A PROGRAMME FOR THE SOLUTION OF THE MONOENERGETICAL TRANSPORT EQUATION IN THE P5 APPROXIMATION FOR A MULTIREGION CYLINDRICAL GEOMETRY

by

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1965

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The report describes a Fortran II code for the IBM 7090 computer which solves the monoenergetical transport equation in the P5 approximation for a multiregion cylindrical geometry.

Special attention is paid to the numerical difficulties encountered in this work.
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1. General description

The code described in this report was written by the author while at the Dragon Project, A.E.E. Winfrith, for the Ferranti Mercury computer.

Because of the difficulty in the use of double precision matrix operations on that computer the code did not work properly there for many difficult cases. Now the programme has been translated into Fortran II for the IBM 7090 computer and it works satisfactorily.

The programme solves the monoenergetical transport equation in the P5 approximation for a multiregion cylindrical geometry. Up to 10 concentrical regions can be considered. The scattering is considered to be isotropic.

Two boundary conditions, reflective and infinite medium, are available, and an air gap may be considered. The equations are solved by the propagation method described for the P3 approximation in ref. 1.

Such methods have the advantage of solving large regions without losing accuracy in cases where numerical techniques might fail, and are quick where large uniform regions are considered.

While the programming of the P3 did not present great numerical difficulties, the P5 required double precision and a tightening up of the procedure of matrix propagation.

Being all analytical the code is very fast (a typical case can run in 15 to 20 seconds) but the numerical difficulties encountered in writing it show that the P5 is about the highest approximation which one can treat with this method.

As a control on the accuracy of the results, the flux at

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each boundary is printed twice with 8 decimal figures, once calculated from the inner side and the other time from the outer side of the boundary. Any difference between the two fluxes is due to numerical inaccuracy.

2. Theory and Code

The expansion of the flux is given by

\[
2\pi\gamma (\mathbf{r}, \mathbf{\perp}) = \frac{1}{2} \psi_{00}(r) + \frac{3}{2} \psi_{11}(r) P_1^1(\eta) \cos \omega + 5 \left[ \frac{1}{2} P_2(\eta) \psi_{02}(r) + \frac{1}{4!} P_2^2(\eta) \cos 2\omega \psi_{22}(r) \right] + 7 \left[ \frac{2}{4!} P_3(\eta) \cos \omega \psi_{13}(r) + \frac{1}{6!} P_3^3(\eta) \cos 3\omega \psi_{33}(r) \right] + 9 \left[ \frac{1}{2} P_4(\eta) \psi_{04}(r) + \frac{2}{6!} P_4^2(\eta) \cos 2\omega \psi_{24}(r) \right] + \frac{1}{8!} P_4^4(\eta) \cos 4\omega \psi_{44}(r) - 11 \left[ \frac{4}{6!} P_5(\eta) \cos \omega \psi_{15}(r) + \frac{2}{8!} P_5^3(\eta) \cos 3\omega \psi_{35}(r) + \frac{1}{10!} P_5^5(\eta) \cos 5\omega \psi_{55}(r) \right] \]

and the expressions for the various moments \( \psi_{mn} \) in the P5 approximation are as follows:

\[
\psi_{00}(r) = \frac{3}{\gamma} \left\{ B_i I_0(\gamma_ir) + C_i K_0(\gamma_ir) \right\}
\]
\[ \varphi_{11}(r) = \frac{1}{2} G_1(\gamma_i) \left[ B_{1} I_{1}(\gamma_i r) - C_{1} K_{1}(\gamma_i r) \right] \]

\[ \varphi_{02}(r) = \sum_{i} - \frac{1}{2} G_2(\gamma_i) \left[ B_{1} I_{0}(\gamma_i r) + C_{1} K_{0}(\gamma_i r) \right] + \]

\[ + \sum_{j=1}^{2} \left[ M_{j} I_{0}(\beta_{jr}) + N_{j} K_{0}(\beta_{jr}) \right] \]

\[ \varphi_{22}(r) = \sum_{i} 3 G_2(\gamma_i) \left[ B_{2} I_{2}(\gamma_i r) + C_{2} K_{2}(\gamma_i r) \right] + \]

\[ + \sum_{j}^{2} \left[ M_{j} I_{2}(\beta_{jr}) + N_{j} K_{2}(\beta_{jr}) \right] \]

\[ \varphi_{13}(r) = \sum_{i} - \frac{3}{2} G_3(\gamma_i) \left[ B_{1} I_{1}(\gamma_i r) - C_{1} K_{1}(\gamma_i r) \right] - \]

\[ - 5 \sum_{j=1}^{2} \left[ M_{j} I_{1}(\beta_{jr}) - N_{j} K_{1}(\beta_{jr}) \right] \]

\[ \varphi_{33}(r) = \sum_{i} 15 G_3(\gamma_i) \left[ B_{1} I_{3}(\gamma_i r) - C_{1} K_{3}(\gamma_i r) \right] - \]

\[ - 30 \sum_{j=1}^{2} \left[ M_{j} I_{3}(\beta_{jr}) - N_{j} K_{3}(\beta_{jr}) \right] \]

\[ \varphi_{04}(r) = \sum_{i} \frac{3}{8} G_4(\gamma_i) \left[ B_{1} I_{0}(\gamma_i r) + C_{1} K_{0}(\gamma_i r) \right] + \]

\[ + \frac{5(\beta^2 - 7)}{12 \beta^2} \sum_{j=1}^{2} \left[ M_{j} I_{0}(\beta_{jr}) + N_{j} K_{0}(\beta_{jr}) \right] + Q I_{0}(\mu r) + R K_{0}(\mu r) \]
\[ \psi_{24}(r) = \sum_i \left( \frac{15}{2} G_4(\gamma_i) \{ B_i I_2(\gamma_i r) + C_i K_2(\gamma_i r) \} - \right.
\sum_j \left( \frac{5}{\beta_j} \right) \{ N_j I_2(\beta_j r) + N_j K_2(\beta_j r) \} + 12 \{ Q I_2(\nu r) + R \} \right) \]

\[ \psi_{44}(r) = \sum_i \left( 105 G_4(\gamma_i) \{ B_i I_4(\gamma_i r) + C_i K_4(\gamma_i r) \} - \right.
\sum_j \left( \frac{70}{\beta_j} \right) \{ M_j I_4(\beta_j r) + M_j K_4(\beta_j r) \} + 24 \{ Q I_4(\nu r) + R \} \}

\[ \psi_{15}(r) = \sum_i \left( \frac{15}{8} G_5(\gamma_i) \{ B_i I_1(\gamma_i r) - C_i K_1(\gamma_i r) \} - \right.
\sum_j \left( \frac{35}{44\beta_j} \right) \{ M_j I_1(\beta_j r) - N_j K_1(\beta_j r) \} - \frac{9}{5} \{ Q I_1(\nu r) - R \} \right.

\[ \psi_{35}(r) = \sum_i \left( -\frac{105}{2} G_5(\gamma_i) \{ B_i I_3(\gamma_i r) - C_i K_3(\gamma_i r) \} + \right.
\sum_j \left( \frac{105}{\beta_j} \right) \{ M_j I_3(\beta_j r) - N_j K_3(\beta_j r) \} - \frac{324}{5} \{ Q I_3(\nu r) - R K_3(\nu r) \} \]
\[ \psi_{55}(r) = \sum_{i} 945 G_{5}(\gamma_{i}) \left[ B_{1} I_{5}(\gamma_{i} r) - C_{i} K_{5}(\gamma_{i} r) \right] + \]
\[ + \sum_{j} \frac{3150}{11} \left( \frac{\beta_{j}^{2} - 7}{\beta_{j}} \right) \left[ M_{j} I_{5}(\beta_{j} r) - N_{j} K_{5}(\beta_{j} r) \right] - \frac{1080}{\nu} \left[ Q I_{5}(\nu r) - R K_{5}(\nu r) \right] \] ............(2a)

where

\[ G_{0} = 1 \quad \alpha = 1 - c \]
\[ G_{1} = \frac{\alpha}{\gamma} \quad c = N^{\circ} \text{of secondaries per collision} \]
\[ G_{n+1} = - \frac{1}{n+1} \left[ \frac{2n+1}{\gamma} G_{n} + n G_{n-1} \right] \] ............(2b)

and \( \gamma_{1}, \gamma_{2}, \gamma_{3} \) are the positive roots of

\[ 25\gamma^{6} - 21(14 + 11\alpha)\gamma^{4} + 35(11 + 34\alpha)\gamma^{2} - 1155\alpha = 0 \]
(see appendix 2)

\( \beta_{1} \) and \( \beta_{2} \) are the positive roots of

\[ \beta^{4} - 18\beta^{2} + 33 = 0 \]
and \( \nu = \sqrt{11} \) ............(2c)

We can consider the flux as a vector whose components are given by the moments \( \psi_{mn} \), so that at a point \( r \) in a given region \( i \), it may be represented by

\[ \psi_{r}^{i} = A_{1}^{r} C_{i} + S_{i} \] ....(3)

\( A_{1}^{r} \) is a \((12 \times 12)\) matrix of modified Bessel functions whose arguments are function of the region number \( i \) and position \( r \).

\( S_{i}^{r} \) is the source vector (as the source is supposed to be isotropic all the components are zero but the first one).
\( \mathbf{C} \) is the vector of the 12 integration constants \([B_1, C_1, B_2, \ldots, Q, R]\) to be determined from the boundary conditions imposed.

If \( v \) is the number of regions and the boundaries are \( r_1, r_2, \ldots, r_n \), the interface conditions of continuity will lead to the following set of \((n - 1)\) equations:

\[
\begin{align*}
A_1^1 C_1 + S_1 &= A_2^1 C_2 + S_2 \\
A_2^2 C_2 + S_2 &= A_3^2 C_3 + S_3 \\
&\vdots \\
A_{n-1}^{n-1} C_{n-1} + S_{n-1} &= A_n^{n-1} C_n + S_n
\end{align*}
\]

By defining \( B_i = (A_{i+1})^{-1} \) we obtain the recurrent formula

\[
C_{i+1} = B_i (A_i^i C_i + S_i - S_{i+1})
\]

which propagates the coefficients \( C_i \) outwards, region by region.

The coefficients \( C_i \) are found by applying the boundary conditions of finiteness of the flux in the centre (the coefficients associated with the \( K \) functions must be zero) and the chosen boundary conditions at the last region. If the last region is infinite, in \( C_n \) the coefficients associated with the \( I \) functions must vanish.

If the last boundary is a reflective boundary the components \( \psi_{11}, \psi_{13}, \psi_{33}, \psi_{15}, \psi_{35}, \psi_{55} \) must vanish [2]. In this latter case the condition will not be applied to \( C_n \) but to \( (A_n^n C_n + S_n) \).

In both cases we start from the centre with a starter \( C_1 (12 \times 1) \) with 6 zero components and we must determine the other 6 in such a way that the \((12 \times 1)\) response \( R \) that we get after propagation has zeros in 6 defined positions.
The response $R$ will be the sum of a homogeneous response $R^*$ (that we would get with all $S_i = 0$) and an effect $b$ of the sources (that we can get by propagating a $C_1$ of all zeros).

$$R = R^* + b$$

If we propagate a matrix $C$ of 6 independent starters (each satisfying to the centre boundary condition) as for example:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

with all $S_i = 0$.

We get an homogeneous response $R^*$. If $\phi$ is a $(6 \times 1)$ vector we will have that

$$C_1\phi \rightarrow R^*\phi + b$$

We need to find $\phi$ such that $R^*\phi + b$ satisfies the outer boundary condition, and $C_1\phi$ will be the starter $C_1$ that solves our problem.

Because of loss of accuracy it is necessary to tighten up the process of obtainment of $R^*$.

To do so we select from $R^*$ (which is a $12 \times 6$ matrix) the six rows which are of interest for imposing the boundary conditions (i.e. the rows corresponding to the coefficients of the...
I functions in case of infinite medium, or the rows corresponding to the 6 components of the flux which must vanish in the case of reflective boundary conditions).

Let us call $Q$ this $6 \times 6$ matrix. If we propagate as a new starter $C'$ the product $C' Q^{-1}$ we will obtain a new $R^*$, out of which we can select a new $Q'$. This new $Q'$ should be a unit matrix if there would be no loss of accuracy in the process. If $Q'$ is not near enough to a unit matrix the loop is continued rep propagating $C' Q'^{-1}$ until a good enough unit matrix is obtained.

At the end we obtain a starter which gives a response $R^*$ which is a unit matrix on the rows of interest for the boundary conditions.

At this point the vector $\phi$ is easily determined because $R^* \phi$ will be a $(12 \times 1)$ vector having the components of $\phi$ on the positions of interest for the boundary conditions.

Because we want the $12 \times 1$ vector $R^* \phi + b$ to have zeros on those positions, the components of $\phi$ will be equal and of opposite sign of the components of $b$ corresponding to those positions.

For example in the case of infinite medium, calling $\phi_1$, $\phi_2$,...,$\phi_6$ the components of $\phi$ we have

$$
R^* = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
$$
so that

\[ R^* \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \end{pmatrix} = -b,
\]

and to satisfy the boundary conditions we will need:

\[ \begin{align*}
\phi_1 &= -b_1 \\
\phi_2 &= -b_3 \\
\phi_3 &= -b_5 \\
\phi_4 &= -b_7 \\
\phi_5 &= -b_9 \\
\phi_6 &= -b_{11}
\end{align*} \]

At this point the programme can calculate the coefficients \( C_i \) in every region. If we call \( C_i^* \) the \((12x6)\) matrices that we got as intermediate results while calculating \( R^* \), and \( b_i \) the corresponding intermediate results obtained while calculating \( b \), we have:

\[ C_i = C_i^* \phi + b_i \]

The flux is then calculated in the requested points.

The output also contains the mean flux in every region and an asymptotic flux and current for extrapolation lengths calculations.
3. References

1. J.R. Askew, R.J. Brissenden
   Mercury programme 560 - Spherical harmonics P₃ approximation - One group, cylindrical geometry.
   AEEW-M116, January 1961

2. J. Tait
   The calculation of the fine structure of the thermal neutron flux in a pile, by the spherical harmonics method.
   A/CONF. 8/P/433 - 1955

4. Acknowledgment

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   The translation into Fortran is largely due to Mr. G. Fassone.
Appendix 1

The air-gap condition

The equations connecting the various moments across the air gap are as follows:

\[ a_{11}(a) = c \psi_{11}(c) \]  
(5.1)

\[ a_{13}(a) = c \psi_{13}(c) \]  
(5.2)

\[ a_{15}(a) = c \psi_{15}(c) \]  
(5.3)

\[ a^3 \{ \psi_{33}(a) - 12 \psi_{11}(a) + 2 \psi_{13}(a) \} = c^3 \{ \psi_{33}(c) - 12 \psi_{11}(c) + + 2 \psi_{13}(c) \} \]  
...... (5.4)

\[ a^3 \{ \psi_{35}(a) - 12 \psi_{11}(a) - 28 \psi_{13}(a) + 12 \psi_{15}(a) \} = c^3 \{ \psi_{35}(c) - - 12 \psi_{11}(c) - 28 \psi_{13}(c) + 12 \psi_{15}(c) \} \]  
...... (5.5)

\[ a^5 \{ \psi_{55}(a) + 1296 \psi_{11}(a) - 336 \psi_{13}(a) + 48 \psi_{15}(a) - - 168 \psi_{33}(a) + 6 \psi_{35}(a) \} = c^5 \{ \psi_{55}(c) + 1296 \psi_{11}(c) - 336 \psi_{13}(c) + 48 \psi_{15}(c) - - 168 \psi_{33}(c) + 6 \psi_{35}(c) \} \]  
...... (5.6)

\[ - \frac{\pi}{4} \psi_{00}(a) + \frac{\pi}{2} \psi_{11}(a) + \frac{5\pi}{32} \psi_{02}(a) - \frac{5\pi}{64} \psi_{22}(a) + \frac{9\pi}{256} \psi_{04}(a) = - - \frac{\pi}{256} \psi_{24}(a) + \frac{\pi}{2048} \psi_{44}(a) = - \frac{\pi}{4} \psi_{00}(c) + \frac{a}{c} \{ \sin^{-1} \frac{a^2}{c^2} + \frac{a}{c} \sqrt{1 - \frac{a^2}{c^2}} \} \psi_{11}(c) + \frac{5\pi}{32} \psi_{02}(c) = - - \frac{15\pi}{64} (1 - \frac{2}{3} \frac{a^2}{c^2}) \psi_{22}(c) - \frac{7}{45} (1 - \frac{a^2}{c^2})^{3/2} \psi_{33}(c) - \frac{9\pi}{256} \psi_{04}(c) = - - \frac{3\pi}{256} (1 - \frac{2}{3} \frac{a^2}{c^2}) \psi_{24}(c) - \frac{\pi}{2048} (15 - 40 \frac{a^2}{c^2} + 24 \frac{a^4}{c^4}) \psi_{44}(c) + + \frac{11}{1260} (1 - \frac{a^2}{c^2})^{3/2} \psi_{35}(c) + \frac{11}{4200} (1 - \frac{a^2}{c^2})^{3/2} (1 - \frac{8}{3} \frac{a^2}{c^2}) \psi_{55}(c) \]  
...... (5.7)
\[-\frac{3\pi}{32} \psi_{00}(a) - \frac{105\pi}{128} \psi_{02}(a) + \frac{5\pi}{256} \psi_{22}(a) + \frac{\pi}{2} \psi_{13}(a) + \] 
\[+ \frac{207\pi}{4096} \psi_{04}(a) - \frac{17\pi}{4096} \psi_{24}(a) - \frac{9\pi}{32768} \psi_{44}(a) = - \frac{3\pi}{32} \psi_{00}(c) - \] 
\[- \frac{105\pi}{128} \psi_{02}(c) + \frac{15\pi}{256} \left(1 - \frac{2}{3} \frac{a^2}{c^2}\right) \psi_{22}(c) + \frac{c}{a} \left\{ \sin^{-1} \frac{a}{c} + \right. \] 
\[+ \frac{a}{c} \sqrt{1 - \frac{a^2}{c^2}} \right\} \psi_{33}(c) - \frac{1}{15} \left(1 - \frac{a^2}{c^2}\right)^{3/2} \psi_{33}(c) + \frac{207\pi}{4096} \psi_{04}(c) - \] 
\[- \frac{513\pi}{4096} \left(1 - \frac{2}{3} \frac{a^2}{c^2}\right) \psi_{24}(c) + \frac{9\pi}{32768} \left(15 - 40 \frac{a^2}{c^2} + 24 \frac{a^4}{c^4}\right) \psi_{44}(c) + \] 
\[+ \frac{11}{210} \left(1 - \frac{a^2}{c^2}\right)^{3/2} \psi_{35}(c) - \frac{11}{6300} \left(1 - \frac{a^2}{c^2}\right)^{3/2} \left(1 - \frac{8}{3} \frac{a^2}{c^2}\right) \psi_{55}(c) \] 
\[\ldots (5.8)\]

\[- \frac{15\pi}{256} \psi_{00}(a) - \frac{1425\pi}{4096} \psi_{02}(c) + \frac{25\pi}{8192} \psi_{22}(a) - \frac{42255\pi}{32768} \psi_{04}(a) + \] 
\[+ \frac{495\pi}{32768} \psi_{24}(a) + \frac{9\pi}{262144} \psi_{44}(a) + \frac{\pi}{2} \psi_{15}(a) = - \frac{15\pi}{256} \psi_{00}(c) - \] 
\[- \frac{1425\pi}{4096} \psi_{02}(c) + \frac{75\pi}{8192} \left(1 - \frac{2}{3} \frac{a^2}{c^2}\right) \psi_{22}(c) - \frac{42255\pi}{32768} \psi_{04}(c) + \] 
\[+ \frac{1485\pi}{32768} \left(1 - \frac{2}{3} \frac{a^2}{c^2}\right) \psi_{24}(c) - \frac{7\pi}{262144} \left(15 - 40 \frac{a^2}{c^2} + 24 \frac{a^4}{c^4}\right) \psi_{44}(c) + \] 
\[+ \frac{c}{a} \left\{ \sin^{-1} \frac{a}{c} + \frac{a}{c} \sqrt{1 - \frac{a^2}{c^2}} \right\} \psi_{15}(c) - \frac{1}{28} \left(1 - \frac{a^2}{c^2}\right)^{3/2} \psi_{35}(c) + \] 
\[+ \frac{1}{2520} \left(1 - \frac{a^2}{c^2}\right)^{3/2} \left(1 - \frac{8}{3} \frac{a^2}{c^2}\right) \psi_{55}(c) \] 
\[\ldots (5.9)\]
\begin{align*}
\psi_{22}(c) &= \frac{375\pi}{128} \left( \frac{1}{3} - \frac{14}{15} \frac{a^2}{c^2} \right) \psi_{22}(c) - 2 \frac{c}{a} \left( 1 - \frac{c^2}{a^2} \right) \{ \sin^{-1} \frac{a}{c} - \frac{c}{a} \sqrt{1 - \frac{a^2}{c^2}} \} \psi_{13}(c) + \\
&\quad \frac{c^3}{a^3} \left( \sin^{-1} \frac{a}{c} - \frac{c}{a} \sqrt{1 - \frac{a^2}{c^2}} \right) \left( 1 + \frac{4}{3} \frac{a^2}{c^2} - \frac{10}{3} \frac{a^4}{c^4} \right) \psi_{24}(c) + \\
&\quad \frac{405\pi}{2048} \psi_{04}(c) - \frac{225\pi}{2048} \left( \frac{1}{3} - \frac{14}{15} \frac{a^2}{c^2} \right) \psi_{24}(c) + \\
&\quad \frac{15\pi}{16384} \left( 35 - 392 \frac{a^2}{c^2} + 312 \frac{a^4}{c^4} \right) \psi_{44}(c) - \frac{11}{945} \left( 1 - \frac{a^2}{c^2} \right)^2 (1 - 17 \frac{a^2}{c^2} + 16 \frac{a^4}{c^4}) \psi_{55}(c)
\end{align*}

\begin{align*}
\psi_{22}(a) &= \frac{105\pi}{64} \psi_{00}(a) + \frac{5775\pi}{1024} \psi_{02}(a) - \frac{1575\pi}{2048} \psi_{22}(a) - \frac{82215\pi}{8192} \psi_{04}(a) - \\
&\quad - \frac{20601\pi}{8192} \psi_{24}(a) + \frac{945\pi}{32768} \psi_{44}(a) + \frac{\pi}{2} \psi_{35}(a) = \frac{105\pi}{64} \psi_{00}(c) + \\
&\quad + 12 \frac{c}{a} \left( 1 - \frac{c^2}{a^2} \right) \{ \sin^{-1} \frac{a}{c} - \frac{c}{a} \sqrt{1 - \frac{a^2}{c^2}} \} \psi_{11}(c) + \frac{5775\pi}{1024} \psi_{02}(c) + \\
&\quad + \frac{175\pi}{2048} \left( 5 - 14 \frac{a^2}{c^2} \right) \psi_{22}(c) + 28 \frac{c}{a} (1 - \frac{c^2}{a^2}) \{ \sin^{-1} \frac{a}{c} - \\
&\quad - \frac{c}{a} \sqrt{1 - \frac{a^2}{c^2}} \} \psi_{13}(c) - \frac{82215\pi}{8192} \psi_{04}(c) + \frac{2289\pi}{8192} (5 - 14 \frac{a^2}{c^2}) \psi_{24}(c) - \\
&\quad - \frac{21\pi}{65536} \left( 35 - 390 \frac{a^2}{c^2} + 312 \frac{a^4}{c^4} \right) \psi_{44}(c) - 12 \frac{c}{a} (1 - \frac{c^2}{a^2}) \{ \sin^{-1} \frac{a}{c} - \\
&\quad - \frac{c}{a} \sqrt{1 - \frac{a^2}{c^2}} \} \psi_{15}(c) + \frac{c^3}{a^3} \sin^{-1} \frac{a}{c} - \frac{c}{a} \sqrt{1 - \frac{a^2}{c^2}} \left( 1 + \frac{4}{3} \frac{a^2}{c^2} - \frac{10}{3} \frac{a^4}{c^4} \right) \psi_{35}(c) + \\
&\quad + \frac{1}{135} \left( 1 - \frac{a^2}{c^2} \right)^2 \left( 1 - 17 \frac{a^2}{c^2} + 16 \frac{a^4}{c^4} \right) \psi_{55}(c)
\end{align*}
where $a$ and $c$ are the inner and outer radii of the air gap.

Those equations can be represented by the matrix equation:

$$X(A_{j-1} C_{j-1} + S_{j-1}) = Y(A_{j+1} C_{j+1} + S_{j+1}) \quad \ldots \ldots (6)$$

where the region $j$ is considered to be the air gap. It is convenient to include the air gap conditions in the chain (4) as part of the propagation \[Ref. 1\]. For this purpose we must find fictitious $A_{j-1}^r$, $A_j^r$, $S_j^r$ such that the relation

$$A_{j-1}^r C_{j-1} + S_{j-1}^r = A_j^r C_j + S_j^r$$

is equivalent to the (6). For that we need:

$$S_j^r = 0 \quad A_{j-1}^r = X^{-1} \quad A_j^r = Y^{-1}$$
Appendix 2

Numerical calculations of the \( G(\gamma) \) functions

The formulae given for the \( G(\gamma) \) functions are not suitable for numerical calculations, because, for very small values of \( \alpha \), \( \gamma^2 \approx 3\alpha \), and as it can easily be seen, in the equation (2b) the term \( 3\alpha - \gamma^2 \) appears.

From the bicubic we can get

\[
(3\alpha - \gamma^2) = \frac{1}{385} \left[ 25\gamma^6 - 21(14 + 11\alpha)\gamma^4 + 1190\alpha\gamma^2 \right]
\]

Having made this substitution we get:

\[
G_1(\gamma) = -\frac{\alpha}{\gamma}
\]

\[
G_2(\gamma) = 0.0324675324\gamma^4 - 0.381818182\gamma^2 - 0.3\gamma\alpha + 1.54545454\alpha
\]

\[
G_3(\gamma) = -0.041322314\gamma^5 + 0.381818182\alpha\gamma^3 + 0.431837856\gamma^3 - 1.4669421\alpha^3
\]

\[
G_4(\gamma) = 0.03047744\gamma^6 - 0.31045134\gamma^4 - 0.28161156\alpha\gamma^4 + 1.007544412\alpha^2
\]

\[
G_5(\gamma) = -0.0138533847\gamma^7 + 0.1411142654\gamma^5 + 0.128005276\alpha\gamma^5 - 0.45797481\alpha^2
\]
### Appendix 3

**Input description**

<table>
<thead>
<tr>
<th>Card</th>
<th>Format</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E 12.5</td>
<td>EPS1</td>
<td>Accuracy on $\gamma$ coefficients</td>
</tr>
<tr>
<td></td>
<td>E 12.5</td>
<td>EPS2</td>
<td>Accuracy on unit matrix</td>
</tr>
<tr>
<td></td>
<td>I 12</td>
<td>NINV1</td>
<td>N° of iterations in 12x12 matrix version</td>
</tr>
<tr>
<td></td>
<td>I 12</td>
<td>NINV2</td>
<td>N° of iterations in 6x6 matrix version</td>
</tr>
<tr>
<td></td>
<td>I 12</td>
<td>IMP</td>
<td>= 0 macroscopic input</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= N microscopic input with a library of N materials</td>
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</table>

If IMP = N then N card 2 are needed

<table>
<thead>
<tr>
<th>Card</th>
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<th>Name</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>E 12.5</td>
<td>TRANSPL</td>
<td>Microscopic transport cross section material I</td>
</tr>
<tr>
<td></td>
<td>E 12.5</td>
<td>ABS(I)</td>
<td>Microscopic absorption cross section material I</td>
</tr>
<tr>
<td></td>
<td>A 12</td>
<td>TITLE(I)</td>
<td>Name of material I</td>
</tr>
<tr>
<td>3</td>
<td>I 12</td>
<td>NREGS</td>
<td>N° of regions ($\leq$ 10)</td>
</tr>
<tr>
<td></td>
<td>I 12</td>
<td>NX</td>
<td>= 0 no effect</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 1 print intermediate data (normally not necessary)</td>
</tr>
</tbody>
</table>

If IMP > 0 then cards 4 and 5 are needed for each region

<table>
<thead>
<tr>
<th>Card</th>
<th>Format</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>E 12.5</td>
<td>RAD(IN)</td>
<td>Outer radius region IN</td>
</tr>
<tr>
<td></td>
<td>E 12.5</td>
<td>SOURCE(IN)</td>
<td>Source (neutrons/unit volume) region IN</td>
</tr>
<tr>
<td></td>
<td>E 12.5</td>
<td>AMAT</td>
<td>N° of materials in region IN</td>
</tr>
<tr>
<td></td>
<td>E 12.5</td>
<td>APRIN(IN)</td>
<td>N° of intermediate print-out point region IN</td>
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</table>

Each card 4 is followed by AMAT card 5.
<table>
<thead>
<tr>
<th>Card</th>
<th>Format</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>I 12</td>
<td>NMAT</td>
<td>Material N° (given by the order in which the material appears in card 2)</td>
</tr>
<tr>
<td></td>
<td>E 12.5</td>
<td>CONC</td>
<td>Concentration</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>If IMP = 0 then card 6 is needed for each region</td>
</tr>
<tr>
<td>6</td>
<td>E 12.5</td>
<td>RAD(I)</td>
<td>Outer radius region I</td>
</tr>
<tr>
<td></td>
<td>E 12.5</td>
<td>FMP(I)</td>
<td>Mean free path region I (negative implies air gap)</td>
</tr>
<tr>
<td></td>
<td>E 12.5</td>
<td>SOURCE(I)</td>
<td>Source (neutrons/unit volume) region I</td>
</tr>
<tr>
<td></td>
<td>E 12.5</td>
<td>ALPHA(I)</td>
<td>N° absorptions/collision region I</td>
</tr>
<tr>
<td></td>
<td>E 12.5</td>
<td>blank</td>
<td>= 0 infinite medium</td>
</tr>
<tr>
<td></td>
<td>E 12.5</td>
<td>APRIN(I)</td>
<td>N° of intermediate print-out p region I</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>= 1 reflective boundary.</td>
</tr>
<tr>
<td>7</td>
<td>I 12</td>
<td>IQ</td>
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</table>
EPS1  ACCURACY ON GAMMA COEFFICIENTS 
EPS2  ACCURACY ON UNIT MATRIX 
NINV1  NO. OF ITERATIONS IN 12X12 MATRIX INVERSION 
NINV2  NO. OF ITERATIONS IN 6X6 MATRIX INVERSION 
NRREGS  NO. OF REGIONS 
NX  =1 NO EFFECT, =1 PRINT INTERMEDIATE DATA 
RAD(IN) OUTER RADIUS REGION IN 
FMP(IN) MEAN FREE PATH REGION IN 
SOURCE(IN) A NEGATIVE MEAN FREE PATH MEANS AIR GAP 
TRANSP(I) MICROSCOPIC LIBRARY SIGMA TRANSP. MATERIAL I 
ABS(I) MICROSCOPIC LIBRARY SIGMA ABSORPTION MAT. I 
TITLE(I) MICROSCOPIC LIBRARY NAME MATERIAL I 
CONC CONCENTRATION 
AMAT NO. OF MATERIALS REGION IN 
NMAT MATERIAL NUMBER (IN LIBRARY) 
DIMENSION A(12,12,10), B(12,12,12), U(12,6,10), 
V(12,6,10), W(6,12,10), C(12,12), 
FMP(10), SOURCE(10), ALPHA(10), AMASS(10), APRIN(10) 
DIMENSION Z(12,12), ZH(12,12) 
DIMENSION Y(1), RADM(1) 
DIMENSION EPS1(1), EPS2(1) 
COMMON A,H,U,VAN,U1,RESP,V,RV,V1,S,C,RAD,FMP,SOURCE,ALPHA,AMASS,A 
IRIN,IR,IRR,IN,N5,N6 
COMMON X,NX 
COMMON EPS1,EPS2,NINV1,NINV2 
1 FORMAT (6E12.0) 
2 FORMAT (6I12) 
3 FORMAT (2F12.5,3I12) 
4 FORMAT (2F12.5,5I12) 
5 FORMAT (1H1,36X,2CH) 
6 FORMAT (1E12.5) 
7 FORMAT (1E16.5) 
8 FORMAT (1E16.5) 
9 FORMAT (2E16.5) 
10 FORMAT (3E16.5) 
11 FORMAT (2E16.5) 
12 FORMAT (15X) 
13 FORMAT (3I12.10) 
14 FORMAT (15X) 
15 FORMAT (15X) 
16 FORMAT (15X) 
17 FORMAT (15X) 
18 FORMAT (15X) 
Tape designation 
N5=5
READ INPUT TAPE N5,4, EPS1, EPS2, NINV1, NINV2, IMP
IF (IMP) 999, 999, 999
999 READ INPUT TAPE N5, 8001, (TRASP(I), ABS(I), TITLE(I), I = 1, IMP)
WRITE OUTPUT TAPE N6, 8032
WRITE OUTPUT TAPE N6, 8033, (TRASP(I), ABS(I), TITLE(I), I = 1, IMP)
READ INPUT TAPE N5, 2, NREGS, NX
IF (IMP) 8060, 8060, 8051
WRITE OUTPUT TAPE N6, 8034
DO P101 IN = 1, NREGS
READ INPUT TAPE N5, 8005, RAD(IN), SOURCE(IN), AMAT, APRIN(IN)
WRITE OUTPUT TAPE N6, 8005, IN, RAD(IN), SOURCE(IN), AMAT
MAT = AMAT + C01
SIGT = 0.0
SIGA = 0.0
DO 8100 I = 1, MAT
READ INPUT TAPE N5, 8007, NMAT, CONC
WRITE OUTPUT TAPE N6, 8008, TITLE(NMAT), CONC
SIGT = SIGT + CONC * TRASP(NMAT)
SITA = SIGA + CONC * ABS(NMAT)
ALPHA(I) = SIGA / SIGT
FMP(IN) = 1.0 / SIGT
WRITE OUTPUT TAPE N6, 8009, ALPHA(IN), FMP(IN)
8100 CONTINUE
GO TO 8052
READ INPUT TAPE N5, 1, (RAD(I), FMP(I), SOURCE(I), ALPHA(I), AMASS(I), APRIN(IN), I = 1, NREGS)
WRITE OUTPUT TAPE N6, 5003, (I, RAD(I), FMP(I), SOURCE(I), ALPHA(I), AMASS(I), APRIN(IN))
NREGS = NREGS - 1
FORM(///18X, ICH REGION = I3//, 12X, OUTER RADIUS = E12.5/
1 17X, MEAN FREE PATH = E12.5//, 18X, SOURCE = E12.5/
2 28H, NO. ABSORPTIONS/COLLISION = E12.5//)
READ INPUT TAPE N5, 2, IQ
DO 20 I = 1, NR
IF (ALPHA(IN) = 0.00001)
20 CONTINUE
WRITE OUTPUT TAPE N6, 221
FORMAT (PH:REGION I5, I9H IS PURE SCATTERER //)
CONTINUE
NR = NREGS - 1
DO 150 INN = 1, NR
INN = INN
READ INPUT TAPE N5, 6500, 6501, 6501
CONTINUE
ALPHA(I) = 0.00001
WRITE OUTPUT TAPE N6, 6502
CONTINUE
RADM = RAD(IN) / FMP(IN)
Y = ALPHA(IN)
CALL MATRIX
DO 23 I=1,12
DO 23 J=1,12
A(I,J,IN)=Z(I,J)

TEST ON AIR GAP
IF(FMP(IN+1))6502,6500,6500
5502 CALL AIRGAP
GO TO 150
5500 CONTINUE

RADM=RAD(IN)/FMP(IN+1)
Y=ALPHA(IN+1)
CALL MATRIX
CALL SIMH(Z, Z1, 12, NINV1)
DO 24 1=1,12
DO 24 J=1,12
B(I,J,IN)=Z1(I,J)
24 CONTINUE

DIRECT MATRICES ARE STORED IN A(I,J,IN) FOR EACH RADIUS IN
INVERTED MATRICES ARE STORED IN B(I,J,IN) FOR EACH RADIUS IN

RADM=RAD(NREGS)/FMP(NREGS)
Y=ALPHA(NREGS)
CALL MATRIX
DO 25 1=1,12
DO 25 J=1,12
A(I,J,NREGS)=Z(I,J)
25 CONTINUE

NOW ALL MATRICES ARE PREPARED
PREPARE FIRST STARTER
IN=1
DO 500 1=1,6
DO 500 J=1,12
U(I,J,IN)=0.0
500 CONTINUE

PROPAGATION OF STARTER
NITER=0
1001 DO 1000 IN=2,NREGS
MATRIX MULTIPLICATION A*U=U1
DO 502 1=1,12
DO 502 J=1,6
1000
PSMAI N

U1(I,J) = 
DO 502 IS = 1,12

U1(I,J) = U1(I,J) + A(I,IS,IN-1)*U(IS,J,IN-1)

MATRIX MULTIPLICATION B*U1 = U

DO 503 I = 1,12
DO 503 J = 1,6
U(I,J,IN) = 1.0
DO 503 IS = 1,12
U1(I,J,IN) = U1(I,J,IN) + B(I,IS,IN-1)*U(IS,J,IN-1)
1003 CONTINUE

IF(I1) 504,505,504

MATRIX MULTIPLICATION A*U = U1

DO 504 I = 1,12
DO 504 J = 1,6
U1(I,J) = U1(I,J)
DO 504 IS = 1,12
U1(I,J,IN) = U1(I,J,IN) + A(I,IS,NREGS)*U(IS,J,NREGS)

N=1
DO 510 K = 1,3
I = I + K
DO 510 KK = 1,K
DO 510 J = 1,6
VAN(M,J) = U1(I,J)
N = N + 1
510 I = I + 1
GO TO 400

N=1
DO 508 K = 1,6
DO 508 J = 1,6
VAN(N,J) = U1(I,J,NREGS)
N = N + 1
508 I = I + 2

VAN(I,J) IS A 6*6 MATRIX OBTAINED SELECTING FROM THE RESPONSE 6 ROWS OF INTEREST FOR THE OUTER BOUNDARY CONDITIONS

CHECK IF VAN(I,J) IS UNIT MATRIX

DO 401 I = 1,6
DO 401 J = 1,6
IF(I-J)402,403,402

RESP(I,J) = VAN(I,J) - 1.0
GO TO 401

RESP(I,J) = VAN(I,J)
RMAX = ABSF(RESP(I,J))
DO 404 I = 1,6
DO 404 J = 1,6
IF(RMAX = RESP(I,J)) 405,404,404
RMAX = RESP(I,J)
CONTINUE

IF(NX)5G02,5G02,5G01
5G01 CONTINUE
DO 5099 KK=1,NREGS
WRITE OUTPUT TAPE N6,2.00,3, KK
2503 FORMAT(I9,17H U MATRIX REGION I5//)
WRITE OUTPUT TAPE N6,2001, ((U(I,J, KK)), J=1,6), I=1,6)
5G00 WRITE OUTPUT TAPE N6,2.00,1
2500 FORMAT(1HC//)
WRITE OUTPUT TAPE N6,2.00,1, ((VAN(I,J), J=1,6), I=1,6)
WRITE OUTPUT TAPE N6,2.00,0, RMAX, EPS2
2504 FORMAT(7H2 RMAX= E12.5//)
WRITE OUTPUT TAPE N6,2.00,2
2502 FORMAT(1H1//)
5G52 CONTINUE

NITER=NITER+1
IF(RMAX-EPS2)409,4,09,407
IF(NITER-1C)4C 8,4C 8,409
CALL SIMH(VAN,RESP,6,NINV2)

MULTIPLICATION OF OLD STARTER BY THE INVERTED VAN(I,J) AND
REPETITION OF THE CYCLE

DO 450 I=1,12
DO 450 J=1,6
U1(I,J)=C.0
DO 450 IS=1,6
U1(I,J)=U1(I,J)+U(I,IS,1)*RESP(IS,J)
DO 451 I=1,12
DO 451 J=1,6
U1(I,J,1)=U1(I,J)
GO TO 1001

WRITE OUTPUT TAPE N6,410,NITER,RMAX,EPS2
410 FORMAT(1//I9// 1 MAXIMUM OFF-DIAGONAL TERM OF UNIT MATRIX AFTER
1 13H ITERATIONS RMAX = E12.5,13H WHILE EPS2 = E12.5///)

PROPAGATION OF A ZERO STARTER TO GET THE EFFECT OF THE SOURCES

DO 550 I=1,12
DO 550 IN=2,NREGS
DO 551 I=1,12
V(I,IN)=0.0
DO 551 IS=1,12
V(I,IN)=V(I,IN)+B(I,IS,IN-1)*V1(IS)
CONTINUE
PSMAIN

IF(12)554,555,554
DO 560 I=1,12
V(I)=0.0
DO 560 IS=1,12
V(I)=V(I)+A(I,IS,NREGS)*V(IS,NREGS)
DO 650 I=1,12
C(I,IN)=C(I,IN)+U(I,IS,IN)*RV(IS)
DO 650 1=1,12
C(I,IN)=C(I,IN)+V(I,IN)
CONTINUE
IF(10)7000,7001,7000
WRITE OUTPUT TAPE N6,7000
FORMAT(21H1 CONTROL ON ACCURACY /33H COEFFICIENTS IN THE LAST
LION /47H THE COEFFICIENTS MARKED WITH * SHOULD VANISH //)
WRITE OUTPUT TAPE N6,7003,(C(I,NREGS),I=1,12)
FORMAT(E13.5,2H */E13.5)
CONTINUE
IF(10)652,653,652
DO 654 I=1,12
C(I,NREGS)=0.0
CONTINUE
WRITE OUTPUT TAPE N6,3000,(RV(I),I=1,6)
FORMAT(2H EFFECT OF THE SOURCES ON A ZERO STARTER //6E20.5//
WRITE OUTPUT TAPE N6,3001
FORMAT(31HC COEFFICIENTS FOR EACH REGION //)
WRITE OUTPUT TAPE N6,3002,(C(I,IN),IN=1,NREGS)
GO TO 3004
WRITE OUTPUT TAPE N6,2000,(C(I,IN),IN=1,J)
WRITE OUTPUT TAPE N6,2000,(C(I,IN),IN=1,J)
CONTINUE
PSMAIN
GO TO 999
END(1,0,0,0,0,1,0,0,0,0,0,0,0,0,0)
ATRI

SUBROUTINE MATRIX

DIMENSION A(12, 12, 10), B(12, 12, 10), U(12, 6, 10),
1 V(12, 12), U(12, 6), RESP(12, 12), V(12, 10),
3 S(12, 10), V(12), C(12, 10),
3 RAD(1), FMP(1), SOURCE(10), ALPHA(1), AMASS(10), APRIN1(10)

DIMENSION Z(12, 10), Z(12, 12)

DIMENSION Y(1), RADM(1)

COMMON A, B, U, V, U, V, S, C, RAD, FMP, SOURCE, ALPHA, AMASS,
1 Rn, IN, TR, IN, IQ, N6

COMMON Z(1), Y, RADM, NREGS

COMMON NX

COMMON EPS1, EPS2, NINV1, NINV2

DIMENSION G1(13), G2(3), G3(3), G4(3), G5(3), T(4)

DIMENSION ARG(6), BS1(6), BS2(6), HSK1(6), HSK3(6), HSK5(6)

DIMENSION AA(1), BB(1), C(1), D(1), E(1), F(1), G(1), H(1), W(1),
1 X(1)

COMMON ARG, BS1, BS2, HSK1, HSK3, HSK5, HS1, HS2, HS1

CALCULATION OF GAMMA COEFFICIENTS

NITER = 0

T(1) = 1

T(2) = 0, 56

T(4) = 11.76 + 55.44 * Y

BB = T(4)

T(1) = T(2)

T(2) = T(3)

T(3) = T(4)

T(4) = 11.76 * T(3) + 9.24 * Y * T(3) - 15.4 * T(2) - 47.6 * Y * T(2) + 46.2 * Y * T(1)

T(1) = T(4) / T(3)

CC = T(1) / H6

D = ABSF(CC)

BB = T(1)

NITER = NITER + 1

IF (CC < EPS1) 13, 55

55 IF (NITER = 50) 2, 2, 56

56 WRITE OUTPUT TAPE N6, 57

57 FORMAT(47HO CONVERGENCE NOT ACHIEVED AFTER 50 ITERATIONS //)

WRITE OUTPUT TAPE N6, 58, D, EPS1

58 FORMAT(5H D = E12.5 ; 12H WHILE EPS1 = E12.5 //)

D = T(1) - 11.76 - 9.24 * Y

E = 15.4 + 47.6 * Y + D * T(1)

F = D * 4.0 * E

G = SQRTF(F)

H = 0, 5, 5

T(2) = E / T(2)

DO 50 I = 1, 1, 3

UU = T(1)

T(1) = SQRTF(UU)

E = UU * UU
MATRIX

\[ F = E \cdot T(1) \]
\[ H = U \cdot T(1) \]
\[ AA = -Y \cdot T(1) \]
\[ BB = -0.324675324 \cdot E - 3.3913181 \cdot UU - 0.3 \cdot UU \cdot Y + 1.54545454 \cdot Y \]
\[ CC = -0.41322314 \cdot F + 0.3618182 \cdot Y \cdot H + 0.31373737 \cdot E - 1.466742 \cdot Y \cdot U \]
\[ WW = -0.324774 \cdot E \cdot UU - 0.3145134 \cdot E - 1.2811563 \cdot Y \cdot E + 1.3754444 \cdot Y \cdot UU \]
\[ XX = -0.1395338 \cdot E \cdot T(1) + 0.141114 \cdot 2 \cdot E - 0.120305276 \cdot E \cdot F - 1.45774631 \]

\[ IH \]
\[ G1(1) = AA \]
\[ G2(1) = BB \]
\[ G3(1) = CC \]
\[ G4(1) = W \]
\[ G5(1) = X \]

CONTINUE

ARGUMENTS OF BESSEL FUNCTIONS

\[ ARG(1) = RADM \cdot T(1) \]
\[ ARG(2) = RADM \cdot T(2) \]
\[ ARG(3) = RADM \cdot T(3) \]
\[ ARG(4) = 2.99101931 \cdot RADM \]
\[ ARG(5) = 0.459737373 \cdot RADM \]
\[ ARG(6) = 3.51662479 \cdot RADM \]

DO 1000 I = 1, 6

BSI1(I) = BI0F(ARG(I))
BSI2(I) = BI1F(ARG(I))
BSK1(I) = BK0F(ARG(I))
BSK2(I) = BK1F(ARG(I))
BSK3(I) = BK2F(ARG(I))
BSK4(I) = BK3F(ARG(I))
BSK5(I) = BK4F(ARG(I))
AA = 0.1 \cdot BSI1(I)
BB = 0.5
SS = 2.0
CC = 0.5 \cdot SS \cdot AA / ARG(I) + BB
BB = AA
AA = CC
SS = SS - 1.0
P2 = AA
P3 = BB

DO 30 J = 1, 11

CC = 2.0 \cdot SS \cdot AA / ARG(I) + BB
BB = AA
AA = CC
SS = SS - 1.0
CC = 0.5 \cdot AA / ARG(I) + BB
CC = RS11(I) / CC
BS112(I) = AA \cdot CC
BS113(I) = BB \cdot CC
BS114(I) = P2 \cdot CC
BS115(I) = P3 \cdot CC

CONTINUE

MATRIX ASSEMBLY
<table>
<thead>
<tr>
<th>i</th>
<th>j</th>
<th>z(i, j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2</td>
<td>375 \times G4(1)</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>375 \times G4(2)</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>375 \times G4(3)</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>2335 \times 315</td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td>0.9112907</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>0.9912907</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
<td>12.0</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td>12.0</td>
</tr>
</tbody>
</table>

...
SIMH

SUBROUTINE SIMH(A, AT, N, ITER)

MATRIX INVERSION SUBROUTINE

A(I,J)  MATRIX TO BE INVERTED
AT(I,J)  INVERTED MATRIX
N  MATRIX ORDER
ITER  NO. OF ITERATIONS IN THE INVERSION

DIMENSION A(12,12), AT(12,12)

DO 4 I = 1, N
  DO 4 J = 1, N
    AT(I,J) = A(J,I)
  4 CONTINUE

DET = 1.
DO 80 CI = 1, ITER
  C = 0.
  DO 10 K = 1, N
    C = C + A(I,K) * AT(K,I)
  10 CONTINUE
  IF(C)200,555,200
  DET = DET * C
  DO 300 J = 1, N
    AT(J,I) = AT(J,I) / C
  300 CONTINUE
  DO 700 J = 1, N
    IF(J-1)400,700,400
    H = 0.
    DO 500 K = 1, N
      H = H + A(I,K) * AT(K,J)
    500 CONTINUE
    AT(K,J) = AT(K,J) - H * AT(K,I)
  700 CONTINUE
  800 CONTINUE

RETURN

WRITE OUTPUT TAPE 6,1000
FORMAT(25H1) MATRIX ILL CONDITIONED //)
CALL EXIT
END
SUBROUTINE AIRGAP
DIMENSION A(12,12,13), B(12,12,13), U(12,6,10),
1 V(12,12), RES(12,12), Y(12,12),
2 R(12,6), V(12,12), S(12,12), C(12,12),
3 RAD(1), EPS(1), SOURCE(1), ALPH(1), AMASS(1), APRIN(1)
DIMENSION Z(12,12), T(12,12)
DIMENSION Y(1), RADM(1)
DIMENSION EPS1(1), EPS2(1)
COMMON A, H, U, V, RV, VI, S, C, RAD, EPS, SOURCE, ALPH, AMASS,
1 INR, IR, IRS, IN, VN, NR
COMMON Z, T, Y, RAD, IMGS,
COMMON IN,
COMMON EPS1, EPS2, VINV1, VINV2
DIMENSION AA(1), BB(1), PI(1), CC(1), D(1), E(1), G(1), H(1), UU(1), VV(1)
1 F(1), H3(1), Z(1)
PI=3.1415926535
AA=RAD(IN)*3
HH=RAD(IN)*5
DO 13 I=1,12
DO 13 J=1,12
13 Z(I,J,1)=AA
Z(I,1,2)=RAD(IN)
Z(1,2,1)=RAD(IN)
Z(I,2,2)=12.*AA
Z(1,2,2)=2.*AA
Z(1,5,1)=AA
Z(I,5,2)=12.*AA
Z(1,5,2)=2.*AA
Z(5,5,1)=AA
Z(5,5,2)=AA
Z(5,2,1)=12.*AA
Z(5,2,2)=2.*AA
Z(6,6,1)=6.*AA
Z(6,6,2)=6.*AA
Z(7,1,1)=PI/4.*5
Z(7,2,1)=PI/2.*4
Z(7,3,1)=156.25*PI
Z(7,4,1)=375.125*PI
Z(7,7,1)=3.351625*PI
Z(7,9,1)=3319.625*PI
Z(7,9,2)=3319.625*PI
Z(8,1,1)=2468.125*PI
Z(9,2,1)=2468.125*PI
Z(9,3,1)=19512.5*PI
Z(9,5,1)=19512.5*PI
Z(3,7,1)=60.37563759*PI
Z(8,9,1)=33.41746166*PI
Z(9,1,1)=33.8959375*PI
Z(9,3,1)=33.8959375*PI
Z(9,4,1)=33.305175791*PI
| Z(5,1) = F | Z(5,2) = -12 * F | Z(5,5) = -28 * F |
| Z(5,10) = 12 * F | Z(6,12) = 27 |
| Z(6,2) = 1206 * Z | Z(6,5) = -336 * Z |
| Z(6,10) = 48 * Z | Z(6,6) = -168 * Z |
| Z(6,11) = 6 * Z |

\[ \begin{align*}
Z(7,2) &= EE/AA \\
Z(7,3) &= (E * P1) / 32 \\
Z(7,4) &= -((15 * P1) / 64) * EE \\
Z(7,6) &= (7 / 45) * DD \\
Z(7,7) &= (9 * P1) / 256 \\
Z(7,9) &= -(P1 / 2048) * (15 - 40 * G + 24 * UU) \\
Z(7,11) &= (11 / 128) * DD \\
Z(7,12) &= (11 / 4288) * CD * (1 - (8 / 3) * G) \\
Z(8,11) &= -((3 * P1) / 32) \\
Z(8,13) &= -(105 * P1) / 128 \\
Z(8,4) &= ((15 * P1) / 256) * EE \\
Z(8,5) &= EE/AA \\
Z(9,2) &= -((1 / 15) * DD) \\
Z(9,7) &= (2079 * P1) / 4096 \\
Z(9,9) &= -(513 * P1) / 4096 * EE \\
Z(9,11) &= (9 / 21) * DD \\
Z(9,12) &= -(11 / 630) * DD * (1 - (8 / 3) * G) \\
Z(9,3) &= -(15 * P1) / 256 \\
Z(9,4) &= (75 * P1) / 8192 * EE \\
Z(9,7) &= (42255 * P1) / 32768 \\
Z(9,8) &= -(1455 * P1) / 32768 * EE \\
Z(9,9) &= -(17 * P1) / 262144 * (15 - 40 * G + 24 * UU) \\
Z(9,10) &= EE/AA \\
Z(9,11) &= (1 / 28) * DD \\
Z(9,12) &= (1 / 2520) * DD * (1 - (8 / 3) * G) \\
Z(10,1) &= 15 * P1 / 16 \\
Z(10,2) &= 12 * A1 / 4 \\
Z(10,3) &= (75 * P1) / 64 \\
Z(10,4) &= (375 * P1) / 128 * (1 / 3 - (14 / 15) * G) \\
Z(10,5) &= (2 * A1) / 4 \\
Z(10,6) &= (1 + (1 / 3)) * G - (10 / 3) * UU) / 16 \\
Z(10,7) &= (4095 * P1) / 2048 \\
Z(10,8) &= -(1225 * P1) / 2048 * (1 / 3 - (14 / 15) * G) \\
Z(10,9) &= (15 * P1) / 6384 * (35 - 392 * G + 312 * UU) \\
Z(10,12) &= -(11 / 945) * BB * (1 - (17 * G) + 16 * UU) \\
Z(11,1) &= 105 * P1 / 64 \\
Z(11,2) &= 12 * A1 / 4 \\
Z(11,3) &= (125 * P1) / 128 \\
Z(11,4) &= (175 * P1) / 2048 \\
Z(11,5) &= (28 * A1) / 4 \\
Z(11,7) &= (82215 * P1) / 8192 \\
Z(11,8) &= ((2289 * P1) / 8192) * (5 - 14 * G) 
\end{align*} \]
\[ z(11, 2) = -((21 \cdot P1) / 65536) \cdot (35 \cdot 398 \cdot G + 312 \cdot UU) \]
\[ z(11, 13) = -((12 \cdot A1 \cdot E) / A4) \]
\[ z(11, 11) = (CC \cdot D \cdot (1 \cdot (4 \cdot 3) \cdot G \cdot (15 \cdot 3) \cdot UU)) / H \]
\[ z(11, 12) = (17 / 132) \cdot (1 - 17 \cdot G + 10 \cdot UU) \]
\[ z(12, 1) = -((94 \cdot P1) / 32) \]
\[ z(12, 2) = (64 / VV) \cdot ((2 \cdot G + UU) \cdot CC \cdot D \cdot (2 \cdot (5 \cdot 3) \cdot G - (1 \cdot 3) \cdot UU) \]
\[ z(12, 3) = (123625 \cdot P1) / 512 \]
\[ z(12, 4) = -((158 \cdot P1) / 128) \cdot (21 \cdot 46 \cdot G) \]
\[ z(12, 5) = (161 / VV) \cdot ((2 \cdot G + UU) \cdot CC \cdot D \cdot (2 \cdot (5 \cdot 3) \cdot G - (1 \cdot 3) \cdot UU) \]
\[ z(12, 6) = -((54 / VV) \cdot (3 \cdot (1 - C) \cdot CC \cdot D \cdot (3 \cdot G - 2 \cdot H \cdot H)) \]
\[ z(12, 7) = -((76545 \cdot P1) / 4192) \]
\[ z(12, 8) = ((567 \cdot P1) / 4096) \cdot (21 \cdot 46 \cdot G) \]
\[ z(12, 9) = -((567 \cdot P1) / 4096) \cdot (63 \cdot 592 \cdot G + 664 \cdot UU) \]
\[ z(12, 10) = (21 / VV) \cdot ((2 \cdot G + UU) \cdot CC \cdot D \cdot (2 \cdot (5 \cdot 3) \cdot G - (1 \cdot 3) \cdot UU) \]
\[ z(12, 11) = (2 / VV) \cdot ((3 \cdot (1 - C) \cdot CC \cdot D \cdot (3 \cdot G - 2 \cdot H \cdot H)) \]
\[ z(12, 12) = (CC \cdot D \cdot (1 \cdot (2 \cdot 3) \cdot G + (2 \cdot 15) \cdot UU + (86 \cdot 15) \cdot H \cdot H - (12 \cdot 1 \cdot 1 \cdot UU \cdot UU) / VV) \]

8201 FORMAT(///(eE26.5))
8301 FORMAT(///19H MATRICES///19H INNER RADIUS)
8403 FORMAT(///19H OUTER RADIUS)
8203 WRITE OUTPUT TAPE 6, 8201
WRITE OUTPUT TAPE 6, 8201, ((0(I,J,N), J=1,6), I=1,12)
WRITE OUTPUT TAPE 6, 8201, ((8(I,J,N), J=7,12), I=1,12)
WRITE OUTPUT TAPE 6, 8201
WRITE OUTPUT TAPE 6, 8200, ((Z(I,J), J=1,6), I=1,12)
WRITE OUTPUT TAPE 6, 8200, ((Z(I,J), J=7,12), I=1,12)
8223 CALL SIMH(21, 12, 11, MV1)
DO 22 J = 1, 12
DO 33 I = 1, 12
22 A(I, J, IN+1) = 71(I, J)
RETURN
END(1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
```
PRINT
SUBROUTINE PRINT
  FLUX(IN) MEAN FLUX REGION IN
  VOLUME(IN) VOLUME REGION IN
  S(1,IN) SOURCE VECTOR REGION IN
  ASFLUX ASYMPOTIC FLUX
  ASCURR ASYMPOTIC CURRENT
  DIMENSION A(12,12,10), B(12,12,12), U(12,6,1),
               V(12,1), S(12,16), C(12,16),
  DIMENSION Z(12,12), ZI(12,12)
  DIMENSION Y(1), RAD(1)
  COMMON A,B,V,S,C,RAD,FMP,SOURCE,ALPHA,AMASS,
               RIN,IR,IRIN,IN,IN6
  COMMON Z, ZI, RAD, REGS
  COMMON NX
  COMMON EPS1,EPS2,NINV1,NINV2
  DIMENSION DELTA(1), FLUX(1), RPRIN(1)
  DIMENSION X(1), Y(1), Z(1), V(1), ASFLUX(1), ASCURR(1),
  DIMENSION ARG(6), HS1(16), HS1(16), BSK4(6), BSK1(6), BSK2(6), BSK3(6)
  DIMENSION ARG, HS1, HS1, BSK4, BSK1, BSK2, BSK3, BSK4, HS1, HS1, BSK1, BSK2, BSK3, BSK4
  DIMENSION VOLUME(10)

  IF(IX=0)

  N=ABS(10)

  WRITE OUTPUT TAPE N6,1
  WRITE OUTPUT TAPE N6,2
  WRITE OUTPUT TAPE N6,22

  FORMAT (11H1) RADIUS NX,4HFLUX,8X,15HASYMPOTIC FLUX,2CH ASYMPT

  IF (N=15)

  IF (N=16)

  IF (N=17)

  IF (N=18)

  IF (N=19)

  IF (N=20)

  IF (N=21)

  IF (N=22)

  IF (N=23)

  IF (N=24)

  IF (N=25)

  IF (N=26)

  IF (N=27)

  IF (N=28)

  IF (N=29)

  IF (N=30)

  CONTINUE
```

PRINT
C MATRIX MULTIPLICATION AND ADDITION V1=A*C+S
D DO 13 I=1,12
D V1(I)=0.0
D DO 13 IS=1,12
D 13 V1(I)=V1(I)+Z(I,IS)*C(IS,IN)
D DO 14 I=1,12
D V1(I)=V1(I)+S(I,IN)
D FLUX=V1(I)
D ASFLUX=Z(1,5)*C(5,IN)+Z(1,6)*C(6,IN)+S(1,IN)
D ASCURR=Z(2,5)*C(5,IN)+Z(2,6)*C(6,IN)
D WRITE OUTPUT TAPE N6,2,RPRIN,FLUX,ASFLUX,ASCURR
D IF(IP-N)52,53,52
D 52 CONTINUE
D 11 RPRIN=RPRIN-Delta
D 10 CONTINUE
D IF(IP)1002,1003,1002
D 1002 WRITE OUTPUT TAPE N6,103G
D 1006 FORMAT(2H) CONTROL ON ACCURACY/29H FLUX AT REFLECTIVE BOUNDARY
D 1/45H THE COMPONENTS MARKED WITH * SHOULD VANISH
D 1003 CONTINUE
D RETURN
D 50 XN=CC1,IN)*BSI11(1)-C(2,IN)*BSK11(1)
D XN=RPRIN=N*XN+RADM/ARG(1)
D UN=CC3,IN)*BSI11(2)-C(4,IN)*BSK12(2)
D UN=RPRIN=N*UN+RADM/ARG(2)
D VN=CC5,IN)*BSI11(3)-C(6,IN)*BSK13(3)
D VN=RPRIN=N*VN+RADM/ARG(3)
D XN=XN+UN+VN
D YN=RAD(IN)**2-RPRIN**2
D YN=2.0*FPRIN1)/YN
D GO TO 51
D 53 WN=CC1,IN)*BSI11(1)-C(2,IN)*BSK11(1)
D WN=RPRIN=N*WN+RADM/ARG(1)
D UN=CC3,IN)*BSI11(2)-C(4,IN)*BSK12(2)
D UN=RPRIN=N*UN+RADM/ARG(2)
D VN=CC5,IN)*BSI11(3)-C(6,IN)*BSK13(3)
D WN=N*WN+UN+WN
D FLUXM(IN)=YN*WN+5(1,IN)
D VOLUME(IN)=3.1415926535*(RAD(IN)*RAD(IN)-RAD(IN-1)*RAD(IN-1))
D WRITE OUTPUT TAPE N6,60,IN,FLUXM(IN),VOLUME(IN)
D 60 FORMAT (RH REGION 13,13H MEAN FLUX = E12.5,4X,9HVOLUME = E12.5
D GO TO 52
D END
The double precision Bessel function subroutines are not included in this listing.

<table>
<thead>
<tr>
<th>REGION</th>
<th>OUTER RADIUS</th>
<th>MEAN FREE PATH</th>
<th>SOURCE</th>
<th>NO. ABSORPTIONS/COLLISION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.12700E+01</td>
<td>0.13825E+01</td>
<td>0.76</td>
<td>0.46710E-00</td>
</tr>
<tr>
<td>2</td>
<td>0.34925E+01</td>
<td>0.10000E+01</td>
<td>0.62</td>
<td>0.10880E-02</td>
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<tr>
<td>3</td>
<td>0.11285E+02</td>
<td>0.24619E+01</td>
<td>0.82</td>
<td>0.10880E-02</td>
</tr>
<tr>
<td>RADIUS</td>
<td>FLUX</td>
<td>ASYMPTOTIC FLUX</td>
<td>ASYMPTOTIC CURRENT</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>-----------------</td>
<td>--------------------</td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.2540</td>
<td>0.0.15028335E+03</td>
<td>-0.085674568E+01</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>0.6080</td>
<td>0.16161099E+03</td>
<td>0.20504069E+03</td>
<td></td>
</tr>
<tr>
<td>0.76</td>
<td>0.7620</td>
<td>0.12261283E+03</td>
<td>0.21278194E+03</td>
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<tr>
<td>1.01</td>
<td>0.1600</td>
<td>0.19057240E+03</td>
<td>0.22886651E+03</td>
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</tr>
<tr>
<td>1.27</td>
<td>0.2000</td>
<td>0.22147449E+03</td>
<td>0.2554876E+03</td>
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</tbody>
</table>

**REGION 1 MEAN FLUX = 0.18320E+03 VOLUME = 0.50671E+01**

<table>
<thead>
<tr>
<th>AIR GAP</th>
<th>FLUX</th>
<th>ASYMPTOTIC FLUX</th>
<th>ASYMPTOTIC CURRENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.49</td>
<td>0.2618</td>
<td>0.26184095E+03</td>
<td>0.26322768E+03</td>
</tr>
<tr>
<td>4.79</td>
<td>0.2809</td>
<td>0.28099620E+03</td>
<td>0.26162069E+03</td>
</tr>
<tr>
<td>6.09</td>
<td>0.2937</td>
<td>0.29374386E+03</td>
<td>0.29398270E+03</td>
</tr>
<tr>
<td>7.39</td>
<td>0.3023</td>
<td>0.30230287E+03</td>
<td>0.3023582E+03</td>
</tr>
<tr>
<td>8.69</td>
<td>0.3078</td>
<td>0.30782378E+03</td>
<td>0.30767512E+03</td>
</tr>
<tr>
<td>9.98</td>
<td>0.3164</td>
<td>0.3164599E+03</td>
<td>0.31362385E+03</td>
</tr>
<tr>
<td>11.28</td>
<td>0.3120</td>
<td>0.31207167E+03</td>
<td>0.3115457E+03</td>
</tr>
</tbody>
</table>

**REGION 3 MEAN FLUX = 0.30154E+03 VOLUME = 0.36178E+03**

**CONTROL ON ACCURACY**

**FLUX AT REFLECTIVE BOUNDARY**
THE COMPONENTS MARKED WITH * SHOULD VANISH

<table>
<thead>
<tr>
<th>FLUX</th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>0.31207E+03</td>
<td>0.23568E-10</td>
<td>0.40619E-00</td>
<td>0.22127E+01</td>
</tr>
<tr>
<td>0.59039E-06</td>
<td>0.31665E-05</td>
<td>0.30952E-00</td>
<td>0.79679E-06</td>
</tr>
<tr>
<td>0.94697E-05</td>
<td>0.17324E-03</td>
<td>0.84697E-05</td>
<td>0.17324E-03</td>
</tr>
</tbody>
</table>
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Alfred Nobel
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