COMPARATIVE STUDY ON THE NON-UNIFORM BURN-UP OF BURNABLE POISON PARTICLES USING ANALOGUE AND DIGITAL COMPUTER TECHNIQUES AND A NEW DEFINITION OF THE SELF-SHIELDING FACTOR

by

W.A. RAAlJMAKERS
(RCN)

1968
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Upon comparison of the results it is possible to conclude that also the first method adequately predicts the absorption characteristics provided the adjustment of the analogue model to some recent developments pertaining to the definition of the self-shielding factor.
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SUMMARY

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KEYWORDS

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POISONING
NEUTRON BEAMS
IRRADIATION
ANALOG SYSTEMS
PROGRAMMING
DIGITAL SYSTEMS
ABSORPTION
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Comparative study on the non-uniform burn-up of burnable poison particles using analogue and digital computer techniques and a new definition of the self-shielding factor

1. Introduction

In the study of the characteristic burn-up of a localized absorber volume (also called burnable poison particle) the main problem exists in evaluating the non-uniform changes with time of the neutron flux and absorption rate pattern inside the particle.

Two different approaches have been made by the author. In an early stage the problem was adapted to the means of solving it, at that time, i.e. to analogue machine computation (ref. 5). This had one immediate drawback. As one knows the analogue computer is only capable of handling one independent variable at the time whereas the problem has physically to be described by at least 2 parameters, due to time and a space dependent effects.

Therefore, it was unavoidable to rule out one of them which, evidently, could only be the space variable. This was done by making certain assumptions as to how the non-uniformity would affect the long term behaviour. To this end it was assumed in the model that at any moment the neutron flux and absorber atom's concentration should be uniform. (This presumption, as will be seen, turned out not too bad at all.)

Recently a digital computer model was constructed based on transport theory. This had the advantage of avoiding, to a certain degree, any mathematical restrictions (see ref. 4).

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The programme calculates the spatial neutron flux (in one or two groups) through the absorbing particle and an environment of fuel material and moderator. Consequently, the burn-up of the absorber and fuel atoms is computed during a time interval after which the whole procedure is repeated.

Obviously, this programme as against the analogue model does take account of the fine structure events i.e. the local change in neutron flux and absorption rate pattern. Details of the theory can be found in the references and will not be given here.

The aim of this report is to show how the results obtained with the two programmes compare. In order to be able to do so the analogue programme as presented in ref. 5 had to be adjusted and rearranged slightly. The most significant modification was the replacing of the curve of the self-shielding factor (fig. 14 in ref. 5, see also figures of the preceding chapter 3) with that given in fig. 3 of the chapter 3. Neglecting this difference i.e. leaving the model in its old form would produce the curves indicated by 0 (old) in the figures following.

In the past the self-shielding factor was developed on a basis assuming a uniform and hence isotropic non-perturbed neutron flux incident on the particle's surface (ref. 5 and curve f in figure 1, chapter.). In a later stage, however, it was found that neither the local neutron flux perturbations nor the non-uniformity of the neutron flux in the surrounding region could be left out in the consistent unit cell homogenization procedure laid down in chapter 2, where it is also shown that the former assumptions are inadmissible in practical cases. This led to a new self-shielding factor (fR in figure 1, chapter 3) which is to be used together with the factor β (L) also developed there.

Use was made of the data for the factor β (L) contained in figure 4, chapter though, as they served as a correction, they were linearized in order to facilitate the machine computations (see below). The quantity at hand, β (L), being the ratio of the neutron flux at the particle-fuel interface and the average fuel flux could actually be taken 1 as well as .97 (see below) without affecting the results to any significant degree nor, at all, the conclusions.

The results obtained are indicated by A (analogue), respectively D for those from the digital calculations.
2. **Theory**

This chapter will deal with the theory of the self-shielding factor. It is known from the theory of neutron transport that when an absorber is introduced into a region where a neutron flux is existing the neutron flux will be perturbed.

First of all, the neutron flux inside the absorber will exhibit a depression. Thus, apart from the eventual perturbation of the neutron flux in the vicinity of the absorber the effectiveness of neutron absorption by the absorber is lower than when each of the absorber atoms would be exposed to the same unperturbed neutron flux.

This effect is called self-shielding.

A way of expressing the neutron absorption in terms of the unperturbed, initially present, neutron flux is through the definition of a so-called self-shielding factor. When this factor \( f \) is defined as

\[
 f = \alpha_p / \beta
\]

in which \( \bar{\alpha}_p \) is the average neutron flux in the absorber and \( \bar{\beta} \) the unperturbed neutron flux outside of it, the total neutron absorption rate by the absorber can be written as

\[
 A_p = V_p \sum_p \bar{\alpha}_p = V_p \sum_p \bar{\beta} \cdot f
\]

\( V_p \) is the volume and \( \sum_p \) the macroscopic absorption cross section of the absorber. \( \sum_p . f \) is then called the effective macroscopic absorption cross section.

Once arrived at this point it is recommendable to make the following remark. A derivation of \( f \) can easily be given for various kinds of absorber geometry under the supposition that the, initially present, neutron flux \( \bar{\beta} \) is not affected or perturbed by the introduction of the absorber. This will be done below in the next paragraph.

In the second paragraph it will be explained in more detail how and where this definition is intrinsically wrong and to which faulty results it may lead. Obviously the crucial point is the assumption of non-perturbation of the neutron flux whereas this in itself is untolerable in most of the circumstances.
The self-shielding factor in idealized situations

References will be made in this paragraph to ref. 1, 2 and 3. At first it is assumed that there exists a region where one has a uniform neutron flux $\Phi$. As explained above the neutron absorption rate $A_p$ is then equal to

$$A_p = \frac{V_p}{\rho} \sum_{p} \Phi f$$

with the above definition for $f$. The problem now is the derivation of the formula for $f$. This will be executed briefly for spherically shaped neutron absorbers with radius $R$.

It can easily be shown (ref. 4) that the neutron absorption rate in such a particle is equal to

$$A_p = \frac{V_p}{\rho} \sum_{p} \Phi f = 4\pi R^2 j_\perp \cdot \left\{ 1 - \frac{1}{2y^2} + \frac{e^{-2y}}{y^2} (y + 1/2) \right\}$$

with $j_\perp$ the neutron current density into the particle

and $y = \Sigma R$ the self-shielding parameter.

Furthermore the number of neutrons entering the particle per second is equal to

$$E_p = 4\pi R^2 j_\perp$$

In the literature one now proceeds as follows. One defines a quantity $a$ as the fraction of neutrons entering that is absorbed hence

$$a = \frac{A_p}{E_p} = \left\{ 1 - \frac{1}{2y^2} + \frac{e^{-2y}}{y^2} (y + 1/2) \right\}$$

and a quantity $a^\perp$ as

$$a^\perp = \lim_{y \to 0} a = \frac{4y}{3}$$

for the present case of spherical absorbers. Next one defines the self-shielding factor $f$ as

$$f = \frac{a^\perp}{a} = \frac{3}{4y} \cdot \left\{ 1 - \frac{1}{2y^2} + \frac{e^{-2y}}{y^2} (y + 1/2) \right\}$$
This is the well-known formula customarily used everywhere in literature in the expression for $A_p$, i.e., for the ratio $\sigma_p/\phi$.

A graphical representation of $f$ vs $y$ is given in figure 1.

An example of the use of this quantity in connection with reactor calculation (burnable poisons) can be found in ref. 5.

Looking once more at the derivation of $f$ and analyzing what has happened actually one finds that $a$ stands for the ratio of the neutron absorption rate in the particle and the neutron in-flux into the particle taking into account as regards the numerator the flux depression within the particle. $a'$ is the same ratio, however, for the case of $y$ approaching zero, that means for the case where the particle has become infinitely small and each atom is exposed to the same originally present and unperturbed flux $\phi$. Hence the numerator of $a'$ can be written as $V_p \sum_p \phi$ as there is no flux depression within the particle any more. Calling the denominator of $a'$ one may find

$$a' = \frac{4V}{3} = \frac{V_p \sum_p \phi}{\phi}$$

thus $a'_d = (4\pi R^2) \frac{\phi}{4}$

Evidently the denominator $a'_d$ which, per definition should be equal to the in-flux of neutrons now has become the in-flux indeed, however, with the restriction that

$$j = \frac{\phi}{4}$$

which is actually so in a medium with uniform neutron flux.

Rewriting now the formula for $f$ one arrives at

$$f = \frac{a}{a'} = \left( \frac{\text{abs. rate with depression}}{\text{in-flux}} \right) \cdot \left( \frac{\text{abs. rate without depression}}{\text{in-flux (uniform)}} \right)$$

$$= \left( \frac{V_p \sum_p \phi}{4\pi R^2 j} \right) \cdot \left( \frac{V_p \sum_p \phi}{4\pi R^2 \phi/4} \right)^{-1} = \left( \frac{\phi}{\phi} \right) \cdot \left( \frac{2}{4} \right)$$
The same expression is obtained for e.g. solid cylinders and slabs. In other words \( f \) is, indeed, equal to the ratio of \( \bar{\Phi}_p \) and \( \Phi \) provided, in the actual case, that

\[
j_\infty = \frac{\Phi}{4}
\]

This, obviously, is never the case and only approximately so in the case of thin, lowly absorbing, particles that would leave an, initially present, uniform flux unaffected. Nevertheless, in literature, \( f \) is simply taken equal to the flux ratio thus neglecting the possibly significant deviation of \( j_\infty \) from \( \Phi/4 \). This neglect is inherently due to the assumption of non perturbed uniform flux for which indeed \( j_\infty = \Phi/4 \).

Physically speaking this is, of course, nonsense. The neutron flux in the vicinity of the absorber must be considered perturbed as a uniform neutron flux implies no gradient hence no net neutron current into the absorber and hence no absorptions taking place at all. Therefore it is stressed that \( j_\infty \neq \Phi/4 \) and may deviate from it considerably (see following paragraph).

2.2. The self-shielding factor in non-idealized situations

From the above it appears that a better self-shielding factor \( f_R \) defined as

\[
f_R = \frac{\bar{\Phi}_p}{\Phi}
\]

should be derived from

\[
f_R = f \left( \frac{j_\infty}{\Phi/4} \right)
\]

Doing so, however, one must focus a little more on the real flux shape in the vicinity of the absorber and on the significance of \( \Phi \) in this regard.

To this end one is invited to study the three figures 2-1, 2-2 and 2-3.

Situation 1 (fig. 2-1) corresponds to a unit cell of a reactor where, for simplicity's sake, only two regions (fuel and moderator) are shown. The flux pattern is indicated as well as the average flux levels in the two regions. A disadvantage factor
factor is defined as

$$F_1 = \frac{\bar{\phi}_f}{\bar{\phi}_m}$$

where \( f \) and \( m \) refer to fuel respectively moderator and subscript 1 to situation 1.

In situation 2 (fig. 2-2) an amount of poison is introduced into the fuel in a uniform manner keeping, again for simplicity's sake but without losing generality, the average fuel flux \( \bar{\phi}_f \) equal to \( \bar{\phi}_{f1} \). Again a disadvantage factor is defined as

$$F_2 = \frac{\bar{\phi}_f^2}{\bar{\phi}_m^2}$$

The neutron absorption rate by the poison may now be written as

$$A_{p2} = V_p \sum_{p2} \bar{\phi}_{p2} = V_f \sum_{p2} \bar{\phi}_{f2}$$

with \( \sum_{p2} \) the volume averaged macroscopic poison absorption cross section.

Situation 3 (fig. 2-3), finally, corresponds to the same unit cell, however, with all the poison lumped together as a tiny central string occupying volume \( V_p3 \). The average fuel flux \( \bar{\phi}_f \) is, again, kept constant, i.e. equal to the previous ones, so

$$\bar{\phi}_{f3} = \bar{\phi}_f = \bar{\phi}_{f1} = \bar{\phi}_f$$

This disadvantage factor now reads \( F_3 = \frac{\bar{\phi}_f}{\bar{\phi}_m} \)

The neutron absorption rate by the poison now becomes

$$A_{p3} = V_p \sum_{p3} \bar{\phi}_{p3} \cdot f_{R3}$$

defining

$$f_{R3} = \frac{\bar{\phi}_p}{\bar{\phi}_f}$$

and \( \sum_{p3} \) as the macroscopic absorption cross section of the poison material itself.

From the above one finds
\[ \frac{A_{p3}}{A_{p2}} = f_{R3} \]

hence \( A_{p3} = A_{p2} f_{R3} \)

with \( A_{p2} = V_{p3} \sum_{p3} \bar{\varphi}_{f3} = V_{p} \sum_{p} \bar{\varphi}_{f} \)

and \( f_{R3} = \frac{\bar{\varphi}_{p3}}{\bar{\varphi}_{f3}} = \frac{A_{p3}}{V_{p} \sum_{p} \bar{\varphi}_{f}} = f_{R} \)

From paragraph I one has

\[ A_{p3} = A_{p} = 4 \pi R^2 j_{-} \left[ 1 - \frac{1}{2y^2} + \frac{e^{-2y}}{y} (y + 1/2) \right] \]

\[ = \frac{16}{3} \pi \sum_{p} R^3 j_{-}.f \]

hence \( f_{R} = f \left( \frac{j_{-}}{\varphi_{f}/4} \right) \)

which can be shown to apply to solid cylinders, slabs and spheres.

This formula for the self-shielding factor as against the one given before takes into account two things: the being non-uniform of the external neutron flux and the fact that at a particular point, in this case the interface between poison and fuel \( j_{-} \neq \varphi_{R}/4 \) where \( \varphi_{R} \) is the neutron flux in that point.

The formula is now rearranged as

\[ f_{R} = f_{R1} \cdot \beta \]

with \( \beta = \frac{\varphi_{R}}{\varphi_{f}} \)

and \( f_{R1} = f(j_{-}/\varphi_{R}/4) \) which can be proven to give
A graph of $f_H$ vs $y$ is given in figure 3 together with a graph (fig. 4) representing $\beta$ also as a function of $\Sigma_p$ calculated by the ECLIPSE programme on a system with spherical geometry (ref. 4) for specific values of particle size and of fuel enrichment and moderator-to-fuel ratio.

It can not be denied that the method presented above gives the only correct way of taking account of the contribution of a lumped poison in cell homogenization procedures. When e.g. the total neutron absorption rate for the entire cell is to be calculated one has

$$A_{cell \ 3} = \frac{A}{f_p} + A_f + A_m$$

$$= f_R (V_p \ Sigma_p \ \bar{\rho}_p) + V_f \Sigma_f \bar{\rho}_f + V_m \Sigma_m \bar{\rho}_m$$

$$= \bar{\rho}_f \left\{ (V_p \Sigma_p)f_{H} + V_f \Sigma_f + V_m \Sigma_m/f_3 \right\}$$

Clearly the difference in result when using $f$ (see next chapter) instead of $f_R$ at this place would be considerable. The latter expression can be modified somewhat more to

$$A_{cell \ 3} = \bar{\rho}_f \left\{ (V_p \Sigma_p)f_{R1}\beta + V_f \Sigma_f + V_m \Sigma_m/f_{mp} \right\}$$

with $f_{mp} = \frac{1}{F_3} = \frac{1}{\alpha F_1}$

with $\alpha = \bar{\rho}_m/\bar{\rho}_3$

As to $\alpha$ it can easily be shown that this quantity may be written as

$$\alpha = \frac{A_{1} - \frac{A_{f1}}{V_m}}{A_{f1} - \frac{A_{f1}}{V_m} + \frac{A_{p3}}{V_m}}$$
where $q$ is the slowing down density in the moderator region in the two situations 1, respectively 3. As such $\alpha$ would be a function of $y$ also. A closer look, however, learns that over the range of $y$ from 3.5 to 0 $\alpha$ changes with only 4%. Considering the fact that $\alpha$ appears in the third term of the last expression for $A_{\text{cell} 3}$, a term which, itself, contributes very little to the whole it may be concluded that for practical cases $\alpha$ may as well be taken 1 all the time.

The final expression for $A_{\text{cell} 3}$ therefore becomes

$$A_{\text{cell} 3} = \frac{V_p}{\Sigma_f} \left\{ \left( V_p \Sigma_f R^1 \beta \right) + V_f \Sigma_f + V_m \Sigma_m \right\}$$

with $f_m = 1/F_1$

A final remark may be given concerning $\beta$. Considering only one poison particle in the centre of a fuel rod $\beta$ would be less than one (see fig. 4) and $f_{R_1}$ should be corrected for it in order to yield $f_R$. Other particles, however, e.g. in the outer regions of the fuel pellet where the neutron flux will be higher than the average, may have $\beta$ values higher than one. Therefore as an approximate average $\beta$ may as well be taken 1 in calculations where the fuel is considered to contain a more or less uniformly distributed lumped poison.
FIG. 2-1

f  m

FIG. 2-2

f+p  m

FIG. 2-3

p  f  m
5. Discussion of the results

A series of 18 examples have been examined the results of which are contained in the figures 2 through 19 of this chapter. They all show the relative absorption rate in the absorber (μA) versus the average irradiation (I) of the fuel region.

Figure 1 gives the two curves for the self-shielding factor f versus the self-shielding parameter \( y = \frac{\Sigma}{2} \)

as used before (ref. 5) respectively \( f_1 \) versus \( y \) as used this time and as proposed in chapter 2 (\( \Sigma \) is the macroscopic thermal neutron absorption cross section; \( d \) is the particle's diameter).

The cases 1 through 8 (figures 2 through 9) were all for natural \( \text{B}_{10} \) as absorber material and for different values of \( d \). The cross section for absorption of the \( \text{B}^{10} \) isotope has been varied corresponding to various possible conditions of the neutron spectrum. Only thermal neutron absorption was considered in these examples. Next 4 cases were selected from the first series this time including epithermal neutron absorption as well (cases 9 through 12, figures 10 through 13).

The last 6 cases (13 through 18, figures 14 through 19) were all for \( \text{Dy}_2\text{O}_3 \) as absorber material taken to consist of 4 isotopes contributing to the thermal as well as epithermal neutron absorption. Natural \( \text{Dy}_2\text{O}_3 \) was assumed except for the very last case which is for \( \text{Dy}_2\text{O}_3 \) enriched in the Dy-164 isotope.

In connection with the linearization of the factor \( \beta(\Sigma) \) (see above) a quantity \( \alpha \) has been defined as

\[
\alpha = \frac{\beta(0) - \beta(\Sigma)}{\Sigma \cdot d/2}
\]

with \( \beta(0) = .97 \) which figure has been obtained from the digital calculations (see ref. 5). Thus it is possible to write

\[
f_{R1} \cdot \beta(\Sigma) = f_{R1} (\beta(0) - dy) \quad \text{(ref. 5)}
\]

according to which the analogue model had to undergo an unimportant rearrangement.
The table below gives all the relevant data used in the treating of the 18 examples.

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<th>Figure</th>
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<th>$E_2$</th>
<th>$E_3$</th>
<th>$E_4$</th>
<th>$\sigma_e$</th>
<th>$\sigma_{e1}$</th>
<th>$\sigma_{e2}$</th>
<th>$\sigma_{e3}$</th>
<th>$\sigma_{e4}$</th>
</tr>
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<tbody>
<tr>
<td>natural</td>
<td>.150</td>
<td>.255</td>
<td>.249</td>
<td>.281</td>
<td>.290</td>
<td>1.0</td>
<td>1.8</td>
<td>6.0</td>
<td>10.0</td>
</tr>
<tr>
<td>enriched</td>
<td>.100</td>
<td>.100</td>
<td>.100</td>
<td>.100</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
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The crosses refer to the Dy$_2$O$_3$ cases for which the following data were used:
The $E$ values are the initial number densities in units of $0.0252 \times 10^{24} \text{cm}^{-3}$; the $\sigma$'s are given in barns ($= 10^{-24} \text{cm}^2$).

The significance of the various symbols is:

- $\sigma_{ti}$ = microscopic thermal absorption cross section for isotope $i$
- $\sigma_{ei}$ = microscopic epithermal absorption cross section for isotope $i$
- $E_i$ = initially relative occurrence of isotope $i$
- $N(0)$ = initially present number density of poison atoms
- $\Sigma(0)$ = initial macroscopic thermal absorption cross section for the poison material
- $d$ = particle's diameter
- $\alpha(0)$ = quantity defined above
- $\gamma$ = ratio of epithermal-to-thermal neutron flux (ref. 5).

The density of the various materials was taken

- a) 2.34 gr. cm$^{-3}$ for B$_4$C
- b) 7.81 gr. cm$^{-3}$ for Dy$_2$O$_3$
4. Conclusions

Looking at the figures one general conclusion may be drawn. It is evident that the analogue programme is quite well able to predict the results otherwise obtainable from the digital programme.

This is an interesting fact as, in practice, analogue computing may be preferred over digital work especially in survey calculations where in a short time many parameters are being studied and varied over wide ranges.

The difference in results for the relative absorption rate mainly consists of an initially slightly faster falling off which, however, is not at all important. It should not be forgotten that the studies on the behaviour of burnable poison are aimed at the possible application of such absorbers as reactivity control materials in nuclear reactor cores. The deviations observed would then be reflected in a slightly different reactivity behaviour of the reactor core.

What is more important is how the absorption characteristics behave after a longer period of time, when also the reactivity of the reactor core has diminished so that any significant difference then would result in e.g. a shortening of the lifetime of the core. In this respect the A and D curves compare satisfactorily certainly when considering all sources of inaccuracy and uncertainty involved in these and other calculations on such events. The biggest deviations occur for DyO₃ where it is hard to judge whether the A curves come out better than the 0 curves. However, in these examples the RA falls off too slowly anyway, i.e. shortening reactor life severely in order to be of any interest at all with respect to its potential use. Considering this fact the A curves are also here well in accordance with the D curves.
<table>
<thead>
<tr>
<th>References</th>
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<th>Authors</th>
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<tr>
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Alfred Nobel
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