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AN EXPERIMENTAL INPUT-OUTPUT DECISION MODEL FOR IRELAND

by

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The object of the present exercise is to develop a model, based on the Input-Output (IO) approach which can be used for forecasting purposes. The model in type will be decisional, as distinct from the "onlooker" or purely prophetic. Decision models are entirely hypothetical though naturally the hypotheses must be reasonable and as restricted as possible. The model is designed to show, in fairly considerable industrial detail, the economic pattern in some future year of reference on the assumption of different rates of increase in GNP. However detailed, the pattern must be consistent in all its parts.

The Curtailed Irish Table

Perhaps the best way to explain the model is, in the first instance, to display an IO table, namely that for Ireland in 1956, in which, for arithmetical convenience, the number of industrial groups has been reduced from the original 36 to 6: groups numbered 2 and 3 have been introduced for their importance in capital formation. The data in the primary input section has also been recast very considerably. As a statistical presentation the figures in Table 1 are not to be taken too seriously. The data are designed for the purpose only of illustrating a method. Nor is it suggested that, even if the figures were correct, results useful for decision-making would emerge for so dimensionally small a table as in 6 industrial groups. The figures in Table 1 are, however, believed to be of the right order of magnitude.

The task facing the analyst is to produce, on various hypothetical bases, involving policy-decisions during the period from base to reference year, tables for the year of reference (which, for purposes of illustration here, will be taken as 1966, i.e. 10 (= T) years from the Irish IO base year 1956) on the lines of the basic IO table.

The Irish IO table is compiled on the "sellers' price" principle. Thus, on the first row, all the figures shown are valued at prices which farmers receive: e.g. agriculture etc sells £64 million at farmers' prices to the food etc industries, £66 million to households, total output being £181 million. Column 1 of the table shows the costs of agriculture; thus agriculture purchases £3 million from engineering at factory prices. A result of this sellers' price treatment is that the cost of transport and distribution of all classes of goods (the principal constituent in line 6 of the table) is very large; for instance, the £117 million in the household expenditure column includes £56 million for transport and trade services, including the transport costs and trade margins for the products of agriculture, industry etc, as well as imports which in the table are valued c.i.f.

The row for imports includes the value of all imports whether these are competitive with home industry or not. In such treatment the practice here differs from that of the Irish IO table which conforms with the more common practice adopted by other countries of assigning competitive imports to the cells pertaining to home production. While the present author is rather doubtful of the competitiveness of most imports into Ireland - is Manitoba wheat competitive with Irish wheat in a normal year? - he is not concerned to make a major point of this issue. It is simply more arithmetically convenient for illustrative purposes to use a single line for imports. The validity of the model to be described is not impaired by the treatment of imports.

There is a considerable departure from the Irish IO table in the primary input section of Table 1. Thus row 1 of this section represents the disposable (i.e. after direct taxation) income of households (by way of employee compensation, dividends and non-corporate profits after tax). Row 2 contains all public authorities income including income from property and entrepreneurship as well as taxes. Thus in the industrial part of the row are included direct taxes on employees, rates on business premises, import and excise duties on materials and products etc. However, the £25 million on the row in the household column is made up, for the greater part, of rates on dwelling-houses and import duties on consumer goods ready for use, i.e. this item is closely associated with imports valued c.i.f. at £59 million and net rent included in the £117 million for services.

Row 4 in the primary input section directs attention to a special difficulty in IO work. In the industry part of the row the figures relate necessarily to companies, for the saving of non-corporate enterprises is, for the greater part, indistinguishable from saving of households and must be included therein, i.e. in the £22 million for households in Table 1. If, as seems likely in the future, the corporate proportionate share in the economy increases, then so will the coefficients pertaining to saving in the industrial sectors.

Row 6 in this section represents profits on externally-owned enterprises to the total of £17 million. The obverse of this item, namely factor income from abroad of £41 million has been arbitrarily assigned altogether to households.

National Accounting Identities

The object of the adjustments (compared with the original IO table) is to enable us to produce directly from Table 1 all the major national accounting identities. It is an invariable feature of IO tables that the totals of corresponding rows and columns in the interindustry section should be identical; see, for example, that the figure of £181 million for agriculture at the end of row 1 agrees with the figure at the end of column 1. In addition in Table 1 the row and column totals for primary input and final demand have been brought into close agreement. Thus household and government income (£401 million and £137 million respectively) coincide with the column totals for expenditure and saving - with a negative entry for government transfers to households (including interest on the public debt) of £55 million. Gross capital formation of £87 million (stock changes have been taken as nil) is financed by saving £38 million, net investment from abroad £21 million (or a total of £59 million shown in the last column of Table 1) and depreciation £28 million. Finally the external account: imports of goods and non-factor services(£194 million) together with factor imports (£17 million) equal non-factor exports (£149 million), factor exports (£41 million) and net investment from abroad (£21 million) or a total of £211 million, shown at the foot of the export column.

Table 2 displays unitary coefficients derived from the data in Table 1. This differs from the more usual table of coefficients in that it covers not only the interindustry sector of the table but also the primary input and final demand parts for reasons which, it is

Table 1. Summary Input-Output Table for Ireland, 19[illegible]6

Input \ Output	Interindustry 1	2	3	4	5	6	Total inter-industry	Final demand: Consumption: House-holds	Gov-ern-ment	Fixed capital	Stock changes	Exports	Total final demand	Output = input
Non-factor input														
1 Agriculture, forestry, fishing	2	0	0	64	2	0	68	66	1	0	0	46	113	181
2 Construction	0	2	0	0	1	2	5	4	11	49	0	0	64	69
3 Engineering	3	4	3	0	2	4	16	8	0	8	0	2	18	34
4 Food, drink, tobacco	14	0	0	22	0	0	36	105	0	0	0	38	143	179
5 Other industry	8	12	4	5	25	7	61	50	2	10	0	26	88	149
6 Services	10	7	3	7	8	7	42	117	46	0	0	32	195	237
Total home	37	25	10	98	38	20	228	350	60	67	0	144	621	849
Imports	10	8	11	22	47	11	109	59	1	20	0	5	85	194
Total non-factor input	47	33	21	120	85	31	337	409	61	87		149	706	1043
Primary input														
1 Disposable household income	120	32	9	19	32	148	360	–	–	–	–	41	41	401
2 Government income	9	2	2	41	21	37	112	25	–	–	–	–	25	137
3 Transfer payments	–1	0	0	–10	0	–6	–17	–55	72	–	–	–	17	0
4 Saving etc	0	1	1	2	3	7	14	22	2	–	–	21	45	59
5 Depreciation	6	1	1	3	4	11	26	–	2	–	–	–	2	28
6 Profits paid abroad (imports)	0	0	0	4	4	9	17	–	–	–	–	–	–	17
Total primary input	134	36	13	59	64	206	512	–8	76	–	–	62	130	643
Input = Output	181	69	34	179	149	237	849	401	137	87		211	811	

Table 2. Unitary Coefficients derived from Table 1 with Formulae for Stock Changes

Input \ Output	Interindustry 1	2	3	4	5	6	Final demand: Households	Government	Fixed capital	Stock changes	Exports
Non-factor input											
1 Agriculture, forestry, fishing	.0110	0	0	.3575	.0134	0	.1521	.0159	0	$.0780Y_1 - 14.11$	.3087
2 Construction	0	.0290	0	0	.0067	.0084	.0092	.1746	.5632	$.0219Y_2 - 1.51$	0
3 Engineering	.0166	.0580	.0882	0	.0134	.0169	.0184	0	.0920	$.0279Y_3 - 1.01$	.0134
4 Food, drink, tobacco	.0773	0	0	.1229	0	0	.2419	0	0	$.0197Y_4 - 3.52$	.2550
5 Other industry	.0442	.1739	.1176	.0279	.1678	.0295	.1152	.0317	.1149	$.0256Y_5 - 3.81$	.1745
6 Services	.0552	.1014	.0882	.0391	.0537	.0295	.2696	.7302	0	$.0295Y_6 - 7.00$	.2148
Total home	.2044	.3623	.2941	.5475	.2550	.0844	.8065	.9524	.7701		.9664
Imports	.0552	.1159	.3235	.1229	.3154	.0464	.1359	.0159	.2299		.0336
Total non-factor input	.2597	.4783	.6176	.6704	.5705	.1308	.9424	.9683			
Primary input											
1 Disposable household income	.6630	.4638	.2647	.1061	.2148	.6245	–				
2 Government income	.0497	.0290	.0588	.2291	.1409	.1561	.0576				
3 Transfer payments	-.0055	0	0	-.0559	0	-.0253					
4 Saving etc	0	.0145	.0294	.0112	.0201	.0295					
5 Depreciation	.0331	.0145	.0294	.0168	.0268	.0464		.0317			
6 Profits paid abroad (imports)	0	0	0	.0223	.0268	.0380					
Total primary input	.7403	.5217	.3824	.3296	.4295	.8692					
Input = Output	1	1	1	1	1	1	1	1	1		1

Table 3. Algebraic Notation of the Model

Output / Input	Interindustry 1	2	...	n	Final demand: Households	Government	Fixed capital	Stock changes	Exports	Output = Input
Industry-input										
1	a_{11}	a_{12}	...	a_{1n}	h_1	g_1	v'_1	$p_1(Y_1 - Y_{10})/T$	x_1	Y_1
2	a_{21}	a_{22}	...	a_{2n}	h_2	g_2	v'_2	$p_2(Y_2 - Y_{20})/T$	x_2	Y_2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	a_{n1}	a_{n2}	...	a_{nn}	h_n	g_n	v'_n	$p_n(Y_n - Y_{n0})/T$	x_n	Y_n
Imports	b_1	b_2	...	b_n	h_m	g_m	v'_m		x_m	
Primary input										
1 Disposable household income	c_{11}	c_{12}	...	c_{1n}	–	–	–	–	(F)	
2 Government income	c_{21}	c_{22}	...	c_{2n}	h_g	–	–	–	–	
3 Transfer payments	c_{31}	c_{32}	...	c_{3n}	(D)	(B)	–	–	–	
4 Saving etc	c_{41}	c_{42}	...	c_{4n}	(S_h)	(S_g)	–	–	(N)	
5 Depreciation	c_{51}	c_{52}	...	c_{5n}	–	d	–	–	–	
6 Profits paid abroad (imports)	c_{61}	c_{62}	...	c_{6n}	–	–	–	–	–	
Input = Output	Y_1	Y_2	...	Y_n	H	G	V'	V"	X	

hoped, will be evident from what follows: in a word this procedure is designed to enable us to bring all the major national accounting entities into our model. It will be noted, by comparison with the entries in Table 1, that subsidies, saving, foreign investment and factor income receivable have been ignored. These elements will be seen to be the strategic variables in the model.

The notation to be used for setting down the equations and identities of the model is displayed in Table 3. Workers in this field have not yet succeeded in evolving a satisfactory algebraic notation for IO work and notationally Table 3 will probably be found to be no exception to this sorry experience. The significance of the symbols may be clear from the illustrative Tables 1 and 2, noting that small letters denote unitary coefficients and capital letters values (in £ million). The number of industrial groups is n (= 6 in the example) and T is the time period between base year and year of reference. The entries in the stock changes column will be explained later. Brackets ( ) around F, D etc indicate that the corresponding values are not deemed included in the values H, G etc at the foot of the column.

Final Demand Categories

It will be useful to consider briefly the various categories of final demand:-

Households. The coefficients h_i cannot be accorded the kind of quasi-predetermination with which the interindustry coefficients a_{ij} are customarily endowed. As is well-known, these coefficients will depend on the *average* level of household expenditure, in accordance with Engel's Law. They are functions of this average level. If the economy is generally advancing at a given rate, say 4%, total household expenditure is likely to rise at about the same rate. If we assume, as we shall, that the proportionate rise in population is the same as that of the labour force, then the values of the h_i will depend on the evolution of the labour force and, therefore, on labour productivity. If the labour force increases at the same rate as GNP then productivity remains constant at its base year level and there is no logical reason for changing the h_i for the year of reference. On the other hand if total household consumption is to increase at the rate of r% and the labour force by s% then household expenditure on average will increase by (r - s)% approximately. It is this (r - s)% or labour productivity which determines the value of the h_i. By way of illustration C. E. V. Leser has kindly supplied the following data (which, however, are to be regarded only as rough approximations at this stage) for the coefficients in 1966 on the assumption of a 3% a head a year growth in total consumption. The "actual" 1956 coefficients are shown for comparison. The 3% increase is consistent with a rise of 4% in total consumption and 1% rise in population.

Table 4. Unitary Patterns of Household Consumption

	1956	1966
Home industry		
1. Agriculture, forestry, fishing	.1614	.1481
2. Construction	.0098	.0115
3. Engineering	.0196	.0263
4. Food, drink, tobacco	.2567	.2355
5. Other industry	.1222	.1113
6. Services	.2861	.3315
Imports	.1442	.1358
Total	1	1

The coefficients for 1956 in Table 4 differ slightly (but proportionately) from those shown in Table 2 since the latter total 0.9424. The marked decline in the proportions for agriculture and food in 1966 is the familiar Engel's phenomenon. When one considers that in the 10 years a rise in consumption of 3% a head a year is equivalent to a rise of 34% the changes are not very marked.

The outcome of the application will depend on the view taken with regard to the productivity increase (r - s)%. It must be deemed advisable to produce answers for all reasonable levels of productivity. As will be pointed out in the concluding section of this paper, national planners using the present model will have a wide choice before them but will have ample opportunities of modifying the coefficients and therefore the original targets of the plan selected as the time-period of the plan advances.

Government expenditure. This is the strategic area over which public authorities have absolute control, in theory at any rate. It is therefore an area in which it would be well to try many experiments with the model. The government pattern as time evolves must be conditioned by actions in the private sector, for example if private saving is insufficient for the plan the government may have to create forced saving by taxation; or, if private investment in certain sectors is insufficient for the attainment of the prescribed targets, government may have to step in. With a large IO table available presented on the lines indicated here the planning authorities could experiment with many alternative patterns with a view to determining the optimal course of action.

Fixed capital formation. At first sight it might appear desirable to evolve formulae for gross fixed capital formation (GFCF) industry by industry consistent with rises from Y_{io} to Y_i between base and reference years in gross output of industry i. From international experience during the post-war period it would in fact be easy to find the relation between rate of increase in each broad industrial sector and the rate of increase in GFCF, even in constant price terms. Such an exercise would be rather different in concept from the more usual incremental net capital-output ratio in which the entities studied are net annual increase in added value and net fixed capital formation (or the net increment in physical capital). The main reason for the difference in approach is that the IO table deals essentially with gross entities though, of course, added value, industry by industry, is derivable from the primary input table as well as net capital formation as the difference between the GFCF column and the depreciation row. In recent years, however, an increasing number of economists tend to favour the GFCF approach as distinct from the net and not only for the reason of the notorious statistical unreliability of depreciation statistics. Such economists take the view that on the alleged mere replacement (i.e. depreciation), as distinct from a net increase of a physical capital good, there is likely to be an increment in productivity because replacements are rarely identical with the goods they purport to replace and are more than likely to incorporate improvements. If one be allowed to assume an arithmetical annual increase in the economy there would be little difficulty in evolving algebraic formulae based on international experience for GFCF, on the lines of the formulae below for stock changes.

The writer is, however, rather sceptical about the value of such an exercise, though he remains open to conviction; and, should another view be taken, there would be no difficulty about changing the model in this, which is a mere detail. He bases his scepticism on the following statistics pertaining to the whole economy of 20 countries during the period 1953-59:-

Country	Ann.av. increase GNP	Av.ratio GFCF to GNP	Country	Ann.av. increase GNP	Av.ratio GFCF to GNP
	%			%	
Germany (F.R.)	6.5	0.219	continued Portugal	3.4	0.155
China (Taiwan)	6.3	0.135	Cyprus	3.1	0.245
Greece	6.3	0.108	Ceylon	2.9	0.117
Austria	6.2	0.216	Norway	2.9	0.305
Italy	5.3	0.205	Denmark	2.8	0.166
Netherlands	4.3	0.235	Belgium	2.5	0.160
France	4.2	0.179	U.S.A.	2.4	0.170
Porto Rico	4.2	0.192	U.K.	2.1	0.146
Sweden	3.6	0.208	Chile	1.7	0.104
Canada	3.4	0.246	Ireland	-0.1	0.147

Source: Based on data in UN Yearbook of National Accounts 1960

Countries are arranged in descending order of rate of increase in GNP. It is true that there are certain regularities in the table and perhaps it is easy to account for some of the low ratios as well as the exceptionally high ones in regard to rates of increase. For instance the low ratios in Greece and China may have been due to increased labour intensity and the high ratio in Norway to investment in shipping which is highly capital-intensive. It will be noted that the U.K. and Irish ratios are practically identical. Theoretically there can be no qualifications (for reasons of differential population changes or otherwise) as to the validity of the well-known formula

$$V = krY$$

where Y is net national product, V net fixed capital formation, k the incremental capital-output rate and r the rate of increase. At any level of the capital-output ratio the value of V/Y should accordingly increase with r. It was really with a view to examining whether such a relationship obtained in fact that the foregoing table was prepared.

It would appear that, at this stage, the most sensible course would be to adopt experimental, but reasonable value or values of the ratio q given by

(1) $$V' = qZ$$

where Z is the gross domestic product given by

(2) $$Z = \sum_{j=1}^{n} c_j Y_j$$

with

$$c_j = \sum_{i=1}^{6} c_{ij}.$$

Furthermore, the value of V' can be distributed proportionately amongst the industries using the formula

(3) $$V_i' = v_i' V'.$$

For the application to Ireland which follows, q will be taken as 0.2. Such a ratio there will allow some margin for manoeuvre. For example, if the demand for economic investment should increase, social investment (e.g. in dwellings) could be postponed, to keep total fixed capital investment within the planned aggregate in the year of reference.

Stock changes. For a growing economy allowance must be made for changes in stock, industry by industry. It would appear reasonable, as certainly it is algebraically convenient, to try to express changes in terms of gross value of output of the industry, the marginal figure of the IO table. For the six industrial groups the relevant figures for 1956 are as follows:-

Industrial group	Gross output 1956	Stock end of 1956	Per-cent-age
	£m	£m	
1 Agriculture	181	141.1	78.0
2 Construction	69	15.1	21.9
3 Engineering	34	10.1	29.7
4 Food, drink, tobacco	179	35.2	19.7
5 Other industry	149	38.1	25.6
6 Services	237	70.0	29.5
Total	849	309.6	36.5

If the gross output of industrial group i be Y_i in the reference year and Y_{io} in 1956, the base year and if the stock ratio p_i be assumed to apply throughout then, in the reference year the increase in stock may be taken as

$$(4) \qquad V_i'' = (Y_i - Y_{io})p_i/T.$$

Admittedly this formula is not very satisfactory in that it assumes an arithmetical rate of increase between base and reference years, whereas one would prefer the geometrical (or "compound interest") hypothesis. The arithmetical formula has the immense advantage that thereby the equations in the IO model displayed below are maintained linear.

Applying the formula to the foregoing data, following are the actual formulae for stock increases in the reference year:-

Formulae for increases in stock in reference year.

	Industrial Group		Increases in stock
1	Agriculture	:	$-14.11 + 0.0780Y_1$
2	Construction	:	$-1.51 + 0.0219Y_2$
3	Engineering	:	$-1.01 + 0.0297Y_3$
4	Food, drink, tobacco	:	$-3.52 + 0.0197Y_4$
5	Other industry	:	$-3.81 + 0.0256Y_5$
6	Services	:	$-7.00 + 0.0295Y_6$

Exports. The coefficients x_i in this column of Table 3 are the least stable in the model. There is no reason to suppose that proportions obtaining in 1956 will obtain in any future year of reference. Clearly the future pattern depends on external demand. Many alternative reasonable patterns may be postulated for exports, however, and the model will supply the whole consequential economic pattern. The model, applied to the detailed IO table, will identify the exports which it is in the country's interest to promote.

The Equations of the Model

With number of industrial groups n and period from base to reference year T the equations (by reference to the notation in Table 3) are as follows:-

(i) Interindustry: $\Sigma_j a_{ij} Y_j + h_i H + g_i G + v_i' V' + p_i (Y_i - Y_{io})/T + x_i X = Y_i$, $i = 1, 2, \ldots, n$

(ii) Gross domestic product: $Z = \Sigma_j c_j Y_j$

(iii) Gross fixed capital formation: $V' = qZ$

(5) (iv) Change in stock: $V'' = \Sigma_i p_i (Y_i - Y_{io})/T$

(v) Household: $\Sigma_j c_{1j} Y_j + F = H + D + S_h$

(vi) Government: $\Sigma_j c_{2j} Y_j + h_g H = G + B + S_g$

(vii) Transfer payments (redistribution): $\Sigma_j c_{3j} Y_j + D + B = C$

(viii) Saving - capital: $\Sigma_j (c_{4j} + c_{5j}) Y_j + S_h + S_g + dG + N = V' + V''$

(ix) External: $\Sigma_j (b_j + c_{6j}) Y_j + h_m H + g_m G + v_m' V' + x_m X = X + F + N$

These equations simply give algebraic expression to the accounting identities of the type shown in Table 1. Amongst the (n + 8) relations one is redundant as will be seen by adding both sides of (i) (n equations) (v), (vi), (vii), (viii) and (ix). On using the unitary properties of the coefficients it will found that the left side is identical with the right side. There are accordingly (n + 7) independent relations in general in the system. Relations (ii), (iii) and (iv) are definitional in character. In any single experiment the coefficients, i.e. the small letters, are going to be regarded as given. The variables are therefore $Y_1, Y_2, \ldots, Y_n$; H, G, V', V'', X, Z, F, D, S_h, S_g, B, N numbering (n + 12). There are accordingly 5 degrees of freedom in the system; 5 variables have to be assigned predetermined values to solve the system; or all or some of them may be additional relationships.

Two Applications to Irish Data

Both will be based generally on the unitary coefficients in Table 2. The object of the calculations was to discover if there were any snags in the working of the model in practice, and to see if it yielded reasonable answers. Otherwise these arithmetical exercises, using a desk multiplying machine, would be quite unnecessary since, even with a large-dimensional IO matrix, the calculations would be trivially simple and speedy on a digital computer. This point will be developed later.

Even though there are 12 variables to be calculated from 12 linear simultaneous equations the system is not as formidable, even for hand calculation, as might at first sight appear. In fact, once the n equations (i) have been solved for the Y_i in terms of H, G, V' and X and the resulting expressions substituted in the remaining equations, these turn out to be of very simple type. They are solved for whatever set of variables one cares to regard as dependent or endogenous. These values are then substituted in the linear expressions for the Y_i yielding the values of these variables.

From now on, the 5 predetermined variables or relations will be termed the <u>instruments</u>. For both applications we shall make the following assumptions:-

(i) that the time period is 10 (= T) years, the base year being 1956 so that the year of reference is 1966;

(ii) that the economy, deemed measured by gross domestic product Z, is advancing by 4% a year, i.e. in 1966 Z is given as

$$\overline{Z} = \overline{Z}_o(1.04)^{10} = 512 \times 1.480245 = 757.9;$$

(iii) from (1) above $\overline{V}'$ is then known as

$$\overline{V}' = q\overline{Z} = 0.2 \times 757.9 = 151.6;$$

(iv) that government current expenditure G will advance at about the same rate as GDP in fact by 50%:

$$\overline{G} = 1.50\overline{G}_o = 1.50 \times 63 = 94.5;$$

(v) that factor income from abroad F (£41 million in 1956) has advanced to

$$\overline{F} = 50;$$

(vi) that government transfer payments (£72 million in 1956) in 1966 are

$$\overline{B} = 80,$$

and that the unitary pattern of government expenditure i.e. the g_i and g_m of Tables 2 and 3 were the same in 1956 and 1966.

In equations (5)(i) the coefficient matrix of the Y_i is

$$P = [(1 - p_i/T)d_{ij} - a_{ij}],$$

where $d_{ij} = 1$ when $j = i$, zero otherwise. On inverting this matrix using the coefficient data in Table 2 we find

$$P^{-1} = \begin{vmatrix} 1.1391 & .0037 & .0026 & .4756 & .0190 & .0007 \\ .0011 & 1.0566 & .0035 & .0011 & .0115 & .0059 \\ .0241 & .0758 & 1.1386 & .0117 & .0214 & .0218 \\ .1027 & .0004 & .0003 & 1.2092 & .0017 & .0001 \\ .0725 & .2439 & .1707 & .0727 & 1.2488 & .0445 \\ .0775 & .1347 & .1110 & .0834 & .0756 & 1.0678 \end{vmatrix}$$

The foregoing data were common to both applications.

Application I.

It was assumed that in 1966 the unitary pattern of household expenditure and exports were those of 1956, i.e. as given in Table 2. In addition the country tries to budget for an export surplus equal to 5% of exports i.e.

$$N = - 0.05X.$$

We now have all the data required to find the values of the Y_i and the macro-economic variables in 1966. Solving for the Y_i from (5)(i) by using the transpose of the inverse matrix $(P^{-1})'$ in the usual way we find for the Y_i

	H	G	V'	X	Const
Y_1 =	.2908	.0199	.0045	.4768	-17.83
Y_2 =	.0131	.1892	.5967	.0039	-1.70
Y_3 =	.0365	.0302	.1499	.0341	-1.88
Y_4 =	.3084	.0018	.0004	.3404	-5.71
Y_5 =	.1899	.1158	.2966	.2707	-6.89
Y_6 =	.3318	.8069	.0948	.2892	-9.47

e.g. the formula for Y_1 reads

$$Y_1 = .2908H + .0199G + .0045V' + .4768X - 17.83.$$

On substituting these values for the Y_i in the remaining equations of the model we find

Eq.	H	X	D	S_h	S_g	V''		
(ii)	.7077	.8479					=	614.7
(iv)	-.0447	-.0604				1	=	-25.6
(v)	-.5107	.6018	-1	-1			=	-156.2
(vi)	.2238	.1871			-1		=	152.5
(vii)	-.0272	-.0290	1				=	-78.3
(viii)	.0102	-.0483		1	1		=	110.7

Ostensibly in 6 variables this system can readily be reduced to two equations, in H and X only, namely (ii) and the sum of (v) - (viii). The values of all the macro variables in the system are then found as follows with the 1956 values inserted for comparison:-

APPLICATION I

Macro-economic variables		1956 £m	1966 £m	Percentage increase %
Z	Gross domestic product	512	757.9*	48*
V'	Gross fixed capital formation	87	151.6*	74*
V''	Change in stock	-	15.1	-
H	Household expenditure	434	542.6	25
F	Factor income from abroad	41	50.0*	22*
S_h	Household saving	22	98.5	348
S_c	Company saving (including depreciation)	40	59.0	48
D	Transfers to households	55	55.7	1
G	Government expenditure (including depreciation)	63(2)	94.5*(3.0)	50*
B	Government transfers	72	80.0*	11*
S_g	Government saving	2	19.8	890
X	Exports	149	272.1	83
M	Imports (including profits paid abroad)	211	307.5	46
N	Import excess (net extern investment in the State)	21	-13.6	-
Individual industries				
Y_1	Agriculture, forestry, fishing	181	272.3	50
Y_2	Construction	69	114.8	66
Y_3	Engineering	34	52.8	55
Y_4	Food, drink, tobacco	179	254.5	42
Y_5	Other industry	149	225.7	51
Y_6	Services	237	339.9	43

*Predetermined instrumental values

Readers who may trouble to cast up the external equation ((5)(ix) in the model) will notice a discrepancy of about £1 million. This (which also appears in Application II) was due to an unfortunate small error in copying the matrix P prior to inversion. Since these calculations are only illustrative, and since the error does not affect the inferences anyway, it did not seem worth while to correct the figures. This experience points, however, to the value of equation (5)(ix) (or whichever equation one selects as redundant) in the model for checking purposes.

It will be noted that two new variables have been introduced, namely company saving S_c and imports M. In fact the whole of the IO table for 1966 could be reproduced from the foregoing data, using the coefficients in Table 2 and we are assured that the table would be consistent in all its parts; for instance sales of agriculture to the food etc industry (£64 million in 1956) would be £97.3 (= 0.3573 x 272.3) million in 1966. A large superstructure has accordingly been built on a very few instrumental values.

The strategy involved in Application I would be a very bad one. There is no prospect that householders could be induced to make such a prodigious increase in saving as indicated, or, in other words, that they would make such a sacrifice in consumption - note the rise of only 25%. The figure of £22 million for saving shown for 1956 is unrealistically low, however: on recent experience the figure should be equivalent to about 10% of household expenditure; the figure of £98.5 million equals 18%. There are other anomalies amongst the macros. On the other hand, for what they are worth, the rises shown in the individual industrial groups seem reasonable enough - apart from one's feeling that the rise in gross agricultural output of 50% in 10 years is beyond the bounds of optimism.

It is highly significant that our broad judgment of the validity of the strategy implied in choice of instruments was based on the macros. It might be argued that it is a point in favour of the present model that the macros are an integral part of it; and the national strategy must be based on the macros.

Application II.

For the second exercise it was decided that household saving S_h would be fixed at 12% of household expenditure, so that an instrumental relation would be

$$S_h = 0.12H,$$

leaving the import balance N endogenous. In addition the Leser pattern of household expenditure* for 1966 was adopted. Also it seemed that export proportions should be modified: the following figures assumed for 1966 should be regarded as purely experimental:-

	Export coefficient x_i	
Industry i	1956	1966
1	.3087	.25
2	0	0
3	.0134	.02
4	.2550	.25
5	.1745	.21
6	.2148	.24
Imports	.0336	.03
	1	1

These two sets of coefficients are believed to be more realistic than those for 1956 used in Application I. The solution proceeds on almost identical lines as before and it may not be necessary to reproduce the details of the calculation. Following are the results:-

*See page 6.

APPLICATION II

Macro-economic variables		1956 £m	1966 £m	Percentage increase %
Z	Gross domestic product	512	757.9*	48*
V'	Gross fixed capital formation	87	151.6*	74*
V''	Change in stock	–	13.2	–
H	Household expenditure	434	570.7	31
F	Factor income from abroad	41	50.0*	22*
S_h	Household saving	22	68.5	211
S_c	Company saving (including depreciation)	40	60.2	51
D	Transfers to households	55	56.0	2
G	Government expenditure (including depreciation)	63(2)	94.5*(3.0)	50*
B	Government transfers	72	80.0*	11*
S_g	Government saving	2	21.3	965
X	Exports	149	245.2	65
M	Imports (including profits paid abroad)	211	306.0	45
N	Import excess	21	11.8	-44
Individual industries				
Y_1	Agriculture, forestry, fishing	181	237.0	31
Y_2	Construction	69	116.2	68
Y_3	Engineering	34	59.0	74
Y_4	Food, drink, tobacco	179	236.5	32
Y_5	Other industry	149	227.4	53
Y_6	Services	237	372.3	57

* Predetermined instrumental values

Application II affords a much more reasonable outcome than does Application I. Amongst the macros the only bizarre figure is that for S_g, government saving, (also large in Application I). This may be regarded as a reflection of the too low instrumental value of £80 million for transfers – will income redistribution be relatively higher in the future than in the recent past? On the other hand, government may have to take over a larger share of economic investment to attain objectives in the future. The rise of only 31% in household consumption is a reminder that no economic advance is conceivable without sacrifice. The external deficit of £11.8 million is of modest dimensions. As regards industries, the showing of agriculture is now much more rational - it may even be conceivable that a ten-year increase of 31% is within the bounds of possibility.

Of course there is no attempt here to discuss the economic implications of the model. The 6 x 6 IO table used for illustrative purposes does not show up what may possibly be bottlenecks to development, in setting targets for individual industries which they may feel is beyond their capacity.

Some Remarks

The proposed decision model is of great simplicity; at the same time the model is a comprehensive one. The writer regards the function of these models as strictly limited and that over-elaboration is at all costs to be avoided. It is possible to be consistent at any level of detail; on the one hand it is quite a useful exercise to speculate on the macro-economic entities alone within the frame-work of the national accounts* (which are equivalent to a 1 x 1 IO table!) but this tells us nothing about the industrial pattern and so the exercise does not constitute blue-print for a Plan. On the other hand if the Plan is prepared in too great detail there is pro tanto a lesser degree of flexibility. The proper course would appear to be to set out the preferred blue-print in somewhat general terms and place oneself in a position to modify its details as the period of the Plan advances, with as little interference as possible

*E.g. the writer's stencilled memoranda entitled "A Simple Macro-economic Growth Model", Parts I-III.

with its rather general lines.

It will be convenient to discuss the model under two broad headings

(1) work to be done on the model itself to make it a better instrument; and

(2) the use of the model for planning.

The two headings are not, of course, distinct. Most of these concluding remarks will, however, be confined to (1).

The object is to produce an IO table for the future year (or years) of reference. Accordingly all the coefficients should ideally be those pertaining to that year which, of course, are unknown. Our best efforts must be directed towards making them realistic in the experimental series proposed. As regards the technical coefficients, i.e. the a_{ij} and b_i of Table 2, the writer recalls a conversation with Wassily Leontief, the inventor of IO, in Harvard some years ago. Leontief was very much exercised with this problem and his approach to "modernization" (or perhaps one might write "futurization") of these coefficients was an interesting one. He and his assistants were trying to set up a coefficient system based on a sample of establishments in the different industries which were founded within the previous five years, on the assumption that these would yield the average pattern for years ahead. This is an approach which it should be easy to try out in this country. Also it might be useful to consult experts in the different industries about the interindustry coefficient matrix of the future.

As regards the import and primary input sections of the IO table, many experiments may be made within the present model. One important question is: what will the effect be of the probably more than proportionate increase in imports of materials for all or some industries under the freer trading conditions in future. Unless exports respond, balance of international payments difficulties will arise. In connection with the examination of this problem there will be no trouble within the model about expanding the single line for imports into as many headings as one desires; or of examining the desirability of promoting import substitution (by home products) on the most efficient lines when this course is considered expedient.

One cannot assume the permanence of the relationship in each industry of the proportion borne by primary input to gross output, i.e. the aggregate coefficients c_j. From a rather cursory examination over a fairly extended period of years in the ratio net output to gross output of the different industries the writer has observed no very marked trend: the data will bear further analysis. The writer has far less objection than most of his colleagues to the heinous practice of extrapolation - when confined to coefficients, as distinct from absolute figures. With the primary input section the categories can be further elaborated within the model. One can easily examine the effect of changing the government income coefficients, for example.

As regards the coefficients in the final demand part of the IO table, the best method would be to try out many alternatives. The household consumption coefficients h_i are probably the most inflexible and therefore predictable; the export coefficients x_i are the most flexible - here especially there must be many experiments to devise the export lines which would be most profitable (in terms of GNP) to push. Inevitably agriculture (with its larger internal factor content) will receive a high export rating. The possibility of devising the optimal export distribution subject to constraints, with GDP (Z) as the preference function, using linear programming, is now being examined in the Institute.

The solution of the model for any given set of initial instrumental values or relations - which, by the way, may be any 5 selected from the whole range of variables in the system - will depend on the pre-assigned values of the coefficients. Hence, if predictive accuracy were required the whole approach would be a hopeless one indeed. This is far from being the case, however; how far remains to be seen. All the figure-work, and a great deal will be required, is but a means towards the end of devising a workable Plan and it may well (and happily!) happen that the best Plan is a more or less invariant to the kind of data (within the limits of reason, of course) that one feeds into the model. The only way to identify the best Plan and to study this problem of coefficient sensitivity is to ask the model a great variety of questions and to examine the results.

Accordingly recourse must be had to a digital computer of suitable capacity. Fortunately, since the model is linear and since the solution of only linear simultaneous equations is involved, it will not be necessary to programme each of the many proposed experiments separately. All the machine companies have a sub-routine for solving linear simultaneous equations; once the data are prepared and set into the machine the complete answer emerges in 2 or 3 minutes. With all the answers before it, the Planning Authority will be in a position to make its recommendation.

Of course this recommendation cannot be made on the showing of the figures alone. In fact, it is for the economists and statisticians of the Plan to propound the questions to the model and, the more important stage, for the economists to examine and to pronounce upon the solutions which emerge.

Another important set of questions centres around the classifications used, especially in the inter-industry part of the IO table. Is it enough that there should be a single heading for "agriculture, forestry, fishing"? Considerable difficulties will be experienced in extending the dimensions of the table: a compromise worth examination might be to extend the number of <u>lines</u> of the present table (retaining the present columnar classification of 36 broad industrial groups).

Labour Aspects

Most approaches elsewhere of the present type contain a production function with a labour constituent. The writer prefers labour (and its classification in desired detail) to be regarded as endogenous in Irish conditions, with a large labour surplus. The set of numerical experiments contemplated must therefore include various assumptions about the labour productivity rate of increase within the bounds of possibility. Of course, with productivity given, the present model can be used to forecast manpower, capital etc provided statistics for these are available in the base year.