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EUROPEAN ATOMIC ENERGY COMMUNITY — EURATOM

TEMPERATURE DEPENDENCE OF CIRCUITS USING TWO EMITTER COUPLED TRANSISTORS

by

L. STANCHI

1963

Joint Nuclear Research center
Ispra Establishment - Italy

Engineering Department
Electronics Service

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LIST OF SYMBOLS

I_A, I_B Terms having dimension of current, used in equations of junction reverse currents.

I_{CO} The collector current when the collector is biased in the reverse direction and the emitter is open.

I_{CS} The collector current when the collector is biased in the reverse direction and the emitter is *dc* short-circuited to the base.

I_{EO} The emitter current when the emitter is biased in the reverse direction and the collector is open.

I_{ES} The emitter current when the emitter is biased in the reverse direction and the collector is *dc* short-circuited to the base.

K Boltzmann constant equal to $1.38 \cdot 10^{-23}$ joules/°K.

q electron charge equal to $1.602 \cdot 10^{-19}$ Coulombs.

T absolute temperature.

V_A a constant for a given semiconductor related to the energy gap by equation (3.2.) measured in electron-volts.

v_{CB} *dc* voltage between collector and base, taken positive if forward biased.

v_{EB} *dc* voltage between emitter and base, taken positive if forward biased.

V_g value of the forbidden gap of energy between the valence and the conduction band, measured in electron volts.

V_{go} actual value of V_g at the absolute zero.

V_{ge} equivalent value of V_g at the absolute zero as it is obtained by the extrapolation of fig. 4.

α_N common base forward current gain.

α_I common base reverse current gain.

β common emitter current gain.

ϱ experimental parameter defined by (4.1) and measured in $°C^{-1}$.

φ theoretical ratio between I_{CO} measured at temperature $T_o + \Delta T$ and the same at the temperature T_0.

ψ actual ratio between I_{CO} measured at the temperature $T_0 + \Delta T$ and the same at the temperature T_0.

TEMPERATURE DEPENDENCE OF CIRCUITS USING TWO EMITTER COUPLED TRANSISTORS

SUMMARY

The parameters of the transistors which are influenced by the temperature are examined. The analysis is made for difference amplifiers and Schmitt triggers and in general for that circuits which have two transistors with the emitters connected together. The theoretical investigations is followed by some results which were obtained on real transistors. As an example is given the application to the Schmitt trigger circuit in order to demonstrate that using simple design criteria and selection of components it is possible to arrive to a remarkable stability without the use of a thermostat beeing requested.

1 — INTRODUCTION

The problem of the variations of the transistor characteristics as a function of the temperature has been object of many works and in the literature many papers can be found. If on the one hand the characteristics of the semiconductors vary with temperature much more than the characteristics of the electronic tubes, on the other hand these variations follow fairly tight the well known laws and for that they can be foreseen and taken into account. The strong variations which can drive to a complete thermal run-away are not inspected but only the little variations of voltage which can occur between the bases of two emitter coupled transistor and which can affect the precision of the circuit, are analyzed.

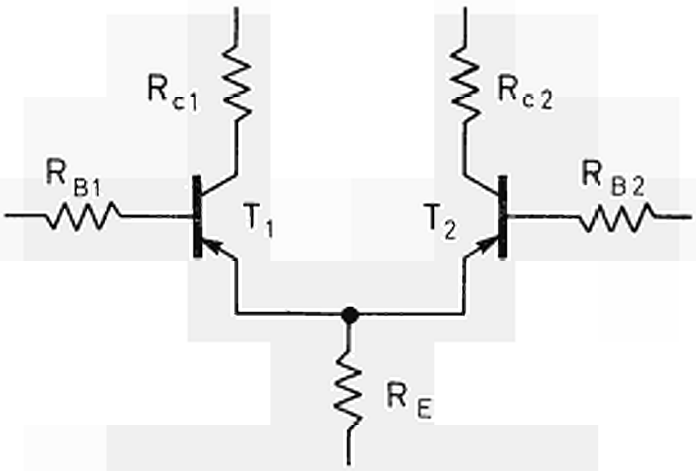


Fig. 1 — Circuit used for the theoretical analysis.

The analysis is made on the circuit of fig. 1 in order to evaluate the total effect of the variations of the temperature on the voltage difference between the bases. The quantities which are influenced by the temperature are examined starting from the equations of the intrinsic transistor.

2 — EFFECT OF REVERSE CURRENTS I_{CO}

Be the transistors at the same temperature. This is not verified if the power dissipation is not negligible and the dissipations are not the same. The effect will be discussed in point 6.

The base current has the value as derived in appendix I : (see list of symbols)

$$I_B = -\frac{1-\alpha_N}{1-\alpha_N\,\alpha_I} I_{EO}\,(e^{qv_{EB}/KT} - 1) + I_{CO}\,\frac{1-\alpha_I}{1-\alpha_N\,\alpha_I} \tag{2.1}$$

The first term of second member is related to the emitter current and the variations with temperature are influenced by the variations of the common emitter current gain β. If the transistor is cut off, that is if $v_{EB} \ll O$, the first term become $I_{EO}\,(1 - \alpha_N)/(1 - \alpha_N\,\alpha_I)$ and can be negligible if α_N is still high and/or I_{EO} is low (as for example for drift transistors). The second term is nearly equal to the saturation reverse current of collector to base junction (emitter open).

The current I_{CO} gives rise to a voltage drop on the base resistances and that voltage is a function of the temperature. The reverse current I_{CO} as given by the junction theory follows the law (1)

$$I_{CO} = I_A \left(\frac{T}{T_0}\right)^3 e^{-qV_g/KT} \tag{2.2}$$

where I_A has the dimension of a current and V_g is the energy gap which varies linearly with temperature in the range of ambient temperature. So it is possible to write

$$V_g = V_{ge} + T\left(\frac{d\,V_g}{dT}\right) \tag{2.3}$$

where V_{ge} is the value of the gap in electron volts as it is found extrapolating to zero the curve V_g *vs* temperature. The influence of reverse currents I_{CO} can be compensated with some precision if two suitable transistors are used. It is necessary to choose two transistors having the same I_{CO} and to design the circuit with equal base resistances.

In practice it is not very easy to match the transistors for equal values of I_{CO}. What it is possible to measure is the sum of various components which add to the true reverse current of the junction and contribute to the meter indication. These components (resistive components, surface leakage currents, channel components etc.) are reported in the literature (2). It is proved by the experience that all those components have variations with temperature which are in the same sense of the theoretical reverse current but are always smaller. Moreover the reverse currents which can be measured are not independent of the voltage. They are unforeseeably influenced by the voltage and this in a manner which is different from sample to sample. (fig. 2). The theoretical value is on the contrary independent of voltage. We call φ the ratio between the theoretical value at the temperature $T + \Delta T$ and the starting value at the temperature T. If we measure the same ratio for actual transistors we obtain a ratio $\psi = \psi(V)$ which has the maximum at the zero voltage (extrapolated value) (fig. 3). The value of ψ is variable from sample to sample. The criterium of selection of the samples must take account of these variations in order to calculate "a priori" the limits of the error. It is not reasonably possible to select transistors having equal $I_{CO} = f(-\,V_{CB})$ and equal variation with temperature but it is necessary to reduce the selection only to one measurement of only one quantity. It comes that the matching is made by measuring the value of I_{CO} for only one value of voltage V_{CB} at one given temperature. So doing there is an error which is composed of three different errors which must be taken into account :

a) error in matching the transistors which will be coupled with a stated percentile error.
b) error for the influence of the temperature in the measurement.
c) error in the variation of I_{CO} with temperature even if the initial values are the same.

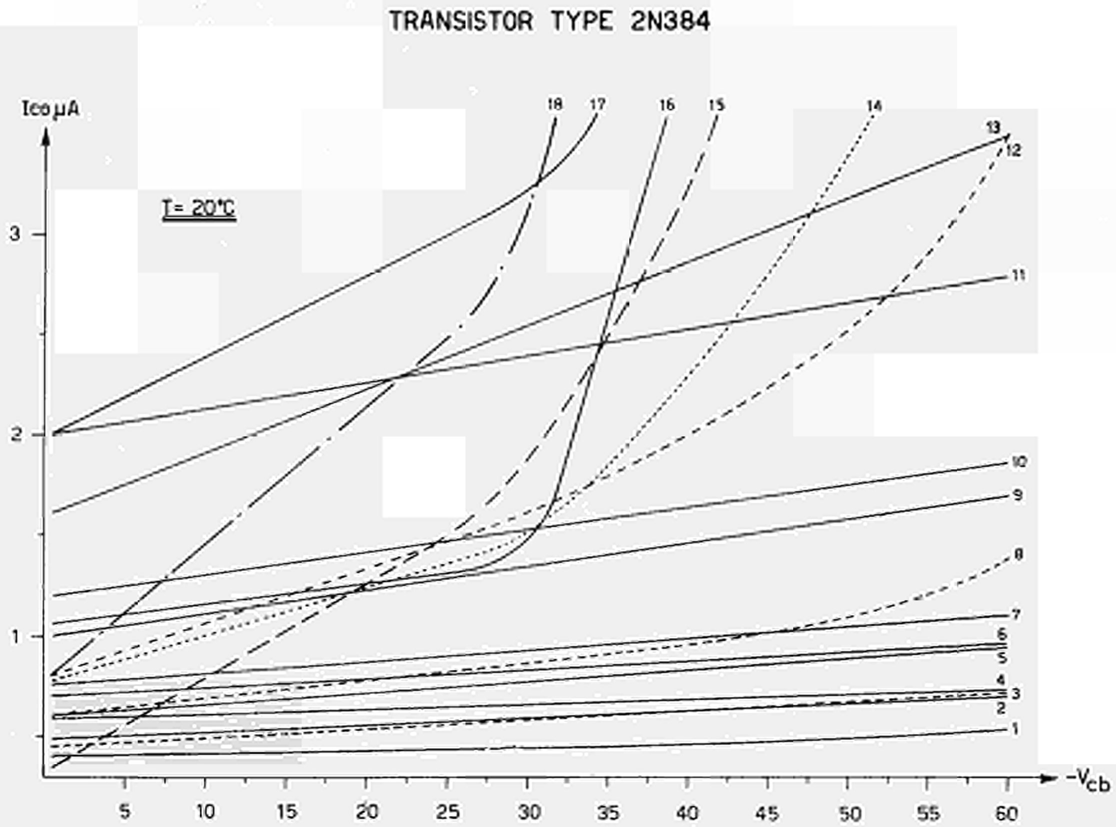


Fig. 2 — Tipical diagram of I_{CO} plotted against $-V_{CB}$ for a batch of transistors 2N 384 of the production RCA of 1959.

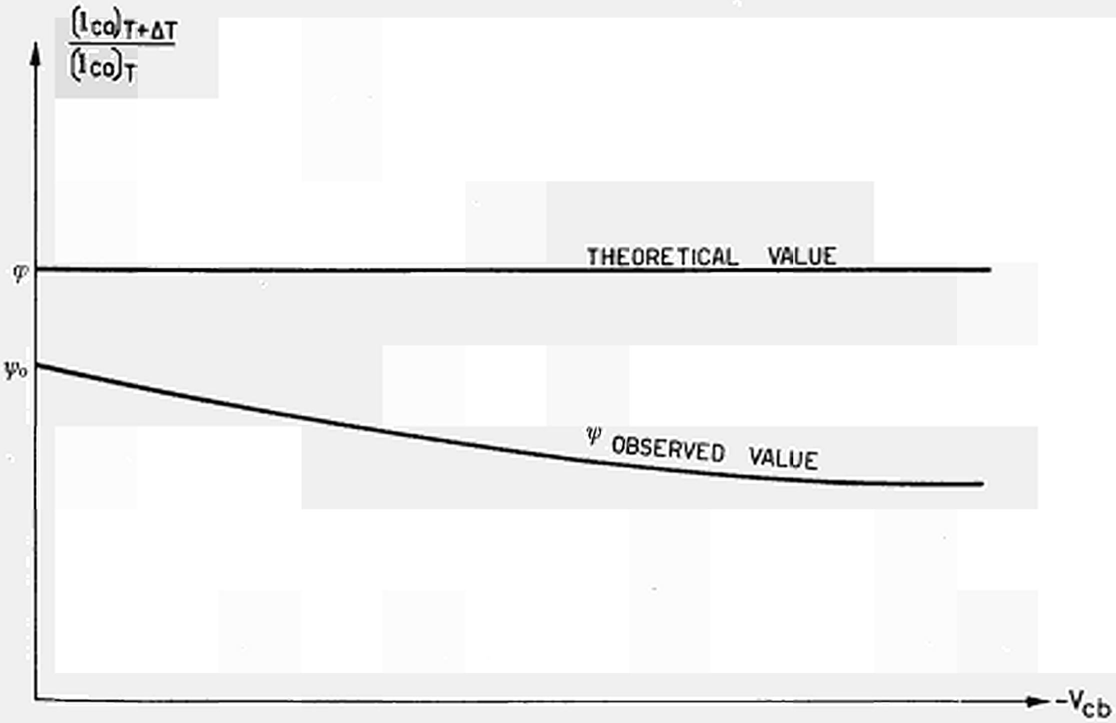


Fig. 3 — Qualitative diagram showing the difference between the theoretical increase (φ) of I_{CO} current for a given increase of temperature ΔT and the actual value ψ as proved by the experience.

3 — VARIATION OF EMITTER-BASE VOLTAGE WITH TEMPERATURE

The variation with temperature of the emitter-base voltage for constant emitter current is reported. Afterwards the same effect is shown for a current which is directly variable with the temperature. The first case is normally encountered in many circuits, the second is found in the Schmitt trigger where the first transistor has the firing current which is proportional to the temperature while the second transistor has almost constant current ([3]). The two cases are not strongly different. The results are obtained by means of the calculation of appendix II.

a) emitter current i_E = const.

$$\left(\frac{dv_{EB}}{dT}\right)_{i_E = \text{const.}} = -\frac{1}{T}\,(V_A - v_{EB}) \tag{3.1}$$

where :

$$V_A = V_{ge} + 3\,\frac{KT}{q} \tag{3.2}$$

is a constant for a given semiconductor.

The equation (3.1) is a very simple one, valid for any type of semiconductor and shows clearly the behaviour of the emitter to base voltage with temperature. It is apparent that v_{EB} decreases when the temperature increases and the smaller is v_{EB} the greater is the variation.

The values of the energy gap are reported in the literature but the autors give slightly different values ([4,5,6,7,8,9]). Extensive discussions of these values can be found in bibl. ([6]) for silicon and ([7]) for germanium. It should be noted that the shapes of the energy gap *vs* temperature curves are representative of the behaviour generally observed in semiconductors, i.e. the rate of change with temperature is approximately constant at the higher temperature but tends towards zero near 0°K. From the curves it is obtained by extrapolation the value of V_{ge} which is the equivalent value of the gap at 0°K as it would be if the approximately right line at ambient tem-

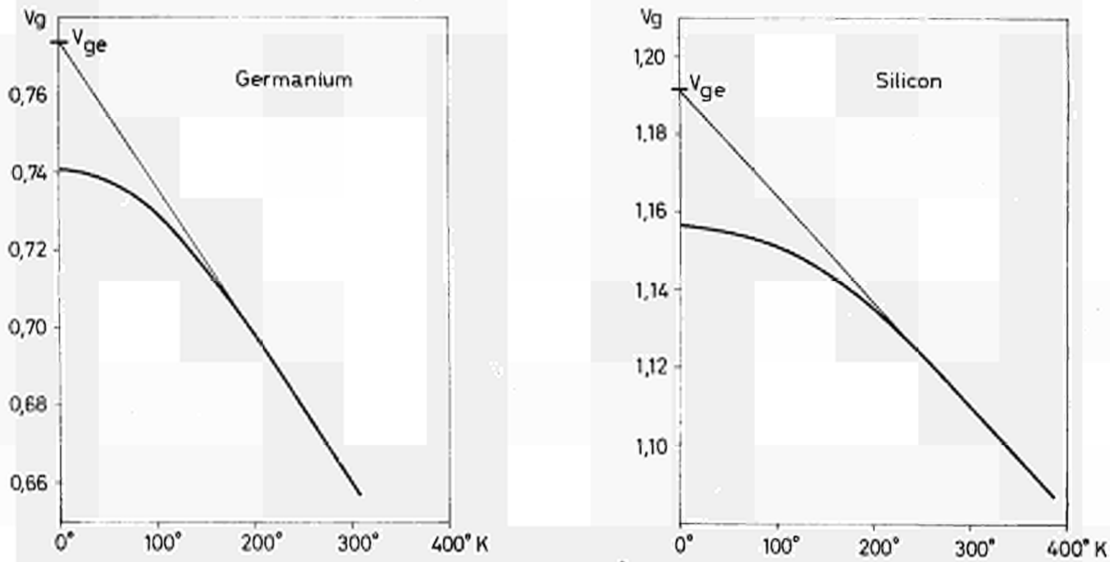


Fig. 4 — Construction for obtaining the equivalent gap V_{ge} at the absolute zero of temperature.

perature would be prolonged up to zero (fig. 4). It is to be noted that the value of the equivalent zero gap V_{ge} is somewhat greater than the true value V_{g0} of the gap at absolute zero. It is obtained for germanium :

$V_{ge} = 774$ mV and $\frac{dV_g}{dT} = -0.38$ mV/°K and for silicon :

$$V_{ge} = 1.192 \text{ V and } \frac{dV_g}{dT} = -0.27 \text{ mV/°K}.$$

b) emitter current proportional to the temperature.

From appendix II :

$$\left(\frac{dv_{EB}}{dT}\right)_{i_E \propto T} = -\frac{1}{T}\left(V_A - v_{EB} - \frac{KT}{q}\right) \tag{3.3}$$

the result is analogous to that for constant current. The term into parenthesis differs from (3.1) of $\frac{KT}{q}$, that is about 26 mV.

The full variation with temperature can be obtained by integrating (3.1) and (3.3). For the circuit with two emitter coupled transistors it is important to know the voltage difference between the bases in a finite interval of temperature. From the equation (B. 13) of the appendix II we have for a variation from T_0 to $T = T_0 + \Delta T$:

a) constant currents

$$\Delta (v_{EB2} - v_{EB1}) = (v_{EB2} - v_{EB1})_{T_0+\Delta T} - (v_{EB2} - v_{EB1})_{T_0} = \\ = \frac{K\Delta T}{q}\left(ln\frac{i_{E2}}{i_{E1}} + ln\frac{I_{ES1}}{I_{ES2}}\right) \tag{3.4}$$

where i_{E2} and i_{E1} are kept constant.

An increase of the emitter current or a decrease in the same ratio of the emitter junction reverse current have the same influence. Obviously if the transistors are equal ($I_{ES1} = I_{ES2}$) and the currents are kept equal there is no variations of the voltage difference between the bases for a variation of temperature.

b) i_{E2} constant and i_{E1} proportional to the temperature. It is obtained in the appendix II

$$\Delta (v_{EB2} - v_{EB1}) = \frac{K \,.\, \Delta T}{q}\left(ln\frac{i_{E2}}{i_{E10}} + ln\frac{I_{ES1}}{I_{ES2}}\right) - \frac{KT}{q}\, ln\frac{T}{T_0} \tag{3.5}$$

where i_{E2} is constant and i_{E10} is the value of i_{E1} at the temperature T_0 and

$$i_{E1} = i_{E10}\frac{T}{T_0} \tag{3.6}$$

If $i_{E2} \gg i_{E10}$ (as in the Schmitt trigger if i_{E10} is the firing current) the first term of (3.5) is greater than the second; equation (3.5) is analogous to (3.4) but the variation is decreased by the second term.

All these results were obtained starting from the theoretical law of the rectification for *pn* junctions deduced by Shockley in his famous paper (10). However some departures from that law were observed and the logarithmic I, *V* caracteristic is not always verified. Theoretical and experimental investigations show a deviation from the Shockley law (11) (12) which is evident for silicon leading to a semilogarithmic slope greater than KT/q volts per *e* (natural log base); on the contrary the germanium follows fairly well the theory up to moderate values of injection currents.

Experimental tests made on germanium transistors (13) proved that the logarithmic law was very well verified on several decades with the semilogarithmic slope perfectly equal to KT/q so that it was possible to use this fact for a pulse multiplier.

4 — VARIATION WITH TEMPERATURE OF THE COMMON EMITTER CURRENT GAIN β

The variation of the *dc* common emitter current gain β causes a variation in the base current and therefore in the base potential. It is here intended β as the ratio between collector current and base current calculated without I_{CO}. For germanium alloy junction transistors the temperature dependence of β obeys fairly well to the following law

$$\beta_T = \beta_{T_0} \cdot e^{\varrho \Delta T} \tag{4.1}$$

The law is experimentally verified in the range between 0°C and 45°C. For higher temperatures the variation of β becomes irregular. The parameter ϱ varies with the current and decreases as the current increases. For transistors 2N384 we found in 1959 a value of ϱ between 0.01 and 0.018°C^{-1}. If the temperature varies from T_0 to T and if we neglect 1 in respect to β the variation in the voltage drop v_d caused by the base current is :

$$\Delta v_d = (v_d)_T - (v_d)_{T_0} = i_E R_B \left(\frac{1}{\beta_T} - \frac{1}{\beta_{T_0}}\right) = \frac{i_E R}{\beta_{T_0}} (1 - e^{-\varrho \Delta T}) \tag{4.2}$$

For the two transistors the voltage difference between the bases is

$$\Delta v_{d2} - \Delta v_{d1} = (1 - e^{-\varrho_1 \Delta T}) \frac{i_1 R_{B1}}{\beta_1 T_0} - (1 - e^{-\varrho_2 \Delta T}) \frac{i_2 R_{B2}}{\beta_2 T_0} \tag{4.3}$$

5 — EFFECT OF I_{CO} OF ONE TRANSISTOR ON THE OTHER

If the transistors are *dc* coupled it is apparent that the variations of I_{CO} of one transistor causes a variation on the base potential of the other. In fact I_{CO} of one transistor acts in opposite sense on the bases of the two transistors and the two variations contribute to increase the voltage difference between the bases. If the circuit is not simmetrical and/or there is only one *dc* coupling the full effect must be taken into account. If the circuit is simmetrical this effect adds to the effect of chapter 2 in the sense of increasing the error due to an imperfect matching of I_{CO} currents.

6 — EFFECT OF THE POWER DISSIPATION ON THE TRANSISTORS

If the transistors drain current, they dissipate. Therefore if the dissipations are not the same there is a difference in the temperature of the two transistors. Therefore there is a temperature effect on I_{CO}, β, and so on. The computation of this effect on I_{CO} is reported below. The other effects are normally negligible.

It is assumed that transistor T_2 dissipates more than transistor T_1. We call I_{CO} the reverse current of transistor T_1. In the transistor T_2 the reverse current is indeed $n\, I_{CO}$ where $n > 1$ is depending on the temperature difference caused by dissipation. Beeing the second derivative of I_{CO} in respect of temperature very little depending on the temperature, it is possible to suppose that both reverse currents will variate in the same ratio for the same variation ΔT of the temperature. So we have after a variation of temperature ΔT :
in transistor T_1 : mI_{CO}
in transistor T_2 : $mn\, I_{CO}$.
The voltage difference between the bases is varied by the quantity :

$$\Delta v_w = I_{CO} R_{B2} (mn - n) - I_{CO} R_{B1} (m - 1)$$

and if the resistances are the same :

$$\Delta v_w = (m - 1)\,(n - 1)\, I_{CO} R \tag{6.1}$$

where R is the resistance which is seen by each base.

7 — APPLICATION TO THE SCHMITT CIRCUIT

The criteria indicated in the preceding paragraphs were applied in the design of the threshold discriminator model DS3 and the single channel discriminator model DC1 of the transistor counting chain of Ispra [10]. The two discriminators are of the Schmitt trigger tipe, the first with *dc* coupling, the latter with *ac* coupling (fig. 5). The discussion on the opportunity of *dc* or *ac* coupling is far from the intention of this report and is widely discussed in the literature. (e.g. [11]). There only the differences in regard of temperature effects will be considered. The simplified schematics of the Schmitt circuits are reported from the complete instruments. The two diodes in series to the emitters absorb reverse voltage and allow the use of drift transistors which demonstrated to be very suitable for that circuit. Moreover the diodes have another task that is they reduce the hysteresis that in a circuit as in fig. 5a would be higher than 100 mV without diodes. Also this discussion is not in the purpose of this paper. The points indicated in the preceding paragraphs are here investigated with relation to the circuits of fig. 5. It is considered a variation of temperature from 20°C to 40°C.

a — Temperature effects of I_{CO}

In paragraph 2, methods for compensating temperature effects of I_{CO} are indicated. Transistors 2N 384 were first used because they have a function $I_{CO} = f(-V_c)$ very regular and of subhorizontal type.

Experimental tests demonstrated that it is sufficient the only measurement of one quantity for the acceptable accuracy. This is the value of I_{CO} for $V_c = -40$ V at a temperature of 20°C and the samples which have in this condition $I_{CO} \leqslant 1 \mu A$ are accepted. The 50% of samples were accepted with this test in 1959. Afterwards the quality of 2N 384 changed and it was no more possible this selection : we obtained some better result with 2N1177. Last year we had good results with Philips AF 118P with 90% of samples which passed the test.

Errors a) and b) of par. 2 were kept less than 5% and error c) results experimentally equal to about 5% for a temperature variation of 20°C. Adding all the errors a total variation of 1 mV max is obtained raising the temperature from 20°C to 40°C.

b — Variation of v_{EB}

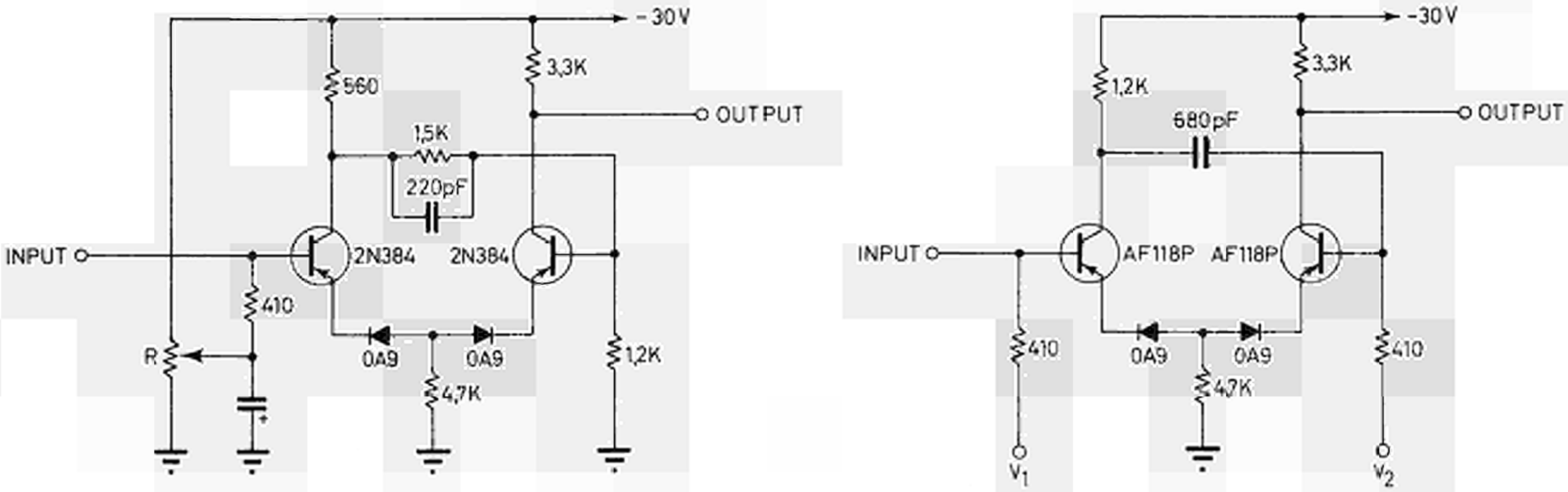
The current i_E of the left transistor in the instant of tiggering was proved by test to be directly proportional to the temperature as indicated in bibl. [3]. By applying equation (3.5) it was obtained a variation of 2.5 mV for the considered increment of temperature.

c — Variations of β

Equation (4.3) can be used but for pulses the contribute of the term due to transistor T_1 is negligible because the current of the first transistor is a signal current which acts on the low impedance of the signal driving point. The second transistor has a steady current of 2 mA and the mean value of ϱ was found to be 0.012°C^{-1}. This gives a variation of 3.6 mV for the considered variation of temperature. Measurements in *dc*, that is with the contribute of the emitter current i^*_{E1} of the first transistor, gave a variation of 3.2 mV.

d — Effect of I_{CO} of one transistor on the other

The first transistor acts on the second one increasing the potential of the second base of 0,8 mV max for the increase of temperature from 20°C to 40°C. This effect is only present in the circuit of fig. 5a and not in that of fig. 5b.



a) b)

Fig. 5 — Simplified schematics of *dc* coupled discriminator model DS 3 and *ac* coupled discriminator DC 1. Resistance *R* and values of the voltage dividers which give V_1 and V_2 are so arranged that for the minimum threshold the two bases in each circuit see the same *dc* resistance.

e — **Effect of power dissipation**

Eq. (6.1) gives a variation of 1.5mV for the stated variation of temperature.

f — **Effect of the variation of the voltage drop on the diodes**

For the diodes the effect is of the same type and magnitude as that due to the variation of v_{EB}. (equation (3.5)) .

g — **Effect of the variation of the emitter current i_{E1} corresponding to triggering**

The current of the first transistor corresponding to the triggering acts on the second transistor base as I_{CO} does but this effect has a sense only if measured in *dc*. For pulses it is necessary to consider the effect of charging the capacitances so that the voltage variation of the base of the second transistor is very little. For *dc* measurements the variations of i_{E1} with temperature following a law as (3.6) drives to a threshold variation of about 1 mV.

The effects due to the variation of temperature present some compensation. In fact an increase of temperature lowers the threshold for the effects b) d) f) g) and increases the threshold for the effects c) and e). The effect of point a) may be in both senses so it must be taken low with the appropriate compensation in itself. The total variation can be evaluated adding all the previous values. A figure of 2.1 mV for the *dc* coupled discriminator model DS 3 and some tenths of mV for the *ac* coupled discriminator model DC 1 were calculated. Experimental tests on 10 samples proved that only one had a variation of 3 mV that is 0.15 mV/°C. All the others were less than 0.1 mV/°C.

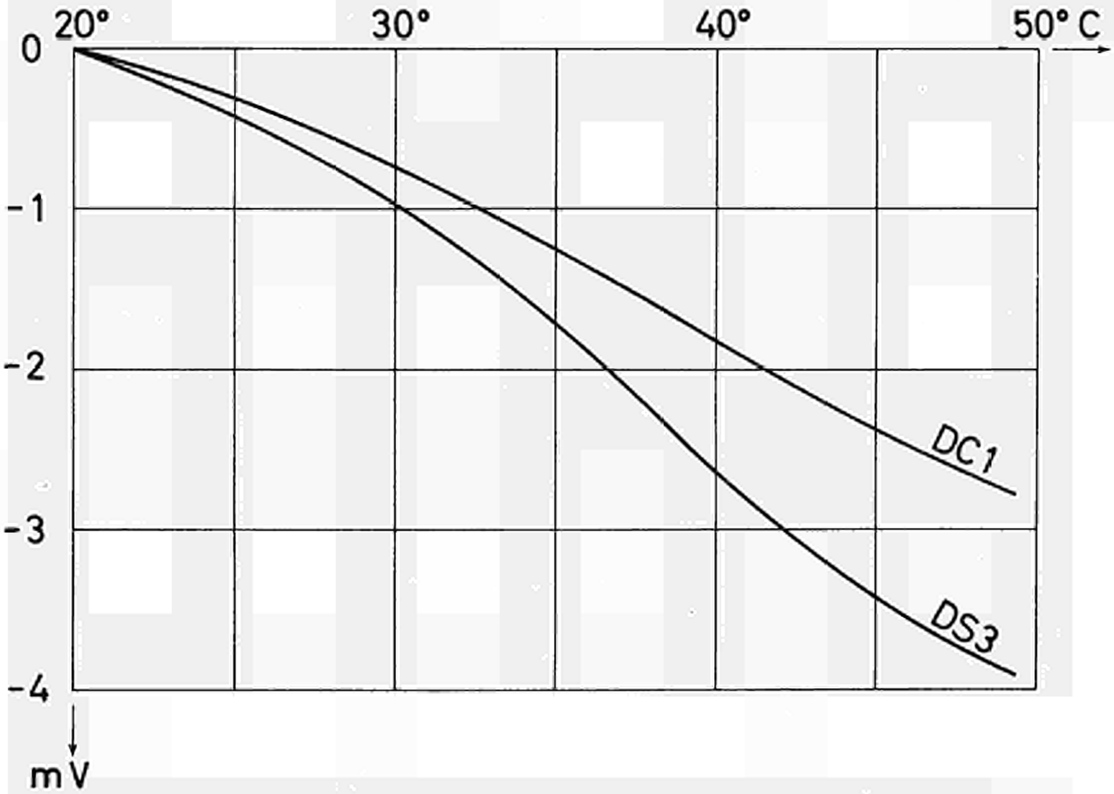


Fig. 6 — Tipical diagram of the threshold variation against temperature of the two discriminators.

Tipical curves representing the total variation for the two types of discriminators used in the transistor counting chain of ISPRA are reported in figure 6.

AKNOWLEDGEMENT

I wish to thank Mr. C. Köchler whose collaboration was precious, not only in deriving with an exceptional care and precision all the experimental data, but also in giving skilful suggestions in every theoretical investigation.

APPENDIX I

The equation (2.1) is immediatly derived writing down the general equations of the intrinsic transistors using the symbols of the IRE standards ([16]) :

$$\begin{cases} i_E = \dfrac{I_{E0}}{1-\alpha_N\alpha_I}\left(e^{qv_{EB}/KT}-1\right) - \dfrac{\alpha_I}{1-\alpha_N\alpha_I} I_{CO}\left(e^{qv_{CB}/KT}-1\right) \\ \\ i_C = -\dfrac{\alpha_N}{1-\alpha_N\alpha_I} I_{EO}\left(e^{qv_{EB}/KT}-1\right) + \dfrac{I_{CO}}{1-\alpha_N\alpha_I}\left(e^{qv_{CB}/KT}-1\right) \end{cases} \tag{A.1}$$

and considering the active region $v_{CB} \ll 0$ where the second terms into paranthesis are -1.

Writing :

$$i_B = -(i_E + i_C) \tag{A.2}$$

equation (2.1) is immediatly obtained.

APPENDIX II

From (A.1) and the following relations

$$I_{ES} = \frac{I_{EO}}{1-\alpha_N\alpha_I} \qquad I_{CS} = \frac{I_{CO}}{1-\alpha_N\alpha_I} \tag{B.1}$$

it is obtained in active region ($v_{CB} \ll 0$)

$$i_E = I_{ES}\left(e^{qv_{EB}/KT}-1\right) + \alpha_I I_{CS} = I_{ES}\, e^{qv_{EB}/KT} - (I_{ES} - \alpha_I I_{CS}) = I_{ES}\, e^{qv_{EB}/KT} - I_{ES}(1-\alpha_N) \tag{B.2}$$

The last term comes from the relation : $\alpha_I I_{CS} = \alpha_N I_{ES}$

The second term in (B.2) is very little and for normal values of current completely negligible. Considering that I_{ES} obeys a law of the type (2.2) :

$$I_{ES} = I_B\left(\frac{T}{T_0}\right)^3 e^{-qV_g/KT} \tag{B.3}$$

it is possible to write :

$$i_E = I_B\left(\frac{T}{T_0}\right)^3 e^{-q/KT\,(V_g - v_{EB})} \tag{B.4}$$

and from that

$$v_{EB} = Vg - \frac{KT}{q}\left(\ln\frac{I_B}{i_E} + 3\ln\frac{T}{T_0}\right) \tag{B.5}$$

and

$$\frac{dv_{EB}}{dT} = \frac{dV_g}{dT} - \frac{K}{q}\left(\ln\frac{I_B}{i_E} + 3\ln\frac{T}{T_0}\right) - \frac{KT}{q}\left(\frac{d\ln I_B/i_E}{dT} + \frac{3}{T}\right) \tag{B.6}$$

Two cases are considered :

a) *Constant emitter current*

$$\frac{dv_{EB}}{dT} = \frac{dV_g}{dT} - \frac{K}{q}\left(\ln\frac{I_B}{i_E} + 3\ln\frac{T}{T_0} + 3\right) \tag{B.7}$$

Adding and substracting V_g :

$$\frac{dv_{EB}}{dT} = \frac{1}{T}\left(Vg - \frac{KT}{q}\ln\frac{I_B}{i_E} - \frac{KT}{q}.\ 3\ln\frac{T}{T_0}\right) - \frac{1}{T}\left(Vg - T\frac{dVg}{dT} + 3\frac{KT}{q}\right) \tag{B.8}$$

Writing remembering (2.3)

$$V_A = V_g - T\frac{dV_g}{dT} + 3\frac{KT}{q} = V_{ge} + 3\frac{KT}{q} \tag{B.9}$$

and remembering (B.5) it is obtained :

$$\frac{dv_{EB}}{dT} = -\frac{1}{T}(V_A - v_{EB}) \tag{B.10}$$

b) *Emitter current proportional to the temperature.*

$$i_E = i_{EO}\frac{T}{T_0} \tag{B.11}$$

Using the same elaborations. from (B.6) :

$$\frac{dv_{EB}}{dT} = \frac{dVg}{dT} - \frac{K}{q}\left(\ln\frac{I_B}{i_E} + 3\ln\frac{T}{T_0}\right) - \frac{KT}{q}\left(-\frac{1}{T} + \frac{3}{T}\right) =$$

$$= \frac{dVg}{dT} - \frac{K}{q}\left(\ln\frac{I_B}{i_E} + 3\ln\frac{T}{T_0} + 2\right) = -\frac{1}{T}\left(V_A - v_{EB} - \frac{KT}{q}\right) \tag{B.12}$$

For the two cases we can calculate the voltage variation between the bases of the circuit of fig. 1. We apply the case a) with two transistors having constant currents and case b) with one transistor having constant current and the other a current proportional to the temperature.

I) both transistors having constant current (case a). Using (B.7) for a variation in temperature from T_0 to $T = T_0 + \Delta T$:

$$\Delta(v_{EB2} - v_{EB1}) = \int_{T_0}^{T}\left(\frac{dv_{EB2}}{dT} - \frac{dv_{EB1}}{dT}\right)dT = \frac{K\Delta T}{q}\left(\ln\frac{i_{E2}}{i_{E1}} + \ln\frac{I_{B1}}{I_{B2}}\right) \tag{B.13}$$

II) Transistor T_2 having constant current and T_1 a current proportional to the temperature as in (B.11). For a variation of temperature from T_0 to $T = T_0 + \Delta T$, using (B.7) and (B.12) :

$$\Delta(v_{EB2} - v_{EB1}) = \int_{T_0}^{T}\left(\frac{dv_{EB2}}{dT} - \frac{dv_{EB1}}{dT}\right)dT =$$

$$= -\frac{K}{q}\int_{T_0}^{T}\left(\ln\frac{I_{B2}}{i_{E2}} + 3\ln\frac{T}{T_0} + 3 - \ln\frac{I_{B1}}{i_{E10}}.\frac{T_0}{T} - 3\ln\frac{T}{T_0} - 2\right)dT = -\frac{K}{q}\int_{T_0}^{T}\left(\ln\frac{I_{B2}}{I_{B1}}.\frac{i_{E10}}{i_{E2}} + 1 + \ln\frac{T}{T_0}\right)dT$$

$$= -\frac{K}{q}\left[T\ln\frac{I_{B2}}{I_{B1}}.\frac{i_{E10}}{i_{E2}} + T + \left(T\ln\frac{T}{T_0} - T\right)\right]_{T_0}^{T} = \frac{K\Delta T}{q}\left(\ln\frac{I_{B1}}{I_{B2}} + \ln\frac{i_{E2}}{i_{E10}}\right) - \frac{KT}{q}\ln\frac{T}{T_0} \tag{B.14}$$

BIBLIOGRAPHY

1. R.D. Middlebrook — "An introduction to Junction Transistor Theory" John Wiley & Sons, Inc. 1957. pag. 174.
2. See for example R.H. Kingston — "Review of Germanium Surface Phenomena". *J. Appl. Phys.* Vol. 27 — Nov. 1956.
3. L. Stanchi — "A transistor Pulse — Height Discriminator — AERE — Trans. 855 (1960) from *Strumentazione e Automazione*. Vol. 6, 273 (1959) and *report CNI*-21.
4. E.M. Conwell — "Properties of Silicon and Germanium II". *Proc. IRE* 46, 1281 — june 1958.
5. R.D. Middlebrook — *op. cit.* pag. 292.
6. G.G. Farlane, T.P. Mc Lean, J.E. Quarrington and V. Roberts — *Phys. Rev.* **111**, 1245 (1958).
7. G.G. Farlane, T.P. Mc Lean, J.E. Quarrington and V. Robert — *Phys. Rev.* **108**, 1377 (1957).
8. Donald Long — "Energy Bands in Semiconductors". *Journal of Appl. Phys.* **33**, 1682 — may 1962.
9. Gärtner W. Wolgang — "Transistors : Principles and Applications" D. Van Nostrand Company Inc. 1960, pag. 25.
10. W. Shockley — "The bell Syst. *Tech. J.* vol 28, 435 — July 1949.
11. C.T. Sah, R.N. Noyce and W.S. Shockley — "Carrier Generation and Recombination in *p-n* Junctions Characteristics" *Proc. IRE* 45, 1228 — sept. 1957.
12. J.L. Moll — "The Evolution of the Theory for the Voltage-Current characteristic of *p-n* Junctions" *Proc. I.R.E.*, **46**, 1076 — june 1958.
13. G. Giannelli and L. Stanchi — "Nonlinear Operations on Pulses by Means of *p-n* Junctions " *Nuclear Instr. & Meth.* vol. 8, 79, July 1960.
14. G. Giannelli, V. Mandl and L. Stanchi — "The Transitor counting Chain of Ispra" *Report CNI*-78 (1961).
15. K. Kandiah — "A Highly Stable Pulse-Height Discriminator". Porc. of the Conf. on Nuclear Electronics Belgrade may 61 edited by I.A.E.A. vol. II, pag. 239.
16. IRE STANDARDS — On letter Symbols for Semiconductor Devices, 1956. *Proc. I.R.E.* 44, 934 — July 1956.