CEPS WORKING DOCUMENT
Thinking ahead for Europe

Does the growth in higher education mean a decline in the quality of degrees?

The role of economic incentives to increase college enrolment rates

Miroslav Beblavý, Mariya Teteryatnikova and Anna-Elisabeth Thum

No. 405 / March 2015

Abstract

In this paper we construct a theory about how the expansion of higher education could be associated with several factors that indicate a decrease in the quality of degrees. We assume that the expansion of tertiary education takes place through three channels, and show how they are likely to reduce average study time, academic requirements and average wages, and inflate grades.

First, universities have an incentive to increase their student body through public and private funding schemes beyond a level where they can keep their academic requirements high. Second, due to skill-biased technological change, employers have an incentive to recruit staff with a higher education degree. Third, students have an incentive to acquire a college degree due to employers' preferences for such qualifications, the university application procedures and through the growing social value placed on education.

We develop a parsimonious dynamic model in which a student, a college and an employer repeatedly make decisions about requirement levels, performance and wage levels. Our model shows that if i) universities have the incentive to decrease entrance requirements, ii) employers are more likely to employ staff with a higher education degree and iii) all types of students enrol in colleges, the final grade will not necessarily induce weaker students to study more to catch up with more able students. In order to re-establish a quality-guarantee mechanism, entrance requirements should be set at a higher level.

ISBN 978-94-6138-453-9
Available for free downloading from the CEPS website (http://www.ceps.eu)

Does the growth in higher education mean a decline in the quality of degrees?

The role of economic incentives to increase college enrolment rates

*Miroslav Beblavý, Mariya Teteryatnikova, Anna-Elisabeth Thum**

Abstract

In this paper we construct a theory about how the expansion of higher education could be associated with several factors that indicate a decrease in the quality of degrees. We assume that the expansion of tertiary education takes place through three channels and show how they are likely to reduce average study time, academic requirements and average wages, and to inflate grades.

Firstly, universities have an incentive to increase their student body through public and private funding schemes beyond a level where they can keep their academic requirements high. Secondly, due to skill-biased technological change, employers have the incentive to recruit staff with a higher education degree. Thirdly, students have the incentive to acquire a college degree due to employers' preferences for higher educational attainment, the university application rules and through the growing social value placed on education.

We develop a parsimonious dynamic model in which a student, a college and an employer repeatedly make decisions about requirement levels, performance and wage levels. Our model shows that if i) universities have the incentive to decrease entrance requirements, ii) employers are more likely to employ staff with a higher education degree and iii) all types of students enrol in colleges, the final grade will not necessarily act as a mechanism to induce weaker students to study more to catch up with more able students. In order to re-establish a quality-guarantee mechanism, entrance requirements should be set at a higher level.

*Corresponding author: Anna-Elisabeth Thum at anna.thum@ceps.eu. We would like to thank Richard Spady, Lucia Kurekova and Marcela Veselkova for interesting comments and insights.

1 Introduction

In this paper we study how the expansion of higher education can result in a decline in the quality of degrees. We measure the quality of degrees through studying time, academic requirements, average wages and the signalling power of grades (which is reduced through grade inflation).

The issue of studying time as an indicator of quality in degrees has recently gained attention. In the United States a decline in full-time college students' studying time has been observed empirically and has itself become a subject, both academically and politically (Arum and Roksa 2011; Babcock and Marks 2010a and Babcock and Marks 2010b; OECD 2012; The Economist 2012; De Vise 2012a and b[1]). Hall (2010) and McInnis and Hartley (2002) find that studying time is decreasing in Australia too. The decrease in studying time in Europe has been less the focus of study than in the US. Most academic and public debate on a possible decline in studying time is national but involves comparison with other European countries (Bekhradnia 2012; Sastry and Bekhradnia 2007; Bekhradnia 2012; Eurostudent 2005, 2008, 2012; BBC news 2007; The Irish Times 2012; Halloran 2011; Robatham 2013). Moreau and Leathwood (2006) find that students' engagement in paid work has increased in England. Bekhradnia (2012:7) finds that in the UK degrees take less time to complete than those in continental Europe and that less time is spent studying per week; a characteristic that mainly seems to apply to post-1992 universities, according to Bekhradnia (2012).

Reduced studying time has been associated with improved learning technology (Babcock and Marks 2010a,b, Darmody and Smyth 2008, Fischer 2011, Gomis-Porqueras et al 2011). However, Babcock and Marks (2010 a, b) note that this can only be part of the reason. Dolton, Marcenaro and Navarro (2003) confirm this by showing that time spent in formal university study is positively related to student performance. Another identified reason is the growing financial pressure on students: as tuition fees increase more students work and thus spend less time studying (Baffoe-Bonnie and Golden 2007; Hall 2010; Purcell 2010; Moreau and Leathwood 2006 and McCaig 2011; Manthei and Gilmore 2005). Findings indicate that there was an increase in part-time over full-time students, especially in Europe. Hakkinen (2006) finds that in Finland 50% of students were working in 2000, which increased the time-to-degree. Working students spent 19 hours per week studying whereas non-working students spent 35 hours per week studying. Eurostudent (2005) finds that more than 50% of students in Europe are employed. In the Netherlands and Estonia the student employment rate is two-thirds. At the same time Eurostudent (2005) provides evidence that students who work for 11 to 15 hours a week spend less time studying. In Germany and Romania the impact is seven hours less and in Bulgaria, the Czech Republic and Lithuania one hour less. Babcock and Marks (2010 a,b) show that even when controlling for part-time students by studying only full-time students, studying time still declines. The increase in part-time students alone therefore does not seem to be a sufficient explanation for the decrease in studying time.

Most of the prominent work in the relevant US literature associates declining studying

[1] The amount of time US secondary students spend on homework has changed very little in the last 20 years. At high school, students did not study more in the 1940s than they did in the 1960-80s (Gill and Schlossmann 2003). However, according to Larson (2001), American children and adolescents spend less time on homework than their Asian or European counterparts and have more free time.

time with a decrease in college requirements and standards. This decrease arises from a set of reasons related to a more vulnerable position of faculty staff (Brint 2011; Clotfelter, 2011; Arum and Roksa 2011; Bok 2006; Johnson 20002; Babcock and Marks 20101; Fischer 2011; Clack 2011): the amount of temporary positions as opposed to permanent positions has increased, publication pressure has risen and student evaluations matter more in securing an academic job. Grade inflation has also been documented and associated with decreasing studying time (Johnson 2003; Rojstaczer 2011; Rojstaczer and Healey 2010). The decline in studying time has also been associated with the fact that college enrolment rates increase and less prepared, less motivated students enter campus (Brint 2011, Bound, Lovenheim and Turner 2009; Clack 2011).

Notwithstanding the wide array of factors that were considered as determinants for the decline in studying time, the increase in college enrolment rates has been accorded less importance in the debate about quality of degrees, but – taken together with a decrease in college requirements – it may have a considerable impact on the time spent studying (Briggs 2010). Indeed, Briggs (2010) predicts a decline in the number of average hours spent studying as the share of students entering college increases, but he has not given a clear theory on how this could actually happen.

The contribution of this paper is to provide a theoretical reasoning of how the expansion of tertiary education – together with a decrease in college requirements – could decrease average time spent studying, average wages attributed to college graduates and contribute to grade inflation. We propose a model in which a university, multiple employers and a student can play a dynamic game with incomplete information. We focus on the undergraduate tertiary level of education since universities are ranked according to teaching quality, entry and achievement requirements and future employability of students[2]. In our model, expansion of education reduces average studying time because expansion turns out to be a low-ability expansion (the number of low-ability students accepted to university, whose studying time is zero, increases while the number of high-ability students stays the same). Machin and McNally (2007:11) already raise this possibility. An expansion towards more low-ability students entering college could be taking place because 1) college requirements decrease because universities are setting requirements that will maximise the pool of students, 2) all students are better off if they get accepted to a university with a high rank and 3) employers are more likely to employ highly skilled individuals[3]. Our model shows that under these three conditions, the grade will not necessarily act as a mechanism to make weaker students study to catch up with more able students. In order to re-establish a quality guarantee mechanism, entry requirements should be set at a higher level.

We choose the UK as an empirical example of our model since students there seem to spend little time studying compared to their continental European counterparts (Eurostudent 2011 and Bekhradnia 2012).

The remainder of the paper is organised as follows: the following section provides empirical evidence of tertiary education expansion and decline in the quality of higher education in the UK. Section 3 delineates the theoretical background for our model. Section 4 presents

[2] With respect to the ranking of universities, we do not address such factors as diversity of faculty research profile, faculty publications and citations, which are relevant determinants for the graduate programs.

[3] By 'highly skilled' individuals we mean those with a university degree.

the theoretical model and its assumptions. Section 5 concludes.

2 Empirical Evidence

In this section we provide empirical evidence on the expansion of tertiary education in the UK since 1950 compared to other European countries, using several indicators of quality of higher education (as mentioned above these are studying time, academic requirements, average wages and the signalling power of grades)in order to empirically support the key variables in our model. In this section we focus on the United Kingdom since students in the United Kingdom seem to be spending little time studying compared to their continental European counterparts (Eurostudent 2011 and Bekhradnia 2012). Furthermore, universities in the United Kingdom might be more strongly affected by the need for a large number of students than continental European universities. The reason for this is that the United Kingdom is among the countries with the largest share of private funding through student fees with 263 in 2009 (Estermann and Pruvot 2011).

2.1 Enrolment in higher education

The expansion of tertiary education needed to be preceded by the universalisation of upper secondary education (Beblavy et al 2012:2). The US was the leading country in the expansion and universalisation[4] of upper secondary education in the 20th century. US enrolment rates in the upper secondary sector rose from 18% in 1910 to 73% in 1940 (Goldin 1998:347). Europe followed this example around 30 years later in the form of government efforts to make upper secondary education available to the masses (Beblavy 2012:2). As a consequence, tertiary education could equally expand (for an excellent review of this expansion see Trow 2007 or Schofer and Meyer 2005) – a phenomenon observed across Europe at different rates since the 1970s or 1980s (OECD 2012, Eurostat 2006 and Machin and McNally 2007). The OECD (2012:9) measures that in 2012 over 503 of university-age young adults are enrolled in higher education in OECD countries. Eurostat (2006) shows that between 1998 and 2006, the number of students enrolled in tertiary education rose from 15 million to 19 million in the EU-27 area. Bratti and de Blasio (2008) provide evidence for Italy. Beblavy et al (2012) provide evidence for increasing enrolment in tertiary education in Germany, Slovakia, Czech Republic, Spain, the United Kingdom and Sweden[5]. In Figure 1 we show enrolment rates in tertiary education in the UK. We can see that enrolment in higher education increased only slightly between 1953 and 1989 but then surged very quickly and grew by about 400% from 5 to 25%. Compared to certain other European countries, expansion in the UK happened rather abruptly as well as rather late. As Figures 2 to 5 show, the expansion of tertiary education occurred at different speeds and dynamics across the various countries. Whereas we can observe a rather steady increase in tertiary enrolment rates in Sweden, the expansion in Spain happened more rapidly and more steeply.

[4] Beblavy et al (2012) define an expansion of education that passes the 80 percent threshold as 'universalisation'.

[5] The graphs show net or gross enrollment rates in tertiary education.

Figure 1: Enrolment rates in tertiary education in the United Kingdom 1953-2009 in percent, *Source: Toubeau (2012)*

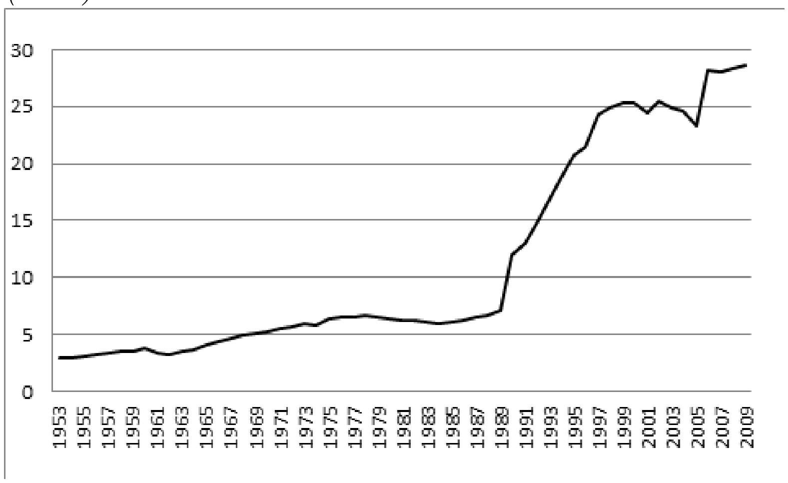
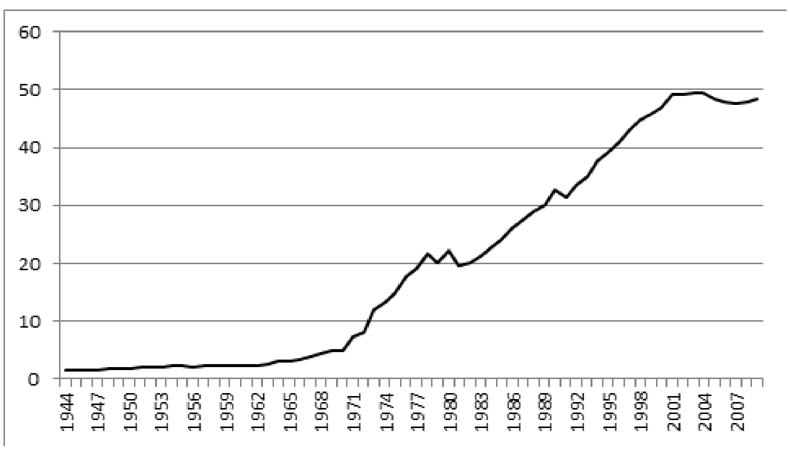


Figure 2: Enrolment rates in tertiary education in Spain 1944-2009 in percent, *Source: Pensiero (2012)*



2.2 Quality of tertiary degrees

As mentioned at the beginning of this paper and of this section, we measure the quality of higher education degrees through four elements: studying time, colleges' academic requirements, average wages and the 'signalling power' of grades. In this section we provide empirical evidence on studying time, academic requirements and the signalling power of grades (though examining grade inflation). We do not analyse wage development in the United Kingdom and leave this discussion to our theoretical model.

Studying time of full-time students decreased in the US from 40 hours a week in 1961 to 27 hours a week in 2007 as Babcock and Marks (2010) show. In Europe, there are no such clear-cut numbers. Europe shows a high heterogeneity across countries in the time students spend studying (Eurostudent 2005, 2008, 2011)[6]. According to the Eurostudent findings, time devoted to studying (classroom time and personal study time) ranges, on the Bachelor

[6] Heterogeneity can also be measured across subjects as well as across universities (Bekhradnia 2012).

Figure 3: Enrolment rates in tertiary education in Sweden 1940-2002 in percent, *Source: Peterson (2012)*

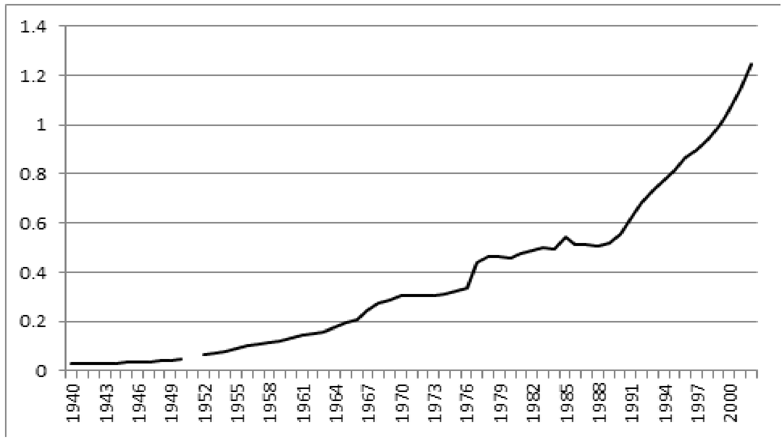
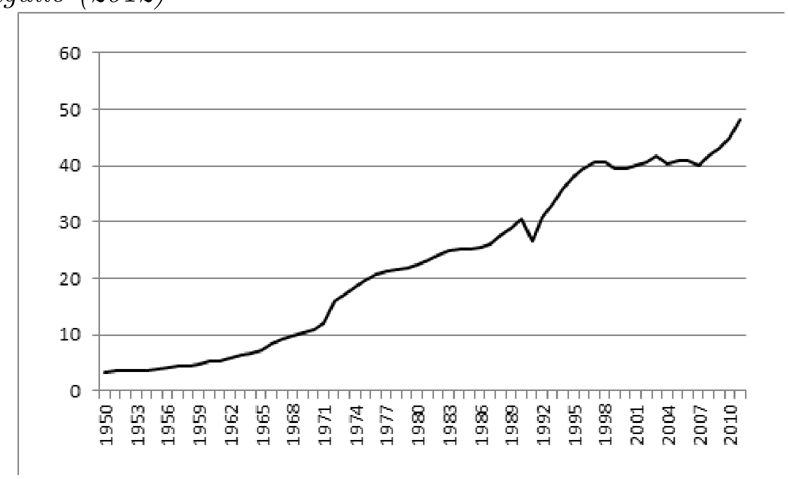


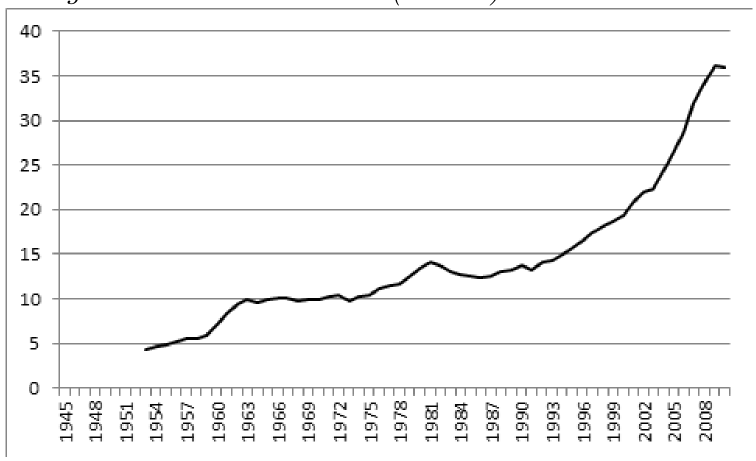
Figure 4: Enrolment rates in tertiary education in Germany 1950-2011 in percent, *Source: Thum and Potjagailo (2012)*



level, from 31 hours in France to 38 hours in Italy, and on Master level from 23 hours in Romania to 39 hours in Sweden. Sastry and Bekhradnia (2007) find that university students in the UK spend 26 hours on teaching and private study in 2006. In Germany, however, a survey of 1000 students in the state of Hessen state that they spend 35 hours for their studies in 2012 (Sparda Bank 2013). These findings are confirmed by Eurostudent (2011) but are challenged by Schulmeister and Metzger (2011) who find that students in Hamburg studied only 26 hours in 2009. How these numbers differ from the past is not clear.

National evidence from the UK attests tothe fact that standards are declining at universities there (University World News 2008, The Guardian 2010). Based mainly on qualitative research, House of Commons (2008-2009) finds that stakeholders and of higher education institutional actors in the UK believe that standards at university have worsened: a university degree is seen as being worth less than before by employers, students and academics. In the report, employers speak of their observation that students appear to be less motivated and have a less than ideal learning approach. Employers are said to focus more now

Figure 5: Decreasing enrolment requirement at university in UK for females - males follow a similar pattern (1996-2001), *Source: Abbott and Leslie (2004) in Toubeau (2012), database 'Universities and Colleges Admissions Service (UCAS)'*

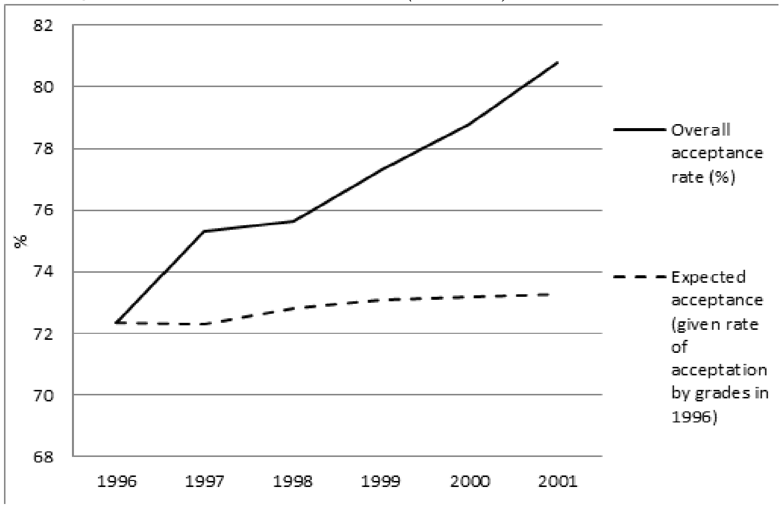


on previous work experience than on degrees when selecting job candidates. Students are quoted as saying that what they had been taught at school early on was now being taught at university (ibid: 112). Academics believe that certain degrees have lost value compared to five, ten or twenty years ago; essays have declined in quality and students appear to be less well selected (House of Commons 2008-2009: 111). In fact, as shown in Figure 6, requirements to enter college have decreased as acceptance rates have an increasing trend. One reason for these higher acceptance rates may be an issue tackled by the House of Commons (2008-2009), which provides evidence of academic staff's interest to increase the number of students admitted[7].

Evidence on grade inflation in the US has been provided by Rojstaczer and Healey (2010) and Rojstaczer (2011), who show that 43% of grades at four-year universities are 'A's. This signifies an increase of 28 percentage points since 1960. In the UK, based on the annually collected Higher Education Statistics Agency (HESA) student record, the House of Commons (2008-2009) finds that between 1994 and 2008 the good honours degrees – understood as an upper second or a first class honours degree – has risen. Yorke (2009) finds that between 1994 and 2002 this rise was most concentrated in the elite 'Russell Group' universities and between 2002 and 2007 the rise was more evenly spread. Among other reasons Yorke, in House of Commons (2008-2009), gives as explanations for an improvement in grades the fact that there might have been changes in the student entry profile of higher education institutions. This has not been empirically proven, however.

[7] A quote on funding issues by academics is given in House of Commons (2008-2009): "More particularly it relates to a tacit understanding amongst university staff that assessment levels and methods shall be geared mainly, if not exclusively, to the need to retain as many students as possible for the subsequent years and for graduation."

Figure 6: Decreasing enrolment requirement at university in UK for females - males follow a similar pattern (1996-2001), *Source: Abbott and Leslie (2004) in Toubeau (2012), database 'Universities and Colleges Admissions Service (UCAS)'*



3 Theoretical Background

In this section we discuss various economic theories that form the backbone of the model presented in section 4. Our model is characterised by three distinguishing features that explain the expansion of tertiary education. We show how education expansion may lead to a potential decrease in studying effort under certain circumstances:

1. The value of high skills on the labour market: employers employ only students who went to university - there is a quest for highly skilled individuals. It is important to note here that by "highly skilled" individuals we refer to those who went to university. They can be of low or high (innate) ability.

2. The role of universities and the value of expansion: universities maximise the number of students accepted since funding obtained by university increases with the number of students.

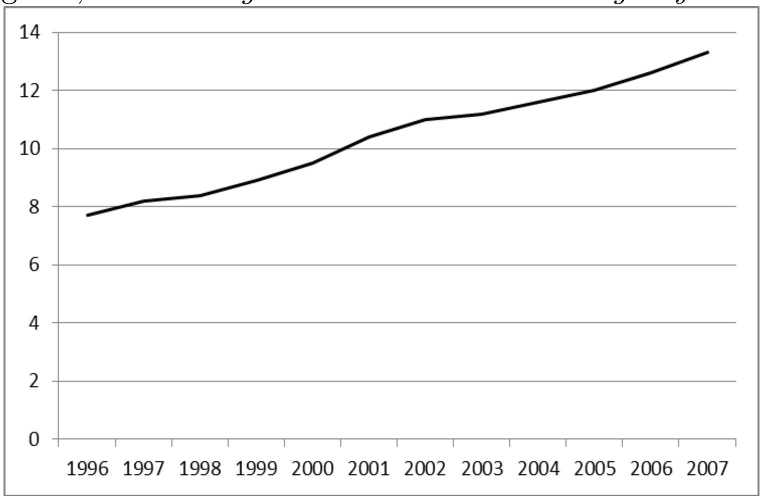
3. The rising importance of university degrees: all potential students are strictly better off if they get accepted to university and if the ranking of this university is high enough to guarantee them a job.

Below we describe the theoretical basis for each of these features of the model.

3.1 The value of high skills on the labour market

The employer employs only students with high skills. He or she is modelled based on a screening model (Stiglitz 1975; Rothschild and Stiglitz 1976; Arrow 1973 and Wiles 1974) - an education economic model in which the employer is mainly interested in high educational attainment when making hiring decisions on a potential employee. In a screening model the

Figure 7: Percentage of upper class first class grades awarded by higher education institutes in the United Kingdom, *Source: Higher Education Statistics Agency*



employer cannot observe the actual ability of potential employees and he or she therefore screens applicants according to their educational attainment to determine an applicant's ability of learning. Hiring applicants with university degrees reduces the employer's risk of hiring someone with a diminished capacity for learning. Implicit characteristics of this model, upon which results are dependent, are: (1) students with higher ability are better in absolute terms; the ranking of abilities is unambiguous (such screening is called hierarchical),

(2) labour supply is inelastic, (3) individuals have perfect information on their own abilities, (4) on-the-job screening is not possible, (5) screening is exact, (6) the information acquired is general, in the sense that it is not firm specific (Stiglitz 1975:286). In the screening model as in Stiglitz (1975) there can be a separating equilibrium or a pooling equilibrium[8] the latter characterises a situation in which all workers receive the same average wage whereas the former characterises a situation in which high ability types receive a high wage and low ability types receive a low wage. In both types of equilibria the existence of the low ability type decreases the net income of the high ability type and the existence of the high ability type will provide the low ability type with at least his or her marginal product (Stiglitz 1975:285). The screening model has several implications: firstly, social returns differ from private returns and are not necessarily positive in the presence of screening. In fact, screening may tend to lead to higher inequality even if the screening mechanism increases the productivity of firms (Stiglitz 1975:299). There is thus a trade-off between distributive versus productivity concerns. Secondly, there is a tendency to over-invest in education, especially in publicly funded schools: education serves mainly as a signal and therefore education systems should choose the least costly method of transmitting this signal. In consequence, the screening model is often used as a theoretical basis to decrease public spending on education (Quiggin 1999). Huang and Capelli (2001), Bedard (2001) and Weiss (1995) offer some empirical evidence of the model and of the importance of higher education for employers.

In our model we add – compared to the Stiglitz and Rothschild (1976) model – a new

[8] In Rothshild and Stiglitz' (1976) screening model the authors show that only the separating equilibrium exists. Similarly, in our model we show that the pooling equilibrium does not exist.

decision-maker, the university, whose requirements affect both the wage contract and studying time. However, given university requirements, the interaction between the students and the employers in each period t is similar to that in other standard models of screening and there is a unique symmetric perfect Bayesian equilibrium (SPBE).11 We are interested in comparative statics, focusing on the effects of university requirements on students' equilibrium studying time, grades and wages. As a further extension in comparison with Stiglitz (1975), we consider an interaction between the decision-makers that is repeated over time, with a new student taking decisions in every period. The equilibrium of this repeated interaction is then defined as a set of players' strategies and beliefs that constitute an SPBE in each period.

The fact that university degrees determine employment and the wage contract can be bedded into a wider (sociological) theory about the expansion of higher education. Theories about the expansion of tertiary education being linked to economic development (reflected by the employers' interests described above) are termed as socioeconomic functionalist explanations (Beblavy et al. 2012:3). These theories are based on the fact that national economic development creates demand for skills and supply is given through resources created through economic development (Collins 1971; Goldin 1998). This assumption is in line with recent findings and predictions that high skills matter and will matter most on the labour market (CEDEFOP 2010).

3.2 The role of universities and the value of expansion

We consider universities as public institutions providing education and making choices according to public choice theory: in public choice models, it is usually assumed that heads of public institutions aim to maximise their own utility. They will promote the benefits of the institutions only in a way that it is beneficial to them personally. Niskanen (1968) argues that heads of institutions will benefit from expansion of their institutions beyond the social optimum. A reason why managers of educational institutions will benefit from an expansion of their institution stems from not-for-profit (NFP) theory (Holland and Ritvo 2008). NFP theory concerns institutions that do not distribute surplus revenues as dividends or profit, such as universities. Surplus revenues should be kept in the institution to serve its goals. Not-for-profit theory states that in the absence of a profit motive, NFPs tend to expand since that is the way for managers to increase their prestige and personal perquisites. It is therefore not necessary for the income of universities to expand linearly with number of students - it may if students pay tuition or if state pays per student - but even if there are more complicated funding mechanisms, if they lead to immediate or eventual expansion of university budget following an increase in number of students, our condition is satisfied so that a linear assumption about the correlation of university funds with the number of students is an acceptable approximation.

3.3 The growing importance of university degrees

The assumption that all potential students are strictly better off if they get accepted to university is based on the features of the labour market, the fact that universities do have the incentive to take up as many students as the government quota – or university capacity –

allows and the fact that the social reputation of a university degree has risen with the opening of universities to a larger population. Rubinson and Fuller (1992) show that educational expansion has also occurred as a result of education having become a status symbol. This theoretical explanation for the expansion of tertiary education can be termed sociological or conflict, competition and organisation theory (Beblavy et al 2012:3). These theories put less emphasis on market powers as an explanatory factor but rather emphasise social status. Due to competition between different status groups for success in education, the expansion of education rises beyond purely functional requirements (Bourdieu and Passeron 1977; Collins 1971 and 1979). In times of high competition among social status groups, as can be the case with a rising population, expansion should be more rapid than in times with less competition (Rubinson and Fuller 1992).

4 The Model

In this section we construct a parsimonious model to explore theoretically how the expansion of tertiary education can decrease studying time.

Consider the following dynamic model. There are three essential entities that make decisions: a student with ability a that is distributed according to a probability distribution function (p.d.f.) f with support $[\underline{a}, \overline{a}]$, a university and employers. Implicitly, there is also a fourth party – a government or an education authority – that imposes restrictions on the university by setting (a) the maximal increment in the number of students per period (the quota of students) and (b) a *rank condition* according to which an expansion of the student pool at the university is only allowed if the rank of the university exceeds a certain threshold. In the model, we take the quota of incoming students and the threshold for the rank as given and fixed over time, so that there is no active role for the government.

Time is discrete, $t \in \{1, 2, ...\}$, and the three decision makers interact repeatedly over time. Each time period a new cohort of students substitutes the previous, so that a new representative student makes decisions at each time t. The timing of events is the following:

1. the university sets the level of requirements, r_t, that determines the entry conditions and the grades;

2. M employers simultaneously select a wage contract, given the university requirements and the grades;

3. students with ability high enough to be accepted to the university choose the studying effort.

Below we describe each of these steps and the interaction between the decisions of the university, students and the employers in more detail. First, the university sets the level of requirements, r_t, that determines the entry conditions and the grades. Only students with ability $a > r_t$ pass the entry requirements and get accepted to the university. Upon acceptance they can guarantee themselves a positive level of utility, provided that the rank of the university is higher than a certain threshold (below which the wages are zero). Other *potential* students, with abilities below r_t, are not accepted and make no study-related decisions;

their utility is set to be zero. Thus, all potential students are strictly better off if they get accepted to the university when the rank of the university is high enough to guarantee them a job, and hence the student pool in this case is represented by all students with abilities in $(r_t, \overline{a}]$. Suppose that at $t = 1$, $\underline{a} \ll r_1 < \overline{a}$. This initial value of requirements is set arbitrarily within the specified range and determines the initial pool of students, N_1. To make things interesting though we assume that r_1 is large enough so that the rank of the university at $t = 1$ is above the threshold R set by the authorities; this condition is formally stated and discussed below.[9] At any $t \geq 2$ the requirements, r_t, are chosen by the university so as to maximize the pool of students, N_t, within the quota assigned by the authorities. Implicit in this is the idea that the larger the number of students, the larger the funding that the university obtains, either on a public or on a private basis. For this reason the university has an incentive to reduce its requirements and accept all students within the quota. Yet, it can only do so as long as with the new reduced standards its rank, $R_t(r_t)$, is above the threshold R set by the authorities. One can think of this as a situation where the authorities care about prestige of the university and in order to promote high standards, they only allow the university to expand when its rank is higher than a certain level.[10]

When the time t requirements are set, the grade of a student with ability a is determined by

$$g_t = \max\{s_t a - r_t, 0\}$$

where s_t is the studying time.

Next, M independent employers simultaneously choose the wage contract. They only employ people who went to the university (students with abilities above r_t), conditional on the fact that the university rank is above the publicly known threshold, that is, $R_t \geq R$. If $R_t < R$, the employers do not employ anyone. The employers are willing to employ students with abilities above a certain level A at wage w_{Ht}, and those with abilities below A – at wage w_{Lt}, where $w_{Ht} > w_{Lt}$.[11] However, the employers do not observe the abilities of students and can only judge them from the grades. They select the wage contracts that may *pool* or *separate* high- and low-ability students, depending on the employers' beliefs about which types of students will choose which grade. The wage levels, w_{Ht} and w_{Lt} are determined via a standard Bertrand competition[12] between employers. Moreover, only two different wages may exist in equilibrium since as we will show, only two different grades – signals of ability, may be observed.

Finally, given the university requirements and the wage contracts of the employers, a student of any ability a decides how much time to allocate to studying. Studying more improves the grade and this may increase the wage if the wage contracts of the employers are designed to separate high- and low-ability students; if the wage contracts are pooling, studying time does not affect the wage. Also, studying reduces the leisure time. The student's

[9] See (1).

[10] Notice that even without the prescription from the authorities, the university can only guarantee itself a positive number of students if its rank is above R. For instance, when students are obliged to pay an amount $\varepsilon > 0$ for their education, no one would apply to the university if the rank is below R. This is clear from the specification of students preferences and wages described below.

[11] We assume the most interesting scenario, when $r_1 < A < \overline{a}$.

[12] In a Bertrand competition, sellers set prices simultaneously (in our case the employers set wages). Buyers take these prices as given and set quantities (in our case students set effort).

utility is increasing in wage and in leisure; therefore, the choice of the amount of studying time depends crucially on the type of the wage contract – pooling or separating. If the contract is pooling, all students tend to choose the same studying time, minimal that is required to receive the grade that grants the employment. If the contract is separating, then high-ability students study more because for them the returns to studying are higher.

More formally, the dilemmas faced at each time period by the university, employers and students, in their chronological order, are described below.

4.1 University decision making

The university's funding is proportional to the number of students. In fact, assuming that the tuition fee per student or a government subsidy per student is fixed at a certain level p, the total tuition fee or subsidy that the university obtains at time t is $p \cdot N_t$, where N_t is the number of students. Therefore, the goal of the university at any period t is to maximize the size of its student pool[13]. However, two restrictions are imposed externally. First, at any requirements level set by the university, its rank, R_t, should not fall below R, the fixed given constant. We will refer to $R_t \geq R$ as the rank condition. Secondly, at each time t the student pool cannot be increased by more than $\frac{\Delta}{\bar{a}-\underline{a}}$, the *quota extension* for the total number of students. This second restriction can arise as a result of regulation by the government or the education authority or it can simply reflect facility constraints at the university, that is unable to extend its studying facilities (such as computers, libraries, study materials and space) faster than at a certain rate. Naturally, $\frac{\Delta}{\bar{a}-\underline{a}}$ should not exceed $1 - N_1$.

Thus, the university chooses its requirements r_t at time t with the objective to expand its student pool subject to two constraints:

$$\begin{aligned} N_t(r_t) &\rightarrow \max_{r_t} \\ \text{s.t. } N_t &\leq N_{t-1}(r_{t-1}) + \frac{\Delta}{\bar{a}-\underline{a}} \\ R_t(r_t) &\geq R \end{aligned}$$

where

$$N_t = \int_{r_t}^{\bar{a}} f(a)da$$

and

$$R_t(r_t) = \gamma \frac{T}{N_t} + \delta r_t + \eta Avw_t + \xi$$

The definition of the rank accounts for the teaching quality as captured by $\frac{T}{N_t}$ – the ratio of the number of teachers to the number of students ($T < 1$ and reasonably $T < N_1$, the initial number of students), the level of entry and grade requirements, r_t, the employment prospects of time t student cohort as captured by the average wage, Avw_t, and other factors – included in constant term ξ. The weights γ, δ and η are all positive constants and $\gamma + \delta + \eta < 1$. The value of R is arbitrary; however, the interesting case is when it is neither too high, nor too low. Namely, assume that R is such that a) initially, at $t = 1$ the rank condition is satisfied,

[13] This assumption is consistent with empirical obseravations delineated in Section 2.

that is $R_1(r_1) > R$ (in fact, let $R_1(r_1) \gg R$), and b) if requirements decline to the lowest possible level, $r_t = \underline{a}$, when all students are accepted to the university, the rank condition does not hold, $R_t(\underline{a}) < R$. Given the definition of the rank above and using the expression for the average wage that will be derived later, this means that

$$\gamma T+\delta\underline{a}+\eta\frac{w_{Ht}\overline{a}-w_{Lt}\underline{a}-A(w_{Ht}-w_{Lt})}{\overline{a}-\underline{a}}+\xi < R \ll \gamma T\frac{\overline{a}-\underline{a}}{\overline{a}-r_1}+\delta r_1+\eta\frac{w_{Ht}\overline{a}-w_{Lt}r_1-A(w_{Ht}-w_{Lt})}{\overline{a}-r_1}+\xi \tag{1}$$

4.2 Employers' decision making

There are M independent employers in the market. For simplicity assume that $M = 2$; the analysis for $M > 2$ can be easily extended. The employers have identical technology that is characterized by the feature that all workers with ability above A (high-ability workers) can perform the work that entitles them to the same wage w_{Ht}. Also, all workers with ability below A (low-ability workers) can do the work that entitles them to the same wage w_{Lt}. Assume that $r_1 < A < \overline{a}$. In equilibrium, each employer seeks to maximize his expected profit, equal to the expected work productivity of the employee net of the wage payment. We assume that the work productivity of a high-ability individual is equal to his ability times constant factor x_H and the work productivity of a low-ability individual is equal to his ability times constant factor x_L. We consider $x_L \leq x_H$ to make sure that the work productivity (and the equilibrium wage) of a high-ability individual is higher than the work productivity (and the equilibrium wage) of a low-ability individual. The wages w_{Ht} and w_{Lt} are determined endogenously, via standard Bertrand competition between the employers.

Since the ability is not observable by the employers, they form beliefs about the ability based on the grade that the job applicant receives at the university. That is, having observed the grade g_t, the employers assign some probability to the fact that the worker is of high-ability type (even for off-the-equilibrium path choices of g_t). Given these beliefs, the employers design the wage contract that either "pools" or "separates" different types of workers, so that wages are allocated to workers consistently with the beliefs.

Consider first the following beliefs of the employers:

- $g_t \geq \bar{g}_t$ corresponds to $a \geq A$
- $g_t < \bar{g}_t$ corresponds to $a < A$

for some threshold grade $\bar{g}_t$. These beliefs lead to a separating contract that offers the high wage w_{Ht} to anyone with the grade $g_t \geq \bar{g}_t$ and the low wage w_{Lt} to anyone with the grade $g_t < \bar{g}_t$. Later, in section 5, we show that the separating beliefs of the type described above, with a uniquely defined level of $\bar{g}_t$, are actually the only equilibrium beliefs of employers.

Moreover, the employment takes place and the wages are paid only if the rank condition is satisfied, $R_t \geq R$. If it is not satisfied, the employers are not interested in employing anyone (wages are zero).

4.3 Student decision making

Given the wage contract of the employers, a representative student with ability $a > r_t$ who is accepted to college at time t, chooses the time for study and for leisure with the objective to maximize his or her utility at time t. If the wage contract is separating, of the type described above, the optimization problem of a student is

$$\begin{aligned} u(l_t, w_t) &= l_t^{\alpha} w_t^{\beta} \rightarrow \max_{l_t} \\ \text{s.t. } l_t &+ s_t = 1 \\ g_t &= \max\{s_t a - r_t, 0\} \\ w_t &= \begin{cases} w_{Ht} & \text{if } R_t(r_t) \geq R \text{ and } g_t \geq \bar{g}_t \\ w_{Lt} & \text{if } R_t(r_t) \geq R \text{ and } g_t < \bar{g}_t \\ 0 & \text{if } R_t(r_t) < R \end{cases} \\ & \alpha, \beta > 0 \end{aligned}$$

As it was mentioned earlier, l_t and s_t denote leisure and studying time, respectively, g_t is the grade, and w_t is the wage. Given the university requirements, r_t, a grade g_t is obtained if a student with ability a studies for s_t units of time. The student is employed only if the rank condition holds and then the wage is either w_{Ht} if the student's grade is above the threshold $\bar{g}_t$ set by the employer, or w_{Lt} if the grade is below this threshold.

4.4 Equilibrium

Equilibrium in period t is a standard symmetric perfect Bayesian equilibrium, where all decision makers achieve their objectives subject to the constraints they face and the beliefs of the employers are a) the same, b) "correct" on the equilibrium path, that is, confirmed by the rational choices of high- and low-ability students. Formally, equilibrium is the combination of requirements r_t set by the university, the beliefs and the wage contract of the employers and studying times s_t chosen by students with all abilities $a \in (r_t, \bar{a}]$ such that:

1. the size of the student pool, N_t, is maximized subject to $N_t \leq N_{t-1}(r_{t-1}) + \frac{\Delta}{\bar{a}-\underline{a}}$ and $R_t(r_t) \geq R$;

2. the employer's beliefs on the equilibrium path are correct, i.e. consistent with the choices of students. If the corresponding wage contract is separating, workers with ability above A receive the wage w_{Ht}, workers with ability below A receive the wage w_{Lt} and both wages are determined via Bertrand competition between the employers; if the wage contract is pooling, all workers receive the same fixed wage, determined via Bertrand competition;

3. the utility of any student with ability $a \in (r_t, \bar{a}]$ is maximized given the employers' beliefs (on and off the equilibrium path), wage contract, the university requirements and the definition of a grade, $g_t = \max\{s_t a - r_t, 0\}$;

4. the beliefs of the employers off the equilibrium path are arbitrary.

Below we characterize the equilibrium in any period $t \geq 2$.[14] As we will demonstrate, this equilibrium is unique.The equilibrium of the entire model is then defined as a situation where in each $t \geq 2$ the SPBE of is played.

5 Results

In the following assume for the sake of simplicity that abilities of students are distributed uniformly on $[\underline{a}, \overline{a}]$, so that f is the p.d.f. of the uniform distribution, $f(a) = \frac{1}{\overline{a} - \underline{a}}$ on $[\underline{a}, \overline{a}]$. Then the c.d.f. is $F(a) = \frac{a - \underline{a}}{\overline{a} - \underline{a}}$ for $a \in (\underline{a}, \overline{a}]$ (0 below $\underline{a}$ and 1 above $\overline{a}$) and the truncated p.d.f. $\widetilde{f}_t$ – with the support on the interval $(r_t, \overline{a}]$ for $\underline{a} \leq r_t < \overline{a}$ – is defined as

$$\widetilde{f}_t(a) = \frac{f(a)}{F(\overline{a}) - F(r_t)} = \frac{1}{\overline{a} - \underline{a}} \cdot 1 / \left(1 - \frac{r_t - \underline{a}}{\overline{a} - \underline{a}}\right) = \frac{1}{\overline{a} - r_t}$$

Our first proposition states that as soon as the threshold A, separating high- and low-ability students is high enough, the beliefs of the employers and the corresponding separating wage contract of the type described above are in fact equilibrium beliefs and contracts in any period t.

Proposition 1 *If the threshold of abilities A is sufficiently high, then the set of strategies and employers' beliefs characterized below constitutes an equilibrium at any $t \geq 2$:*

1. *The university requirements are defined recursively by $r_t = \max\{r_{t-1} - \Delta, r^*\}$, where r^* is the* unique *solution of $R_t(r_t) = R$. Given the initial requirements r_1, r_t is defined uniquely in every $t \geq 2$:*

$$\begin{aligned} r_t &= r_1 - (t-1)\Delta \qquad \textit{if } r_1 - (t-1)\Delta > r^* \\ r_t &= r^* \qquad\qquad\qquad\quad \textit{otherwise} \end{aligned}$$

2. *The beliefs of the employers are such that all students with the grade $g_t \geq \bar{g}_t$ are of high ability (with $a \geq A$) and all the other students are of low ability (with $a < A$), where the threshold grade, $\bar{g}_t$, is equal to*

$$\bar{g}_t = A\left(1 - \left(\frac{w_{Lt}}{w_{Ht}}\right)^{\frac{\beta}{\alpha}}\right) - r_t$$

The corresponding separating wage contract offers the high wage w_{Ht} to students with $g_t \geq \bar{g}_t$ and the low wage w_{Lt} to students with $g_t < \bar{g}_t$. The wages, w_{Ht} and w_{Lt}, are equal to

$$\begin{aligned} w_{Ht} &= \frac{x_H}{2}(\overline{a} + A) \\ w_{Lt} &= \frac{x_L}{2}(r_t + A) \end{aligned}$$

[14] Recall that period 1 requirements are set exogenously and predetermine the initial size of the student pool N_1.

3. The studying time of the student of any ability $a < A$ is 0, while the studying time of the student of any ability $a \geq A$ is

$$\bar{s}_t = \frac{\bar{g}_t + r_t}{a} = \frac{A\left(1 - \left(\frac{w_{Lt}}{w_{Ht}}\right)^{\frac{\beta}{\alpha}}\right)}{a}$$

These studying times lead to $g_t = 0$ and $g_t = \bar{g}_t$, respectively. Both types of students accept the offered wage contract and earn wages w_{Ht} (if of high type) and w_{Lt} (if of low type).

The next proposition states the "uniqueness" result. Namely, it claims that the separating equilibrium described in Proposition 3.1 is unique and no pooling equilibrium exists.

Proposition 2 *At any $t \geq 2$, there exists no equilibrium in which the wage contract of the employers is pooling.*

The proofs of Proposition 3.1 and 3.2 are provided in the Appendix.

Thus, the unique equilibrium outcome at any time $t \geq 2$ is such that (a) the university sets the requirements that are by Δ lower than the ones in the previous period, as soon as this level does not fall below the lowest acceptable threshold, r^*; (b) the employers offer the separating wage contract that allocates high wage to students of high ability and low wage to students of low ability, and the wages are determined *à* la Bertrand; (c) the students either do not study or study just enough to obtain the threshold grade $\bar{g}_t$ and accept the wage contract of the employers.

It is easy to see that the long-run equilibrium value of requirements is r^*, the value at which the rank condition holds as equality, $R_t(r^*) = R$. This induces the long-run value of the threshold grade

$$\bar{g}^* = A\left(1 - \left(\frac{w_L^*}{w_H^*}\right)^{\frac{\beta}{\alpha}}\right) - r^* = A\left(1 - \left(\frac{x_L}{x_H}\right)^{\frac{\beta}{\alpha}}\left(\frac{r^* + A}{\bar{a} + A}\right)^{\frac{\beta}{\alpha}}\right) - r^*$$

The long-run studying time of any low-ability student is 0, while long-run studying time of any high-ability student is

$$\bar{s}^* = \frac{A\left(1 - \left(\frac{w_L^*}{w_H^*}\right)^{\frac{\beta}{\alpha}}\right)}{a} = \frac{A\left(1 - \left(\frac{x_L}{x_H}\right)^{\frac{\beta}{\alpha}}\left(\frac{r^*+A}{\bar{a}+A}\right)^{\frac{\beta}{\alpha}}\right)}{a}$$

We are interested in the effects that the university requirements have on equilibrium studying time, grades and wages. Notice that the equilibrium wage of high-ability students (with $a \geq A$) is constant, unaffected by r_t, while the wage of low-ability students (with $a < A$) is increasing in r_t, that is, it declines when r_t declines. This is so because Bertrand competition between the employers who have separating beliefs induces them to set w_{Ht} equal to the expected productivity of the high-type students, $x_H E(a \mid a \geq A)$, and w_{Lt} equal to the expected productivity of the low-type students, $x_L E(a \mid r_t \leq a < A)$. The former conditional expectation is constant since the pool of high-ability graduates, with

$a \geq A$, remains the same as requirements decline. On the other hand, the latter conditional expectation moves together with r_t: smaller r_t leads to a larger pool of low-ability graduates, hence their expected ability and wage become smaller.

The equilibrium grade of low-ability students is constant and equal to 0, while the equilibrium grade of high-ability students, $\bar{g}_t$, increases as requirements become lower.

Finally, the equilibrium studying time of low-ability students remains fixed at 0, while the equilibrium studying time of high-ability students, $\bar{s}_t$, increases as r_t declines. The reason why the studying time of high-ability students increases has to do with the fact that as r_t declines, declining low-to-high wage ratio, $\frac{w_{Lt}}{w_{Ht}}$, creates incentives for low-ability students to study and receive high wage, i.e. imitate high-ability students. For this not to happen, the studying time required to earn high wage increases, by just enough to discourage imitation by low types.

These effects of the requirements on equilibrium studying time, grades and on wages induce prediction for the effects on the average/expected equilibrium studying time, Avs_t, the average/expected equilibrium grade, Avg_t, and the average/expected equilibrium wage, Avw_t, of time t cohort of students. Clearly, the average wage declines because the wage and the number of high-ability students are fixed, while the wage of low-ability students decreases and the number of low-ability students becomes larger.

On the other hand, the effects of declining university requirements on the average grade and average studying time are not so straightforward. Comparative statics results (derived in the Appendix) suggest that as the requirements decline, the average grade increases and the average studying time decreases, with the latter being true at least as soon as (i) the ratio of productivity factors, $\frac{x_L}{x_H}$, is sufficiently low and/or (ii) the ratio of students' preferences for leisure or wages $\frac{\beta}{\alpha}$ is sufficiently small and high types constitute only a relatively small proportion of the total student pool. Intuitively, as requirements become weaker, the average grade increases because the threshold grade, $\bar{g}_t$, increases and hence, even though the number of high-ability students remains the same, their grades rise and this turns out to overcompensate an increase in the pool of low-ability students whose grade is zero. Similarly, the decline in average studying time is a result of an increase in the number of low-ability students, whose studying time is zero. This effect turns out to outweigh an increase in the studying time of high-ability students, at least as soon as the impact of the declining low-to-high wage ratio (that increases $\bar{s}_t$) is muted by low productivity factors ratio, $\frac{x_L}{x_H}$, and/or if students' preferences allocate sufficiently higher weight to leisure than to wages (so that $\bar{s}_t$ of high-ability students does not increase much) *and* high types constitute only a relatively small proportion of the total student pool.[15]

The proposition below summarizes these conclusions:

Proposition 3 *The equilibrium of the model is characterized by the following comparative statics:*

1. *The equilibrium wage of high-ability students, w_{Ht}, is constant, independent of r_t, while the equilibrium wage of low-ability students, w_{Lt}, decreases when r_t declines.*

[15] Notice that the restriction on A to be sufficiently high has been actually imposed already earlier, in order to guarantee the existence of the separating equilibrium in this model.

2. *The equilibrium studying time of low-ability students is fixed at* 0*, while the equilibrium studying time of high-ability students increases when r_t declines. As more low-ability students enter university, the increased studying time of the high ability students will not compensate the growing number of low ability students and average studying time will decline.*

3. *The equilibrium grade of low-ability students is fixed at 0, while the equilibrium grade of high-ability students increases when r_t declines.*

4. *The average equilibrium wage, Avw_t, decreases and the average equilibrium grade, Avg_t, increases when r_t declines. The average equilibrium studying time, Avs_t, decreases at least as soon as (i) $\frac{x_L}{x_H}$ is sufficiently small and/or (ii) $\frac{\beta}{\alpha}$ is sufficiently small and A is sufficiently large.*

6 Conclusion

In this paper we constructed a theory on how the expansion of higher education could be associated with several factors that indicate a decline in the quality of degrees. We focused on how the expansion of higher education – together with a decrease in university requirements resulting from budgetary considerations – might be associated with a decline in average studying time (student effort), grade inflation and a decrease in average wages provided to university graduates. We constructed a model in which a university, multiple employers and a student play a dynamic game with incomplete information. This model can be interpreted as an extension of the Rothschild and Stiglitz (1976) and Stiglitz (1975) model of screening in which the employer needs to employ a mechanism to sort employees into high-performing or low-performing workers. Other previous theoretical models also form the basis of our model: universities are modelled as non-for-profit organisations in which managers are interested in expanding their institutions.

By the means of our model we show that students' average studying time, average wages as well as the signalling power of grades decline because college enrolment rates increase beyond a level, at which academic requirements can be kept at a high level. In our model the expansion of education reduces average studying time because expansion is modelled as low-ability expansion (the number of low-ability students accepted to university, whose optimal studying time is shown to be zero, is shown to increasewhile the number of high ability students stays the same). Machin and McNally (2007:11) already raise this possibility. This is the case because(1) college academic requirements decrease as a consequence of universities setting requirements so as to maximise the pool of students, (2) all students are better off if they get accepted to a university with a high rank and (3) employers award a high wage to those with a high grade.

Our model shows that under these circumstances, the grade will not necessarily act as a mechanism to induce weaker students to study and catch up with more able students. In order to re-establish a quality guarantee mechanism, entrance requirements should be set at a higher level.

References

[1] Abbott, A. and D. Leslie (2004), "Recent Trends in Higher Education Applications and Acceptances", Education Economics, Vol. 12, No. 1, pp. 67-86.

[2] Alderman, G. (2010), "Why university standards have fallen", The Guardian, 10 March (http://www.guardian.co.uk/commentisfree/2010/mar/10/universities-standards-blair-target).

[3] Arrow, K. (1973), "Higher education as a filter", Journal of Public Economics, Vol. 2, No. 3, pp.193-216.

[4] Arum, R. and J. Roksa (2011), Academically Adrift: Limited Learning on College Campuses, Chicago: University of Chicago Press.

[5] Babcock, P. and M. Marks (2010a), "Leisure College, USA: The decline in student study time", Education Outlook, No. 7, American Enterprise for Public Policy Research, August (http://www.aei.org/publication/leisure-college-usa).

[6] Babcock, P. and M. Marks (2010b), "The falling time cost of college: Evidence form half a century of time use data", The Review of Economics and Statistics, Vol. 93, No. 2, pp. 468-478.

[7] Baffoe-Bonnie, J. and L. Golden (2007), "Work-Study: Time Use Trade-offs, Student Work Hours and Implications for Youth Employment Policy", SSRN eLibrary, 26 December, (http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1078688).

[8] BBC News (2007), "Students in England 'work less'. Undergraduates in England spend less time studying per week than their European counterparts, a report says", BBC, 24 September, (http://news.bbc.co.uk/2/hi/uk_news/education/7011121.stm).

[9] Beblavy, M., M. Veselkova, N. Pensiero, E. Peterson, A.-E. Thum and S. Toubeau (2012), "Lessons from the expansion of upper secondary education for the expansion of tertiary education", NEUJOBS Working Paper D4.4.1., July (http://www.neujobs.eu/sites/default/files/publication/2013/02/4.4.1%20NEUJOBS%20Working%2

[10] Bedard, K. (2001), "Human Capital versus Signaling Models: University Access and High School Dropouts", Journal of Political Economy, Vol. 109, No. 4, pp. 749-775.

[11] Bekhradnia, B., C. Whitnall and T. Sastry (2006), "The academic experience of students in English universities", HEPI report, October (http://www.hepi.ac.uk/wp-content/uploads/2014/02/27Academicexperienceofstudents.pdf).

[12] Bekhradnia, B. (2012), "The Academic Experience of Students in English Universities", HEPI report (http://www.hepi.ac.uk/wpcontent/uploads/2014/02/AcademicExpereinceStudentsEnglishUnivesitie

[13] Bourdieu, P. and J.-C. Passeron (1977), Reproduction of education, society and culture, London: Sage.

[14] Brint, S. (2011), Undergraduate Student Orientations in the United States: Academically Adrift? Bildung und Erziehung, Vol. 65, No. 2, pp. 195–207.

[15] Bok, D. (2006), Our Under-Achieving Colleges: A Candid Look at How Much Students Learn and Why They Should be Learning More, Princeton: Princeton University Press.

[16] Bound, J., M. Lovenheim and S. Turner (2009), "Why have college completion rates declined? An analysis of changing student preparation and collegiate resources", NBER Working Paper No. 15556, Cambridge, MA: National Bureau of Economic Research, December.

[17] Briggs, W. (2010), "Students studying less: Shocking report", (http://wmbriggs.com/blog/?p=2605).

[18] CEDEFOP (2010), Skills Supply and Demand in Europe; Medium-Term Forecast up to 2020, Luxemburg: Publications Office of the European Union.

[19] Clack, J.W. (2011), Studying and Grades: When Less is More and More is Less, Bioscene: Journal of College Biology Teaching, Vol. 37, No. 1, pp. 29-30.

[20] Clotfelter, C. (2011), Big-Time Sports in American Universities, Cambridge: Cambridge University Press.

[21] Collins, R. (1971), "Functional and conflict theories of educational stratification", American Sociological Review, Vol. 36, No. 6, pp. 1002-1019.

[22] Collins, R. (1979), The Credential Society: A Historical Sociology of Education and Stratification, New York: Academic.

[23] Darmody, M. and E. Smyth (2008), "Full-time students? Term-time employment among higher education students in Ireland," Journal of Education and Work, Vol. 21, No. 4, pp. 349-362.

[24] De Vise, D. (2012a), "Is college too easy? As study time falls, debate rises", The Washington Post, 21 May, 2012, (http://articles.washingtonpost.com/2012-05-21/local/35456815_1_college-students-study-time-students-report).

[25] De Vise, D. (2012b), "College students spending less time studying", The Boston Globe, 23 May (http://www.bostonglobe.com/news/nation/2012/05/22/college-students-spending-less-time-studying/mnU3XDU5TVUs8VFpzXk8TP/story.html).

[26] Dolton, P., O. Marcenaro and L. Navarro (2003), "The effective use of student time: a stochastic frontier production function case study", Economics of Education Review, Vol. 22, No. 6, pp. 547-560.

[27] Estermann, T. and B. Pruvot (2011), Financially Sustainable Universities II; European universities diversifying income streams, EUA publications (http://www.eua.be/Libraries/Publications_homepage_list/Financially_Sustainable_Universities_

[28] Eurostat (2006), Youth in Europe: A Statistical Portrait, Eurostat Statistical Books, (http://epp.eurostat.ec.europa.eu/cache/ITY_OFFPUB/KS-78-09- 920/EN/KS-78-09-920-EN.PDF).

[29] Eurostudent (2005), Eurostudent Report 2005: Social and Economic Conditions of Student Life in Europe (http://www.eurostudent.eu/results/reports).

[30] Eurostudent (2008), Social and Economic Conditions of Student Life in Europe, Synopsis of indicators I Final report I Eurostudent IV 2005-2008 (http://www.eurostudent.eu/download_files/documents/EIV_Synopsis_of_Indicators.pdf).

[31] Eurostudent (2011), Social and Economic Conditions of Student Life in Europe; Synopsis of indicators I Final report I Eurostudent IV 2008-2011, (http://www.eurostudent.eu/download_files/documents/EIV_Synopsis_of_Indicators.pdf).

[32] Eurostudent (2012), The European Higher Education Arena in 2012: Bologna Process Implementation Report, (http://eacea.ec.europa.eu/education/eurydice/documents/thematic_reports/138EN.pdf).

[33] Fischer, M. (2010), Eight theories on why college students are studying less, The Atlantic Wire, 6 Jul (http://www.thewire.com/entertainment/2010/07/8-theories-on-why-college-kids-are-studying-less/23827/).

[34] Gill, B. and S. Schlossmann (2003), "A nation at rest: The American way of homework", Educational Evaluation and Policy Analysis, Vol. 25, No. 3, pp. 319-337.

[35] Goldin, C. (1988), "America's graduation from high school: The evolution and spread of secondary schooling in the twentieth century", The Journal of Economic History, Vol. 58, No. 2, pp. 345-374.

[36] Gomis-Porqueras, P., J. Meinecke and J. Rodrigues-Neto (2011), "New Technologies in Higher Education: Lower Attendance and Worse Learning Outcomes? Agenda, Vol. 18, No. 1, pp. 69-83.

[37] Guile, D. and T. Griffiths (2010), "Learning through work experience", Journal of Education and Work, Vol. 14, No. 1, pp. 113-131.

[38] Hakkinen, I. (2006), "Working while enrolled in a university: does it pay?" Labour Economics, Vol. 13, No. 2, pp. 167-189.

[39] Hall, R. (2010), "The work-study relationship: experiences of full-time university students undertaking part-time employment", Journal of Education and Work, Vol. 23, No. 5, pp. 439-449.

[40] Halloran, A. (2011), Danes study less, and spend less time in class, University of Copenhagen: University Post, 16 June, (http://universitypost.dk/article/danes-study-less-and-spend-less-time-class).

[41] Holland, T. and R. Ritvo (2008), Nonprofit Organizations, New York: Columbia University Press.

[42] House of Commons: Innovation, Universities, Science and Skills Committee (2009), "Students and universities", Eleventh Report of Session 2008-09, Vol. 1, 20 July (http://www.publications.parliament.uk/pa/cm200809/cmselect/cmdius/170/170i.pdf).

[43] Huang, F. and P. Capelli (2001), "Employee screening: Theory and evidence", NBER Working Papers No. 12071, March (http://www.nber.org/papers/w12071).

[44] Johnson, V. (2002), "Teacher course evaluations and student grades: An academic tango", Chance, Vol. 15, No. 3, pp. 9-16 (https://stat.duke.edu/~dalene/chance/chanceweb/153.johnson.pdf).

[45] Johnson, V. (2003), Grade inflation: A crisis in college education, New York: Springer Verlag.

[46] Larson, R. (2001), "How U.S. children and adolescents spend time: What does it (and doesn't) tell us about their development?" Current Directions in Psychological Science, Vol. 10, No. 5, pp. 160-165.

[47] Machin, S. and S. McNally (2007), "Tertiary education systems and labour markets", OECD Thematic Review of Tertiary Review, Education and Training Policy Division (http://www.oecd.org/education/innovation-education/38006954.pdf).

[48] Manthei, R. and A. Gilmore (2005), "The effect of paid employment on students' lives", Education and Training, No. 47, Vol. 3, pp. 202-215.

[49] McCaig, C. (2011), "Trajectories of higher education system differentiation: Structural policy-making and the impact of tuition fees in England and Australia", Journal of Education and Work, Vol. 24, Vol. 1-2, pp. 7-25.

[50] McInnis, C. and R. Hartlet (2002), Managing study and work: The impact of full-time study and paid work on the undergraduate experience in Australian universities, Department of Education, Science and Training: University of Canberra.

[51] Moreau, M.-P., and C. Leathwood. (2006), "Balancing Paid Work and Studies: Working (-Class) Students in Higher Education," Studies in Higher Education, Vol. 31, No. 1, pp. 23-42.

[52] Niskanen, W.A. (1968), "The peculiar economics of bureaucracy", American Economic Review, Vol. 58, No. 2, pp. 293-305.

[53] OECD (2012), How Marks and Education Policies Shape Students' Ambitions, OECD, Paris, France.

[54] Purcell, K. (2010), "Flexible employment, student labour and the changing structure of the UK labour market in university cities", Texto Para Discussão No. 007, Centro de Estudos da Metropole

(http://www.fflch.usp.br/centrodametropole/antigo/static/uploads/007.2010%20-%20Purcell.pdf).

[55] Quiggin, J. (1999), “Human capital theory and education policy in Australia”, Australian Economic Review, Vol. 32, No. 2, pp. 130-44.

[56] Robotham, D. (2013), “Lessons at the school of waiting tables: Working while studying”, The Independent, 9 May (http://www.independent.co.uk/incoming/lessons-at-the-school-of-waiting-tables-working-while-studying-8609766.html).

[57] Rojstaczer, S. (2011), Grade inflation at America Colleges and Universities, (http//www.gradeinflation.com).

[58] Rojstaczer, S. and C. Healey (2010), Grading in American Colleges and Universities, Teachers College Record, No. 15928.

[59] Rothschild, M. and J.E. Stiglitz (1976), “Equilibrium in competitive insurance markets: An essay on the economics of imperfect information”, Quarterly Journal of Economics, Vol. 90, No. 4, pp. 630-649.

[60] Rubinson, R. and B. Fuller (1992), “Specifying the Effects of Education on National Economic Growth”, in B. Fuller and R. Rubinson (eds), The political construction of education, New York: Praeger.

[61] Schofer, E. and J. Meyer (2005), “The Worldwide Expansion of Higher Education in the Twentieth Century”, American Sociological Review, Vol. 70, No. 6, pp. 898-920.

[62] Schulmeister, R. and C. Metzger (2011), Die Workload im Bachelor: Zeitbudget und Studierverhalten: Eine empirische Studie, Muenster, Germany: Waxmann.

[63] Sparda Bank (2013), “Studierende und Geld, Die erste Internet-Generation der “Digital Natives” im Studium”, Sparda Studie 2012/2013 (https://www.sparda-hessen.de/pdf/sparda-hessen/SpardaStudie2012-2013.pdf).

[64] Stiglitz, J. (1975), The Theory of "Screening", Education, and the Distribution of Income, The American Economic Review, Vol. 65, No. 3, pp. 283-300.

[65] The Economist (2012), “Not what it used to be: American universities represent declining value for money to their students”, The Economist, 1 December (http://www.economist.com/news/united-states/21567373-american-universities-represent-declining-value-money-their-students-not-what-it).

[66] The Irish Times (2012), “Irish pupils spend less time on maths”, 11 September, (http://www.irishtimes.com/newspaper/breaking/2012/0911/breaking38.html).

[67] Trow, M. (2007), Reflections on the Transition from Elite to Mass to Universal Access: Forms and Phases of Higher Education in Modern Societies since WWII, Springer International Handbooks of Education, Vol. 18, pp 243-280 (http://link.springer.com/chapter/10.1007%2F978-1-4020-4012-2_13).

[68] University World News (2008), "Standards in decline at many universities", Issue No. 33, 22 June (http://www.universityworldnews.com/article.php?story=20080620085336838).

[69] Usher, A. (2009), "Ten Years Back and Ten Years Forward: Developments and Trends in Higher Education in Europe Region", UNESCO Forum on Higher Education in the Europe, 21-24 May, Bucharest (http://www.canedproject.ca/publications/2009/0905_UNESCO.pdf).

[70] Weiss, A. (1995), "Human capital versus signalling explanations of wages", The Journal of Economic Perspectives, Vol. 9, No. 4, pp. 133-154.

[71] Wiles, P. (1974), "The correlation between education and earnings: the external test-not-content hypothesis", Higher Education, Vol. 3., pp. 43-58.

[72] Yorke, M. (2009), "Trends in honors degree classifications, 1994-5 to 2006-7, for England, Wales and Northern Ireland", The Higher Education Academy. (http://www.heacacademy.ac.uk/assets/documents/.../Yorke_trends.doc).

Appendix

Proofs

The proofs of both propositions below are analogous to proofs of the existence and uniqueness of the separating equilibrium in Rothschild and Stiglitz (1976).

Proof of Proposition 3.1

First, suppose that $R_t \geq R$ and consider the optimal choice of students with any ability $a \in (r_t, \overline{a}]$, given the employers' beliefs and the offered wage contract. Students with abilities below and above A will select their studying time, s_t, so that their utility is maximized. First, observe that the minimal studying time needed to obtain the threshold grade $\bar{g}_t$ is

$$\bar{s}_t = \frac{\bar{g}_t + r_t}{a}$$

Then the student who chooses $s_t < \bar{s}_t$ will actually set $s_t = 0$ (and receive $g_t = 0$) because the grade is costly and until it reaches $\bar{s}_t$, there are no benefits to increasing s_t, given the employers' wage contract. Similarly, any student who chooses $s_t \geq \bar{s}_t$ will in fact set $s_t = \bar{s}_t$ (and receive $g_t = \bar{g}_t$), since further increases would merely incur costs with no corresponding benefits. Everyone will therefore set either $s_t = 0$ or $s_t = \bar{s}_t$.

Given the wage contract and the fact just deduced, if the employers' beliefs are to be consistent, then students with ability $a < A$ must set $s_t = 0$, while students with ability $a \geq A$ must set $s_t = \bar{s}_t$. To make sure that this is the case and the threshold grade $\bar{g}_t$ does separate $a \geq A$ and $a < A$, it needs to be such that the incentive-compatibility constraints hold for any $a \in (r_t, \overline{a}]$. In other words, the low-ability types should have no incentives to imitate the high-ability types and the other way round. Namely, it must be that

$$\begin{aligned} u(1-\bar{s}_t, w_{Ht}) &> u(1, w_{Lt}) \text{ for any } a \geq A \\ u(1, w_{Lt}) &> u(1-\bar{s}_t, w_{Ht}) \text{ for any } a < A \end{aligned}$$

that is,

$$\begin{aligned} \left(1 - \frac{\bar{g}_t + r_t}{a}\right)^{\alpha} w_{Ht}^{\beta} &> 1 \cdot w_{Lt}^{\beta} \text{ for any } a \geq A \\ 1 \cdot w_{Lt}^{\beta} &> \left(1 - \frac{\bar{g}_t + r_t}{a}\right)^{\alpha} w_{Ht}^{\beta} \text{ for any } a < A \end{aligned}$$

Both inequalities are fulfilled if and only if at $a = A$ they hold as equalities:[16]

$$\left(1 - \frac{\bar{g}_t + r_t}{A}\right)^{\alpha} w_{Ht}^{\beta} = 1 \cdot w_{Lt}^{\beta}$$

This equality identifies $\bar{g}_t$ *uniquely* for any given w_{Lt}, w_{Ht}, A and r_t:

$$\bar{g}_t = A\left(1 - \left(\frac{w_{Lt}}{w_{Ht}}\right)^{\frac{\beta}{\alpha}}\right) - r_t$$

[16] Since with continuously distributed abilities $a = A$ has measure 0, the fact that for $a = A$ the inequality is not strict "does not matter". This is so in a sense that the probability that a student with ability A prefers to not study and obtain low wage – the behavior associated with $a < A$– is zero.

Now, the proof of the equilibrium still requires (1) to show that as long as A is sufficiently high, no pooling contract chosen by the competitive employer can break the candidate separating equilibrium, (2) to find the equilibrium wages, (3) to specify the equilibrium strategy of the university. Consider each of these points in turn.

(1) Notice that in equilibrium Bertrand competition over wages results in zero profits for each employer. Consider the deviation by one of the employers to the pooling wage contract that is the best (among all pooling contracts) for the students and allows the employer to make positive profits. This best pooling contract offers the single wage that is equal to the average/expected wage in the whole population of students (minus small $\varepsilon > 0$), and this wage is paid to a student with *any* grade $g_t \geq 0$. The average wage is equal to

$$\begin{aligned} Avw_t &= \int_{r_t}^{A} w_{Lt}\widetilde{f}_t(a)da + \int_{A}^{\overline{a}} w_{Ht}\widetilde{f}_t(a)da = \frac{w_{Lt}(A-r_t)}{\overline{a}-r_t} + \frac{w_{Ht}(\overline{a}-A)}{\overline{a}-r_t} = & (2) \\ &= \frac{w_{Ht}\overline{a} - w_{Lt}r_t - A(w_{Ht}-w_{Lt})}{\overline{a}-r_t} & (3) \end{aligned}$$

We show that even this best pooling contract cannot be accepted by both types of students, high- and low-ability, as soon as A is sufficiently high. Indeed, at least some high-ability students will *not* accept this contract if

$$u(1, Avw_t) < u(1-\bar{s}_t, w_{Ht}) \text{ for some} a \geq A$$

or

$$\left(\frac{w_{Ht}\overline{a} - w_{Lt}r_t - A(w_{Ht}-w_{Lt})}{\overline{a}-r_t}\right)^{\beta} < \left(1 - \frac{A\left(1-\left(\frac{w_{Lt}}{w_{Ht}}\right)^{\frac{\beta}{\alpha}}\right)}{a}\right)^{\alpha} w_{Ht}^{\beta} \text{ for some} a \geq A$$

It is easy to see that this condition holds for any $w_{Ht} > w_{Lt}$ if $A = \overline{a}$ and $a > A$. So, for A large enough, high-ability students do not accept the pooling contract and prefer the separating contract instead. Then the deviation to the pooling wage contract is not profitable, and the candidate separating wage contract is an equilibrium.

(2) With separating wage contract, the expected profit of each employer from employing a high-ability student is equal to

$$E(\pi_t \mid a \geq A) = x_H E(a \mid a \geq A) - w_{Ht}$$

Similarly, the expected profit from employing a low-ability student is equal to

$$E(\pi_t \mid r_t \leq a < A) = x_L E(a \mid r_t \leq a < A) - w_{Lt}$$

Both employers have the same technology, hence, the standard Bertrand argument leads to zero profits and

$$\begin{aligned} w_{Ht} &= x_H E(a \mid a \geq A) = x_H \int_A^{\overline{a}} a \frac{1}{\overline{a} - A} da = \frac{x_H}{2}(\overline{a} + A) \\ w_{Lt} &= x_L E(a \mid r_t \leq a < A) = x_L \int_{r_t}^{A} a \frac{1}{A - r_t} da = \frac{x_L}{2}(r_t + A) \end{aligned}$$

where $\frac{1}{\overline{a}-A}$ and $\frac{1}{A-r_t}$ are the truncated probability density functions with the support on $[A, \overline{a}]$ and (r_t, A), respectively.

(3) Finally, consider the optimal choice of requirements by the university at time t. Given the objective of the university to increase its funds by maximizing the pool of students, the university reduces the requirements by "as much as possible" since this maximizes $N_t = \int_{r_t}^{\overline{a}} f(a)da = \frac{\overline{a}-r_t}{\overline{a}-\underline{a}}$. Therefore, as soon as requirements are above r^*, the level at which the rank condition just holds, the university cuts them back.

Notice that the level of requirements r^* exists and is unique. Indeed, consider that using the expression for the average wage in (2) and the expression for N_t, the rank of the university at time t can be written as:

$$\begin{aligned} R_t(r_t) &= \gamma \frac{T}{\frac{\overline{a}-r_t}{\overline{a}-\underline{a}}} + \delta r_t + \eta \frac{w_{Ht}\overline{a} - w_{Lt} r_t - A(w_{Ht} - w_{Lt})}{\overline{a} - r_t} + \xi = \\ &= \gamma \frac{T(\overline{a} - \underline{a})}{(\overline{a} - r_t)} + \delta r_t + \eta \frac{w_{Ht}\overline{a} - w_{Lt} r_t - A(w_{Ht} - w_{Lt})}{\overline{a} - r_t} + \xi \end{aligned}$$

All terms in the sum (apart from ξ) are increasing in r_t, that is, the rank of the university moves together with the requirements. Therefore, the lowest level of requirements for which the rank condition holds is such that $R_t(r_t) = R$. Due to the fact that $R_t(r_t)$ is monotonically increasing in r_t and R satisfies (1), there exists a *unique* solution of $R_t(r_t) = R$ in $(\underline{a}, r_1)$, which we denote by r^*.

So, for any $t \geq 2$ as soon as $r_{t-1} < r^*$ ($r_1 < r^*$ due to the second inequality in (1)) the optimal strategy of the university is to set

$$r_t = \max\{r_{t-1} - \Delta, r^*\}$$

$r_t = r_{t-1} - \Delta$ guarantees that $N_t = N_{t-1}(r_{t-1}) + \frac{\Delta}{\overline{a}-\underline{a}}$, i.e. the quota of students at time t is used fully and the number of students at t is maximized:

$$\begin{aligned} \frac{\overline{a} - r_t}{\overline{a} - \underline{a}} &= \frac{\overline{a} - r_{t-1}}{\overline{a} - \underline{a}} + \frac{\Delta}{\overline{a} - \underline{a}} \\ r_t &= r_{t-1} - \Delta \end{aligned}$$

On the other hand, if $r_{t-1} = r^*$, then the requirements are not adjusted anymore.[17]

[17] In particular, this means that r^* is the long-run value of the university requirements at which the rank condition is just satisfied.

Proof of Proposition 3.2

Consider the candidate pooling equilibrium. The best (for the students) pooling wage contract offers the wage that is equal to the average/expected wage in the whole population of students (see (2)), paid irrespectively of a student's grade. The profits of both employers are zero.

Below we show that there exists another contract such that the deviation by any of the employers to this contract is profitable. Consider the contract that offers $w_t = w_{Ht}$ (minus small $\varepsilon > 0$) if $g_t \geq \widehat{g}_t$ and $w_t = w_{Lt}$ if $g_t < \widehat{g}_t$ for some $\widehat{g}_t > 0$. We prove that there exists $\widehat{g}_t$ such that this contract is accepted by high-ability students but rejected by low-ability students, so that the deviating employer raises positive profits.

The contract is accepted by high-ability students, who choose the studying time $\widehat{s}_t = \frac{\widehat{g}_t + r_t}{a}$ (just enough to receive the grade $\widehat{g}_t$) as soon as

$$u(1, Avw_t) < u(1 - \widehat{s}_t, w_{Ht}) \text{ for any } a \geq A$$

Equivalently,

$$\left(\frac{w_{Ht}\overline{a} - w_{Lt}r_t - A(w_{Ht} - w_{Lt})}{\overline{a} - r_t}\right)^{\beta} < \left(1 - \frac{\widehat{g}_t + r_t}{a}\right)^{\alpha} w_{Ht}^{\beta} \text{ for any } a \geq A \tag{4}$$

On the other hand, the contract is rejected by low-ability students, who choose the pooling wage contract with $s_t = 0$ (and $g_t = 0$), as soon as

$$u(1, Avw_t) > u(1 - \widehat{s}_t, w_{Ht}) \text{ for any } a < A$$

Equivalently,

$$\left(\frac{w_{Ht}\overline{a} - w_{Lt}r_t - A(w_{Ht} - w_{Lt})}{\overline{a} - r_t}\right)^{\beta} < \left(1 - \frac{\widehat{g}_t + r_t}{a}\right)^{\alpha} w_{Ht}^{\beta} \text{ for any } a < A \tag{5}$$

Both conditions, (4) and (5) are satisfied if and only if they hold as equality at $a = A$. Solving this equality for $\widehat{g}_t$ produces

$$\widehat{g}_t = A\left(1 - \left(\frac{w_{Ht}\overline{a} - w_{Lt}r_t - A(w_{Ht} - w_{Lt})}{(\overline{a} - r_t)w_{Ht}}\right)^{\frac{\beta}{\alpha}}\right) - r_t$$

Hence, the deviating contract breaks the candidate pooling equilibrium.

Proof of Proposition 3.3

We only need to prove part 4 of the proposition. Namely, we need to study the effect of r_t on Avg_t and Avs_t. First, notice that

$$\begin{aligned}
Avg_t &= \int_A^{\overline{a}} \overline{g}_t \widetilde{f}_t(a)da = \overline{g}_t \int_A^{\overline{a}} \frac{1}{\overline{a} - r_t} da = \overline{g}_t \frac{\overline{a} - A}{\overline{a} - r_t} = \left[A\left(1 - \left(\frac{x_L}{x_H}\right)^{\frac{\beta}{\alpha}} \left(\frac{r_t + A}{\overline{a} + A}\right)^{\frac{\beta}{\alpha}}\right) - r_t\right] \frac{\overline{a} - A}{\overline{a} - r_t} \\
Avs_t &= \int_A^{\overline{a}} \overline{s}_t \widetilde{f}_t(a)da = \int_A^{\overline{a}} \frac{\overline{g}_t + r_t}{a} \frac{1}{\overline{a} - r_t} da = \int_A^{\overline{a}} \frac{A\left(1 - \left(\frac{w_{Lt}}{w_{Ht}}\right)^{\frac{\beta}{\alpha}}\right)}{a} \frac{1}{\overline{a} - r_t} da = \\
&= \frac{A\left(1 - \left(\frac{w_{Lt}}{w_{Ht}}\right)^{\frac{\beta}{\alpha}}\right)}{\overline{a} - r_t} (ln\overline{a} - lnA) = \frac{A\left(1 - \left(\frac{x_L}{x_H}\right)^{\frac{\beta}{\alpha}} \left(\frac{r_t + A}{\overline{a} + A}\right)^{\frac{\beta}{\alpha}}\right)}{\overline{a} - r_t} (ln\overline{a} - lnA)
\end{aligned}$$

Consider the partial derivatives of Avg_t and Avs_t with respect to r_t and evaluate their sign. Simple algebra leads to

$$\frac{\partial Avg_t}{\partial r_t} = \frac{\overline{a}-A}{(\overline{a}-r_t)^2}\left[A\left(\frac{x_L}{x_H}\right)^{\frac{\beta}{\alpha}}\left(\frac{r_t+A}{\overline{a}+A}\right)^{\frac{\beta-\alpha}{\alpha}}\left(-\left(\frac{r_t+A}{\overline{a}+A}\right)-\frac{\beta}{\alpha}\frac{\overline{a}-r_t}{\overline{a}+A}\right)+A-\overline{a}\right]<0$$

$$\frac{\partial Avs_t}{\partial r_t} = \frac{(r_t+A)^{\frac{\beta-\alpha}{\alpha}}}{(\overline{a}-r_t)^2}\frac{A\,(ln\overline{a}-lnA)}{(\overline{a}+A)^{\frac{\beta}{\alpha}}}\left[\left(\frac{\overline{a}+A}{r_t+A}\right)^{\frac{\beta}{\alpha}}(r_t+A)-\left(\frac{x_L}{x_H}\right)^{\frac{\beta}{\alpha}}(r_t+A)-\frac{\beta}{\alpha}\left(\frac{x_L}{x_H}\right)^{\frac{\beta}{\alpha}}(\overline{a}-r_t)\right]$$

The sign of $\frac{\partial Avs_t}{\partial r_t}$ is positive, at least as soon as (i) $\frac{x_L}{x_H}$ is sufficiently small and/or (ii) $\frac{\beta}{\alpha}$ is sufficiently small and A is sufficiently large.