EUR 3291.e

EUROPEAN ATOMIC ENERGY COMMUNITY - EURATOM

THEORY OF A NEUTRON POLARIMETER

by

R. MISENTA

1967

Joint Nuclear Research Center
Ispra Establishment - Italy

Reactor Physics Department
Experimental Neutron Physics

This report is on sale at the addresses listed on cover page 4

at the price of FF 6.– FB 60.– DM 4.80 Lit. 750 Fl. 4.30

When ordering, please quote the EUR number and the title, which are indicated on the cover of each report.

Printed by SMEETS
Brussels, April 1967

EUR 3291.e

THEORY OF A NEUTRON POLARIMETER by R. MISENTA

European Atomic Energy Community - EURATOM
Joint Nuclear Research Center - Ispra Establishment (Italy)
Reactor Physics Department - Experimental Neutron Physics
Brussels, April 1967 - 38 Pages - 2 Figures - FB 60

An experimental arrangement is proposed for the separation of those neutrons which have been scattered from an initially polarized beam of thermal neutrons without change of their initial spin direction, from those which have reversed their spin direction. This proposed neutron polarimeter consists of a polarizing crystal and of an analysing system. The polarizing crystal monochromatizes and polarizes the incident neutron beam from the reactor. The analysing system analyses the polarization and the energy of the scattered beam. It has two different states for the polarization analysis. In the «parallel state» only those neutrons pass the analysing system and reach the detector which have maintained,

EUR 3291.e

THEORY OF A NEUTRON POLARIMETER by R. MISENTA

European Atomic Energy Community - EURATOM
Joint Nuclear Research Center - Ispra Establishment (Italy)
Reactor Physics Department - Experimental Neutron Physics
Brussels, April 1967 - 38 Pages - 2 Figures - FB 60

An experimental arrangement is proposed for the separation of those neutrons which have been scattered from an initially polarized beam of thermal neutrons without change of their initial spin direction, from those which have reversed their spin direction. This proposed neutron polarimeter consists of a polarizing crystal and of an analysing system. The polarizing crystal monochromatizes and polarizes the incident neutron beam from the reactor. The analysing system analyses the polarization and the energy of the scattered beam. It has two different states for the polarization analysis. In the «parallel state» only those neutrons pass the analysing system and reach the detector which have maintained,

EUR 3291.e

THEORY OF A NEUTRON POLARIMETER by R. MISENTA

European Atomic Energy Community - EURATOM
Joint Nuclear Research Center - Ispra Establishment (Italy)
Reactor Physics Department - Experimental Neutron Physics
Brussels, April 1967 - 38 Pages - 2 Figures - FB 60

An experimental arrangement is proposed for the separation of those neutrons which have been scattered from an initially polarized beam of thermal neutrons without change of their initial spin direction, from those which have reversed their spin direction. This proposed neutron polarimeter consists of a polarizing crystal and of an analysing system. The polarizing crystal monochromatizes and polarizes the incident neutron beam from the reactor. The analysing system analyses the polarization and the energy of the scattered beam. It has two different states for the polarization analysis. In the «parallel state» only those neutrons pass the analysing system and reach the detector which have maintained,

and in the ‹antiparallel state› only those neutrons which have reversed their initial spin direction. In addition to the polarization analysis the energy of those neutrons which pass the analysing system, is measured by this system.

The theory of a neutron polarimeter is developed. The influence of deviations of the polarization and analysing efficiencies from 1 on the measuring accuracy of the polarization degree of the scattered beam is discussed and correction factors are derived.

The separate measurement of the intensities of neutrons which have been scattered with and without spin flip allows the separation of coherently and incoherently scattered neutrons in neutron diffraction experiments, and in the determination of phonon dispersion relations and frequency distribution functions. Besides these new kinds of experiments all kinds of experiments are possible, e.g. measurement of the neutron spin flip probability of elements, depolarization studies in magnetic materials which have been performed up to now by the ‹double transmission method›.

and in the ‹antiparallel state› only those neutrons which have reversed their initial spin direction. In addition to the polarization analysis the energy of those neutrons which pass the analysing system, is measured by this system.

The theory of a neutron polarimeter is developed. The influence of deviations of the polarization and analysing efficiencies from 1 on the measuring accuracy of the polarization degree of the scattered beam is discussed and correction factors are derived.

The separate measurement of the intensities of neutrons which have been scattered with and without spin flip allows the separation of coherently and incoherently scattered neutrons in neutron diffraction experiments, and in the determination of phonon dispersion relations and frequency distribution functions. Besides these new kinds of experiments all kinds of experiments are possible, e.g. measurement of the neutron spin flip probability of elements, depolarization studies in magnetic materials which have been performed up to now by the ‹double transmission method›.

and in the ‹antiparallel state› only those neutrons which have reversed their initial spin direction. In addition to the polarization analysis the energy of those neutrons which pass the analysing system, is measured by this system.

The theory of a neutron polarimeter is developed. The influence of deviations of the polarization and analysing efficiencies from 1 on the measuring accuracy of the polarization degree of the scattered beam is discussed and correction factors are derived.

The separate measurement of the intensities of neutrons which have been scattered with and without spin flip allows the separation of coherently and incoherently scattered neutrons in neutron diffraction experiments, and in the determination of phonon dispersion relations and frequency distribution functions. Besides these new kinds of experiments all kinds of experiments are possible, e.g. measurement of the neutron spin flip probability of elements, depolarization studies in magnetic materials which have been performed up to now by the ‹double transmission method›.

EUR 3291.e

EUROPEAN ATOMIC ENERGY COMMUNITY - EURATOM

THEORY OF A NEUTRON POLARIMETER

by

R. MISENTA

1967

Joint Nuclear Research Center
Ispra Establishment - Italy

Reactor Physics Department
Experimental Neutron Physics

SUMMARY

An experimental arrangement is proposed for the separation of those neutrons which have been scattered from an initially polarized beam of thermal neutrons without change of their initial spin direction, from those which have reversed their spin direction. This proposed neutron polarimeter consists of a polarizing crystal and of an analysing system. The polarizing crystal monochromatizes and polarizes the incident neutron beam from the reactor. The analysing system analyses the polarization and the energy of the scattered beam. It has two different states for the polarization analysis. In the «parallel state» only those neutrons pass the analysing system and reach the detector which have maintained, and in the «antiparallel state» only those neutrons which have reversed their initial spin direction. In addition to the polarization analysis the energy of those neutrons which pass the analysing system, is measured by this system.

The theory of a neutron polarimeter is developed. The influence of deviations of the polarization and analysing efficiencies from 1 on the measuring accuracy of the polarization degree of the scattered beam is discussed and correction factors are derived.

The separate measurement of the intensities of neutrons which have been scattered with and without spin flip allows the separation of coherently and incoherently scattered neutrons in neutron diffraction experiments, and in the determination of phonon dispersion relations and frequency distribution functions. Besides these new kinds of experiments all kinds of experiments are possible, e.g. measurement of the neutron spin flip probability of elements, depolarization studies in magnetic materials which have been performed up to now by the «double transmission method».

Table of Contents

I. Introduction (*)

By scattering a beam of polarized neutrons on a target the degree of polarization in the scattered beam is changed since a fraction of the neutrons maintains whereas the other fraction changes its original spin orientation. This depolarization phenomena occurs by scattering of neutrons on free nuclei, molecules, liquids or solids. The degree of depolarization depends on properties of the nuclei and for thermal neutrons in addition on properties of the molecules and of the liquid and solid state. The probability that a neutron which is scattered by a certain scattering angle maintains or changes its original spin direction is related to the coherent and incoherent scattering cross section of the free nuclei.

In performed experiments the polarization degree of the scattered beam has been measured by the "double transmission method", e.g. in measurements of the ratio between incoherent and total scattering cross sections of free nuclei [1] or in studies of the depolarization of neutrons by their passage through magnetized ferromagnetic materials [2]. In the "double transmission method" the polarization degrees of the incident and scattered beam are measured directly without separating the neutrons which have been scattered without and with spin flip.

It is the purpose of this paper to describe the principle of an experimental arrangement by which it is possible to measure separately in the scattered beam the intensity of neutrons which are scattered with spin flip and the intensity of neutrons which are scattered without spin flip. The polarization degree is easily calculated from these two intensities. This experimental arrangement which is called neutron polarimeter, in analogy to a polarimeter in optics, can be used for depolarization measurements which were performed up to now with "the double transmission method." In addition, the possibility to separate with a neutron polarimeter the neutrons scattered with and without spin flip and in consequence to separate the coherent and incoherent part of the scattered beam allows a number of novel applications.

(*) Manuscript received on February 2, 1967.

These novel applications are the separation of the part of neutrons which are scattered incoherently due to a spin flip, from that part which are scattered coherently in elastic and inelastic scattering of thermal neutrons by liquids and solids.

II. Principle of the Neutron Polarimeter

Figure 1 shows a schematic diagram of a neutron polarimeter. The neutrons are monochromatized and polarized by the crystal P. The neutrons scattered by a certain scattering angle enter the analysing system. Only neutrons with a given energy and in **one** of the two polarization directions pass the analysing system and are counted by the detector. For the determination of the polarization degree the analysing system has the two different states

- analysing system "in the parallel state"
- analysing system "in the antiparallel state."

The detector measures in these two states of the analysing system the following intensities:

1. Analysing system "in the parallel state"

   Only those neutrons reach the detector which have maintained their initial spin orientation. The intensity which corresponds to this fraction of the neutron flux is called "intensity in the parallel state" and abbreviated by I_p.

2. Analysing system "in the antiparallel state"

   In this state of the analysing system only those neutrons reach the detector which have changed their initial spin orientation in the scattering process. This intensity is called "intensity in the antiparallel state" and abbreviated by I_a.

In addition, the total intensity I_t of the neutrons scattered into the detector can be measured independently of their polarization.

The polarization of the beam scattered into the analysing system is calculated from the measured intensities by

(II-1) $$P_m = \frac{I_p - I_a}{I_p + I_a}$$

A polarized incident neutron beam is being depolarized by scattering on a sample. The depolarization factor D_f is given by

(II-2) $$D_f = \frac{D^2\sigma_{wo} - D^2\sigma_w}{D^2\sigma_{wo} + D^2\sigma_w}$$

where the abbreviations

$D^2\sigma_{wo} = \frac{d^2\sigma_{wo}}{d\Omega dE}$ and $D^2\sigma_w = \frac{d^2\sigma_w}{d\Omega dE}$ have been used for the double differential cross section without (Index wo) and with (Index w) spin flip during the scattering process. The depolarization factor D_f assumes as maximum possible value + 1 for $D^2\sigma_w = 0$, i.e. the scattering sample induces no spin flip and as negative value - $^1/_3$, i.e. the spins of $^2/_3$ of all scattered neutrons have been flipped during the scattering process.

The depolarization factor D_f and the measured polarization P_m of the scattered beam are equal

(II-3) $$P_m = D_f$$

under the condition that the polarization efficiency of the polarizer and the analysing efficiency of the analysing system are equal to 1. In this case also the following relations are valid

(II-4) $I_p = F\ D^2\sigma_{wo} \qquad I_a = F\ D^2\sigma_w$,

where the proportionality factor F comprises all factors as solid angles, reflectivities of the polarizing and analysing crystal and efficiency of the detector*). The factor F is equal for the two intensities.

The double differential cross sections without and with spin flip are related to the coherent and incoherent cross section by

(II-5a) $D^2\sigma_{wo} = D^2\sigma^{coh} + \frac{1}{3} D^2\sigma^{inc}$

(II-5b) $D^2\sigma_w = \frac{2}{3} D^2\sigma^{inc}$.

---

*) Due to the deviations of the polarization efficiency and the analysing efficiency from 1, the intensity in the parallel state will contain also a term with the cross section $D^2\sigma_w$, and the intensity in the antiparallel state a term with $D^2\sigma_{wo}$. This additional term determines the measuring accuracy of each of the two intensities. In chapter IV.B the dependence of the measuring error on the efficiencies is derived, and the obtainable measuring accuracy is discussed.

The coherently and incoherently scattered intensities

(II-6a) $$I_{coh} = F\ D^2 \sigma^{coh}$$ and

(II-6b) $$I_{inc} = F\ D^2 \sigma^{inc}$$

can be separated by measuring with a neutron polarimeter the intensities in the parallel and antiparallel state. The separation is obtained by combining the relations II-4, II-5 and II-6

(II-7a) $$I_{coh} = I_p - \frac{1}{2} I_a$$

(II-7b) $$I_{inc} = \frac{3}{2} I_a.$$

III. Descriptions of the Neutron Polarimeter

The neutron polarimeter is developed from a normal triple axis spectrometer by adding to the monochromater crystal the property to polarize the incident neutron beam,and using instead of an analyser crystal which determines only the energy of the scattered neutrons,an analysing system which analyses in addition to the energy the polarization state of the scattered beam. For the polarizer and for the analysing system,the techniques used by Nathans, Shull, Shirane and Andresen [3] for the measurement of the magnetic scattering of iron and nickel are applied.

A) General Description

The neutron beam from the reactor is collimated, and impinges the polarizing crystal. The reflected beam is polarized in a direction determined by the direction of the magnetic field which magnetizes the polarizer, and to a degree which depends on the polarization efficiency of the crystal. The partly polarized beam passes through a second collimator which is magnetized and maintains a uniform magnetic field in the same sense as the polarizing field. The neutrons impinge the sample and are scattered with or without change of their spin orientation. The neutrons which are scattered into a certain scattering angle pass a third collimator which is also magnetized and enter into the analysing system. In front of the analysing crystal is a coil which is a part of the spin flipping unit. This unit reverses the spin direction of the neutrons traversing the coil when it is turned on.

If the analysing crystal is set with a glancing angle ϑ_A and the spin flipping unit is turned off,the analysing system is in "the parallel state for neutrons whose wave length correspond to the glancing angle ϑ_A." In this parallel state the analysing crystal scatters those neutrons whose wave length fulfill the Bragg condition for the glancing angle ϑ_A and which have been scattered without spin flip, i.e. whose polarization corresponds to the initial polarization direction, through a fourth collimater into the detector. Due to the deviation of the polarization efficiencies of the polarizing and analysing crystal from 1, also a fraction of neutrons which have been scattered with spin flip will reach the detector .

If the analysing crystal is maintained at the same glancing angle ϑ_A and the spin flipping unit is turned on, the analysing system is in "the antiparallel state for neutrons whose wave length correspond to the glancing angle ϑ_A." In this antiparallel state the analysing crystal scatters those neutrons into the detector whose wave length fulfill the Bragg condition and which have been scattered with spin flip, i.e. whose polarization has been reversed with reference to the initial polarization direction. Due to the deviations of the polarization efficiencies and of the polarization reversal efficiency from 1, also a fraction of neutrons which have been scattered without spin flip will reach the detector.

The initial energy of the neutrons, the analysing energy and the scattering angle of the neutron polarimeter can be changed in the same way as for a triple axis spectrometer. A uniform magnetic field in the same sense as the polarizing field is maintained on the path between polarizing crystal and sample, and between sample and analysing crystal in order to prevent transitions from one neutron spin state to the other, which would diminish the measuring accuracy.

B) Polarizing Crystal and Analysing System

As polarizing and as analysing crystal, single crystals of cobalt-iron with 92 atomic per cent are used. For the reflections from the (111) plane of such crystals polarizing efficiencies between 99 and 100 per cent have been measured in magnetic fields above 2 k.oersteds [3].

For the measurement of the intensity, I_a, in the antiparallel position, the spins of the scattered neutrons are reversed in the region between scattering sample and analysing crystal by means of the magnetic resonance method [4, 5]* . For this purpose a radiofrequency field can be applied to the coil in front of the analysing crystal. The radiofrequency must be chosen to resonate with the Larmor precession frequency of the neutrons in the steady collimating field, and the amplitude of the oscillating magnetic field must be properly adjusted to suit the neutron transit time through the coil. The reversal of polarization direction is produced by switching on the radiofrequency field. Polarization reversal efficiencies of 99 per cent can be achieved.

---

*) Another method to reverse the spin direction of the scattered neutrons is the gradually turning of the magnetic guide field along the beam. This method has the advantage that it can be applied for neutrons of different wave length without change of any parameter. (K. Abrahams, O. Steinsvoll, P.J.M. Bongaarts and P.W. de Lange, Rev. sci. Inst. 33, 524 (1962))

IV. Types of Measurements, Measuring Error and Measuring Time

A) Types of Measurements

The measurement of the elastically or inelastically scattered intensity in the parallel and antiparallel state of the analysing system, I_p and I_a, is the basis for the applications of the neutron polarimeter. The measured intensities are used in different ways, depending on the type of experiment.

1. Measurement of Depolarization

For the determination of the depolarization factor for elastic and inelastic scattering, the corresponding intensities in the parallel and antiparallel state of the analysing system are measured. The polarization of the scattered beam is calculated according to (II-1).

2. Measurement of cross section ratios

The two cross section ratios $D^2\sigma^{inc}/D^2\sigma^{coh}$ and $D^2\sigma^{inc}/D^2\sigma_t$ for free nuclei are of interest. These ratios give together with known values of the coherent or total cross section the incoherent cross section. The two ratios depend on the elastically scattered intensities I_a, and I_p, in the following way

$$\frac{D^2\sigma^{inc}}{D^2\sigma^{coh}} = 3\,\frac{I_a}{2I_p - I_a} \qquad \text{(IV-1a)}$$

$$\frac{D^2\sigma^{inc}}{D^2\sigma_t} = \frac{3}{2}\,\frac{I_a}{I_p + I_a} \,. \qquad \text{(IV-1b)}$$

Instead of measuring the cross section ratios $D^2\sigma^{inc}/D^2\sigma^{coh}$ or $D^2\sigma^{inc}/D^2\sigma_t$, it is also possible to measure the unknown incoherent cross section of a substance in comparison to the known incoherent cross sections of a reference substance. The intensity in the antiparallel state of the analysing system is measured for the two substances. The unknown incoherent cross section σ_x^{inc} is obtained by the relation

$$\sigma_x^{inc} = \frac{3}{2} \frac{I_{acx}}{I_{acst}} \sigma_{wst}.$$

3. Separation of coherent and incoherent part of the scattered intensity

The neutron polarimeter offers the possibility

- to suppress the background due to incoherent scattering *) of the neutrons by measuring with the analysing system in the parallel state if only coherent scattering should be observed, or
- to separate the coherently and incoherently scattered part by measuring with the analysing system in the parallel and antiparallel state, and calculating the coherent and incoherent intensity according to relations (II-7).

---

*) It should be evident from the effects used for the separation of the coherent and incoherent part of the scattered intensity, that only that incoherent part can be separated which is due to spin incoherent scattering. A suppression of the incoherent scattering which is due to isotopes in the sample, is only possible by separating the isotopes and using monoisotopic samples.

Since also the energies of the scattered neutrons are analysed, the neutron polarimeter allows to measure separately the four parts of the total intensity due to

coherently elastically, and

incoherently elastically scattered neutrons,

coherently inelastically, and

incoherently inelastically scattered neutrons.

B) Measuring Error of the intensities in the parallel and antiparallel state.

In chapter III it has been mentioned that in the parallel state of the analysing system also a fraction of neutrons which have been scattered with spin flip, will reach the detector, and vice versa, due to the deviation of the efficiencies from 1. The error in the measured parallel and antiparallel intensity which is caused by these deviations is discussed in this section. All other quantities as coherent and incoherent cross sections, cross section ratios or depolarization factors are obtained from the intensities measured in the two states. The error of those quantities can be calculated from the errors of the two intensities.

The intensities I_p and I_a, measured with a polarimeter that has the efficiencies P_1 and P_2, for the polarization of the polarizing and analysing crystal, and f for the spin reversal unit are calculated as a function of the ratio r between incoherent and coherent cross section, $r = D^2\sigma^{inc}/D^2\sigma^{coh}$.

Using the relations (A-6) for the measured intensities and the relations (II-5) between the cross sections with and without spin flip and the coherent and incoherent cross sections, the measured intensities are

$$(\text{IV-2a}) \qquad I_p = \frac{1}{3}\left[3k_1 D^2 \sigma^{coh} + (k_1+2k_2)D^2\sigma^{inc}\right]$$

$$(\text{IV-2b}) \qquad I_a = \frac{1}{3}\left[(2k_1'+k_2')D^2\sigma^{inc} + 3k_2'D^2\sigma^{coh}\right].$$

The polarimeter coefficients k_1, k_2 and k_1', k_2' are given by the relations (A-7) or (A-8). For a neutron polarimeter with the efficiencies 1, they assume the values $k_1 = k_1' = 1$ and $k_2 = k_2' = 0$. With the relations (IV-2) the relative errors are obtained

$$(\text{IV-3a}) \qquad \frac{\Delta I_p}{I_{p,th}} = \frac{I_p - I_{p,th}}{I_{p,th}} = \frac{3(k_1-1) + r(k_1+2k_2-1)}{3+r}$$

$$(\text{IV-3b}) \qquad \frac{\Delta I_a}{I_{a,th}} = \frac{1}{2}\,(2k_1'+k_2'-2) + \frac{3}{2}\,k_2'\,\frac{1}{r}$$

where r is the ratio between incoherent and coherent cross section.

For the further evaluation it is assumed that the polarization efficiencies of the polarizing and analysing crystal and also the polarization reversal efficiency are 98 per cent, $P_1 = P_2 = f = 0.98$. As mentioned in paragraph III.B, these efficiencies are in the reach of the experimental possibilities. The values of the polarimeter coefficients calculated with these efficiencies are

$k_1 = 0.99 \quad k_2 = 0.025 \quad k_1' = 0.97 \quad k_2' = 0.04$.

These numerical values are inserted into the relations (IV-3)

$$\text{(IV-4a)} \qquad \frac{\Delta I_p}{I_{p,th}} = \frac{-0.03 + 0.04r}{3+r}$$

$$\text{(IV-4b)} \qquad \frac{\Delta I_a}{I_{a,th}} = -0.01 + \frac{0.06}{r}$$

The relative deviations $\Delta I/I_{th}$ (IV-4) have been plotted in Fig. 2 as function of the cross section ratio r. From the relation (IV-4a) it is seen that for the whole range of the cross section ratio the relative deviation $\Delta I_p/I_{p,th}$ for the parallel intensity is between -0.01, the value for $r = 0$ and 0.04, the value for $r = \infty$. The relative deviation for the antiparallel intensity $\Delta I_a/I_{a,th}$ is between +0.05, the value for $r = 1$,and -0.01 the value for $r = \infty$. It is seen from these values that the high polarization efficiencies from CoFe-crystals and the high efficiency of the magnetic resonance method for the spin reversal, allow measurements of the parallel and antiparallel intensity with a precision of some per cents for cross section ratios r larger than 1. Only the relative deviation $\Delta I_a/I_{a,th}$ for the antiparallel intensity increases for cross section ratios r smaller than 1 rapidly.

A check of the measuring accuracy which can be obtained with a neutron polarimeter is the measurement of the intensities in the parallel and antiparallel state and the subsequent calculation of the cross section ratio $D^2\sigma^{inc}/D^2\sigma^{coh}$, and comparing the measured cross section ratio with the value calculated from the known cross sections *).

C) Required Measuring Times

The required measuring time for a neutron polarimeter is crudely estimated, using as a basis the measuring times required for the CNEN-triple axis spectrometer at the reactor ISPRA-I [6]. This spectrometer used in an inelastic scattering experiment for the determination of phonon dispersion curves of Zn as monochromating and analysing crystal Al in the (111) plane [7]. The angular divergencies of the collimators were 20′, 25′, 70′ and 35′, in the sequence from the reactor to the detector. The cross sections of the collimators were about 50 mm × 50 mm. The flux at the source surface was about 3×10^{13} n/cm^2 sec. Measuring times between 15 and 45 min. were required to obtain between 300 and 700 counts on the peak of a typical neutron group.

---

*) The measurement of a completely coherent and a predominantly incoherent scatterer and the information on the polarization resolution which is obtained from these measurements,**are** discussed in appendix II.

The measuring time required for an inelastic scattering experiment with a neutron polarimeter on a high flux reactor is estimated very crudely by taking the ratio between the flux at the source surface of the reference reactor Φ_{ref}, and the flux at the surface of the high flux reactor Φ_{HF}.

$$t_{m,HF} = 2\ t_{m,ref} \frac{\Phi_{ref}}{\Phi_{HF}}$$

where $t_{m,HF}$ and $t_{m,ref}$ the measuring times at the high flux and the reference reactor. The polarized neutron flux which impinges the sample in a neutron polarimeter is reduced by 1/2 with reference to the unpolarized neutron flux which would be incident under the same conditions, since the polarizing crystals reflect only neutrons of one spin state. This factor doubles the measuring time of a neutron polarimeter with reference to the triple axis spectrometer. The value of $\Phi_{HF} = 10^{15}$ n/cm^2 sec gives

$$t_{m,HF} = 0.06\ t_{m,ref}$$

and as a very rough estimate between 1 and 3 minutes for the measuring times required with a neutron polarimeter for an inelastic scattering experiment at a high flux reactor. This time is required for one angular setting of the neutron polarimeter and for one of the two intensities I_a or I_p.

The measuring times have been estimated in an extremely crude way, but the way of estimating should be sufficient to demonstrate that the operation of a neutron polarimeter at a high flux reactor is in the potentialities of these reactors.

V. Applications of the Neutron Polarimeter

From the possible applications of the neutron polarimeter are considered in more detail:

- the determination of incoherent nuclear cross sections by measurement of cross section ratios,
- the separation of coherent and incoherent scattering in neutron diffraction and in the measurements of dispersion relations and frequency distribution functions.

In addition to these applications there are some others, e.g.

- the measurement of the change of the total nuclear spin of molecules associated with excitation or deexcitation of molecular rotational levels by the scattered neutrons,
- the investigation of neutrons scattered without and with spin flip by magnetic materials.

A) Measurement of incoherent nuclear cross sections.

The incoherent nuclear cross section can be obtained from the intensities I_a and I_p by either one of the ratios $D^2\sigma^{inc}/D^2\sigma^{coh}$ or $D^2\sigma^{inc}/D^2\sigma_t$, given by the relations (IV-1) together with the known value of either the coherent or the total cross section *). The ratio $I_a/(I_p + I_a)$ appearing in the ratio $D^2\sigma^{inc}/D^2\sigma_t$ is the spin flip

---

*) The scattering of thermal neutrons by free nuclei is isotropic. For this reason the total coherent and incoherent cross sections are used in this paragraph.

probability Q **) given by

$$Q = \frac{2}{3}\,\frac{\sigma_{inc}}{\sigma_{inc} + \sigma_{coh}} .$$

In order to obtain the incoherent nuclear scattering cross section from one of the ratios (IV-1) it is essential that in the parallel intensity I_p are no contributions due to crystalline effects. The influence of crystalline scattering on the intensity I_p can be eliminated by using a large enough energy of the incident polarized beam to avoid Bragg scattering.

B) Suppression of background due to incoherent elastic and to inelastic scattering in neutron diffraction.

Absolute intensity measurements of the Bragg reflections in diffraction of neutrons by powdered polycrystalline samples with hydrogenous compounds are hampered by the large background scattering which is mainly due to the high incoherent scattering cross section of hydrogen. One possibility used up to now is the investigation of the deuterated compound, e.g. to use ND_4Br [9] instead of NH_4Br for the measurement.

**) The method to measure the spin flip probability in order to determine the incoherent nuclear scattering cross section was suggested by Schwinger and Rabi [8] and applied by Meyerhof and Nicodemus [1] to the measurement of the incoherent scattering cross section of phosphorus by means of the "double transmission method."

Recently Caglioti and Pompa [10] have reduced the background by a factor between 1.2 and 3 in an investigation on non-deuterated ammonium bromide, NH_4Br, by observing only elastically scattered neutrons by the use of a triple axis spectrometer. By the use of a neutron polarimeter for diffraction experiments, a further reduction of the background by a factor of about 3 can be obtained by measuring the intensity due to elastically scattered neutrons as function of the scattering angle with the analysing system in the parallel state. In this way the measured intensity is proportional to the cross section $D^2\sigma_{wo}$ given by (II-5.a). The background can be corrected further by measuring also the intensity in the antiparallel state of the analysing system, I_a, and by subtracting the remaining incoherent part from the intensity I_p according to relation (II-7.a).

The reduction of the background by one of these methods should be sufficient to allow the measurements of diffraction patterns of powdered polycrystalline samples even with high concentrations of hydrogen.

C) Separation of the coherent and incoherent scattering for the measurement of dispersion relations and frequency distribution functions

Dispersion relations have been determined up to now only from substances with a small incoherent cross section compared to the coherent one [11]. In substance with an appreciable incoherent scattering cross section, the observation of coherent scattering, involving modes of frequencies with a peak in the frequency distribution function, may be obscured by the incoherent scattering [12].

Frequency distributions have been determined up to now from substances with a predominantly incoherent cross section [11]. By use of one of the methods described in paragraph IV.A-3 it is possible

- to measure the frequency distribution also for scatterers with a coherent part in the cross section,
- and to measure the dispersion relation also for scatterers with an incoherent part in the cross section.

For the determination of the dispersion relation either only the intensity in the parallel state of the analysing system, I_p, is measured, or the intensity in the antiparallel state is also measured as function of the different angular settings according to the conventional methods of triple axis spectrometers, e.g. the constant Q-methods [13]. By measuring only the parallel intensity the incoherent background is reduced. By measuring the parallel and the antiparallel intensity the inelastically coherent intensity can be calculated by means of the relation (II-7).

For the determination of the frequency distribution function, the energy distribution of the neutrons scattered by a fixed scattering angle is measured with the analysing system in the antiparallel state. In order to obtain the frequency distribution function from the measured intensity I_a which is related to the incoherent double differential cross section, the evaluation proceeds as usual [14].

References

[1] W.E. MEYERHOF, D.B.NICODEMUS
Phys. Rev., 82, 5, 1951

[2] e.g.: M. BURGY, D.J. HUGHES, J.R. WALLACE, R.B. HELLER, W.E. WOOLF
Phys. Rev., 80, 953, 1951

[3] R. NATHANS, C.G. SHULL, G. SHIRANE, A. ANDRESEN
J. Phys. Chem. Solids, 10, 138 (1959)

[4] L.W. ALVAREZ, F. BLOCH
Phys. Rev., 57, 111 (1940)

[5] I.I. RABI, J.R. ZACHARIAS, S. MILLMAN, P. KUSCH
Phys. Rev., 53, 318 (1936)

[6] G. CAGLIOTI, E. DE AGOSTINO, F. MARSILI, A. PAOLETTI, U. PELLEGRINI, F.P. RICCI
Nuovo cimento, Supplemento 1 al Vol. 23, Serie X, 17 (1962)

[7] G. BORGONOVI, G. CAGLIOTI, J.J. ANTAL
Phys. Rev., 132, 683 (1963)

[8] J. SCHWINGER, I.I. RABI
Phys. Rev., 51, 1003 (1937)

[9] H.A. LEVY, S.W. PETERSON
J. Amer. Chem. Soc., 75, 1536 (1953)

[10] G. CAGLIOTI, F. POMPA
submitted for publication in "Nuovo Cimento"

[11] For a review see G. DOLLING and A.D.B. WOODS
"Thermal vibration of crystal lattices" in
P.A. EGELSTAFF "Thermal Neutron scattering", Academic
Press London and New York, 1965, p. 193 - 244

[12] Ref. 11, p. 231

[13] e.g.: P.K. JYENGAR "Crystal Diffraction Techniques"
in P.A. EGELSTAFF "Thermal Neutron Scattering",
Academic Press London and New York, 1965, p. 123

[14] e.g.: R. HAAS, W. KLEY, K.H. KREBS and R. RUBIN
in "Inelastic scattering of Neutrons in Solids and
Liquids", Vol. II, p. 145, IAEA Vienna 1963

Appendix I

Derivation of the intensity relations

The relations (II-4) for the intensity in the parallel and in the antiparallel state have been given for the case that the efficiencies of the polarizing crystal and of the analysing system are 100 per cent. In this appendix the relations for the two intensities are derived, taking into account the deviation of the polarization efficiency of the polarizing and of the analysing crystal, and the deviation of the polarization reversal efficiency from 1.

In order to prepare the derivation, first two introductory cases are considered:

1) An unpolarized neutron beam is reflected by a polarizing crystal which has the polarization efficiency P_1. What are the fractions of neutrons of the reflected beam which are in the parallel and antiparallel spin state, with reference to the polarization direction of the crystal?

2) A partly polarized neutron beam is reflected by an analysing crystal which has the polarization efficiency P_2. How enter the fractions c_{i+} and c_{i-} of the incident neutron beam into the relation for the reflected flux?

For the case 1) the reflected neutron flux Φ_r is divided in its two components with parallel and antiparallel spin state

$$\Phi_r = c_{r+}\Phi_r + c_{r-}\Phi_r.$$

A crystal with a polarization efficiency P_1 produces from an unpolarized beam a beam with the polarization

$$P_1 = \frac{c_{r+} - c_{r-}}{c_{r+} + c_{r-}} .$$

From the two relations

$$c_{r+} = \frac{1 + P}{2} \quad \text{and} \quad c_{r-} = \frac{1 - P}{2}$$

is obtained.

For the case 2) the incident neutron flux is divided into its two components c_{l+} and c_{l-}

$$\Phi_l = c_{l+}\Phi_l + c_{l-}\Phi_l$$

The total reflected intensity is

$$\Phi_r = (\sigma_+ c_{l+} + \sigma_- c_{l-})\Phi_l$$

or

$$\Phi_r = (c_{l+} + R\, c_{l-})\sigma_+ \Phi_l$$

where

σ_+, σ_-	are the scattering cross sections of the crystal for the neutrons polarized parallel (σ_+) or antiparallel (σ_-) to the polarization direction of the crystal.
R	is the ratio between the two different scattering cross sections $R = \sigma_-/\sigma_+$.

From the definition of the intrinsic polarization efficiency of a crystal

$$P = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

it follows that R is related to the polarization efficiency P of the analysing crystal by

$$R = \frac{1 - P}{1 + P} .$$

If the spins of the incident neutrons are reversed by the spin flipping unit before the neutrons impinge the analyser crystal, then the total reflected intensity is given by

$$\Phi_{r,f} = (c_{i-} + R_2 c_{i+})\sigma_+ \Phi_i .$$

After these introductory remarks the intensity relations are derived for the polarization efficiencies P_1 of the polarizing crystal, and P_2 of the analysing crystal and the polarization reversal efficiency f of the spin flipping unit. In order to derive the relations the neutron flux Φ_n at different points n in the polarimeter is divided into its two components

$$\Phi_n = c_{n+}\Phi_n + c_{n-}\Phi_n . \qquad \text{(A-1)}$$

After this division has been made for the flux which impinges the scattering sample, $n = 2$, and for the flux behind the scattering sample, $n = 3$, the relations for the intensities scattered into the detector with the analysing system in the parallel and in the antiparallel state are obtained.

The fractions c_+ and c_- after the polarizer are

(A-2a.b) $$c_{2+} = \frac{1 + P_1}{2} \qquad c_{2-} = \frac{1 - P_1}{2} \, .$$

In the scattered beam the fractions are

(A-3a) $$c_{3+} = \frac{1 + P_1}{2} D^2\sigma_{wo} + \frac{1 - P_1}{2} D^2\sigma_w$$

(A-3b) $$c_{3-} = \frac{1 - P_1}{2} D^2\sigma_{wo} + \frac{1 + P_1}{2} D^2\sigma_w \, .$$

The signification of the two parts in each of the fractions c_{3+} and c_{3-} are evident from the definitions of $D^2\sigma_{wo}$, $D^2\sigma_w$ and from the significations of $(1+P_1)/2$ and $(1-P_1)/2$.

The intensity I_p which is measured by the detector with the analysing system in the parallel state is

(A-4a) $$I_p = (c_{p+} + c_{p-})\Phi_3$$

where

(A-4b) $$c_{p+} = c_{3+} = \frac{1 + P_1}{2} D^2\sigma_{wo} + \frac{1 - P_1}{2} D^2\sigma_w$$

(A-4c) $$c_{p-} = R_2 c_{3-} = R_2\left[\frac{1 - P_1}{2} D^2\sigma_{wo} + \frac{1 + P_1}{2} D^2\sigma_w \right]$$

(A-4d) $$R_2 = \frac{1 - P_2}{1 + P_2}$$

where P_2 is the polarization efficiency of the analysing crystal.

The intensity I_a, i.e. the intensity when the spin flipping unit is on and the spins are flipped with the polarization reversal efficiency f, is

$$(A\text{-}5a) \qquad I_a = [c_{a+} + c_{a-}]\Phi_a$$

where

$$(A\text{-}5b) \qquad c_{a+} = c_{3-}f + c_{3+}(1-f)$$

$$(A\text{-}5c) \qquad c_{a-} = R_2[c_{3+}f + c_{3-}(1-f)].$$

f is the fraction of neutrons which change their spin direction during the transit through the spin flipping unit and (1-f) is the fraction which maintains its spin direction.

After the coefficients of the relations (A-4) and (A-5) have been grouped according to the cross sections $D^2\sigma_{wo}$ and $D^2\sigma_w$, the final relations for I_p and I_a are obtained *):

$$(A\text{-}6a) \qquad I_p = D^2\sigma_{wo}k_1 + D^2\sigma_w k_2$$

$$(A\text{-}6b) \qquad I_a = D^2\sigma_w k_1' + D^2\sigma_{wo}k_2' \,.$$

---

*) The proportionality factor F which has been introduced in the relations (II-4) for the intensities I_a and I_a, has not been included in the formulas of the two appendices since it is not needed in this context.

The polarimeter coefficients k_1 and k_2 in relation (A-6a) are

$$k_1 = \frac{1}{2}\left[(1+P_1) + R_2(1-P_1)\right] \tag{A-7a}$$

$$k_2 = \frac{1}{2}\left[(1-P_1) + R_2(1+P_1)\right] \tag{A-7b}$$

and the coefficients k_1' and k_2' in relation (A-6b) are

$$k_1' = \frac{1}{2}\left\{(1-P_1) + 2P_1 f + R_2\left[(1+P_1) - 2P_1 f\right]\right\} \tag{A-7c}$$

$$k_2' = \frac{1}{2}\left\{(1+P_1) - 2P_1 f + R_2\left[(1-P_1) + 2P_1 f\right]\right\} \tag{A-7d}$$

Since the efficiencies are close to 1 the deviations from 1 are introduced and the polarimeter coefficients are calculated by neglecting quantities which are small of second order.

$$k_1 = 1 - \frac{1}{2}\,\Delta P_1 \tag{A-8a}$$

$$k_2 = \frac{1}{2}\left(\Delta P_1 + \Delta P_2\right) \tag{A-8b}$$

$$k_1' = 1 - \frac{1}{2}\,\Delta P_1 - \Delta f \tag{A-8c}$$

$$k_2' = \frac{1}{2}\left(\Delta P_1 + \Delta P_2\right) + \Delta f\ . \tag{A-8d}$$

Appendix II

Increase of the measuring accuracy by correcting the measured intensities I_p and I_a for the polarization and spin reversal efficiencies.

The relations (A-6) for the parallel and antiparallel intensity can be used to increase the measuring accuracy by correcting the measured intensities, I_p and I_a, for the deviation of the polarization efficiencies and the spin reversal efficiency from 1. The relations for I_p can be written in a form more convenient for the correction, whereas the relations for I_a are maintained in the form given by (A-6b) *)

$$\text{(A-6a)} \qquad I_p = D^2 \sigma_{wo} k_p$$

$$\text{(A-6b)} \qquad I_a = D^2 \sigma_w k_1' + D^2 \sigma_{wo} k_2'$$

where

$$\text{(A-9a)} \qquad k_p = k_1 \left[1 + \frac{D^2 \sigma_w}{D^2 \sigma_{wo}} \; \frac{k_2}{k_1} \right] ,$$

or using the fact that the deviations of the efficiencies from 1 are small

$$\text{(A-9b)} \qquad k_p = \left(1 - \frac{1}{2} \Delta P_1\right)\left[1 + \frac{D^2 \sigma_w}{D^2 \sigma_{wo}} \frac{1}{2} \left(\frac{3}{2} \Delta P_1 + \Delta P_2\right)\right] .$$

---

*) Contrary to the cross section $D^2 \sigma_{wo}$ the cross section $D^2 \sigma_w$ can become zero. For this reason it is not possible to put the intensity I_a into the same form as the intensity I_p and to use the same correction procedure.

The corrected intensity in the parallel state I_{pc}, is obtained by dividing the measured intensity I_p by the known correction factor k_p

(A-10a) $$I_{pc} = \frac{I_p}{k_p} = D^2 \sigma_{wo}.$$

The corrected intensity in the parallel state is used to correct the intensity in the antiparallel state

(A-10b) $$I_{ac} = \frac{1}{k_1'} \left(I_a - I_p \frac{k_2'}{k_p} \right) = D^2 \sigma_w.$$

The remaining errors, if the measured intensities are corrected for the dėviation of the efficiencies from 1, are difficult to estimate. Theoretically they are zero. In reality these errors will depend on the precision by which the efficiencies can be measured,and on time dependent fluctuations of these efficiencies which may be due to different effects.

For the calculations of the polarimeter coefficients k_1, k_2, k_1' and k_2' the value of the efficiencies P_1, P_2 and f are needed. For the calculation of k_p the ratio $D^2\sigma_w/D^2\sigma_{wo}$ is required. The polarization efficiencies P_1 and P_2 and the spin reversal efficiency f can be measured by the usual techniques applied in experiments with polarized neutrons [e.g. ref. 3]. The value of the ratio $D^2\sigma_w/D^2\sigma_{wo}$ can either be estimated, the maximum value being 2, or it can be approximated by the ratio of the measured intensities I_a/I_p

A determination of this ratio which takes into account the deviation of the efficiencies from 1 is possible from the measured intensities. The measured polarization P_m of the scattered beam

$$P_m = \frac{I_p - I_a}{I_p + I_a}$$

is corrected with a correction factor k_D for the deviation of the efficiencies

$$P_{mc} = P_m \cdot k_D$$

where

$$k_D = \frac{1}{P_1 P_2 f \left[1 - \frac{P_m(1-f)}{f}\right]}$$

or

$$k_D = [1 + \Delta P_1 + \Delta P_2 + \Delta f(1+P_m)] \quad .$$

The corrected polarization degree P_{mc} is equal to the depolarization factor D_f, $P_{mc} = D_f$ and the ratio $D^2\sigma_w/D^2\sigma_{wo}$ is calculated from the depolarization factor

$$\frac{D^2\sigma_w}{D^2\sigma_{wo}} = \frac{1 - D_f}{1 + D_f}$$

Another possibility to determine the polarization efficiencies P_1 and P_2, and the spin reversal efficiency **f**, is the measurement of the intensities with the analysing system in the parallel and antiparallel state of a completely coherent scatterer, i.e. a monoisotopic element with the nuclear spin zero, e.g. carbon-12, and a predominantly incoherent scatterer, e.g. hydrogen. For the completely coherent scatterer the cross section $D^2\sigma_w$ is theoretically zero. The cross section ratio $D^2\sigma_w/D^2\sigma_{wo}$ of a predominantly incoherent scatterer has a value close to 2. The ratios of the antiparallel intensity for these two scatterers s coh and s inc, is given by

$$\text{(A-11a)} \quad \left(\frac{I_a}{I_p}\right)_{s\ coh} = \frac{k_2'}{k_1}$$

$$\text{(A-11b)} \quad \left(\frac{I_a}{I_p}\right)_{s\ inc} = \alpha \left\{1 - \Delta f(1 - \frac{1}{\alpha}) + \frac{1}{2}(\frac{1}{\alpha} - \alpha)(\Delta P_1 + \Delta P_2)\right\}$$

With the measured ratios $(I_a/I_p)_{s\ coh}$ and $(I_a/I_p)_{s\ inc}$ and the known value of $\alpha = (D^2\sigma_w/D^2\sigma_{wo})_{s\ inc}$ the sum of the deviations of the polarization efficiencies $(\Delta P_1 + \Delta P_2)$ and the deviation of the spin reversal efficiency Δf can be calculated by rewriting the relations (A-11)

$$\text{(A-12a)} \quad \left(\frac{I_a}{I_p}\right)_{s\ coh} = \Delta f + \frac{1}{2}\ (\Delta P_1 + \Delta P_2)$$

$$\text{(A-12b)} \quad \frac{1}{\alpha}\left(\frac{I_a}{I_p}\right)_{s\ inc} - 1 = -\ (1 - \frac{1}{\alpha})\ \Delta f + \frac{1}{2}\ (\frac{1}{\alpha} - \alpha)(\Delta P_1 + \Delta P_2).$$

The values of k_2 and k_2' are easily obtained from the values of Δf and of the sum $(\Delta P_1+\Delta P_2)$. With the assumptions that ΔP_1 and ΔP_2 are equal, also the values of k_1 and k_1' can be calculated. These assumptions should give a sufficient accuracy.

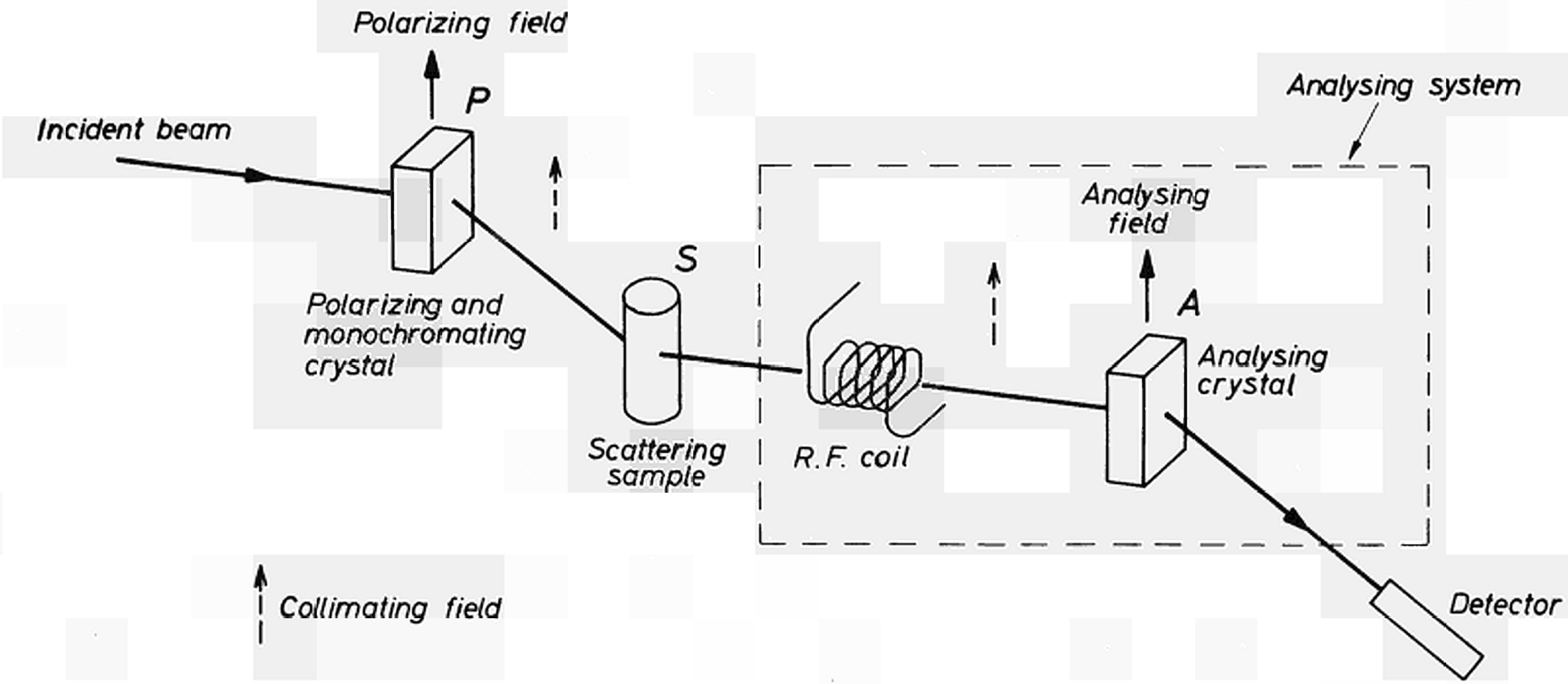

Fig. 1 : A schematic diagram of a neutron polarimeter

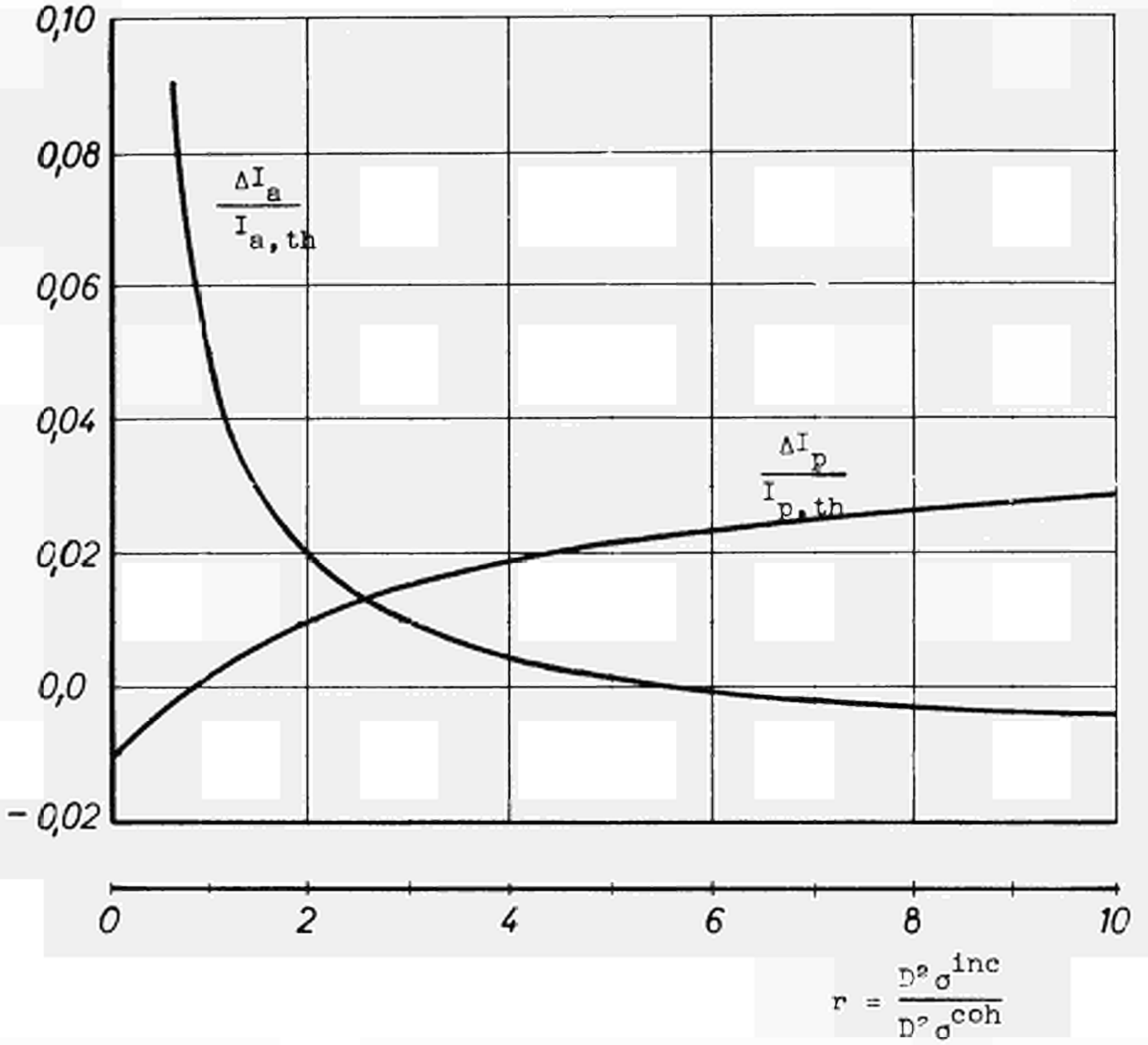

Fig. 2 Relative errors $\Delta I/I_{th}$ of the intensities in the parallel and antiparallel analysing state as function of the cross section ratio $r = D^2\sigma^{inc}/D^2\sigma^{coh}$.
The relative errors are calculated for neutron polarimeter with the assumed efficiencies of the polarizing and analysing crystal and the spin reversal unit of 98 per cent.
0.04 is the asymptotic value of $\Delta I_p/I_{p,th}$, and - 0.01 of $\Delta I_a/I_{a,th}$ for $r \to \infty$.

To disseminate knowledge is to disseminate prosperity — I mean general prosperity and not individual riches — and with prosperity disappears the greater part of the evil which is our heritage from darker times.

Alfred Nobel

SALES OFFICES

All Euratom reports are on sale at the offices listed below, at the prices given on the back of the front cover (when ordering, specify clearly the EUR number and the title of the report, which are shown on the front cover).

OFFICE CENTRAL DE VENTE DES PUBLICATIONS DES COMMUNAUTES EUROPEENNES
2, place de Metz, Luxembourg (Compte chèque postal Nº 191-90)

BELGIQUE — BELGIË
MONITEUR BELGE
40-42, rue de Louvain - Bruxelles
BELGISCH STAATSBLAD
Leuvenseweg 40-42, - Brussel

DEUTSCHLAND
BUNDESANZEIGER
Postfach - Köln 1

FRANCE
SERVICE DE VENTE EN FRANCE DES PUBLICATIONS DES COMMUNAUTES EUROPEENNES
26, rue Desaix - Paris 15e

ITALIA
LIBRERIA DELLO STATO
Piazza G. Verdi, 10 - Roma

LUXEMBOURG
OFFICE CENTRAL DE VENTE DES PUBLICATIONS DES COMMUNAUTES EUROPEENNES
9, rue Goethe - Luxembourg

NEDERLAND
STAATSDRUKKERIJ
Christoffel Plantijnstraat - Den Haag

UNITED KINGDOM
H. M. STATIONERY OFFICE
P. O. Box 569 - London S.E.1

EURATOM — C.I.D.
51-53, rue Belliard
Bruxelles (Belgique)