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STUDIES ON THE PARAMETRIC REPRESENTATION OF FAST REACTOR SPECTRA

by

G. BLÄSSER, E. DIANA

1963

Joint Nuclear Research Center
Ispra Establishment - Italy

Reactor Physics Department
Applied Mathematical Physics Service

Text presented at the «VIII Congresso Nucleare»
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STUDIES ON THE PARAMETRIC REPRESENTATION OF FAST REACTOR SPECTRA

SUMMARY

The possibility of representing the spectrum in a fast assembly as a superposition of two parts, one being proportional to the spectrum of fission neutrons, the other being a PEARSON-type I-curve, is discussed. Cross sections of the main materials in a fast reactor averaged over this spectrum agree quite well with the averaged cross sections as calculated by multigroup methods.

1. Introduction

Up to the present time, fast reactor calculations have been based on the multigroup approach, starting from basic physical data such as cross sections as functions of energy. For thermal reactors, this method is often replaced by a few-group theory based on integral data. Besides reducing the amount of computer time, the integral approach uses information which usually is more easily obtained by the experiments. In the field of fast reactors many cross-sectional data are still unknown, or at least not known with the required precision. Yet, the integral method was seldom used for fast systems, since it was believed that results obtained in this way for one system do not apply to different systems because of the changes in the spectrum. However, if it is possible to represent the spectrum by a small number of parameters, one can use an integral approach for each parameter configuration and extend thereby the information gained from integral experiments to a large extent. In this report we study only the possibilities of representing the spectra in unreflected spherical uranium fuelled fast assemblies by some parametric curves.

2. Method of analysis

We calculated the spectrum $\underline{\phi}$ in the center of the fast assemblies by a multigroup code (ZOOM-7090 version) using 16 groups and the cross sections as given by YIFTAH, OCKRENT and MOLDAUER[1]. We assumed, heuristically, that the spectrum $\underline{\phi}$ can be written as superposition of a part that is proportional to the spectrum χ of the emitted fission neutrons and a residual spectrum $\underline{\psi}$. The constant of proportionality F was fixed by the further assumption, that the residual spectrum $\underline{\psi}$ is negligible in the highest energy group (corresponding to $E > 3.67$ MeV).

If i denotes the group index and i=1 corresponds to the highest group, we have (Φ_i giving the number of neutrons in the group i.) :

$$(1) \qquad \Phi_i = F\chi_i + \Psi_i$$

and

$$(2) \qquad F = \frac{\Phi_1}{\chi_1}$$

Furthermore, it turned out to be convenient to normalize the residual spectrum Ψ ; we call this normalized spectrum ρ_i :

$$(3) \qquad \rho_i = \frac{\Psi_i}{\sum_i \Psi_i}$$

and we also introduce :

$$(4) \qquad \alpha = \frac{F}{\sum_i \Psi_i}$$

Thus up to a constant factor A (which is determined by the total flux level or the power of the reactor) the spectrum is represented in the form :

$$(5) \qquad \Psi_i = A\,(\alpha\chi_i + \rho_i)$$

Since χ is a known spectrum which is the same for all reactors the differences between the different reactor types are contained only in the factor α and in the form of the reduced spectrum ρ . The latter might be characterized in terms of its mean lethargy and its higher moments :

$$(6) \qquad \bar{u} = \sum_i u_i \rho_i$$

$$(7) \qquad \mu_n = \sum_i (u_i - \bar{u})^n \rho_i$$

where u_i is the value of lethargy attributed to the group i.

In statistical literature, one often finds the following problem : given the moments up to the fourth, find a distribution curve with these moments . The simplest approach is that using the K. PEARSON system of frequency curves[2]. There one classifies the frequency curves in the eight types according to the values of β_1 and β_2 where :

$$(8) \qquad \beta_1 = \mu_3^2 / \mu_2^3$$

$$(9) \qquad \beta_2 = \mu_4 / \mu_2^2$$

We have adopted this system and tried to replace ρ by such frequency curves, thus retaining of all the information included in ρ only the information contained in its first four moments.

3. Results of the calculations

We calculated the items mentioned in § 2 for the following assemblies :

a) pure uranium metal assemblies with an enrichment varying between 94% and 10%

b) uranium metal assemblies with an enrichment of 50%, containing sodium up to 50% in volume

c) one assembly containing 50% (vol.) Na, 20% steel and 30% uranium metal (50% enr.)

d) uranium oxide assemblies with an enrichment between 75% and 25%

e) one uranium oxide (50% enriched) assembly with 50% (vol.) Na

f) one uranium oxide (50% enriched) assembly with 50% (vol.) of Na and 20% (vol.) of steel

g) uranium carbide assemblies with an enrichment between 75% and 25%

h) one uranium carbide (50% enriched) assembly with 50% (vol.) Na

i) one uranium carbide (50% enriched) assembly with 50% (vol.) of Na and 20% (vol.) of steel.

The most striking result has been a linearity in the dependence of the parameter α on the enrichment down to enrichments of about 15-20% (which up to now we were not able to derive directly) and the smallness of the variation of the mean lethargy and the second moment with the change in core composition. It turned out, furthermore, that all spectra could be fitted by type I curves of the PEARSON system. The expression for a type I frequency curve is :

$$(10) \qquad \rho = \text{const.} \left(1 + \frac{x}{a_1}\right)^{m_1} \left(1 - \frac{x}{a_2}\right)^{m_2}$$

Table I

Values of α, $\bar{u}$, μ_2, μ_3, μ_4, calculated for a number of fast assemblies

case	α	$\bar{u}$	$\mu 2$	$\mu 3$	$\mu 4$
a) 94% enr.	1.327	2.921	0.771	0.152	1.667
75% "	1.043	2.917	0.741	0.194	1.575
50% "	0.701	2.937	0.707	0.249	1.532
25% "	0.375	3.029	0.708	0.332	1.668
10% "	0.165	3.251	0.816	0.483	2.243
b) 30% vol. Na	0.759	2.972	0.758	0.211	1.772
50% " "	0.617	2.978	0.757	0.251	1.780
c)	0.406	3.008	0.880	0.341	2.465
d) 75% enr.	0.561	2.965	1.033	0.274	2.990
50% enr.	0.416	3.123	0.087	0.369	3.508
25% enr.	0.262	3.443	1.301	0.634	5.156
e)	0.382	3.222	1.186	0.441	4.269
f)	0.249	3.354	1.371	0.772	6.005
g) 75% enr.	0.830	3.014	0.955	0.319	2.634
50% enr.	0.576	3.107	0.997	0.456	3.053
25% enr.	0.331	3.321	1.190	0.767	4.521
h)	0.504	3.172	1.081	0.502	3.643
i)	0.309	3.243	1.255	0.744	5.111

where the relation between the parameters m_1, m_2, a_1, a_2 is given in the appendix and where x is the difference between the lethargy value in question and the mode value, i.e. the value of lethargy for which the distribution attains its maximum value.

4. Calculation of averaged cross sections

The calculation of averaged cross sections gives us also a good test of the accuracy with which our parametric representation of the reduced spectrum reproduces the real spectrum; we can perform this averaging process in two different ways, using either the real multigroup spectrum or its parametric representation as weight function. This shall now be discussed in more detail.
The first and the most direct way to calculate the cross sections averaged over the reduced spectrum is the following.

$$\bar{\sigma} = \sum_{i}^{16} \sigma_i \rho_i \qquad (11)$$

where σ is a generic cross section, σ_i are its group values as given in ref. 2 and the ρ_i are the normalized multigroup values that appear in eg. (11)

The second way is

$$\bar{\sigma}^* = \sum_{i}^{16} \sigma_i \rho_i^* \Big/ \sum_{i}^{16} \rho_i^* \qquad (12)$$

where the σ_i are the same as above and ρ_i^* is the integral of $\rho(u)$ over the lethorgy interval corresponding to the group i

$$\rho_i^* = \int_{\Delta u_i} \rho(u)\, du \qquad (13)$$

In order to test the accuracy with which our parametric representation fits the reduced spectrum we have compared the values of and for a certain number of cross sections and some reduced spectra chosen to cover the whole range spectra examined.

For example we report in table II the results of this comparison for a fast critical assembly fuelled with 25% enriched uranium oxyde.

Table II

	U^{235}	U_{238}	Pu_{239}	Pu_{240}
$\bar{\nu}_{fr}$	8.453	8.419	8.662	8.419
$\bar{\nu}^{*}_{fr}$	8.466	8.429	8.665	8.429
$\bar{\nu}_{c}$	0.3387	0.2021	0.3359	0.3535
$\bar{\nu}^{*}_{c}$	0.3393	0.2027	0.3375	0.3546
$\bar{\nu}_{f}$	1.5653	0.0255	1.7891	0.3607
$\bar{\nu}^{*}_{f}$	1.5694	0.0266	1.7918	0.3662
$\bar{\gamma}_{f}$	3.8721	0.0695	5.2701	1.1588
$\bar{\gamma}^{*}_{f}$	3.8827	0.0725	5.2790	1.1774

The difference between the two values reaches a maximum of 4.3% for the threshold fission cross section of U^{238} and is much smaller for the other cross sections.
In the other spectra examined the results of such a comparison are still better than in this one.

5. Influence of the spatial variation of the spectrum

We calculated all our spectra at the center of the fast assemblies and, therefore, it is useful to know the influence of the spatial variation of the reduced spectrum on the averaged cross sections. This influence is certainly great near the boundary of unreflected assemblies, but, in actual reactors, it will be reduced by the presence of a reflector. Therefore it seemed reasonable to us to study this effect in a reflected assembly.
For this study we have choosen a core containing 54% (vol.) sodium, 14% steel, 32% uranium metal (46% enriched) and a reflector containing 20% sodium, 10% steel, 70% natural uranium metal.
The spatial variation is important only for the threshold fission cross section of Uranium 238.

6. Conclusion

The results of these calculations seem to indicate the feasability of two-group calculations for fast reactors, using two overlapping groups; the first of these has a fission spectrum, the second a spectrum given by eq. (10). The absorption and transport cross sections in the two groups will be obtained by integrating the elementary cross sections over the respective spectra. The transfer cross section from group 1 to group 2 has to be determined simply from the requirement, that the ratio of the two fluxes far from boundaries be equal to α.

This reduction of the number of groups permits the use of more elaborate methods such as two-dimensional S_N-calculations and can thus lead to a more precise description of the reactor than one-dimensional multigroup calculations. According to MARCHUK [4] it leads to almost the same results if one calculates a fast reactor by one-group transport (P_3) theory using suitably averaged cross sections (which he obtained from 25-group P_1-calculations) or if one calculates the reactor directly by using a 25-group P_3-calculation, but the amount of labour involved in the second procedure is much larger.

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(1) S. YIFTAH, D. OKRENT, P.A. MOLDAUER : Fast reactor cross sections - Pergamon Press, Oxford 1960

(2) E.S. PEARSON and H.O. HARTLEY : Biometri tables for statisticians I, p. 79, Cambridge University Press, 1958

(3) S. VIANELLI : Prontuari per calcoli statistici, p. 1334, Abbaco Palermo-Roma, 1959

(4) G.I. MARCHUK : Proceedings of the Vienna Conference 1962 on the "Physics of Fast and intermediate Reactors", Vol. II, p. 19 ff.

Appendix

According to VIANELLI[3], the position of the mode and the parameters of the type I distribution curve are expressed in terms of the moments by :

$$\text{Mode} = \bar{u} - \frac{1}{2} \cdot \frac{\mu_3}{\mu_2} \cdot \frac{r+2}{r-2}$$

where

$$r = \frac{6\,(\beta_2 - \beta_1 - 1)}{6 + 3\beta_1 - 2\beta_2} \qquad \beta_1 = \mu_3^2 / \mu_2^3 \quad \beta_2 = \mu_4 / \mu_2^2$$

$$m_1 = \frac{1}{2}\left\{ r-2-r\,(r+2)\sqrt{\frac{\beta_1}{\beta_1\,(r+2)^2 + 16\,(r+1)}} \right\}$$

$$m_2 = \frac{1}{2}\left\{ r-2+r\,(r+2)\sqrt{\frac{\beta_1}{\beta_1\,(r+2)^2 + 16\,(r+1)}} \right\}$$

$$a_1 = \frac{m_1\ b}{m_1 + m_2} = \frac{m_1\ b}{r-2}$$

$$a_2 = \frac{m_2\ b}{m_1 + m_2} = \frac{m_2\ b}{r-2}$$

where

$$b = a_1 + a_2 = \frac{1}{2}\sqrt{\mu_2} \cdot \sqrt{\beta_1\,(r+2)^2 + 16\,(r+1)}$$