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**COMPARISON OF DIFFERENT METHODS
FOR THE KINETICS
OF AN ESSOR REACTOR TYPE**

by

G. FORTI, C. RINALDINI and E. VINCENTI

1966



ORGEL Program

Joint Nuclear Research Center
Ispra Establishment - Italy

Reactor Physics Department
Reactor Theory and Analysis

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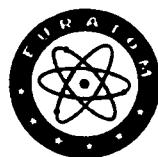
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SUMMARY

With the aim of investigating the kind of representation needed in the kinetics calculation of a reactor made up of three quite different regions, like the ESSor reactor, three approaches were used : a point model method, a nodal method subdividing the reactor in three spatial regions, a direct numerical solution of the time dependent diffusion equations. The results lead to the conclusions that, for a large category of transients, the point model is a good representation of the ESSor reactor.

Introduction

The neutron kinetics behaviour of the thermal reactors is often investigated by a point-model representation. This representation implies that the neutron flux distribution during the transient is not appreciably changed due to the absorber movement which is initiating the excursion, or to the resulting dynamic effects (faster accumulation of neutrons in some regions than in others).

The Essor reactor has in this respect some particular features. Three quite different regions may be distinguished in it, they are: the central cylindrical region which contains the Orgel test channels, natural Uranium or slightly enriched fuel elements, organic cooled and heavy water moderated; the annular feeding zone containing fully enriched, plate type fuel elements, heavy water cooled and moderated; the peripheral region which is the radial heavy water reflector in the innermost part of which control, shim and safety rods are operating (see table page 8).

Such unusual strong heterogeneity of the Essor reactor may introduce the suspect that the conventional point-model representation might be a poor one.

The investigation of this possibility is the purpose of the present report.

Two violent power excursions of the Essor reactor have been taken in consideration, the first corresponding to a reactivity step of 700 p.c.m. and the second to a reactivity step of 1500 p.c.m.. The problem has been solved at three different levels of exactitude:

1) by the usual point-model method.

Here the power distribution is assumed to be unchanged while the power level varies with the time.

2) by a three-regions nodal method.

Here the reactor is subdivided in three spatial regions and the average flux in each of them is tracked during the transient, irrespective of the changes in flux shape inside each individual region.

3) by a direct numerical solution of the time dependent diffusion equation.

From the numerical results it will be seen that the point-model method is sufficiently exact for most of the transients foreseen for the Essor reactor and only for the very large reactivity step of 1500 p.c.m. the difference between the point-model and the more sophisticated calculation may be of some relevance.

Manuscript received on March 8, 1966.

1. The point-model method

The equations to be solved are:

$$\frac{dn}{dt} = \frac{\delta}{\ell} (1-\beta) n + \sum_i \lambda_i c_i ; \quad \frac{dc_i}{dt} = \frac{\beta_i}{\ell} n - \lambda_i c_i$$

($i = 1, \dots, 6$, index of groups of delayed neutrons)

δ and ℓ (reactivity and mean life) have been determined by the perturbation method with the formulae

$$\delta = \frac{\int \varphi_i^* \delta \Sigma_a \varphi_i dv}{\int \varphi_i^* v \Sigma_f \varphi_i dv} ; \quad \ell = \frac{\frac{1}{v_e} \int \varphi_i^* \varphi_i + \frac{1}{v_e} \int \varphi_i^* \dot{\varphi}_i}{\int \varphi_i^* v \Sigma_f \varphi_i}$$

based on static calculations of the fluxes in two groups.

For simplicity sake we have considered only perturbations on the Σ_a .

For the static calculations we have used the code Equipoise 3 which gives the fluxes and adjoint fluxes in two groups.

Of course the imprecision introduced by the perturbation theory is no more negligible when the considered perturbation is too great. In the case of a perturbation introduced as a step this imprecision can be avoided by determining directly the proper value k-eff (from which $\delta = \frac{K_{eff}-1}{K_{eff}}$) with a static calculation made with the absorption cross-section Σ_a already perturbed.

2. The nodal method

The cylindrical reactor ESSOR can be subdivided into 3 regions. The two inner regions consist of multiplying media and the outer region is a reflector.

Without considering the terms containing the delayed neutrons precursor the introduction of which is trivial, the kinetics of the three regions can be expressed by a system of differential equations

$$\frac{dn_1}{dt} = a_{11}n_1 + a_{12}n_2$$

(1)

$$\frac{dn_2}{dt} = a_{21}n_1 + a_{22}n_2 + a_{23}n_3$$

$$\frac{dn_3}{dt} = a_{32}n_2 + a_{33}n_3$$

where n_1, n_2, n_3 are the neutron densities in each region.

This system has a tridiagonal matrix of coefficients, because we suppose that only the adjacent regions are coupled through the neutron currents. A system with a complete coupling between all the regions, with a full matrix of coefficients, was given only for multiplying media by Avery Geneva Conf. (1958)

The determination of the coefficients is the most difficult part of this problem, and can be done with different methods

a) Average diffusion coefficients

The neutron current between adjacent regions is supposed to be proportional to the difference of the average fluxes in the two regions, the diffusion coefficient depends on the geometry and on the physical characteristics of the two media

$$J_{ik} = A_{ik}(n_i - n_k)$$

(For the determination of A_{ik} see Kelber NSE II 285 (1961))

The system can be written in the following form:

$$(2) \quad \frac{dn_1}{dt} = (K_1 - 1) \sum a_{1i} \cdot V \cdot n_i + \frac{A_{12}}{V_1} (n_2 - n_1)$$

$$\frac{dn_2}{dt} = \frac{A_{21}}{V_2} (n_1 - n_2) + (K_2 - 1) \sum a_{2i} V n_i + \frac{A_{32}}{V_2} (n_3 - n_2)$$

$$\frac{dn_3}{dt} = \frac{A_{23}}{V_3} (n_2 - n_3) - \frac{A_{34}}{V_3} n_3 - \sum a_{3i} V n_i$$

V_i are the volumes of the regions and the term with A_{34} represents the external leakage. This method however, when applied to regions of great volume, is imprecise due to the non exact determination of the neutron currents. In the original report of Kelber only cases of sufficiently small regions were considered. Because of this, with the physical constants which render the reactor critical, there is no condition of equilibrium for the system of equations. The condition of equilibrium may be forced by changing the multiplication factors in order to make equal to zero the determinant of the system. The way followed by GAAA (*), in the study of the dynamics of ESSOR, is to take the correct value of n_1/n_2 (first equation) as it should be at criticality and to determine K_1 and K_2 by putting the determinant equal zero. The value of n_3 determined by the system of equations in equilibrium condition is considerably different from the value obtained with a static calculation.

This does not produce remarkable alterations in the dynamic evolution. The value of K_1 and K_2 are however fictitious and depend strongly on the static distribution of fluxes corresponding to the criticality condition chosen as reference. These values have to be kept constant during the transient and this is not correct, considering that the physical conditions change. As K_1 and K_2 have no real physical meaning, it is impossible to perturb directly these magnitudes. The δK_1 or δK_2 corresponding to a given reactivity ρ must be determined by a perturbation method based on independent static calculations.

b) Determination of the coefficients with static calculations

As in the method of Kelber, we assume that the neutron-currents are proportional to the difference of the average fluxes in the regions. The system of equations is always as in (2). $(K_1 - 1) \sum a_1 v$ and $(K_2 - 1) \sum a_2 v$ have now their real physical value. The time derivatives are put equal to zero. The coupling coefficients must satisfy the system when n_1 , n_2 , n_3 have the values given by a static calculation. The system of equations obtained in this way, gives the correct ratios of fluxes in the case of equilibrium chosen as reference.

(*) In the frame of the design studies of the ESSOR reactor.

By this method the coefficients of the diagonal terms have their real physical meaning and the perturbations may be introduced directly. The values of coefficients are strongly dependent on the distribution of fluxes corresponding to the criticality condition chosen as reference. It has been verified that the ratios of fluxes n_1/n_3 , n_2/n_3 obtained with the system of equations, using the same coupling coefficients and introducing compensating perturbations in different regions (without altering the equilibrium) are remarkably different from the values of ratios obtained with a corresponding static calculation with compensating perturbations. With perturbations of the order of 300 pcm we obtained differences of about 20 % on the variation of the flux ratios.

c) Determination of the coefficients with static calculations of fluxes and adjoint fluxes

It is possible to determine the coefficients of the system (1) using static calculations of fluxes and adjoint fluxes, without considering the neutron currents.

In the system (1) let us consider n_1 , n_2 , n_3 as the total number of neutrons in the corresponding regions and not as before as the neutron density.

The system can be written in matrix form

$$A \ N = \frac{d \ N}{d \ t}$$

with $A = A_o + \delta A$

where A_o is the matrix of the coefficients at equilibrium, and δA is a perturbation matrix which we will assume to be diagonal.

Once N_o is determined, in an equilibrium case of reference, by means of a static calculation, it will be

$$A_o \ N_o = 0$$

If the regions are n (in our example of ESSOR $n = 3$). This system will give n equations for the $3n-2$ coefficients A_{ik} of the tridiagonal system (capital letters are here used to indicate that the system refers to the integrated fluxes in the regions as said before).

Let us consider now the adjoint flux N_o^* defined by the equation

$$A^T N_o^* = 0$$

where A^T is the transposed of A .

It will then be

$$N_o^* A N = N_o^* \delta A N = \frac{d}{dt} (N_o^* N)$$

The scalar product $N_o^* N$ represents the total importance of all the neutrons of the system, and its temporal evolution is determined by the equation

$$\frac{d}{dt} (N_o^* N) = \int_{\text{reactor}} \phi_o^* \delta A \phi(t) dv$$

Using the perturbation theory, neglecting terms of the second order, we can make the substitution $\phi(t) = \phi_o(x) \cdot T(t)$

ϕ_o^* and ϕ_o may be determined with a static calculation (also with more than one group). ϕ_o^* and ϕ_o have to be considered as vectors.

If we localize the perturbation in one of the three regions, we can determine the N_{oi}^*

Let us consider for instance a perturbation on the Σ_{a_i} thermal in a two groups model. It will be

$$N_{oi}^* \delta \Sigma_{a_i} v N_i = \delta \Sigma_{a_i} v \int_i \phi_i^* \phi_i dv$$

and from this at the time $t = 0$

$$(3) \quad N_{oi}^* = \frac{1}{N_i} \int_i \phi_i^* \phi_i dv$$

The normalization of the fluxes in the static calculation is of no significance because we are only interested in the ratios of the importances N_{oi}^* in the different regions.

The importance N_{oi}^* in (3) assume a different expression according to the type of perturbation considered.

It is also possible in particular cases to calculate the integrals using the fluxes corresponding to the initial distribution and the adjoint fluxes corresponding to an asymptotic distribution characteristic of the

perturbation introduced. It is possible to obtain in this way a greater precision of the perturbation method. But for this it is impossible to give precise rules, one should decide in each case according to the physics of the problem to be treated.

The determination of the adjoint fluxes N^* gives $n-1$ new equations for the determination of the coefficients A_{ik} . These equations in matrix form are $A_o^T N^* = 0$ (only $n-1$ of the n equations are independent, because the relation $\det A_o^T = 0$ was already implicit in the n equations $A_o N_o = 0$).

As the coefficients are $3n-2$ we still need $n-1$ relations to have the system completely defined.

This relations can be obtained defining the physical constants of the multiplying regions. Let us suppose for instance to have 2 multiplying regions and a reflector as third region, it will be

$$A_{11} + A_{21} = \frac{1}{\sum_{a_1} v_1} (K_1 - 1)$$

$$A_{12} + A_{22} + A_{32} = \frac{1}{\sum_{a_2} v_2} (K_2 - 1)$$

(K_i may be corrected for axial leakage introducing a transverse buckling).

This method has the advantage over other methods to utilize completely all the informations which can be deduced from detailed static calculations also with many groups.

3) The direct numerical solution of the time dependent diffusion equations

The system of the two groups diffusion equations is

$$\frac{1}{\nu_1} \frac{d\varphi_1}{dt} = D_1 \nabla^2 \varphi_1 - \frac{D_1}{\tau} \varphi_1 + K(1-\rho) \sum_a \varphi_2 + \sum_i \lambda_i C_i$$

$$\frac{1}{\nu_2} \frac{d\varphi_2}{dt} = D_2 \nabla^2 \varphi_2 - \sum_a \varphi_2 + \rho \frac{D_1}{\tau} \varphi_1$$

$$\frac{dC_i}{dt} = K\beta_i \sum_a \varphi_2 - \lambda_i C_i$$

This system is solved numerically by the finite difference method, in cylindrical geometry, one dimension R, by means of the code COSTANZA-CIL, which will be described in an other report.

4) Numerical results

The Reactor ESSOR consists of a central region, of the Orgel type, with fuel rods of natural UC, with organic coolant and D_2O as moderator; of an outer region with fuel elements made of an alloy of aluminium and U enriched in 235 at 20 %, the coolant and the moderator are both D_2O . This core is surrounded by a reflector of D_2O . The control rods are located in the reflector in the vicinity of the core. We have considered four regions the physical constants of which are reported in the following table:

Region	D_1	D_2	τ	ρ	Σ_a	K/ρ
Orgel (Radius = 52 cm)	1.34	0.812	117.3	0.873	0.00359	1.1877
Driver (R = 64 cm)	1.32	0.904	140.0	1.0	0.008755	1.751
Control (R = 79 cm)	1.21	0.85	121.36	1.0	0.00171	0
Reflector (R = 119 cm)	1.21	0.85	121.36	1.0	$0.671 \cdot 10^{-4}$	0

Six groups of delayed neutrons (including photo neutrons) have been considered. They have the following constants.

Group	β	λ
1	$0.264 \cdot 10^{-3}$	3.003
2	$0.73 \cdot 10^{-3}$	1.13
3	$0.25 \cdot 10^{-2}$	0.301
4	$0.276 \cdot 10^{-3}$	0.278
5	$0.124 \cdot 10^{-2}$	0.111
6	$0.176 \cdot 10^{-2}$	0.03

The transverse buckling was supposed to be the same for all the regions.

The rods were simulated by an equivalent diffused poison to be introduced in the third region.

Its criticality value has been determined by a static calculation in two groups and two dimensions R, Z made with the code Equipoise 3.

The transverse buckling was determined by a criticality search made with the code Wanda in radial geometry with the same physical constants as for Equipoise.

For the spatial dynamic calculation, the critical poison in region 3 was furtherly corrected using a subroutine incorporated in the code COSTANZA which makes an automatic search of criticality. This was done in order to compensate for small differences due to the different point mesh. In fact in Costanza, to reduce the machine time, we adopted a mesh of only 21 points.

Fig. 1 contains the evolution of the flux, averaged on the whole reactor, caused by a step variation of the absorption cross section in the Orgel region.

This corresponds to a step of reactivity of $\rho = 1500$ pcm. Curve 1 is obtained with the spatial dynamic. Curve 2 with the nodal method in three points (for this we used the code SAHYB made by Mr. D'Hoop and Mr. Monterosso - CETIS).

Curve 3 a was obtained with the point model method (code Airek), using a reactivity value obtained from the variation $\delta \sum_{\omega} = 0.0001394 \text{ cm}^{-1}$ in Orgel by means of perturbation formulae based on the flux distribution of Equipoise at criticality. Curve 3 b was obtained, always with the point model (Airek) using a reactivity obtained directly, as proper value of the already perturbed system, by means of code Wanda. Fig. 2 contains the evolution of the average flux caused by a step in Orgel corresponding to 700 pcm.

Curve 1 is made with the spatial method, curve 2 with the point method. Fig. 3 contains the evolution of the average flux caused by a step of 1500 pcm followed by scram. The scram signal takes place when the average flux has reached the 130 % of its nominal value, the scram itself begins 0.2 sec later. A maximum negative value of -3000 pcm will be reached. The negative reactivity is introduced gradually according to the curve (given by GAAA) in which the 65 % of the max. value is reached 0.3 sec after the beginning of the scram.

Curve 1 is obtained with the spatial method, the negative reactivity is introduced as an equivalent poison in region 3. Curve 2 is obtained with the point method.

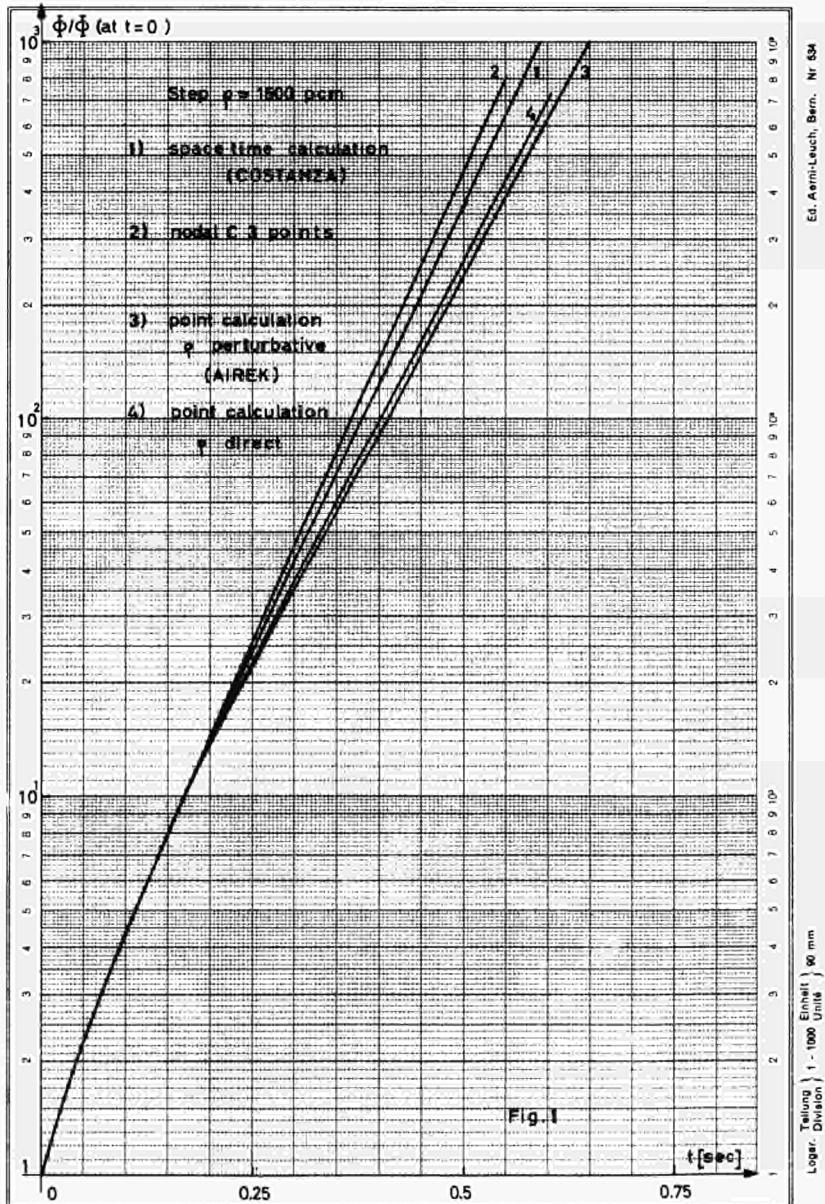
Fig. 4 represents the evolution due to the same perturbation as in Fig. 2 followed by a scram as in Fig. 3.

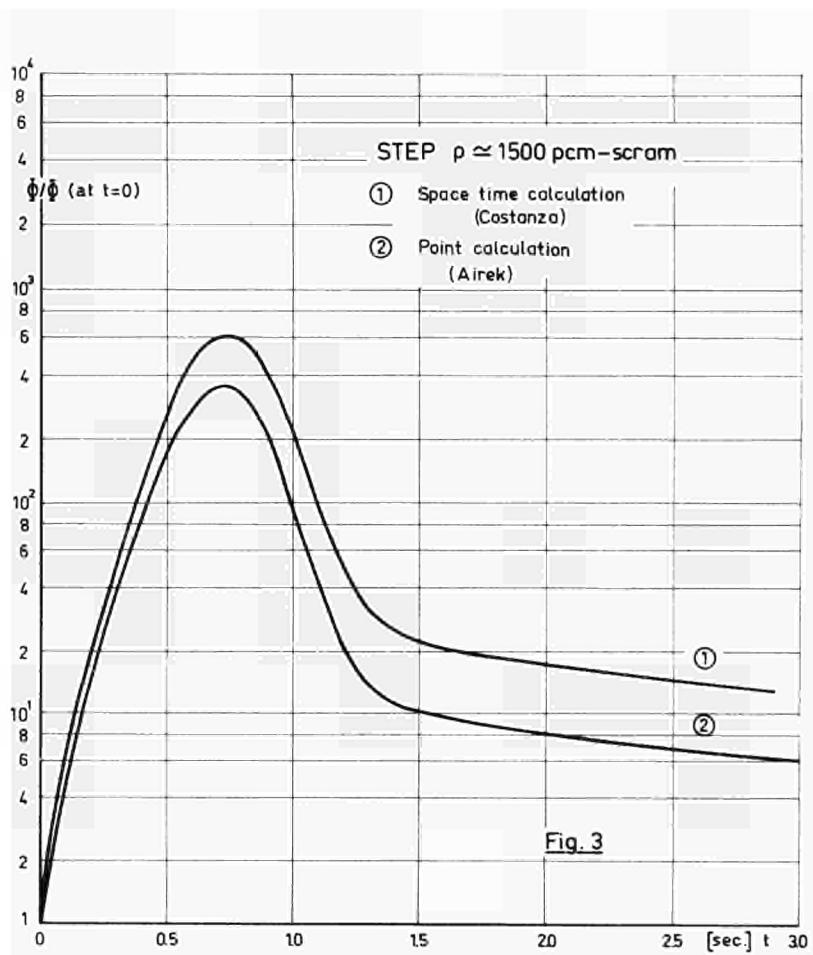
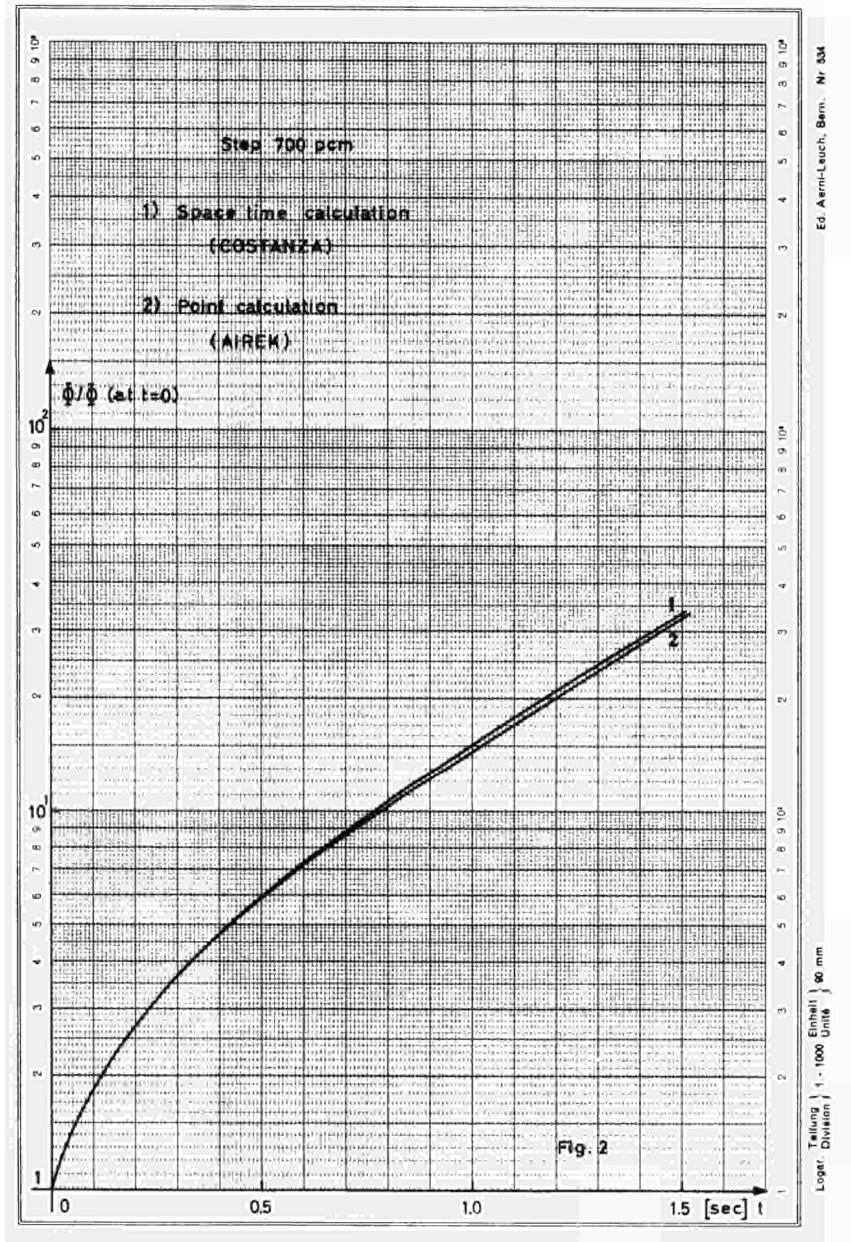
Fig. 5 contains the spatial distribution of the flux, with the spatial method and with the nodal method in 3 points, at successive instants of the transient caused by a perturbation as in Fig. 1.

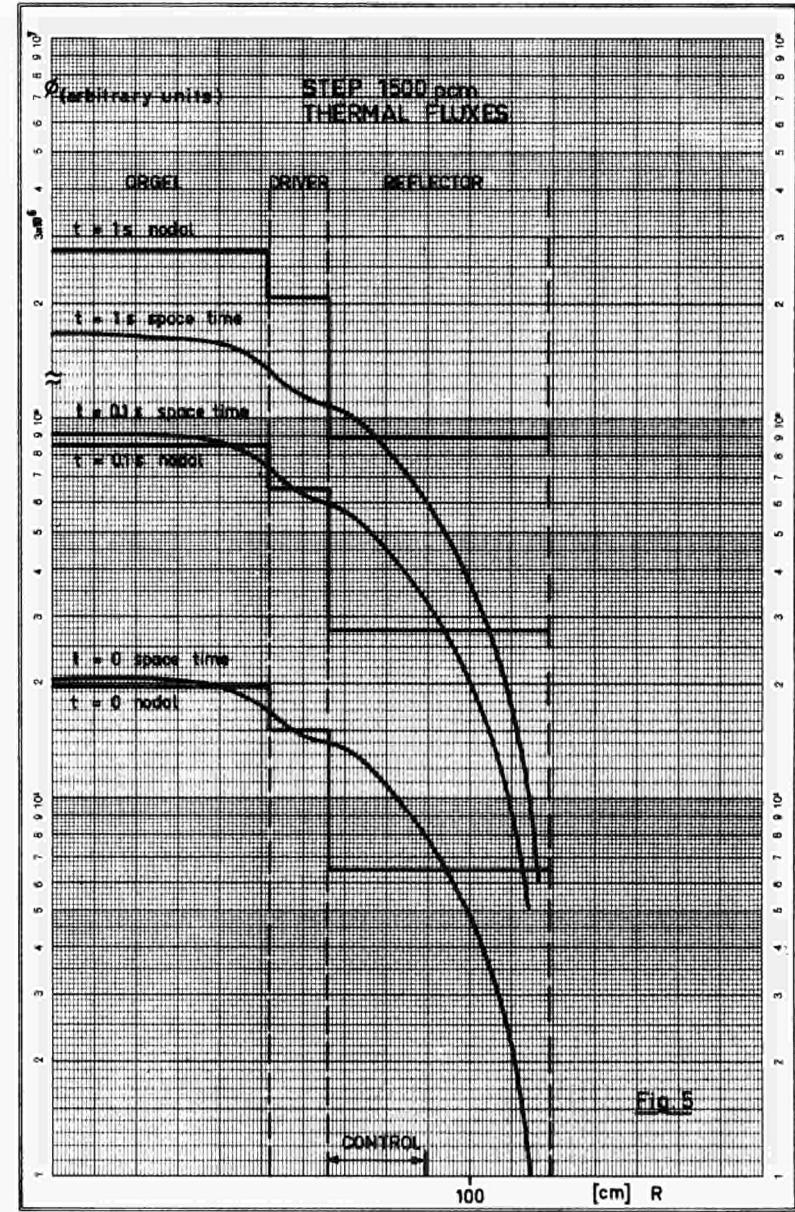
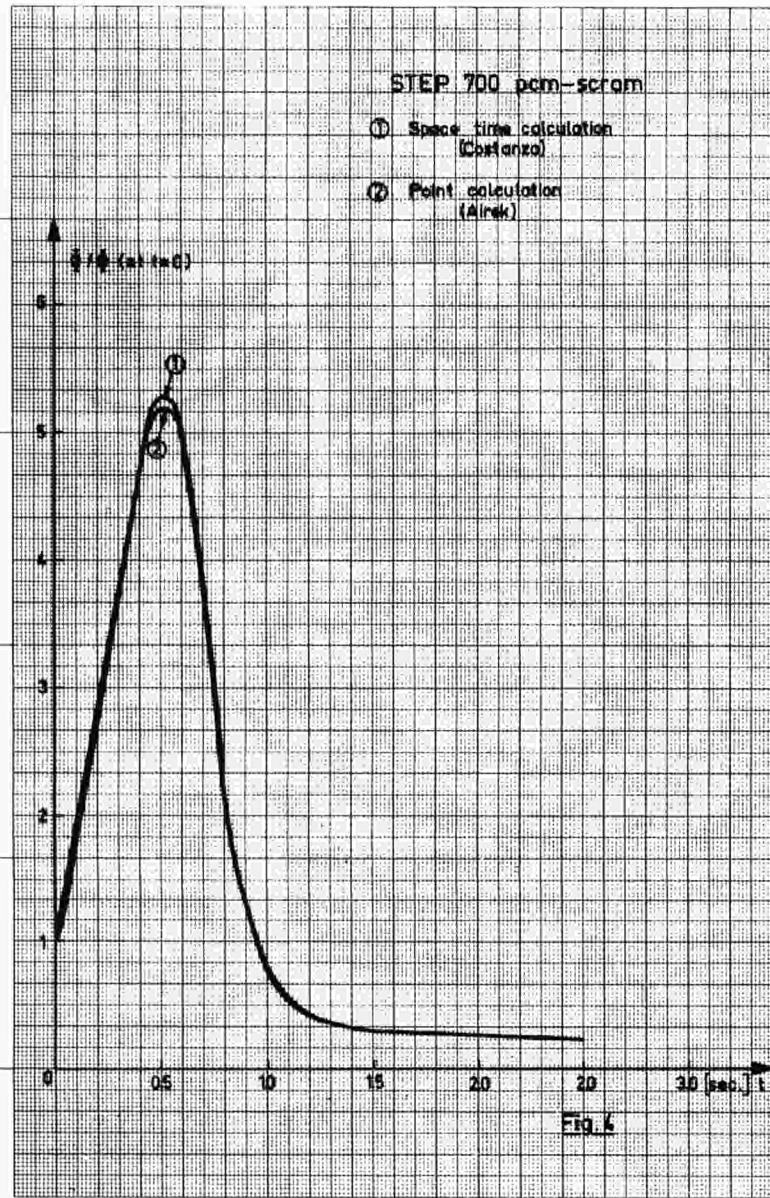
Calculations made for perturbations up to 300 pcm show that, up to that level of perturbation the differences between the three methods are of no practical interest.

Conclusion

From these results we may deduce that, for a reactor of the size of ESSOR, the point method is adequate to represent transients caused by reactivity up to one dollar. For more serious accidents, which are however unusual, the spatial method is the best. Intermediate methods as the nodal are not adequate unless a sufficiently great number of points is considered, but in this case the times of calculation are of the same order of the time requested by the spatial method.







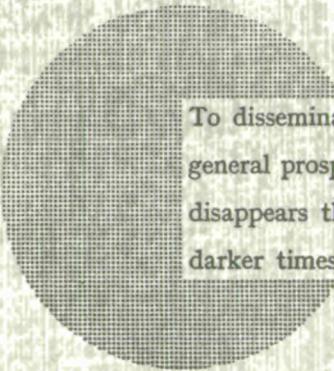
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To disseminate knowledge is to disseminate prosperity — I mean general prosperity and not individual riches — and with prosperity disappears the greater part of the evil which is our heritage from darker times.

Alfred Nobel

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