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THE KINETIC THEORY OF A FAST REACTOR PERIODICALLY PULSED BY REACTIVITY VARIATIONS

by

G. BLÄSSER, R. MISENTA and V. RAIEVSKI

1964

Joint Nuclear Research Center
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The authors' names are listed in alphabetical order.

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The inhour equation obtained by the second method shows the influence on the inhour equation of the time intervals during which the reactor is subcritical. For the cases where the influence of these time intervals is negligible the equivalence between the two methods of derivations is shown.

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CONTENTS

I — INTRODUCTION 5

II — DERIVATIONS OF KINETIC EQUATIONS OF A PERIODICALLY PULSED FAST REACTOR 7

A) Derivation of kinetic equations for the mean values of the neutron density and the delayed emitters 7

B) Derivation of the kinetic equations for the mean neutron density and the density of the delayed emitters 9

III — EQUIVALENCE OF THE TWO DERIVATIONS 12

IV — MULTIPLICATION FACTOR, STATIONARITY CONDITION, AND IN-HOUR EQUATION OF A PERIODICALLY PULSED REACTOR. . . . 14

KEY TO THE FIGURES

Fig. 1 : The multiplication factor $\varkappa$ for the mean power as function of ε_m, and the reactivity R as function of the difference $(\varepsilon_m - \varepsilon_{mo})$ for the frequencies 10 and 100 sec^{-1}.

Fig. 2 : The roots ω for reactivities R between +1 and —1 of the inhour equations

$$R = \Sigma \frac{\beta_i}{\beta} \frac{\omega}{\omega + \lambda_i}$$

and $$R = \Sigma \frac{\beta_i}{\beta} \left\{ \frac{\omega}{\omega + \lambda_i} - \frac{\lambda_i}{\omega + \lambda_i} \left(\frac{\omega T}{c^{\omega T} - 1} - 1 \right) \right\}$$

for $T = 1$ sec.

Fig. 3 : The increase of the mean power of a periodically pulsed fast reactor for the multiplitation factor $\varkappa = 2$ calculated with the two different forms of the inhour equation given in fig. 2 for the two periods $T = 0.1$ and 1.0 sec.

THE KINETIC THEORY OF A FAST REACTOR PERIODICALLY PULSED BY REACTIVITY VARIATIONS

I — INTRODUCTION

The power of a fast reactor can be pulsed by increasing the multiplication factor above the prompt critical value for a short time interval. In a periodically pulsed fast reactor the multiplication factor is above the prompt critical value only for a short time interval, and is subcritical during the largest part of the period. The first periodically pulsed reactor has been constructed and operated in the USSR [1]. Bondarenko and Staviskii [2] have established the kinetic theory of a periodically pulsed reactor for the stationary state, i.e. for the state in which the reactor produces power pulses with amplitudes independent of time, and for the nearly stationary state.

These authors have derived a condition for the stationary state in establishing the kinetic equations for the time variation of the mean power and by determining the condition for which the mean power does not vary. According to this condition the stationary state of a pulsed reactor can only be realized for one pulse frequency by one single value of the prompt critical reactivity ε_{mo}.

I-1 $$\frac{M(\varepsilon_{mo})}{T}\beta = 1$$

For a chosen reactor the function M is proportional to the energy developed during one power pulse and depends on the reactivity ε_m. In the case the prompt critical reactivity deviates from the equilibrium value ε_{mo} the time variation of the mean power is described by a system of equations

I-2.1 $$\bar{n}(t) = \tau \frac{M(\varepsilon_m)}{T} \sum_i \lambda_i c_i(t)$$

I-2.2 $$\frac{dc_i}{dt} = -\lambda_i c_i(t) + \frac{\beta_i}{\tau}\bar{n}(t)$$

where $\bar{n}(t)$ and $\bar{c}_i(t)$ are values of the density of the neutrons and the density of the delayed neutron emitters averaged over one period.

τ is the mean lifetime of the prompt neutrons, β_i, λ_i, and β have the usual signification. Bondarenko and Staviskii have remarked that the time variation of the mean power of a periodically pulsed fast reactor is comparable to the time variation of the power of an ordinary reactor if a fictive value β_{pulsed} is introduced for the fraction of the delayed neutrons.

Stiévenart has shown in 1962 [1] that the assumption of exponential solutions for the equations I-2 for $\bar{n}$ and $\bar{c}_i$ give for the exponents the equation

$$\frac{M(\varepsilon_m) - M(\varepsilon_{mo})}{M(\varepsilon_m)} = \Sigma \frac{\beta_i}{\beta} \frac{\omega}{\omega + \lambda_i}$$

The ratio $M(\varepsilon_m)/M(\varepsilon_{mo})$ is the ratio between the energy developed in one pulse for the reactivity ε_m to the energy developed in one pulse for the reacticity ε_{mo} which corresponds to the equilibrium value. If the variation of M is reduced to a variation of ε_m by developing the function M around the stationary value ε_m and by defining the β_{pulsed} value, the inhour equation for a periodically pulsed reactor is similar to the equation of an ordinary reactor.

$$\beta_{\text{pulsed}} = \frac{M(\varepsilon_m)}{\left(\frac{\partial M}{\partial \varepsilon_m}\right)_{\varepsilon_{mo}}} \qquad \frac{\Delta\varepsilon_m}{\beta_{\text{pulsed}}} = \Sigma \frac{\beta_i}{\beta} \frac{\omega}{\omega + \lambda_i}$$

The value β_{pulsed} is not a constant of the reactor but depends on certain parameters of the reactor, and varies with the prompt reactivity and the pulse frequency.

A new method of derivation (Chapter II, Part A) of the inhour equation of a periodically pulsed fast reactor is given by writing the usual kinetic equation for the power averaged over one period and defining a mean multiplication factor $\bar{k}$ which is the ratio :

$$\frac{\text{number of neutrons produced during one period}}{\text{number of neutrons absorbed during one period}}$$

The derivation in Chapter II.A uses like the derivations by Bondarenko and Staviskii and by Stiévenart the assumption that the time variation of the density of the delayed neutron emitters is constant during one period and can be replaced by the mean value.

Taking into account the formation of delayed neutron emitters during the power pulse, and the decay during the time interval between two power pulses, an inhour equation is derived in Chapter II Part B by using the property of a periodically pulsed fast reactor that the total number of fissions during one pulse and the power averaged over one period is proportional to the source strength of the delayed emitters at the beginning of the pulse (Chapter II, Part B). By this proportionality a multiplication factor $\varkappa$ for the mean power is introduced which is the ratio

$$\frac{\text{number of delayed neutron emitters produced during one period}}{\text{number of delayed neutron emitters decayed during one period}}$$

The roots ω of this inhour equation show the influence of the time interval when the reactor is subcritical. For small deviations from the stationarily pulsed state and for pulse periods in the order of 0.1 sec or smaller, the exact inhour equation gives the inhour equation which has been derived by assuming that the density of the delayed emitters is constant during one period.

The equivalence of the two methods of derivation is shown in Chapter III. The result of numerical calculations of the multiplication factor and the roots ω of the two inhour equations are given in Chapter IV.

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[1] Internal report of Belgonucléaire, Brussels.

II — DERIVATION OF THE KINETIC EQUATIONS OF A PERIODICALLY PULSED FAST REACTOR

II-A. — Derivations of the kinetic equations for the mean values of the neutron density and of the delayed emitters (1)

The kinetic behaviour of a periodically pulsed reactor — like that of any other reactor — is described by the equations

II-1.1
$$\frac{dn}{dt} = \frac{k(t)\,(1 - \beta) - 1}{\tau}\, n + \sum_i \lambda_i c_i$$

II-1.2
$$\frac{dc_i}{dt} = - \lambda_i c_i + \beta_i \frac{k}{\tau} n$$

Now the multiplication factor is a periodic function of time

$$k(t) = k(t - \mathrm{T})$$

By integrating the equations over one period T, permuting the order of integration and differentiation and by dividing by T, the following kinetic equations for the mean values are obtained :

II-2.1
$$\frac{d\bar{n}}{dt} = \frac{\bar{k}(t)(1 - \beta) - 1}{\tau}\, \bar{n} + \Sigma \lambda_i \bar{c}_i$$

II-2.2
$$\frac{d\bar{c}_i}{dt} = - \lambda_i \bar{c}_i + \beta_i \frac{\bar{k}}{\tau} \bar{n}$$

Here the mean values are defined by the relations

II-3.1
$$\bar{n}(t) = \frac{1}{\mathrm{T}} \int_{t-\mathrm{T}}^{t} n(s)ds$$

II-3.2
$$\bar{c}_i(t) = \frac{1}{\mathrm{T}} \int_{t-\mathrm{T}}^{t} c_i(s)ds$$

II-3.3
$$\bar{k}(t) = \frac{1}{\mathrm{T}} \int_{t-\mathrm{T}}^{t} k(s)\, \frac{n(s)}{\bar{n}(t)}\, ds$$

With the definition

II-3.4
$$\bar{\rho} = \frac{\bar{k} - 1}{\bar{k}}$$

and the kinetic equations for the mean values the inhour equation for the mean reactivity is obtained :

II-4
$$\bar{\rho} = \frac{\omega\tau}{\bar{k}} + \Sigma \beta_i \frac{\omega}{\omega + \lambda_i}$$

(1) This derivation of the kinetic equations for the mean values was given by V. Raievski in an internal report ISP-202R printed in 1962.

The equations (II-2) can be given another form if one notes that for a fast reactor τ is very small. The integrated power (number of fissions) over one period is given by

(II-5) $$W = \frac{1}{\nu}\frac{1}{\tau}\int_{t-T}^{t} k(s)n(s)ds = \frac{1}{\nu}\frac{\bar{k}\bar{n}}{\tau}T$$

from which it follows that

$$\bar{n} = \frac{1}{\bar{k}}\frac{W}{\nu T}\tau = 0(\tau)$$

Thus in eq. (II-2) we find that all terms are 0 (1) except the term $\frac{d\bar{n}}{dt}$ which is 0 (τ). If, therefore, one neglects the lifetime τ as compared with the time scales encountered in the normal operation of the pulsed fast reactor one obtaines instead of the eq. (II-2.1) the eq.

(II-6.1) $$0 = \frac{\bar{k}(1-\beta)-1}{\tau}\bar{n} + \Sigma\lambda_i\bar{c}_i$$

while the eq. (II-2.2) remains unchanged.

The inhour equation that follows from eq. (II-2.2) and (II-6.1) is

(II-6.2) $$\bar{\rho} = \Sigma\beta_i \frac{\omega}{\omega+\lambda_i}$$

which differs from eq. (II-4) by the absence of the small term $\frac{\omega\tau}{\bar{k}}$ as was to be expected since we neglected terms of order τ.

For the calculation of the mean multiplication factor $\bar{k}(t)$ equation II-1.1 is solved over one period with the approximation that the mean density of the delayed neutron emitters is constant during the period and corresponds to the value at the time (t — T).

II-7.1 $$\frac{dn}{ds} = \frac{\varepsilon(s)}{\tau}n + \Sigma\lambda_i c_i(t-T) \qquad t-T \leqq s \leqq t$$

$\varepsilon(s)$ is defined by the equation

$$\varepsilon(s) = k(s)\,(1-\beta)-1$$

Without assuming a special function for the change of the reactivity the solution of the equation II-7.1 can be written

II-7.3 $$n(s) = \Sigma\lambda_i c_i(t-T)\,\eta(s)$$

II-7.4 $$\eta(s) = \left\{C + \int e^{-\frac{1}{\tau}\int \varepsilon(s)ds}ds\right\}e^{\frac{1}{\tau}\int \varepsilon(s)ds}$$

With the solution of this equation the multiplication factor $\bar{k}$ and the reactivity for the mean power can be written

II-8.1 $$\bar{k}(t) = \frac{1}{T}\int_{t-T}^{t} k(s)\frac{\eta(s)}{\bar{\eta}(t)}ds$$

II-8.2 $$\bar{\rho} = \frac{1}{\bar{k}(t)}\frac{1}{T}\int_{t-T}^{t}[k(s)-1]\frac{\eta(s)}{\bar{\eta}(t)}dt$$

The physical meaning of $\bar{k}$ is given by the ratio

$$\bar{k} = \frac{\text{number of neutrons produced during one period}}{\text{number of neutrons absorbed during one period}}$$

since the numerator of this ratio is given by $\int_{t-T}^{t} \frac{k(s)n(s)}{\tau} ds$ while the denominator is simply $\int_{t-T}^{t} \frac{n(s)}{\tau} ds$.

II-B — Derivation of the kinetic equation for the mean neutron density of the delayed emitters [1]

The kinetic equation for the neutron density II-1.1 is integrated over one period, under the approximations that the density of the delayed emitters during the pulse is constant and equal to the value at the beginning of the pulse, and that in the remaining interval of the period each group of delayed emitters decays with its own decay constant. The number of delayed neutron precursors which have been formed during the power pulse are added to the density which has been present at the beginning of the power pulse, and this sum forms the initial condition for the time interval where the multiplication factor is below 1. For the *p-th* pulse which occurs during the time $p\mathrm{T}-t_0$ to $p\mathrm{T}+t_0$ the following equations are obtained for the time dependence of the neutron density $n(s)$, the number $\Delta c_i(\mathrm{I}_p)$ of precursors formed in each group during the power pulse, and the mean neutron density during a period $\bar{n}(p\mathrm{T})$

II-9.1 $$n(s) = \sum_i \lambda_i c_i(p\mathrm{T} - t_0)\eta(s) \qquad \text{(during the pulse)}$$

II-9.2 $$n(s) = \frac{\tau}{\varepsilon_0} \sum_i \lambda_i \left\{ c_i(p\mathrm{T}_-) + \Delta c_i(\mathrm{I}_p) \right\} e^{-\lambda_i s} \qquad \text{(after the pulse)}$$

II-9.3 $$\Delta c_i(\mathrm{I}_p) = \beta_i \mathrm{M} \sum_i \lambda_i c_i(p\mathrm{T}_-)$$

II-9.4 $$\mathrm{M} = \frac{1}{\tau} \int_{-t_0}^{+t_0} k(s)\eta(s)ds$$

II-10.1 $$\bar{n}(p\mathrm{T}) = \left\{ \tau \frac{\mathrm{M}}{\mathrm{T}} + \frac{\tau}{\varepsilon_0} \right\} \sum_i \lambda_i c_i(p\mathrm{T}_-)$$

In a periodically pulsed fast reactor the production of delayed neutron emitters in each group is split in two terms, one term is the production during a power pulse, and the second term is the production in the time interval between two power pulses.

II-10.2 [2] $$\frac{dc_j}{dt} = - \lambda_j c_j + \beta_j \left\{ \frac{k_0}{\varepsilon_0} \sum_i \lambda_i c_i(t) + \mathrm{M} \sum_i \lambda_i \sum_m c_i(t) \delta(t - m\mathrm{T}) \right\}$$

---

[1] The derivation of the kinetic equations taking into account the formation and decay of the delayed emitters was given by R. Misenta in an internal report (ISPRA-453) printed in 1963.

[2] By formal solution over one period the differential equation (II-10.2) can be transformed into a difference equation linking the densities of delayed emitters at corresponding time. This procedure is of advantage for a numerical treatment of the kinetics of a pulsed reactor.

Assuming exponential functions as solutions

$$c_j = A_j e^{\omega t}$$

for the differential equation II-10.2 gives the equation

II-11.2 $$\omega A_j e^{\omega t} = - \lambda_j A_j e^{\omega t} + \beta_j \left\{ \frac{k_0}{\varepsilon_0} \Sigma \lambda_i A_i e^{\omega t} + M \sum_i \lambda_i \sum_m A_i e^{\omega t} \delta(t - mT_-) \right\}$$

by integrating this equation over t from 0 to the time pT_- just preceeding the p *th* pulse and performing the summation over m, a system of 6 homogeneous equations is obtained :

II-11.3 $$[\omega + \lambda_j (1 - \beta_j B)] A_j - \beta_j B \sum_{i \neq j} \lambda_i A_i = 0$$

II-11.4 $$B = \frac{k_0}{\varepsilon_0} + \frac{M}{T} \frac{\omega T}{e^{\omega T} - 1}$$

The condition that the determinant of this system is zero, gives an equation for ω

II-12 $$1 = \left\{ \frac{k_0}{\varepsilon_0} + \frac{M}{T} \frac{\omega T}{e^{\omega T} - 1} \right\} \sum_i \beta_i \frac{\lambda_i}{\omega + \lambda_i}$$

This equation is the inhour equation for a periodically pulsed reactor taking into account the formation of the delayed neutron emitters during the power pulse and the decay of the emitters in the time interval between two power pulses.

For the discussion the two cases are considered that the deviation from the stationarily pulsed state is small, and that the multiplication factor in the time interval between two power pulses is small compared to one.

case 1 : $$\omega T \ll 1 ; \qquad \frac{\omega T}{e^{\omega t} - 1} \approx 1$$

case 2 : $$\frac{k_0}{\varepsilon_0} \ll \frac{M}{T} \frac{\omega T}{e^{\omega T} - 1}$$

In the first case the inhour equation can be written

$$1 = \left\{ \frac{k_0}{\varepsilon_0} + \frac{M}{T} \right\} \Sigma \beta_i \frac{\lambda_i}{\omega + \lambda_i}$$

For $\omega = 0$ the condition is obtained

$$\left\{ \frac{M}{T} + \frac{k_0}{\varepsilon_0} \right\} \beta = 1$$

By subtracting β on each side and with the definitions

II-13.1 $$\varkappa = \left\{ \frac{M}{T} + \frac{k_0}{\varepsilon_0} \right\} \beta ; \qquad R = \frac{\varkappa - 1}{\varkappa}$$

the relation is obtained

II-13.2 $$R = \Sigma \frac{\beta_i}{\beta} \frac{\omega}{\omega + \lambda_i}$$

In the second case the equation can be written

$$1 = \frac{M}{T} \frac{\omega T}{e^{\omega T} - 1} \Sigma \beta_i \frac{\lambda_i}{\omega + \lambda_i}$$

For $\omega = 0$ the condition is obtained

$$\frac{M}{T} \beta = 1$$

and with the definition

II-14.1 $$\varkappa = \frac{M}{T} \beta \qquad R = \frac{\varkappa - 1}{\varkappa}$$

the inhour equation is given by

II-14.2 $$R = \Sigma \frac{\beta_i}{\beta} \left\{ \frac{\omega}{\omega - \lambda_i} - \frac{\lambda_i}{\omega + \lambda_i} Q(\omega T) \right\}$$

II-14.3 $$Q(\omega T) = \frac{\omega T}{e^{\omega T} - 1} - 1$$

The significance of the multiplication factor $\varkappa$ is seen by inserting the relation

II-9.3 $$M = \frac{\Delta c_i(I_p)}{\beta_i \Sigma \lambda_i c_i(pT_-)}$$

into the relation II-14.1 for the multiplication factor

$$\varkappa = \frac{\Delta c}{\Sigma c_i(pT_-) \lambda_i T}$$

The multiplication factor $\varkappa$ is the ratio

$$\frac{\text{number of delayed neutron emitters formed during one pulse}}{\text{number of delayed emitters decaying during one period}}$$

if the production of delayed neutron emitters in the interval between two power pulses is neglected. Taking into account this production, as it is done in the relation II-13.1 the significance is obtained

$$\frac{\text{number of delayed neutron emitters formed during the period}}{\text{number of delayed neutron emitters decaying during one period}}$$

The multiplication factor $\varkappa$ for the mean power of a periodically pulsed reactor is introduced because the average power of one period is proportional to the source strength of the delayed emitters $\Sigma \lambda_i c_i(pT_-)$ at the beginning of one period.

This proportionality shows itself also in the inhour equations II-13.2 and II-14.2. The reactivity $\rho = \frac{k - 1}{k}$ in the inhour equation of an ordinary reactor (as well as the mean reactivity $\bar{\rho}$ entering in eq. II-6.2)

$$\rho = \Sigma \beta_i \frac{\omega}{\omega + \lambda_i}$$

tends against β for $\omega \to \infty$, $\lim_{\omega=\infty} \rho = \beta$. The limit for the reactivity R in the inhour equation for a periodically pulsed reactor is 1, $\lim_{\omega=\infty} R = 1$. This difference is due to the proportionality of the average power of one period to the source strength of the delayed emitter at the beginning of the period and the fact that a distinction which corresponds to prompt and delayed neutrons does not exist for the delayed emitters.

III — EQUIVALENCE OF THE TWO DERIVATIONS [1]

Starting from the kinetic equation II-1, two different definitions of a mean multiplication factor of a periodically pulsed reactor have been given

II-3.3 $$\bar{k}(t) = \frac{1}{T\bar{n}(t)} \int_{t-T}^{t} k(s)n(s)ds$$

and

II-13.1 $$\varkappa = \frac{M}{T}\beta + \frac{\beta}{\varepsilon_0}$$

The multiplication factor $\bar{k}(t)$ is based on an average of the ordinary multiplication factor weighed by the ratio $n(s)/\bar{n}(t)$, instantaneous power to mean power. The multiplication factor $\varkappa$ is based on the ratio production to decay of delayed neutron emitters during one period. Together with the corresponding kinetic equation for the mean power II-2 and II-10, the two different inhour equations are obtained

II-6.2 $$\bar{\rho} = \Sigma\beta_i \frac{\omega}{\omega + \lambda_i}$$

and

II-13.2 $$R = \Sigma \frac{\beta_i}{\beta} \frac{\omega}{\omega + \lambda_i}$$

To show the equivalence between the two derivations equation II-6.1 is re-written in the from

III-3.1 $$\bar{n} = -\frac{\tau}{\bar{k}(1-\beta) - 1} \Sigma\lambda_i\bar{c}_i$$

or

II-3.2 $$\bar{k}\bar{n} = \frac{\tau}{\beta - \bar{\rho}} \Sigma\lambda_i\bar{c}_i$$

Using equation II-5 equation III-3.2 can be written

III-3.3 $$\frac{W}{T}\nu\tau = \frac{\tau}{\beta - \bar{\rho}} \Sigma\lambda_i\bar{c}_i$$

or

II-3.4 $$\frac{1}{\Sigma\lambda_i\bar{c}_i}\left(\frac{W}{T}\nu\right) = \frac{1}{\beta - \bar{\rho}}$$

---

[1] The equivalance of the results obtained by the two methods of derivation was shown by G. Blässer in an internal report (ISPRA-439) printed in 1963.

Equation III-3.4 is used as definition of $\bar{\rho}$ in terms of the mean number of neutrons per period

$$\overline{N} = \left(\frac{W}{T} \nu\right) \frac{1}{\Sigma \lambda_i c_i}$$

By arranging equation III-3.4 the mean reactivity is obtained as function of $\overline{N}$

III-4.1
$$\bar{\rho} = \frac{\overline{N}\beta - 1}{\overline{N}}$$

III-4.2
$$\bar{\rho} = \beta \frac{\overline{N}\beta - 1}{\overline{N}\beta}$$

comparing the definition II-9.4 of M and the definition of W the equivalence is obtained

III-5.1
$$W = \frac{1}{\nu} \left\{ M + \frac{T}{\varepsilon_0} \right\} \Sigma \lambda_i c_i(pT)$$

or

III-5.2
$$\overline{N} = \frac{M}{T} + \frac{k_0}{\varepsilon_0}$$

With the definition II-13.1 for $\varkappa$ the relation for the mean reactivity can be written

$$\bar{\rho} = \beta R$$

where

$$R = \frac{\varkappa - 1}{\varkappa}$$

The inhour equation II-4

$$\bar{\rho} = \Sigma \beta_i \frac{\omega}{\omega + \lambda_i}$$

can be written

$$R = \Sigma \frac{\beta_i}{\beta} \frac{\omega}{\omega + \lambda_i}$$

which is identical with the inhour equation II-13.2.

By the way this equivalence has been obtained it can be summarized that the kinetic behaviour of a periodically pulsed reactor can be either described by the equation II-10 with the definition II-9.4 for M or by the system of equations [1]

III-3.1
$$\bar{n} = \frac{\tau}{1 - \bar{k}(1 - \beta)} \Sigma \lambda_i \bar{c}_i$$

---

[1] It can also be shown that the two equations III-3.1 and II-2.2 for the mean neutron density and the mean density of the delayed emitter lead together with the definition II-3.3 of the mean multiplication factor $\bar{k}$ to a similar system of equations as the system I-2 given by Bondarenko and Staviskii and to the same inhour equation. By inserting the definition II-3.3 of the mean multiplication factor into the equation III-3.1 and II-2.2 the equations

$$\bar{n} = \tau \sum_i \lambda_i \bar{c}_i \left\{ (1 - \beta) \frac{A}{T} + 1 \right\}$$

$$\frac{d\bar{c}_i}{dt} = - \lambda_i \bar{c}_i + \beta_i \frac{A}{T} \sum_i \lambda_i c_i$$

II-2.2 $$\frac{dc_i}{dt} = -\lambda_i c_i + \beta_i \frac{\bar{k}}{\tau}\bar{n}$$

II-3.3 $$\bar{k} = \frac{1}{\bar{n}(t)}\frac{1}{T}\int_{t-T}^{t} k(s)n(s)ds$$

or

II-13.1 $$\bar{k} = 1 + \beta\frac{\varkappa - 1}{\varkappa}$$

and

$$\varkappa = \frac{M}{T}\beta + \frac{\beta}{\varepsilon_0}$$

II-9.4 $$M = \frac{1}{\tau}\int_{-t_0}^{+t_0} k(s)\eta(s)ds$$

where the function $\eta(s)$ is the solution of the differential equation II-7.1.

IV — MULTIPLICATION FACTOR, STATIONARITY CONDITION, AND INHOUR EQUATION OF A PERIODICALLY PULSED REACTOR

In Chapter II the multiplication factor $\varkappa$ for the mean power of a periodically pulsed reactor

II-13.1 $$\varkappa = \frac{M}{T}\beta + \frac{\beta}{\varepsilon_0}$$

II-9.4 $$M = \frac{1}{\tau}\int_{-t_0}^{+t_0} k(s)\eta(s)ds$$

has been derived. In order to evaluate the multiplication factor numerically, it is necessary to solve equation II-7.1 for a given reactivity shape. As reactivity shape three different functions have been used :

a rectangular function, a triangular function (Stiévenart, private communication), and a parabolic function (Bondarenko and Staviskii [2], Stiévenart [1]. The parameters which characterize these funtions and the obtained analytical expressions for M are given below.

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are obtained with the definition

$$A = \frac{1}{\tau}\int_{t-T}^{t} k(s)\eta(s)ds$$

The function $\eta(s)$ is the solution of the differential equation II-7.1 normalized to the source. Exponentila function as solutions lead immediately to the inhour equation

$$\frac{\frac{A}{T}\beta - 1}{\frac{A}{T}\beta} = \sum_i \frac{\beta_i}{\beta}\frac{\omega}{\omega + \lambda_i}$$

The equivalence between $\frac{A}{T}\beta$ and $\varkappa$ is evident from the definition of A and the definition II-13.1 of $\varkappa$.

[1] Internal report Belgonucléaire, 1962.

Rectangular function :

IV-1.1 $$\varepsilon(s) = \varepsilon_m \qquad 0 \leq s \leq \delta$$

IV-1.2 $$M(\varepsilon_m) = \frac{\tau}{\varepsilon_m^2} \exp\left(\frac{\varepsilon_m}{\tau} \delta\right)$$

Triangular function :

IV-2.1 $$\varepsilon(s) = \begin{cases} \varepsilon_m + \dfrac{\varepsilon_0 + \varepsilon_m}{\delta} s & -\delta \leq s \leq 0 \\ \varepsilon_m - \dfrac{\varepsilon_0 + \varepsilon_m}{\delta} s & 0 \leq s \leq \delta \end{cases}$$

IV-2.2 $$M(\varepsilon_m) = \frac{\delta}{\varepsilon_m + \varepsilon_0} \exp\left\{\frac{\delta}{\tau} \frac{\varepsilon_m^2}{\varepsilon_m + \varepsilon_0}\right\}$$

Parabolic function :

IV-3.1 $$\varepsilon(s) = \varepsilon_m - B^2 s^2$$
$$B = \alpha^{1/2} v$$

IV-3.2 $$M = \pi \frac{\exp\left\{\dfrac{4}{3} \dfrac{\varepsilon_m^{1/2}}{B\tau}\right\}}{B\varepsilon_m}$$

In order that the reactor works in a stationarilly pulsed state, i.e. the state in which the amplitudes of the power pulses are independent of time the multiplication factor $\varkappa$ for the mean power must be equal to 1, or the reactivity R must be equal to 0.

IV-4.1 $$\varkappa = 1, \quad R = 0$$

Using the expression II-3.1 for the mean multiplication factor, the stationarity condition can be written

IV-4.2 $$\beta M(\varepsilon_{m_0}) = T\left(1 - \frac{\beta}{\varepsilon_0}\right)$$

This stationarity condition relates to each period T_0 one and only one value of the prompt critical reactivity ε_{mo} which the reactor works in the stationarily pulsed state. If the reactivity ε_m is larger than the reactivity ε_{m_0}, the multiplication factor is larger than 1. Fig. 1 shows $\varkappa$ as a function of the reactivity ε_m and the reactivity R as a function of the difference $(\varepsilon_m - \varepsilon_{m_0})$ for a parabolic reactivity wave and the set of parameters :

$\tau = 2 \times 10^{-8}$ sec, $k_0 = 0.9744$ or $\varepsilon_0 = 5\beta$, $\alpha = 7 \times 10^{-4}$ cm^{-2} and $v = 314$ m/sec,

for the two pulse frequencies $N = 10$ sec^{-1} and $N = 100$ sec^{-1},

If the multiplication factor is larger than 1, the reactor deviates from the stationarily pulsed state according to the following relation

$$\bar{n}(p) = \frac{\tau}{\beta} \varkappa \sum_i \lambda_i \sum_m \alpha_m e^{\omega m(pT - t_0)}$$

The exponents ω are solutions of the inhour equation

II-14.2 $$R = \sum_i \frac{\beta_i}{\beta} \left\{\frac{\omega}{\omega - \lambda_i} - \frac{\lambda_i}{\omega + \lambda_i} \left(\frac{\omega T}{e^{\omega T} - 1} - 1\right)\right\}$$

and the coefficients α_m allow the adjustment of the solution to the initial conditions of the reactor. For small deviations of the reactivity from the stationarity value the reactivity R for a parabolic reactivity signal can be developped :

IV-5.1
$$\left(\frac{\partial \varkappa}{\partial \varepsilon_m}\right)_{\varepsilon m_0} = \left(1 - \frac{\beta}{\varepsilon_0}\right)\left(\frac{2\varepsilon_{m_0}{}^{1/2}}{B\tau} - \frac{1}{\varepsilon_{m_0}}\right)$$

Thus the inhour equation can be written

IV-5.2
$$\left(\frac{\partial \varkappa}{\partial \varepsilon_m}\right)_{\varepsilon m_0} \cdot \Delta\varepsilon_m = \sum_i \frac{\beta_i}{\beta} \frac{\omega}{\omega + \lambda_i}$$

or

IV-5.3
$$\frac{\Delta\varepsilon_m}{\beta_{\text{pulsed}}} = \sum_i \frac{\beta_i}{\beta} \frac{\omega}{\omega + \lambda_i}$$

with the definition

IV-5.4
$$\beta_{\text{pulsed}} = \frac{1}{\left(\frac{\partial \varkappa}{\partial \varepsilon_m}\right)_{\varepsilon m_0}}$$

In writing the inhour equation IV-5.2 in the form

IV-5.5
$$\beta\left(\frac{\partial \varkappa}{\partial \varepsilon_m}\right)_{\varepsilon m_0} = \left(\frac{\partial \bar{\rho}}{\partial \varepsilon_m}\right)_{\varepsilon m_0} = \sum_i \beta_i \frac{\omega}{\omega + \lambda_i}$$

it corresponds to the inhour equation of an ordinary reactor. But the left hand side shows that a periodically pulsed reactor is, for the same deviation $\Delta\varepsilon_m$ of the ordinary multiplication factor from the stationary value, more sensitive by a factor $\beta(\partial\varkappa/\partial\varepsilon_m)_{\varepsilon m_0}$.

In the table the reactivity ε_{m_0}, the sensitivities $(\partial\varkappa/\partial\varepsilon_m)$ and $\beta(\partial\varkappa/\partial\varepsilon_m)$ are given. The sensitivity $\beta(\partial\varkappa/\partial\varepsilon_m)$ has the value 20.4 for 10 pulses/sec and 12.6 for 100 pulses/sec.

In order to see the influence of the pulsing period on the kinetics, the inhour equation II-14.2 has been solved for reactivities between —1 and +1, for one case in which the term $Q(\omega T)$ has been set equal to 0, and for a second case with the exact equation for a pulse period $T = 1$ sec. Fig. 2 shows the influence of the pulse period on the rootes ω.

In fig. 3 the time dependence of the mean power of a periodically pulsed reactor is given for a value of the multiplication factor $\varkappa$ of 2.0 corresponding to the reactivity $R = 0.5$. The multiplication factor $\varkappa = 2$ corresponds to a deviation of ε_m from the stationarity value ε_{m_0} by $\Delta\varepsilon_m = 18 \times 10^{-5}$ for 10 pulses/sec, and by $\Delta\varepsilon_m$ 26×10^{-5} for 100 pulses/sec.

N sec^{-1}	ε_{m_0}	$\left(\frac{\partial\varkappa}{\partial\varepsilon_m}\right)_{\varepsilon m_0}$	$\beta\left(\frac{\partial\varkappa}{\partial\varepsilon_m}\right)_{\varepsilon m_0}$	$\Delta\varepsilon_m(\varkappa = 2)$
10	1.53×10^{-3}	3.3×10^3	20.4	18×10^{-5}
100	0.89×10^{-3}	2.0×10^3	12.6	26×10^{-5}

TABLE. — The reactivity ε_{m_0}, the sensitivity $(\partial\varkappa/\partial\varepsilon_m)_{\varepsilon m_0}$ and the deviation of the reactivity $\Delta\varepsilon_m = \varepsilon_m - \varepsilon_{m_0}$ for $\varkappa = 2$ for the pulse frequencies $N = 10$ and 100 sec^{-1}.

The values have been used with a parabolic reactivity shape and the parameters $\tau = 2 \times 10^{-8}$ sec, $k_0 = 0.9744$ or $\varepsilon_0 = 5\,\beta$, $\alpha = 7 \times 10^{-4}$ cm^{-2}, $\upsilon = 314$ m/sec.

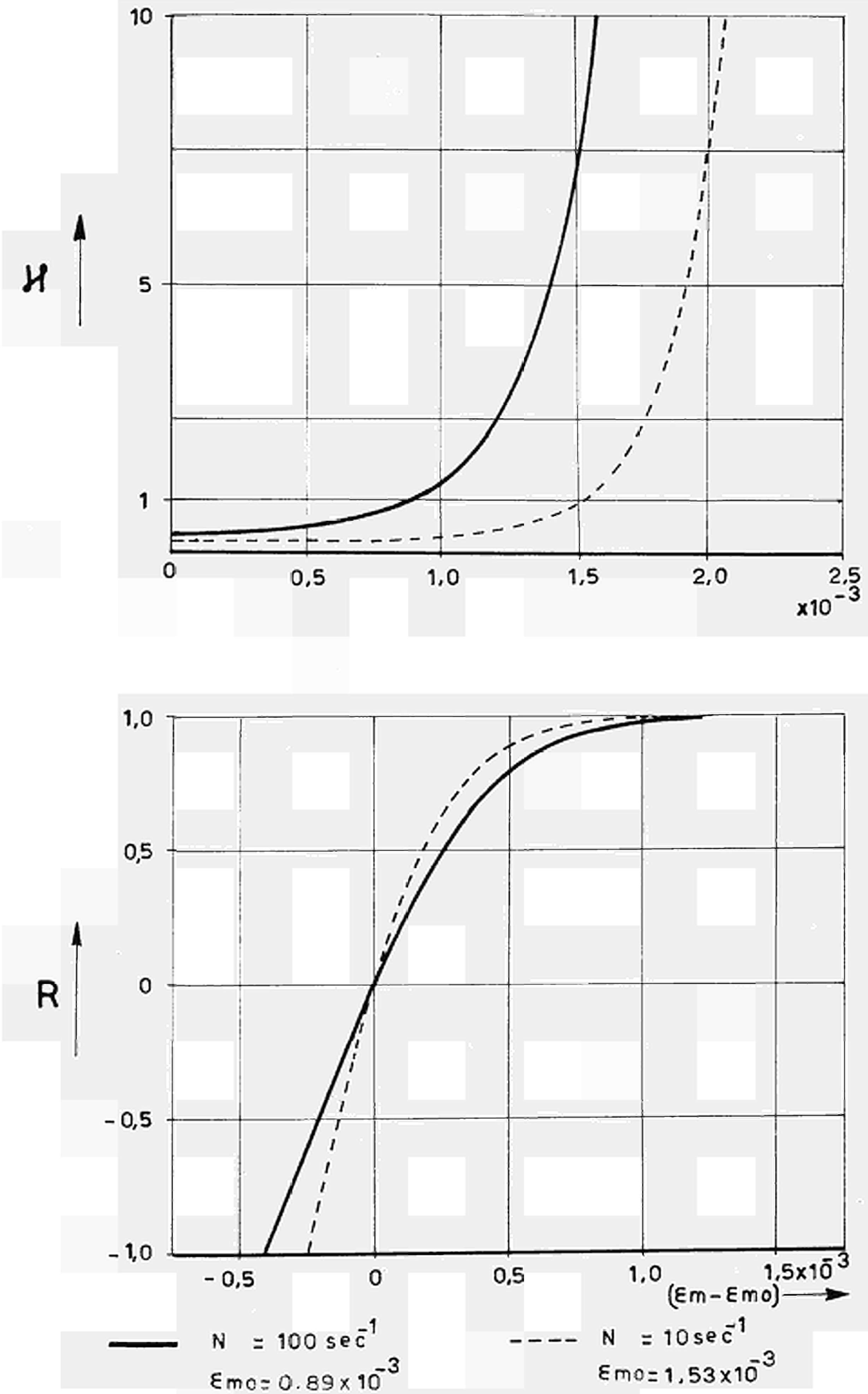

Fig. 1. — Multiplication factor κ and Reactivity R.

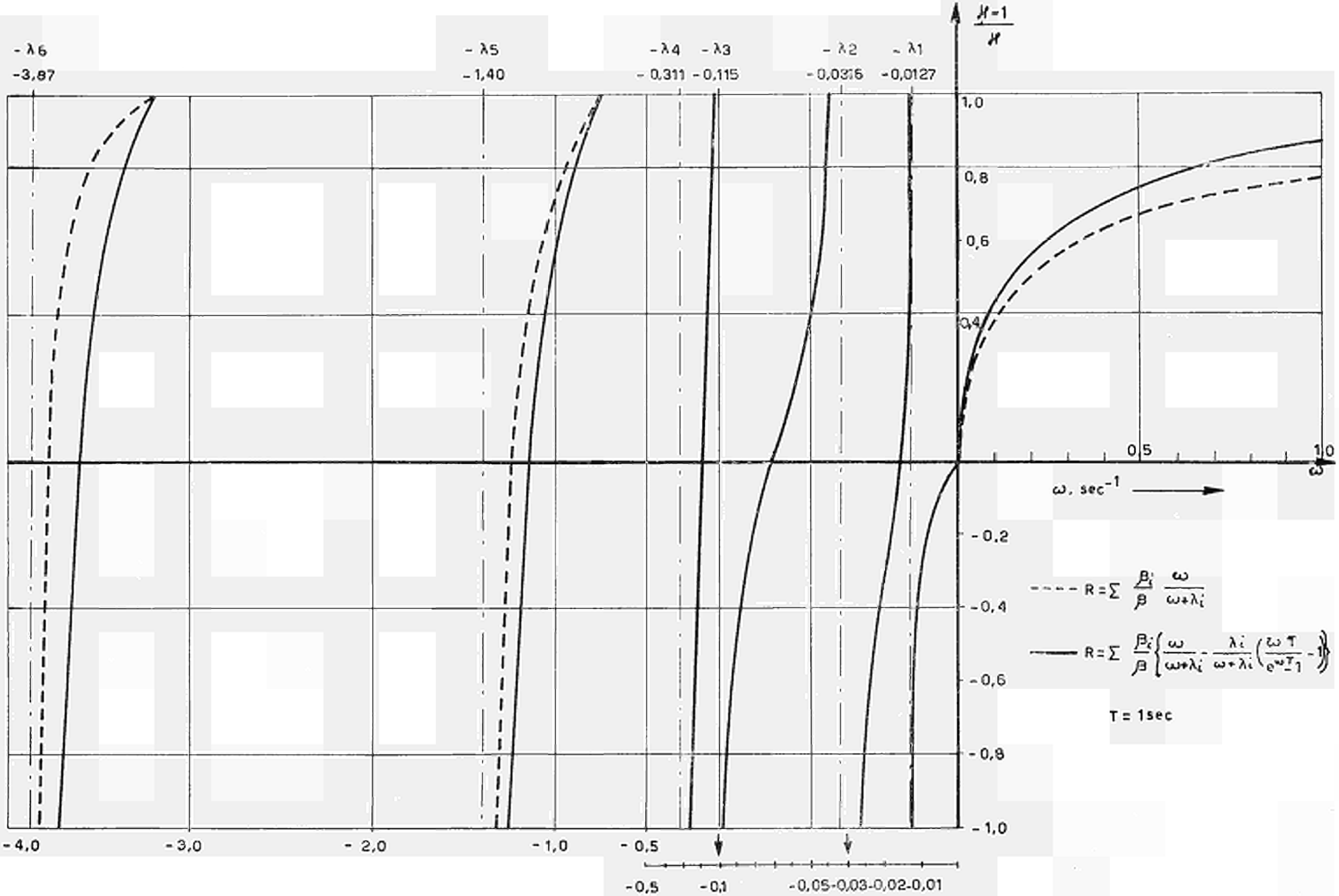

Fig. 2. — Roots ω of the inhour equations.

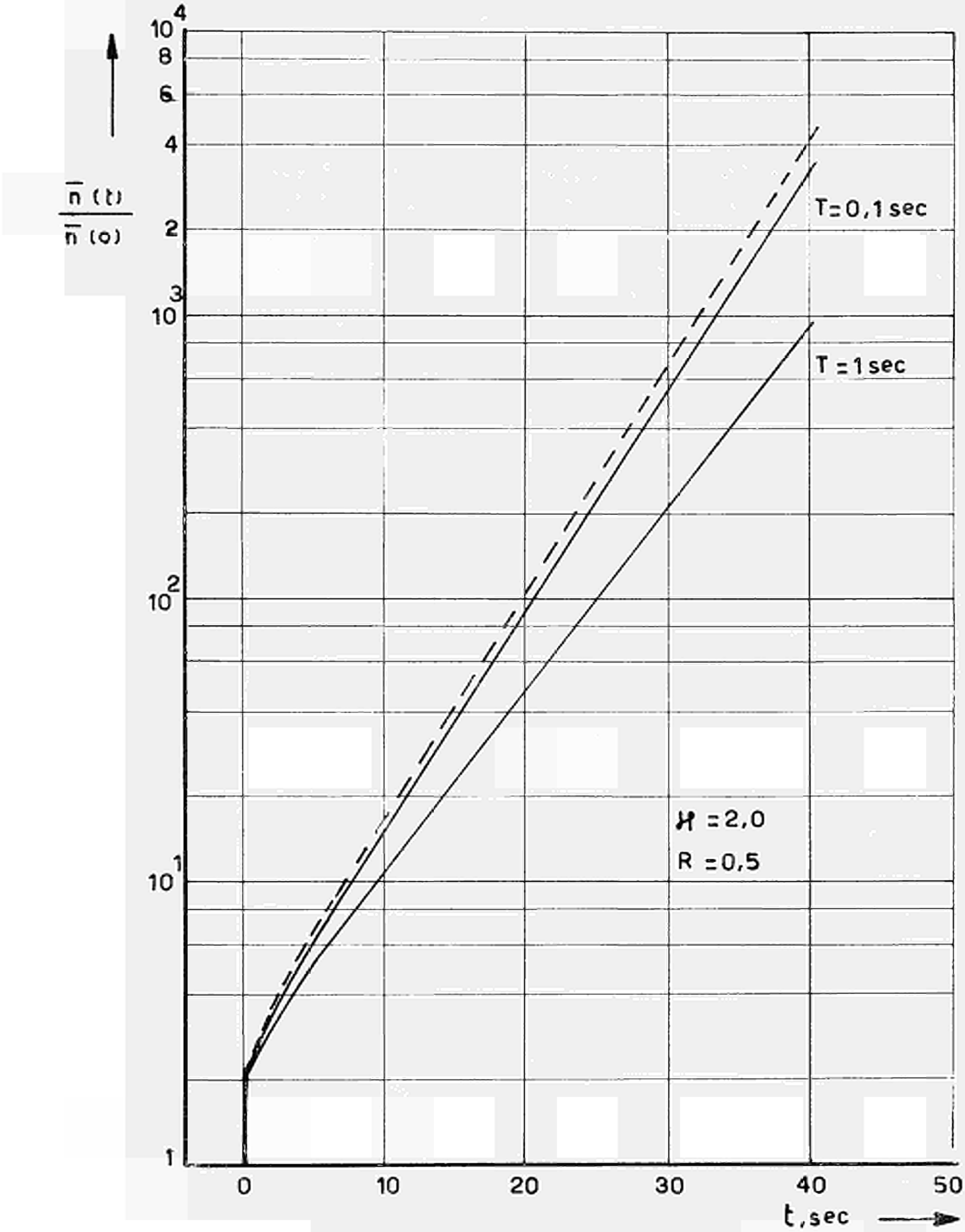

Fig. 3. — Increase of the mean power for $\varkappa = 2$.

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