i}\int_0^{a} dx \int_0^{\alpha_i} \frac{\Sigma_i ds}{\Sigma_i} \int dy \int_{(\phi^*)} d\phi^* e^{i(m'-m)\phi^*} \int_0^{\pi} d\theta^* e^{-\frac{\Sigma_i x}{\sin\theta^*}} \sum_{K=0}^{\frac{1}{2}(n-m)} \sum_{K'=0}^{\frac{1}{2}(n'-m')} \sum_{\ell=0}^{K+K'} c_\ell \sin^{(m+m'+2\ell)}$$

$$= \frac{1}{\Sigma_i^2 V_i} \int dy \int_{(\phi^*)} d\phi^* e^{i(m'-m)\phi^*} \left[\alpha_i Ki_{(2+m+m'+2\ell)}(0) - Ki_{(3+m+m'+2\ell)}(\alpha_i) + Ki_{(3+m+m'+2\ell)}(0) \right]$$

The odd-odd collision-probability is completely analogous. If the body is not convex, further terms of the form (39), (40) must be added to take into account neutrons which leave and re-enter the body.

Since the probabilities $\Delta_{ij}^{(n',m')\to(n,m)}$ are equal to zero for $[(n-m)+(n'-m')]$ odd, the odd modes of the flux will form a set independent from that of the even modes. This is a result of the initial assumption concerning the axial extent supposed infinite, of the geometrical system. A look at equ. (16) shows that the first even mode (0, 0) is related to the total flux while the first odd mode (1, 0) is connected with the axial current, which cannot enter the solution of the problem set here. It will thus be sufficient to consider the set of balance equations (24) corresponding to $(n+m)$, $(n'+m')$ both even, the collision probabilities being given by equ. (40).

4. Formulation in terms of real quantities

The formulation given in the previous section presents the inconvenience of involving complex quantities which may introduce unneccessary complications in the numerical calculations. Although the objection is of minor importance, it is easily circumvented.

We note first that the total flux and the components of the current in the plane of reference are given by:

$$\phi(\vec{r},E) = \int_{(\vec{\Omega})} d\vec{\Omega}\,.\,\phi(\vec{r},E,\vec{\Omega}) = \frac{\phi^{0,0}}{H_0^0} \tag{43}$$

$$j_x(\vec{r},E) = \int_{(\vec{\Omega})} d\vec{\Omega}\,.\,\phi(\vec{r},E,\vec{\Omega}) \sin\theta\cos\phi = \frac{1}{2H_1^1}\left[\phi^{1,1} - \phi^{1,-1}\right] \tag{44}$$

$$j_y(\vec{r},E) = \int_{(\vec{\Omega})} d\vec{\Omega}\,.\,\phi(\vec{r},E,\vec{\Omega}) \sin\theta\sin\phi = \frac{i}{2H_1^1}\left[\phi^{1,1} + \phi^{1,-1}\right] \tag{45}$$

This suggests that we rewrite the expansions (6) and (7) in the form

$$\phi(\vec{r},E,\vec{\Omega}) = \sum_{n=0}^{\infty}\left[\phi^{\binom{0}{n0}}(\vec{r},E)\,Y^{0}_{n0} + \sum_{m=1}^{n}\left(\phi^{\binom{1}{nm}}(\vec{r},E)\,Y^{1}_{nm} + \phi^{\binom{2}{nm}}(\vec{r},E)\,Y^{2}_{nm}\right)\right] \quad (46)$$

$$S(\vec{r},E,\vec{\Omega}) = \sum_{n=0}^{\infty}\left[S^{\binom{0}{n0}}(\vec{r},E)\,Y^{0}_{n0} + \sum_{m=1}^{n}\left(S^{\binom{1}{nm}}(\vec{r},E)\,Y^{1}_{nm} + S^{\binom{2}{nm}}(\vec{r},E)\,Y^{2}_{nm}\right)\right] \quad (47)$$

where

$$Y^{0}_{n0} = (H^{0}_{0})^{2}\,P^{0}_{n}(\cos\theta) \quad (48)$$

$$Y^{1}_{nm} = 2(H^{m}_{n})^{2}\,P^{m}_{n}(\cos\theta)\cos(m\phi) \quad (49)$$

$$Y^{2}_{nm} = 2(H^{m}_{n})^{2}\,P^{m}_{n}(\cos\theta)\sin(m\phi) \quad (50)$$

$$\phi^{\binom{0}{n0}} = \frac{\phi^{n,0}}{H^{0}_{n}} \quad (51)$$

$$\phi^{\binom{1}{nm}} = \frac{\phi^{n,m} + (-1)^{m}\phi^{n,-m}}{2H^{m}_{n}} \quad (52)$$

$$\phi^{\binom{2}{nm}} = i\,\frac{\phi^{n,m} - (-1)^{m}\phi^{n,-m}}{2H^{m}_{n}} \quad (53)$$

$$S^{\binom{0}{n0}} = \frac{S^{n,0}}{H^{0}_{n}} \quad (54)$$

$$S^{\binom{1}{nm}} = \frac{S^{n,m} + (-1)^{m}S^{n,-m}}{2H^{m}_{n}} \quad (55)$$

$$S^{\binom{2}{nm}} = i \cdot \frac{S^{n,m} - (-1)^{m} S^{n,-m}}{2 H_{n}^{m}} \tag{56}$$

All of these quantities are real as can easily be verified by direct substitution of the expressions (6), (7) noting that:

$$P_{n}^{-m} = (-1)^{m} \frac{(n-m)!}{(n+m)!} P_{n}^{m} \tag{57}$$

$$H_{n}^{-m} = \frac{(n+m)!}{(n-m)!} H_{n}^{m} \tag{58}$$

Inserting the definitions (51)-(56) into the balance equation (24) we find:

$$\begin{aligned}\phi_{j}^{\binom{k}{nm}}(E) = \sum_{i} \sum_{n'=0}^{\infty} \Bigg\{ & \Delta_{ij}^{\binom{0}{n'0} \to \binom{k}{nm}}(E) \left(S_{i}^{\binom{0}{n'0}} + \int_{0}^{\infty} \Sigma_{n',i}(E' \to E)\, \phi_{i}^{\binom{0}{n'0}}(E')\, dE' \right) \\ & + \sum_{m'=1}^{n'} \sum_{k'=1}^{2} \left[\Delta_{ij}^{\binom{k'}{n'm'} \to \binom{k}{nm}}(E) \left(S_{i}^{\binom{k'}{n'm'}}(E) + \int_{0}^{\infty} \Sigma_{n',i}(E' \to E)\, \phi_{i}^{\binom{k'}{n'm'}}(E')\, dE' \right) \right] \Bigg\}\end{aligned} \tag{59}$$

where the collision probabilities are given by

$$\begin{aligned}\Delta_{ij}^{\binom{0}{n'0} \to \binom{0}{n0}} &= \frac{H_{n'}^{0}}{H_{n}^{0}} \Delta_{ij}^{(n',0) \to (n,0)} \\ &= \frac{2}{\Sigma_{i} \Sigma_{j} V_{j}} \cdot \frac{H_{n'}^{0}}{H_{n}^{0}} \int_{(\phi^{*})} d\phi^{*} \int dy \sum_{\kappa=0}^{\frac{1}{2}n} \sum_{\kappa'=0}^{\frac{1}{2}n'} \sum_{l=0}^{\kappa+\kappa'} c_{l}\, Ki_{(3+2l)}^{n}(\alpha_{\phi^{*},y})\end{aligned} \tag{60}$$

$$\Delta_{ij}^{\binom{1}{n'm'}\to\binom{0}{n0}} = \frac{H_{n'}^{m'}}{H_n^{0}}\left[\Delta_{ij}^{(n',m')\to(n,0)} + (-1)^{m'}\Delta_{ij}^{(n',-m')\to(n,0)}\right]$$

$$= \frac{4}{\bar{Z}_i\bar{Z}_jV_j}\cdot\frac{H_{n'}^{m'}}{H_n^{0}}\int_{(\phi^*)} d\phi^*\cos(m'\phi^*)\int dy\sum_{K=0}^{\frac{1}{2}n}\sum_{K'=0}^{\frac{1}{2}(n'-m')}\sum_{l=0}^{K+K'} c_l\, Ki^*_{(3+m'+2l)}(\alpha_{\phi^*},y) \qquad (61)$$

$$\Delta_{ij}^{\binom{2}{n'm'}\to\binom{0}{n0}} = i\,\frac{H_{n'}^{m'}}{H_n^{0}}\left[-\Delta_{ij}^{(n',m')\to(n,0)} + (-1)^{m'}\Delta_{ij}^{(n',-m')\to(n,0)}\right]$$

$$= \frac{4}{\bar{Z}_i\bar{Z}_jV_j}\cdot\frac{H_{n'}^{m'}}{H_n^{0}}\int_{(\phi^*)} d\phi^*\sin(m'\phi^*)\int dy\sum_{K=0}^{\frac{1}{2}n}\sum_{K'=0}^{\frac{1}{2}(n'-m')}\sum_{l=0}^{K+K'} c_l\, Ki^*_{(3+m'+2l)}(\alpha_{\phi^*},y) \qquad (62)$$

$$\Delta_{ij}^{\binom{0}{n'0}\to\binom{1}{nm}} = \frac{H_{n'}^{0}}{2H_n^{m}}\left[\Delta_{ij}^{(n',0)\to(n,m)} + (-1)^{m}\Delta_{ij}^{(n',0)\to(n,-m)}\right]$$

$$= \frac{2}{\bar{Z}_i\bar{Z}_jV_j}\,\frac{H_{n'}^{0}}{H_n^{m}}\int_{(\phi^*)} d\phi^*\cos(m\phi^*)\int dy\sum_{K=0}^{\frac{1}{2}(n-m)}\sum_{K'=0}^{\frac{1}{2}n'}\sum_{l=0}^{K+K'} c_l\, Ki^*_{(3+m+2l)}(\alpha_{\phi^*},y) \qquad (63)$$

$$\Delta_{ij}^{\binom{1}{n'm'}\to\binom{1}{nm}} = \frac{H_{n'}^{m'}}{2H_n^{m}}\left[\Delta_{ij}^{(n',m')\to(n,m)} + (-1)^{m}\Delta_{ij}^{(n',m')\to(n,-m)} + (-1)^{m'}\Delta_{ij}^{(n',-m')\to(n,m)} + (-1)^{m+m'}\Delta_{ij}^{(n',-m')\to(n,-m)}\right]$$

$$= \frac{4}{\bar{Z}_i\bar{Z}_jV_j}\cdot\frac{H_{n'}^{m'}}{H_n^{m}}\int_{(\phi^*)} d\phi^*\cos(m\phi^*)\cos(m'\phi^*)\int dy\sum_{K=0}^{\frac{1}{2}(n-m)}\sum_{K'=0}^{\frac{1}{2}(n'-m')}\sum_{l=0}^{K+K'} c_l\, Ki^*_{(3+m+m'+2l)}(\alpha_{\phi^*},y) \qquad (64)$$

$$\Delta_{ij}^{\binom{2}{n'm'}\to\binom{1}{nm}} = i\,\frac{H_{n'}^{m'}}{2H_n^{m}}\left[-\Delta_{ij}^{(n',m')\to(n,m)} - (-1)^{m}\Delta_{ij}^{(n',m')\to(n,-m)} + (-1)^{m'}\Delta_{ij}^{(n',-m')\to(n,m)} + (-1)^{m+m'}\Delta_{ij}^{(n',-m')\to(n,-m)}\right]$$

$$= \frac{4}{\bar{Z}_i\bar{Z}_jV_j}\cdot\frac{H_{n'}^{m'}}{H_n^{m}}\int_{(\phi^*)} d\phi^*\cos(m\phi^*)\sin(m'\phi^*)\int dy\sum_{K=0}^{\frac{1}{2}(n-m)}\sum_{K'=0}^{\frac{1}{2}(n'-m')}\sum_{l=0}^{K+K'} c_l\, Ki^*_{(3+m+m'+2l)}(\alpha_{\phi^*},y) \qquad (65)$$

$$\Delta_{ij}^{\binom{0}{m'o}\to\binom{2}{mm}} = i\,\frac{H^{o}_{m'}}{2H^{m}_{m}}\left[\Delta_{ij}^{(m',o)\to(m,m)} - (-1)^{m}\Delta_{ij}^{(m',o)\to(m,-m)}\right]$$

$$= \frac{2}{\Sigma_i\Sigma_j V_j}\cdot\frac{H^{o}_{m'}}{H^{m}_{m}}\int_{(\phi^*)} d\phi^*\sin(m\phi^*)\int dy\sum_{K=0}^{\frac{1}{2}(m-m)}\sum_{K'=0}^{\frac{1}{2}m'}\sum_{l=0}^{K+K'} c_l\, Ki^{*}_{(3+m+2l)}(\alpha_{\phi^*},y) \tag{66}$$

$$\Delta_{ij}^{\binom{1}{m'm'}\to\binom{2}{mm}} = i\,\frac{H^{m'}_{m'}}{2H^{m}_{m}}\left[\Delta_{ij}^{(m',m')\to(m,m)} - (-1)^{m}\Delta_{ij}^{(m',m')\to(m,-m)} + (-1)^{m'}\Delta_{ij}^{(m',-m')\to(m,m)} - (-1)^{m+m'}\Delta_{ij}^{(m',-m')\to(m,-m)}\right]$$

$$= \frac{4}{\Sigma_i\Sigma_j V_j}\cdot\frac{H^{m'}_{m'}}{H^{m}_{m}}\int_{(\phi^*)} d\phi^*\sin(m\phi^*)\cos(m'\phi^*)\int dy\sum_{K=0}^{\frac{1}{2}(m-m)}\sum_{K'=0}^{\frac{1}{2}(m'-m')}\sum_{l=0}^{K+K'} c_l\, Ki^{*}_{(3+m+m'+2l)}(\alpha_{\phi^*},y) \tag{67}$$

$$\Delta_{ij}^{\binom{2}{m'm'}\to\binom{2}{mm}} = \frac{H^{m'}_{m'}}{H^{m}_{m}}\left[\Delta_{ij}^{(m',m')\to(m,m)} - (-1)^{m}\Delta_{ij}^{(m',m')\to(m,-m)} - (-1)^{m'}\Delta_{ij}^{(m',-m')\to(m,m)} + (-1)^{m+m'}\Delta_{ij}^{(m',-m')\to(m,-m)}\right]$$

$$= \frac{4}{\Sigma_i\Sigma_j V_j}\cdot\frac{H^{m'}_{m'}}{H^{m}_{m}}\int_{(\phi^*)} d\phi^*\sin(m\phi^*)\sin(m'\phi^*)\int dy\sum_{K=0}^{\frac{1}{2}(m-m)}\sum_{K'=0}^{\frac{1}{2}(m'-m')}\sum_{l=0}^{K+K'} c_l\, Ki^{*}_{(3+m+m'+2l)}(\alpha_{\phi^*},y) \tag{68}$$

Of course, as above, only the modes corresponding to ($m+m$), ($m'+m'$) both even need be retained for the present problem. The collision probabilities are related, as will appear from a look at the above equations, through the reciprocity equation

$$\Delta_{ij}^{\binom{k'}{m'm'}\to\binom{k}{mm}} = \left[\frac{H^{\binom{k'}{m'm'}}}{H^{\binom{k}{mm}}}\right]^{2}\Delta_{ij}^{\binom{k}{mm}\to\binom{k'}{m'm'}} \tag{69}$$

where $H^{\binom{o}{mo}} = H^{o}_{m}$

$H^{\binom{k}{mm}} = \sqrt{2}\,H^{m}_{m} \quad (k=1,2)$

There are thus altogether only six independent types of probabilities.

6. The Zeroth and First Order Approximations

For $n = 0, 1$ it will be sufficient to evaluate the mutually independent probabilities, derived from the set of equs. (60)-(68),

$$\Delta_{ij}^{\left({}^{0}_{00}\right)\to\left({}^{0}_{00}\right)} = \frac{1}{4\pi \Sigma_i \Sigma_j V_j} \int\limits_{(\phi^*)} d\phi^* \int dy \, Ki_3^*(\alpha_{\phi^*,y}) \tag{70}$$

$$\Delta_{ij}^{\left({}^{1}_{11}\right)\to\left({}^{0}_{00}\right)} = \frac{3}{2\pi \Sigma_i \Sigma_j V_j} \int\limits_{(\phi^*)} d\phi^* \cos\phi^* \int dy \, Ki_4^*(\alpha_{\phi^*,y}) \tag{71}$$

$$\Delta_{ij}^{\left({}^{2}_{11}\right)\to\left({}^{0}_{00}\right)} = \frac{3}{2\pi \Sigma_i \Sigma_j V_j} \int\limits_{(\phi^*)} d\phi^* \sin\phi^* \int dy \, Ki_4^*(\alpha_{\phi^*,y}) \tag{72}$$

$$\Delta_{ij}^{\left({}^{1}_{11}\right)\to\left({}^{1}_{11}\right)} = \frac{3}{2\pi \Sigma_i \Sigma_j V_j} \int\limits_{(\phi^*)} d\phi^* \cos^2\phi^* \int dy \, Ki_5^*(\alpha_{\phi^*,y}) \tag{73}$$

$$\Delta_{ij}^{\left({}^{2}_{11}\right)\to\left({}^{1}_{11}\right)} = \frac{3}{2\pi \Sigma_i \Sigma_j V_j} \int\limits_{(\phi^*)} d\phi^* \sin\phi^* \cos\phi^* \int dy \, Ki_5^*(\alpha_{\phi^*,y}) \tag{74}$$

$$\Delta_{ij}^{\left({}^{2}_{11}\right)\to\left({}^{2}_{11}\right)} = \frac{3}{2\pi \Sigma_i \Sigma_j V_j} \int\limits_{(\phi^*)} d\phi^* \sin^2\phi^* \int dy \, Ki_5^*(\alpha_{\phi^*,y}) \tag{75}$$

The balance equations for the isotropic case reduce to

$$\phi_j^{\left({}^{0}_{00}\right)}(E) = \sum_i \Delta_{ij}^{\left({}^{0}_{00}\right)\to\left({}^{0}_{00}\right)}(E) \left[S_i^{\left({}^{0}_{00}\right)}(E) + \int_0^\infty \Sigma_{s,i}(E'\to E)\, \phi_i^{\left({}^{0}_{00}\right)}(E')\, dE' \right] \tag{76}$$

whereas, for linear anisotropy, we have the set

$$\begin{aligned}\phi_j^{\binom{0}{00}}(E) = \sum_i \Big\{ & \Delta_{ij}^{\binom{0}{00}\to\binom{0}{00}}(E)\Big[S_i^{\binom{0}{00}}(E) + \int_0^\infty \Sigma_{0,i}(E'\to E)\,\phi_i^{\binom{0}{00}}(E')\,dE'\Big] \\ & + \Delta_{ij}^{\binom{1}{11}\to\binom{0}{00}}(E)\Big[S_i^{\binom{1}{11}}(E) + \int_0^\infty \Sigma_{1,i}(E'\to E)\,\phi_i^{\binom{1}{11}}(E')\,dE'\Big] \\ & + \Delta_{ij}^{\binom{2}{11}\to\binom{0}{00}}(E)\Big[S_i^{\binom{2}{11}}(E) + \int_0^\infty \Sigma_{1,i}(E'\to E)\,\phi_i^{\binom{2}{11}}(E')\,dE'\Big]\Big\}\end{aligned} \tag{77}$$

$$\begin{aligned}\phi_j^{\binom{1}{11}}(E) = \sum_i \Big\{ & \Delta_{ij}^{\binom{0}{00}\to\binom{1}{11}}(E)\Big[S_i^{\binom{0}{00}}(E) + \int_0^\infty \Sigma_{0,i}(E'\to E)\,\phi_i^{\binom{0}{00}}(E')\,dE'\Big] \\ & + \Delta_{ij}^{\binom{1}{11}\to\binom{1}{11}}(E)\Big[S_i^{\binom{1}{11}}(E) + \int_0^\infty \Sigma_{1,i}(E'\to E)\,\phi_i^{\binom{1}{11}}(E')\,dE'\Big] \\ & + \Delta_{ij}^{\binom{2}{11}\to\binom{1}{11}}(E)\Big[S_i^{\binom{2}{11}}(E) + \int_0^\infty \Sigma_{1,i}(E'\to E)\,\phi_i^{\binom{2}{11}}(E')\,dE'\Big]\Big\}\end{aligned} \tag{78}$$

$$\begin{aligned}\phi_j^{\binom{2}{11}}(E) = \sum_i \Big\{ & \Delta_{ij}^{\binom{0}{00}\to\binom{2}{11}}(E)\Big[S_i^{\binom{0}{00}}(E) + \int_0^\infty \Sigma_{0,i}(E'\to E)\,\phi_i^{\binom{0}{00}}(E')\,dE'\Big] \\ & + \Delta_{ij}^{\binom{1}{11}\to\binom{2}{11}}(E)\Big[S_i^{\binom{1}{11}}(E) + \int_0^\infty \Sigma_{1,i}(E'\to E)\,\phi_i^{\binom{1}{11}}(E')\,dE'\Big] \\ & + \Delta_{ij}^{\binom{2}{11}\to\binom{2}{11}}(E)\Big[S_i^{\binom{2}{11}}(E) + \int_0^\infty \Sigma_{1,i}(E'\to E)\,\phi_i^{\binom{2}{11}}(E')\,dE'\Big]\Big\}\end{aligned} \tag{79}$$

In equs. (76)-(79)

$$\phi^{\binom{0}{00}}(E) = \phi(E) \tag{80}$$

$$\phi^{\binom{1}{11}}(E) = j_x(E) \tag{81}$$

$$\phi^{\binom{2}{11}}(E) = j_y(E) \tag{82}$$

For reactor eigenvalue problems

$$S^{\left(\substack{0\\00}\right)} = S^{\left(\substack{1\\11}\right)} = S^{\left(\substack{2\\11}\right)} = 0 \tag{83}$$

$$\bar{\Sigma}_0(E'\to E) = \lambda\chi(E)\nu(E')\bar{\Sigma}_f(E') + \Sigma_{\Delta,0}(E'\to E) \tag{84}$$

$$\bar{\Sigma}_1(E'\to E) = \Sigma_{\Delta,1}(E'\to E) \tag{85}$$

The quantity $\Delta_{ij}^{\left(\substack{0\\i0}\right)\to\left(\substack{0\\j0}\right)}$ is related to the usual first-flight collision probability through the equation

$$\Delta_{ij}^{\left(\substack{0\\i0}\right)\to\left(\substack{0\\j0}\right)} = \frac{V_i}{\bar{\Sigma}_j V_j} P_{ij} \tag{86}$$

Similarly, Benoist's(16) radial collision probability $P_{ij,r}$ is given by:

$$2P_{ij,r} = \frac{\bar{\Sigma}_j V_j}{V_i}\left[\Delta_{ij}^{\left(\substack{1\\i1}\right)\to\left(\substack{1\\j1}\right)} + \Delta_{ij}^{\left(\substack{2\\i1}\right)\to\left(\substack{2\\j1}\right)}\right] \tag{87}$$

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Alfred Nobel

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