

EUROPEAN ATOMIC ENERGY COMMUNITY — EURATOM Brussels (Belgium)

CENTRE D'ETUDE DE L'ENERGIE NUCLEAIRE — C.E.N. Mol (Belgium)

MEASUREMENT OF THE THERMAL NEUTRON ABSORPTION CROSS SECTION OF BORON BY MEANS OF A TIME OF FLIGHT TECHNIQUE

by

A. DERUYTTER (CEN-SCK) G. DEBUS, K. LAUER, H. MORET, A. PROSDOCIMI (CBNM, Euratom)

MAY 1962



CENTRAL BUREAU FOR NUCLEAR MEASUREMENTS - EURATOM - GEEL CENTRE D'ETUDE DE L'ENERGIE NUCLEAIRE - CEN - MOL

Report on Contract No 002.59.8 - AMPB

LEGAL NOTICE

This document was prepared under the sponsorship of the Commission of the European Atomic Energy Community (Euratom).

Neither the Euratom Commission, its contractors nor any person acting on their behalf :

- 1º Makes any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this document, or that the use of any information, apparatus, method, or process disclosed in this document may not infringe privately owned rights; or
- 2° Assumes any liability with respect to the use of, or for damages resulting from the use of any information, apparatus, method of process disclosed in this document.

This report can be obtained, at the price of Belgian Francs 30, from : PRESSES ACADEMIQUES EUROPEENNES, 98, chaussée de Charleroi, Brussels (Belgium).

Please remit payments :

- to BANQUE DE LA SOCIETE GENERALE (Agence Ma Campagne) account Nº 964.558;
- to BELGIAN AMERICAN BANK and TRUST COMPANY
 New York account Nº 121.86;
- to LLOYDS BANK (Foreign) Ltd, 10 Moorgate London E.C.2;

giving the reference : EUR 12.e — Measurement of the thermal neutron absorption cross-section of boron by means of a time of flight technique.

EUR 12.e

MEASUREMENT OF THE THERMAL NEUTRON ABSORPTION CROSS SECTION OF BORON BY MEANS OF A TIME OF FLIGHT TECHNIQUE DEBUS, K. LAUER, H. MORET, BY A. DERUYTTER (CEN-SCK), G. A. PROSDOCIMI (CBNM, EURATOM)

Central Bureau for Nuclear Measurements-EURATOM-Geel. Centre d'Etude de l'Energie Nucleaire-CEN-Mol. Report on contact nº 002.59.8 - AMPB. May 1962 - pages 41 + fig. 10.

A precise determination of the neutron absorption cross-section of boron in the thermal energy region, 0.006 - 0.082 eV, has been undertaken by means of time-of-flight technique.

Two samples of deuterated boric acid solution in heavy water were measured, containing respectively 19.81 % and 96.51 % of B¹⁰.

The following results were obtained (at 0.0253 eV) :

Sample nº 1 : $\sigma_a = (761 \pm 2)b;$ Sample nº 2 : $\sigma_a = (3703 \pm 9)b;$

from which the following values for the absorption cross section of B10 are respectively derived :

 $\sigma_{a} (B^{10}) = (3840 \pm 11)b;$

 $\sigma_{a}(B^{10}) = (3837 \pm 9)b.$

Their weighted mean is equal to $(3838 \pm 10)b$.

الممعد سالم المناجم التارين الماليا المالي والموم و سارد مدامه مراجز مراجر

EUR 12.e

MEASUREMENT OF THE THERMAL NEUTRON ABSORPTION CROSS SECTION OF BORON BY MEANS OF A TIME OF FLIGHT TECHNIQUE BY A. DERUYTTER (CEN-SCK), G. DEBUS, K. LAUER, H. MORET, A. PROSDOCIMI (CBNM, EURATOM)

Central Bureau for Nuclear Measurements-EURATOM-Geel.

Centre d'Etude de l'Energie Nucleaire-CEN-Mol. Report on contract nº 002.59.8 - AMPB.

May 1962 - pages 41 + fig. 10.

A precise determination of the neutron absorption cross-section of boron in the thermal energy region, 0.006 - 0.082 eV, has been undertaken by means of time-of-flight technique.

Two samples of deuterated boric acid solution in heavy water were measured, containing respectively 19.81 % and 96.51 % of B10,

The following results were obtained (at 0.0253 eV) : Sample n° 1 : $\sigma_a = (761 \pm 2)b$; Sample n° 2 : $\sigma_a = (3703 \pm 9)b$;

from which the following values for the absorption cross section of B¹⁰ are respectively derived :

 $\sigma_{a} (B^{10}) = (3840 \pm 11)b;$ $\sigma_{a} (B^{10}) = (3837 \pm 9)b.$

Their weighted mean is equal to (3838 ± 10) b.

EUR 12.e

MEASUREMENT OF THE THERMAL NEUTRON ABSORPTION CROSS SECTION OF BORON BY MEANS OF A TIME OF FLIGHT TECHNIQUE BY A. DERUYTTER (CEN-SCK), G. DEBUS, K. LAUER, H. MORET, A. PROSDOCIMI (CBNM, EURATOM)

.

Central Bureau for Nuclear Measurements-EURATOM-Geel. Centre d'Etude de l'Energie Nucleaire-CEN-Mol. Report on contact nº 002.59.8 - AMPB. May 1962 - pages 41 + fig. 10.

A precise determination of the neutron absorption cross-section of boron in the thermal energy region, 0.006 - 0.082 eV, has been undertaken by means of time-of-flight technique.

Two samples of deuterated boric acid solution in heavy water were measured, containing respectively 19.81 % and 96.51 % of B^{10} .

The following results were obtained (at 0.0253 eV) : Sample n° 1 : $\sigma_a = (761 \pm 2)b;$ Sample n° 2 : $\sigma_a = (3703 \pm 9)b;$ from which the following values for the absorption cross section of B¹⁰ are respectively derived :

 $\sigma_{\rm a} ({\rm B}^{10}) = (3840 \pm 11){\rm b};$

 $\sigma_{\rm a} ({\rm B}^{10}) = (3837 \pm 9){\rm b}.$

Their weighted mean is equal to $(3838 \pm 10)b$.





EUROPEAN ATOMIC ENERGY COMMUNITY — EURATOM Brussels (Belgium)

CENTRE D'ETUDE DE L'ENERGIE NUCLEAIRE — C.E.N. Mol (Belgium)

MEASUREMENT OF THE THERMAL NEUTRON ABSORPTION CROSS SECTION OF BORON BY MEANS OF A TIME OF FLIGHT TECHNIQUE

by

A. DERUYTTER (CEN-SCK) G. DEBUS, K. LAUER, H. MORET, A. PROSDOCIMI (CBNM, Euratom)

MAY 1962



CENTRAL BUREAU FOR NUCLEAR MEASUREMENTS - EURATOM - GEEL CENTRE D'ETUDE DE L'ENERGIE NUCLEAIRE - CEN - MOL

Report on Contract No 002.59.8 - AMPB

·

·

The present report describes a precision measurement of the absorption cross section of boron for thermal neutrons by a joint group of scientists from CEN-SCK, Mol, Belgium and from Euratom's Central Bureau for Nuclear Measurements, Geel, Belgium.

This programme was started at the end of 1959, at a time when the boron absorption cross-section values available presented discrepancies much larger than the errors quoted. Because of the importance of boron as a standard for neutron cross-section measurements, it was decided to try to improve the absorption cross-section measurement, as a supplement to the chemical and isotopic investigations on boron, already started in view of the establishment at the Central Bureau for Nuclear Measurements of a stock of standard boron.

Recent results on the boron absorption cross section have been obtained either for a neutron energy above 0.018 eV (ORNL-group) or with a crystal spectrometer (Columbia-group). It was felt that a measurement in an energy range centered around 0.025 eV and by a time-of-flight technique would be a useful addition to the available data.

I should like to thank the CEN-SCK-scientists, especially M. M. Nève de Mévergnies, head of the neutron physics department, for the very pleasant cooperation within the joint group.

J. SPAEPEN.

CONTENTS

.

,

1 —	INTRO	DUCTION.											
	1.1 —	Present status of the measurer	nents	•		•	•						7
	1.2 —	Purpose of the measurement						•				•	9
	1.3 —	Experimental method		•	•			•		•	•	•	10
2 —	SAMPL	E PREPARATION.											
	2.1 —	Boron sample preparation .								•			12
		2.1.1 — Preparation of nature	al bo	ron	solu	tion							12
		2.1.2 — Preparation of the en	riched	d bo	ron	solu	tion	•		•			12
		2.1.3 — Checking of purified	prod	łuct									12
	2.2 —	Boron content determination										•	13
	2.3 —	Mass-spectrometric determination	on	•									13
	2.4. —	Samples specifications .									•		14
		2.4.1 — Heavy water and so	lution	deı	nsiti	es							14
		2.4.2 — Measuring cells .	•									•	14
		2.4.3 — Sample positioning									•	•	16
3 —	EXPER	IMENTAL APPARATUS.											
	3.1 —	Slow chopper						•	•				17
		3.1.1 — Rotor					•						17
		3.1.2 — Electronic equipment	t.				•						18
		3.1.3 — Detectors											18
	3.2 —	Neutron energy calibration .											18
		3.2.1 — Method											18
		3.2.2. — Calibration measuren	nents	for	the	bor	on i	cross	-sect	ion	mea	su-	
		rements	•	•	•	•	•		•	•	•	•	. 20
4 —	MEASU	JREMENTS AND RESULTS.											
	4.1 —	Measurement of neutron train	nsmiss	sion									21
	4.2 —	Experimental data processing											24
	4.3 —	Results and conclusions										•	27
APPI	ENDIX	I — Flight path definition									•	•	28
APPI	ENDIX	II — Correction for multiple	scat	terin	ıg							•	29
APPI	ENDIX	III — Systematic errors .						•				•	30
Refe	ences												35

KEY TO THE FIGURES

Fig.	1 — Present status of the thermal cross section of natural B	9
Fig.	2 — Measuring cell and neutron diaphragm	
	a) Quartz cell	15
	b) Perspex holder	15
	c) Cadmium window	15
Fig.	3 — Four place sample changer	17
Fig.	4 — Transmission cut-off for the plane (110) of α -Fe	19
Fig.	5 — Transmission of (110) α -Fe after adjustment (1) and during the	
	boron runs (2)	20
Fig.	$6 - BF_3$ counters characteristics, (type 12 EB70/G)	21
Fig.	7 — U^{235} fission monitor characteristics	22
Fig.	8 — K coefficient of the boron absorption cross-section	26
Fig.	9 — Flight path termination in the detector	28
Fig.	10 — Multiple collisions inside the sample	29

KEY TO THE TABLES

Tab.	I — Status of measurements on boron thermal cross-section	ı.	•	•	36 - 37
Tab.	II — Counting in the time-of-flight analyser and monitor .		•		38 - 39
ТаЬ.	III — Experimental conditions for the slow chopper		•		40
Tab.	IV — Boron total cross-section versus time-of-flight				41

MEASUREMENT OF THE THERMAL NEUTRON ABSORPTION CROSS-SECTION OF BORON BY MEANS OF TIME OF FLIGHT TECHNIQUE

SUMMARY.

A precise determination of the neutron absorption cross-section of boron in the thermal energy region, 0.006 - 0.082 eV, has been undertaken by means of time-of-flight technique.

Two samples of deuterated boric acid solution in heavy water were measured, containing respectively 19.81 % and 96.51 % of B^{10} .

The following results were obtained (at 0.0253 eV) :

Sample n° 1 : $\sigma_a = (761 \pm 2)b$ Sample n° 2 : $\sigma_a = (3703 \pm 9)b$,

from which the following values for the absorption cross section of B¹⁰ are respectively derived :

 $\sigma_{a} \ (B^{10}) = (3840 \ \pm \ 11)b$

 $\sigma_{a} (B^{10}) = (3837 \pm 9)b$

Their weighted mean is equal to $(3838 \pm 10)b$.

1. INTRODUCTION (*)

1.1 — Present status of the measurements.

The special importance of boron as a standard thermal neutron absorber is due both to the high absorption cross-section of its isotope of mass 10 in the reaction B^{10} (n, α) Li⁷ and to its rigorous $1/\sqrt{E}$ behaviour versus the energy *E* of neutrons, a phenomenon which has been checked up to values of the order of 10^4 eV.

About 20 % of B^{10} is present in the natural element with fluctuations generally attributed to the geographical origin of the ore; thus, mention of the origin should always be made in the case of "natural" boron.

The measuring procedures applied up to now consist, in general, of neutron transmission measurements, applying either time-of-flight methods (chopper and pulsed accelerators) or using monochromatic neutrons (crystal spectrometer). Some measuring procedures were based on the mean life of neutrons pulsed in an absorbent diffuser; some others, applying the pile-oscillator method, made it possible to draw comparisons between various boron samples. Measurements of the mean life of neutrons, although more complicated from the standpoint of interpretation, give the absorption cross-section, whereas transmission measurements, being directly related to the total cross-section, require that the scattering cross-section is known.

^(*) A. Prosdocimi, Euratom.

Table I gives for natural boron and its isotope B^{10} a complete and up-to-date picture of the results either published by other experimenters or obtained in the course of this investigation, expressed in terms of the absorption cross-section for 2,200 m/s neutrons (or with an energy of 0.0253 eV).

Before proceeding to a comparative analysis, a special remark should be made about the isotopic ratio of B^{10} to B^{11} in natural boron. In fact, the value of the absorption cross-section becomes significant only when such a parameter as can be obtained by means of mass-spectrometric determination is considered.

The main feature of the measurements made up to 1958 was that they were, in scope, restricted to natural boron, stocks of which had been prepared previously in various laboratories, and intended to carry out comparative measurements of the various stocks available. Determination of the isotopic ratio of B^{10} were carried out separately on the whole stock but not during each cross-section measurement.

It was thus ascertained that standard ANL - BNL boron from California contained (19.8 \pm 0.2) % of B^{10}, ref. 11 and 15. For other boron supplies, the absorption cross-section ratios were found to be as follows:

Harwel/ANL-BNL	=	1.012 ± 0.003	ref.	4 and	16
Oxford/Harwell	=	1.001 ± 0.003	ref.	6	
Stockholm/ANL-BNL	=	0.997 ± 0.007	ref.	9	

A different trend noticeable in the recent measurements, made from 1960 onwards, consisted in determining the isotopic ratio of every sample subjected to neutron transmission, affording the possibility of utilizing samples with a higher B^{10} enrichment. Accordingly, the aim in view is now to arrive at a conclusive determination of the absorption cross-section of isotope B^{10} and to make the definition of the various boron standards either available or being established dependent upon the determination of their isotopic ratio. It is the desire to achieve this purpose which forms the basis of the experiment described in this report. The values of σ_A normalized for B^{10} and energy 0.0253 eV have thus been determined and are given in Table I for the purpose of comparison only.

1.2 — Purpose of the measurement.

If a comparison is made between the various values obtained for the thermal absorption cross-section of the so-called "natural boron", taking account of their respective statistical deviations (Fig. 1), the differences observed among them are not always justified by the level of accuracy attained during the measurements. These differences should be attributed to the presence of systematic errors which cannot always be clearly singled out or evaluated. It is with the main purpose of minimizing the causes of systematic error that we have resumed boron cross-section measurements, being particularly careful to achieve the same degree of accuracy in all the experimental steps contributing to the final result. It was necessary te cut down to essentials the data to be taken into account in the earlier works of previous research workers.

The essential factors affecting the accuracy of the results were as follows :---

- measurement of B¹⁰ isotopic ratio using a mass-spectrometer; this measurement should be made with the same boron material as used for the experiment;
- measurement of the boron content by chemical analysis, ascertaining at the same time its degree of chemical purity, in particular with respect to high neutron cross-section elements;
- defining both the uniform thickness of the absorbent sample subjected to neutron transmission, this being obtained by accurate selection of materials, and the shape and dimensions of the container;



FIG. 1. --- Present status of the thermal cross-section of natural B.

- establishing as neutron transmission of the boron sample a suitable value to be calculated on the basis of results selected on account of their statistical reliability and producing a high count likely to reduce the statistical error inherent in the measurement.

In the course of the experiment, many other precautions proved necessary, some of the most important of which were :

- elimination from the boron material of any trace of hydrogen; this was achieved by using deuterated compounds only, and performing all handling operations in controlled atmosphere;
- use of boron with high B^{10} enrichment in order to render negligible other sections of secondary importance in relation to the predominant value of σ_A (B^{10});
- direct measurement of the transmission of the heavy water used as solvent in the preparation of the boron in order to correct for the effects of any density variation in the B_2O_3 solutions;
- integral monitoring of the neutron beam in order to offset any intensity fluctuation of the neutron flux;
- alternate cycle exposure of the samples to the neutron beam in order to offset the longterm drift to which the experimental conditions could be subject;

- -- strictly accurate re-positioning of the sample in the neutron beam by means of an automatic device;
- break-down into sub-totals of the count of the neutrons transmitted so as to check the statistical distribution of the measurements.

1.3 — Experimental method.

The measuring procedure followed was based on the transmission of a collimated neutron beam through a test specimen of uniform thickness d placed in the direction of the beam and made of a material with a total macroscopic cross-section Σ_{T} . As a result of being transmitted through the test specimen, the neutron beam of intensity I_o is attenuated to an intensity I according to the relation

$$I = I_0 \exp\left(-d\Sigma_T\right)$$
^[1]

In order to obtain a homogeneous distribution of the boron atoms in the specimen, the boron is utilized in the form of a B_2O_3 — heavy water solution kept in pure quartz containers with flat parallel walls. In these conditions, and with a container wall thickness δ , the neutron beam attenuation is:

$$I = I_0 \exp \left[--d \Sigma_T (B_2O_3) - d \Sigma_T (D_2O) - \delta\Sigma_T (SiO_2)\right]$$
[2]

It is enough, then, to repeat the transmission measurement with the same container, filled this time with heavy water only,

$$I' = I_0 \exp \left[-d \Sigma_T (D_2 O) - \delta \Sigma_T (SiO_2)\right]$$
[3]

to derive from [2] and [3] the macroscopic cross-section of B2O3,

$$\Sigma_{\rm T} (B_2 O_3) = 1/d \cdot \ln (I'/I).$$
 [4]

In turn, the B_2O_3 total macroscopic cross-section is composed of the following terms, $N(B_2O_3)$ being the number of molecules per unit cf volume.

$$\Sigma_{T} = N (B_{2}O_{3}) [2\sigma_{A} (B) + 2\sigma_{S} (B) + 3\sigma_{T} (O)]$$
[5]

in wich

 σ_{A} is the absorption microscopic cross-section

 σ_s is the scattering microscopic cross-section

 σ_{T} is the total microscopic cross-section.

The numerical values of the individual terms are, according to the current bibliographical references (ref. 17), given in "barn" units (i.e., 10^{-24} cm²) and at the energy $E_0 = 0.0253$ eV,

Atom	σ _A	σs	στ
B10	3848 ± 38	4.0 ± 0.5	
B ¹¹	0.005 ± 0.003	4.4 ± 0.3	
0	_	_	$4.24 ~\pm~ 0.02$

As in the present experiment the measurable quantity is that given between brackets in [5], it was necessary, in order to determine σ_A (B), to make a hypothesis on the remainning terms. In line with refs. 7 and 18, it was assumed that.

$$2\sigma_{s}$$
 (B) + $3\sigma_{T}$ (O) = 21 barns [6]

The contribution of these terms to σ_{τ} (B₂O₃) amounts to 1.3 % in the case of natural boron and to 0.3 % in the case of boron with 96 % B¹⁰ enrichment, so that any uncertainty involved cannot appreciably influence the results.

The transmission experiment was performed at the same time and in identical experimental conditions on 4 samples held in identical quartz cells :

- a) 1.67 % solution of B_2O_3 in heavy water, (19.8 % of B^{10});
- b) 0.38 % solution of B_2O_3 in heavy water, (96.5 % of B^{10});
- c) heavy water, (compensation sample);
- d) helium gas, (compensation sample).

From relative comparisons, the following measurements can be obtained :

a/c — total cross-section of B_2O_3 with 19.8 % of B^{10} enrichment;

b/c — total cross-section of B_2O_3 with 96.5 % of B^{10} enrichment;

c/d — total cross-section of heavy water.

This latter factor is designed to introduce the density correction as described in chapter 4.

The neutron beam was provided by the thermal column of the BR-1 reactor at Mol (power 4 MW), which has a Maxwellian spectrum with a characteristic temperature of 58° C.

Given the dependence of σ_A (B) on the neutron energy, the "time-of-flight" technique was applied to the transmission measurement using a thermal neutron chopper and a multi-channel analyser both of wich are described in chap. 3.

Two series of measurements, each one over 25 channels, covered the following energy ranges :

they were made to overlap at the conventional thermal energy of 0.0253 eV.

The value of σ_A (B) for neutrons of velocity 2,200 m/s is derived from the assumption that the 1/v law applies to boron within the energy range investigated. In other words, according to this hypothesis, the quantity.

$$K = \sigma_A (B)/\tau = [\sigma_T (B_2O_3) - 21 \text{ barns}]/2\tau$$
 [7]

should be independent of the energy (or of the time-of-flight τ). After having established the maintenance of such uniformity within the limits of the statistical deviations inherent in single measurements, we may finally obtain :

$$\sigma_{A} (B)_{2200} = K \cdot \tau_{2200}$$
 [8]

2. SAMPLE PREPARATION

2.1 — Boron sample preparation (*).

2.1.1 — Preparation of natural boron solution.

As raw material, we used boric anhydride (a 5 kg batch of pure anhydride Merck, p. A.) supplied to us with the following analysis report :

B ₂ O ₃ content :	99.4 %
Desiccation loss :	0.32 %
Chloride (Cl):	approx. 0.0003 %
Sulphate (SO ₄) :	approx. 0.003 %
Silicate (SiO ₂):	арргох. 0.0008 %
Phosphate (PO ₄):	approx. 0.0005 %
Copper (Cu):	approx. 0.00006 %
Lead and Zinc (as lead):	approx. 0.0005 %
Iron (Fe):	approx. 0.00007 %
Calcium (Ca):	less than 0.001 %
Arsenic (As):	practically free
Water :	maximum 0.32% (eliminated through desiccation)

An aliquot part of the Merck boric anhydride was oven-dried at 800-900° for about 6 hours in a platinum capsule to produce an anhydride, the water content of which was assumed to be of the order of 0.1 %. It was then dissolved in hot heavy water. After cooling down to approximately 0° C, the re-crystallized product was collected on a Gooch filter. The resulting chemical compound, corresponding to the formula B_2O_3 , x H_2O , y D_2O was then exposed for 8 hours in an exsiccator to a dry nitrogen stream at 20°C to dehydrate it to a compound intermediate between D_3BO_3 and DBO_2 . This compound contained negligible traces of $H_3BO_3 + HBO_2$, and has dissolved in heavy water to obtain a solution containing about 2 % of B_2O_3 . Quartz containers were used for all handling operations and all operation performed in a dry base.

2.1.2 — Preparation of the enriched boron solution.

The B¹⁰ enriched compound was supplied by the Oak Ridge National Laboratory in the form of boric acid. It was dried at 20° C for 4 hours in an exsiccator in a dry nitrogen stream, re-crystallized from its heavy-water solution and dried again for 8 hours in a dry nitrogen stream. The product obtained, which corresponded to a formula identical to the one given for the natural boron product, was then dissolved in heavy water to obtain a solution containing approximately 0.5 % of B₂O₃. As in the first case, all the handling operations were carried out in quartz containers in a dry-box.

2.1.3 — Checking of purified product.

Both spectrum and chemical analysis confirmed the data given in the supplier's analysis report, demonstrating the absence in both natural and enriched boron compounds of harmful quantities of elements having high neutron cross-sections.

^(*) K. F. Lauer, Euratom.

2.2 — Boron content determination (*).

The two solutions referred to above were titrated to determine their exact boric acid contents. The titration method used was based on the neutralization by sodium hydroxide of the highly acidity on boric acid-mannitol complex; the end points of the reaction were detected by an accurate potentiometric method, (ref. 19).

By this method, the end point is localised by recording in the neighbourhood of the end point the derivative of successive potential changes resulting from successive additions of very small quantities of NaOH(0,1n) of the order of 0.015 ml.

The total quantity of 0,1n NaOH used for titrating on known weight of H_3BO_3 solution is given by the volume dispensed between the first end point, corresponding to the neutralization of a few drops of hydrochloric acid added to the medium in the presence of the boric acid at the start of titration, and the end point corresponding to the final neutralization point of the boric acid after mannitol addition.

The normality of the NaOH solution was accurately determined by standardization with: a) HC1 [concentration determined by gravimetric analysis (AgC1)], b) Potassium and phtalate (NBS) and benzoic acid (NBS).

On the other hand, the titration method was found to be in agreement within 0,04 % with the gravimetric analysis method used to determine the boric acid by weight [calcium borate] (ref. 20), this is less then the standard deviation (95 % security) of 0.045 % of the gravimetric method.

The results obtained were calculated by using the evaluation of the molecular weight of boric anhydride, the atomic weight of boron derived from its isotopic composition determined by mass spectrometry. The results are given in grams of boric anhydride per 100 g. of solution.

The various data regarding the solutions have been grouped in the following table :

	Natural Boron	Enriched Boron
Isotopic composition as shown by the mass		
spectrometry laboratory	% B^{10} =19.81 ± 0.02	% B^{01} =96.515 ± 0.013
Molecular weight of boric anhydride calculated		
on the basis of the isotopic content	69.6040 ± 0.0004	68.0696 ± 0.0002
Concentration of the solution in g. of B_2O_3 per		
100 g. of solution	1.67712 ± 0.00018	0.38591 ± 0.00011

2.3 — Mass spectrometric determination (**).

A 60° deflection, 12 inch radius mass spectrometer was used for the isotopic analysis. The filaments for the triple filament source were fabricated of tantalum ribbon (0.0035 mm. x 0.7 mm.). These filaments were mounted on kovar terminals. A few drops of a concentrated solution of HBO₃ — NaOH mixture were loaded on the side filaments and evaporated to dryness in air. After mounting the filaments in the ion source the mass spectrometer was evacuated to $2 \cdot 10^{-6}$ mm Hg or better.

The filaments were heated by means of a d.c. electric current and $Na_2BO_2^+$ ions were obtained. On the recorded spectrum the $Na_2B^{11}O_2^+$ peak was corrected by — 0.074 % of the $Na_2B^{10}O_2^+$ peak (correction necssary for the contribution of the ion $Na_2B^{10}O^{16}O^{17}$ to the $Na_2B^{11}O^{16}O^{16}$ peak).

^(*) K. F. Lauer, Euratom.

^(**) G. Debus, Euratom.

Focussing of the two masses was obtained by a constant acceleration voltage (7000 V) and magnetic scanning.

The isotopic composition of a sample of normal boron was determined by measuring twenty loadings (each of 10 scans or more).

The final results were :

 B^{10} : (19.81 ± 0.04) % (95 % probability for the standard deviation of the mean); B^{11} : (80.19 ± 0.04) %.

A sample of enriched boron was measured (14 loadings of 10 scans or more). The result was:

 B^{10} : (96.515 \pm 0.022) % (95 % prob. of stand. dev. of mean);

B¹¹: (3.485 ± 0.022) %.

A small amount of mass discrimination may influence the absolute value of these results. This mass discrimination is presently being investigated.

2.4 — Samples specifications (*).

2.4.1 — Heavy water and solution densities.

A batch of heavy water was completely distilled in a vacuum distillation apparatus, the first and last fraction being collected in a big flask, while the middle fraction was collected in a glass test tube containing a calibrated float made in fused quartz. Handling and distillation were performed in a special dry-box with a relative humidity better than 5 %. In order to carry out a density measurement the test tube was transferred to a high precision thermostat. The temperature is adjusted to a value at which the float is in kinetic equilibrium with the surrounding liquid, i.e. the densities of liquid and float are equal. From this temperature the density of D_2O at 25°C is derived by means of the known density of the float (ref. 21),

 $\rho_0 (25^{\circ}C) = 1.104485 \text{ g/cm}^3.$

To determine the densities of the natural and enriched boric acid solutions the filling tube of the cells were provided with a reference mark. By weighing the cells empty and filled with distilled water the volume capacity of each cell was measured. After the filling procedure the cells were weighed again. From these values the densities were then calculated.

Cell nº	Filling	Density at 25° C
3	He	
4	D_2O	$(1.104485 \pm 0.000001) \text{ g/cm}^3$
1	nat. boron solut.	(1.1162 ± 0.0001) »
6	enr. boron solut.	(1.1072 ± 0.0001) »

2.4.2 — Measuring cells.

The cells (Fig. 2), are made of fused quartz. The diameter and cell length d are 52 and 20 mm respectively. The cell length was specified with an accuracy of + 0.010 mm. These dimen-

^(*) H. Moret, Euratom.



sions could be determined with a method specially designed for this purpose : steel balls were dropped into the cells and magnetically clamped against the inner sides ot the windows; by means of a measuring microscope the distance between the balls was measured. From this the cell lengths and their uniformity could be determined with an accuracy of \pm 5 μ (ref. 22). From the topography of the cells, the average cell lengths were calculated. Over a central area of 40 mm diameter the maximum deviations from the average value are smaller than \pm 10 μ . The cell lengths and the thicknesses of the windows result :

Cell nº	Cell length	Total thickness of the two windows
3	(2.005 ± 0.001) cm	(0.395 ± 0.002) cm
4	(2.004 ± 0.001) »	(0.390 ± 0.002) ►
1	(2.003 ± 0.001) »	(0.393 ± 0.001) *
6	(2.005 ± 0.001) *	(0.397 ± 0.002) »

The vacuum tightness of the cells has been checked too. The filling of the cell was carried out in the controlled atmosphere of the dry-box. All the necessary equipment was left in the dry-box during 24 hours. Then solution was transferred from the flask into the cells by means of hypodermic syringes. The fourth cell was flushed and filled with helium. After careful removal of air bubbles all the sample cells were sealed with a plastic cap.

2.4.3 — Sample positioning.

In order to place the sample cells in the neutron beam each of them was fitted in a holder made from perspex, according to fig. 2.

The cells are thus completely shielded by cadmium foils of 0.3 mm thickness, apart from the aperture of 40 mm diameter determining the diaphragm of the neutron beam through the cell. The apertures have been measured with a travelling microscope to within \pm 0.02 mm with the following results:

Sample holder n°	Diaphragm diameter
1	(4.006 ± 0.001) cm
2	(4.004 ± 0.002) »
3	(4.001 ± 0.002) »
4	(4.000 ± 0.002) »
(from BCMN Certificate E-42)	I
·	

A sample changer, (fig. 3), designed for a positioning accuracy of \pm 0.1 mm has four mounting holes fitting the cell holders (ref. 23). This apparatus has the function to interchange automatically, after a preset time, the four samples in the same position of the neutron beam. It was set on the mounting rail of the slow chopper and aligned with the axis of the neutron beam already determined by a flux mapping. Also the windows of the cells were placed perpendicular to the same axis with a precision of \pm 0.003 rad. At the same time the flight path length was measured with a telescopic jig adjusted to equal length with the distance between the rotor and the counter bank of the slow chopper. The jig length was measured in the laboratory with a cathetometer. The length of the flight path was, ref. 24,

 $L_{o} = (194.70 \pm 0.02)$ cm.



3. EXPERIMENTAL APPARATUS. (*)

3.1 - Slow chopper.

3.1.1 - Rotor.

The rotor of the chopper is of the well known Fermi type and consists of a stock of plates, alternatively Cd and Al. Each Cd-Al-Cd set forms a neutron transmitting slit, the Al being the slit core and the Cd the walls. The slit width is 1 mm, its length 59 mm, and its

(*) A. J. Deruytter, CEN-SCK.

dimension parallel to the rotation axis is 170 mm. The rotation axis is horizontal. All slits have the same dimensions; in this way they also have uniform dynamic transmission characteristics, which makes corrections easier.

The chopper is driven by a two-phase asynchronous motor, which is powered by a variable frequency oscillator and output amplifier. Motor speed is variable between 900 and 15.000 revolutions per minute. The stability of this speed over short periods is 0.1 %; over long periods (several days) 0.5 %.

From the different parameters one derives for the minimum burst time about 20 μ s, and for the maximum length of the flight path 2.8 m to avoid overlapping of bursts. In general, however, a flight path of only 2 m is used.

3.1.2 — Electronic equipment (ref. 25)

The timing pulses are generated by a 2 MHz quartz oscillator. After passage through a gate, opened by the start signal t_0 , the frequency is divided down to 100 kHz, allowing a good precision for the time determination and a minimum channel width of 10 μ s. The start signal of the time-base is obtained from a collimated beam of light reflected into a photomultiplier by two mirrors on the rotor axis; there is a phase difference of 180° between them. Between the start signal and the opening of the first counting channel a delay may be inserted ranging from 0 to 1000 channel widths. Channel widths are provided ranging from 10 μ s to 1000 μ s in multiples of 10 μ s and from 100 μ s to 200 μ s in multiples of 20 μ s.

At the end of the delay period the 25 counting channels are successively opened. Neutron counts arriving during this period are stored in a magnetic core memory. After completion of a measurement, a read-out cycle is generated during which the data are printed out on paper tape. Afterwards a new sample is inserted and a new memory cycle started. All these operations are fully automatic.

3.1.3 — Detectors.

The detection system consists of BF_3 -counters made by 20th Century Cy., type 12EB70, (1 inch diameter) with their axis perpendicular to the beam direction and at equal distances from the rotor.

3.2 — Neutron energy calibration (ref. 24,26).

For the accurate cross-section work reported here a careful neutron energy calibration was of the utmost importance. So the time of flight of the neutrons from the centre of the rotor to the detectors should be known with good accuracy. To achieve this, the start signal of the timing circuit must exactly coincide with the maximum of the burst intensity at the centre of the rotor; the distance from the rotor-axis to the detection point of the neutron in the detector and the time elapsed from the start signal to the moment of detection must be determined with care. Possible systematic errors and their influence on the calibration precision are discussed in Appendix III. Here the calibration method is described and the results and final accuracy are discussed.

3.2.1 — Method.

To calibrate the wavelength scale we measured the transmission of some polycrystalline materials. They show a sharp break in the coherent scattering cross-section at some typical wave-lengths. This break is broadened by the resolution of the apparatus. Before and after the break the cross-section varies slowly. For these parts of the transmission curve weighted best straight line fits are calculated. The position of the break is taken at a flight-time such that areas A and B are equal in fig. 4. For a symmetric break this condition yields a flight-time corresponding to a transmission $T_c = \frac{1}{2} (T_A + T_B)$, where T_A and T_B are the transmissions deduced from the extrapolated straight line fits. In both cases of symmetric and non symmetric break the errors on the individual transmission values determine the accuracy of the measured break position. To describe the experimental points a normalized gaussian resolution function :

$$R(t-t') = \frac{1}{a\sqrt{\pi}} \exp \left[\frac{-(t-t')^{a}}{a^{a}}\right]$$

was applied with $a = \frac{\Delta t_0}{2 \sqrt{\ln 2}}$. Δt_0 is the full width at half height of the resolution

function.

To get good transmission values a very stable beam monitor is required. A 4π fission chamber (ref. 27) flow counter with a 0.1 mg/cm² U²³⁵ — layer was used in the direct beam between the pile collimator and the rotor.

In fig. 4 the cut-off for the (110) planes in α Fe is represented for a rotor speed of 100 revolutions per second. The calculated resolution broadened cut-off for the experimental



value of the break-position is shown as a full line. For two fictive break positions shifted over 0.5 % with respect to the determined position, similar curves are shown. The full line fits the experimental points much better. The statistical accuracy of the experimental points is not very high but even then one concludes that errors of 0.2 % can easily be detected.

The accuracy of the method is limited by the statistical errors on the points of the transmission curve for the break considered. A statistical error of 0.2 % for the transmission values seems a reasonable minimum with an acceptable measuring time.

A polycrystal has to be selected with precisely known lattice constants. The accuracy found in crystallographic tables is mostly sufficient. However Gould et and reported a deviation for the graphite break at 6.70 Å of about 0.9 % due to the dependence of the «d»-value of graphite upon the conditions under which it was crystalized. In our graphite sample we did not find such a shift.

19

For simplicity of analysis a symmetrical break with mean transmission of about 0.5 should be used. Attention should also be paid to the units used for the lattice parameters for there is a difference between them $d_{grating} / d_s = 1.00204$.

Looking at the previously discussed systematical errors and at the error of the calibration method a final total error of 0.2 % was accepted. This was also the mean deviation between the experimental and calculated cut-off positions for several polycrystals as reported in (ref. 26).

3.2.2 — Calibration measurements for the boron cross-section measurements.

Before the boron cross section measurements were started, the rotor bearings were replaced. After this operation the light-source PM adjustment was disturbed, so that a delay occured between the start pulse from the PM and the real start pulse of the time base at the moment of maximum neutron intensity at the center of the rotor. During the boron measurements nothing was changed, as this time shift could easily be corrected for by two calibration runs : one in the actual conditions of the boron measurements and one after the proper adjustments were made, (fig. 5).

For these calibration runs the (110)-planes of a sample of α -iron powder were used. The sample has a mean transmission of about 0.5, and the break is almost symmetrical.

Both runs were taken with a different flight-path length :

Run 1 : $L = (191.91 \pm 0.02)$ cm, and Run 2 : $L = (194.70 \pm 0.02)$ cm.



FIG. 5. - Transmission of (110) = Fe after adjustement (1) and during the boron runs (2).

Both measurements were made with a rotor speed of 100 revolutions per second. The break position was determined as discussed in 3.2.1, and an error of 0.2 % was accepted for both break positions. The time difference between them yields the time shift after correction for the difference in flight-path length.

The result is : t_0 -shift = -(123 ± 3) µs. The negative sign means that the total flight time in the boron measurements has to be reduced by this amount.

Some of the boron measurements were made at an other rotor speed. The error however results from an angle difference and should be exactly proportional to the rotor angular speed. In order to check this dependence a graphite sample was run at 50.6 revolutions per second and the shift of the C(002) planes was calculated as $-(238 \pm 10) \mu s$, in agreement with the previous value assuming angular velocity dependence of the t₀-shift.

4.1 - Measurement of neutron transmission.

A series of preliminary experiments was performed to check whether some of the experimental conditions are satisfied i.e. :

the stability of the neutron detectors used as either main time of flight counters (BF₃ proportianal counters) or neutron beam intensity monitors, (4 π geometry fission cham-



21



FIG. 7. --- U²⁵ fission monitor characteristics.

ber). Both of them offer a suitable plateau in the range of working conditions with respect to either the anode voltage or the discrimination threshold (Fig. 6 and 7); the counter readings were repeatedly subjected to tests of statistical acceptability;

- a preliminary transmission test was carried out with samples 8 mm-thick; this proved valuable as a means of defining and improving the measurement conditions, which were found satisfactory when applied to 20 mm-thick test specimens;
- the quartz cells and Cd diaphragms delimiting the effective section of the neutron beam

have been subjected to comparative "blank" transmission tests in order to check their degree of identity and select those most suitable for filling with the boron preparation.

The identical nature of the diaphragms was checked within 0.1 % accuracy, this limit being imposed simply by the statistical counting fluctuations. As a result, a group of five quartz containers proved to be identical to 0.2 % about the mean transmission value for a beam of thermal neutrons

T=92.5 %

For each of the samples exposed to the neutron beam, and for every flight time analysis channel, the A counts recorded by the detectors and m counts recorded by the flux monitor were totalised; the two measurements were then repeated placing a 0.5 mmthick Cd shield in front of the samples, the respective A' and m' count being recorded. This second measurement provides a background count of the thermal neutrons which reach the detector without passing through the test specimen, together with a background count of the epicadmium neutrons (E > 0.5eV) contained in the beam and counted at their energy or which are slowed on passing through the sample. Accordingly, count normalized to the monitor to be used for the calculations is

$$N = (A/m - A'/m')$$
 [1]

or adopting the following quantitative symbols:

Material	Density	Macroscopic Cross-Section	Count	Thickness
$B_2O_3 + D_2O$ solution	ρ	Σ	N	d
D ₂ O	ρο	Σ ₀	\mathbf{N}_{0}	d ₀
D_2O in the solution	ρ1	Σ_1		
B_2O_3 in the solution	ρ ₂	Σ_2	—	
Helium gas			$\overline{\mathbf{N}}$	

(the macroscopic cross-section of helium is disregarded, being in the ratio of $5 \cdot 10^{-5}$ in relation to that of D₂O), there exist among them the following relations, in which c is the weight concentration of the B₂O₃ in the solution;

$$\rho_{1} + \rho_{2} = \rho$$

$$c = \rho_{2} / (\rho_{1} + \rho_{2})$$

$$\Sigma_{1} + \Sigma_{2} = \Sigma$$

$$\Sigma_{0} / \Sigma_{1} = \rho_{0} / \rho_{1}$$

from which we can derive

$$\Sigma_2 = \Sigma - \frac{\rho}{\rho_o} (1 - c) \Sigma_0$$
[2]

Bearing in mind the proportionality between the neutron current intensity and the detector counts, [1] of 1.3 gives

$$N_0 = \overline{N} \exp \left(-\Sigma_0 \, d_0\right)$$
 [3]

$$N = \overline{N} \exp\left(-\Sigma d\right)$$
 [4]

thus from [2], [3] and [4] we can derive

$$\Sigma \tau (B_2 O_3) = \Sigma_2 = \frac{1}{d} \ln (\overline{N}/N) - \frac{\rho}{\rho_0} (1-c) \frac{1}{d_0} \ln (\overline{N}/N_0)$$
 [5]

The total number of counts made per sample and per channel in the course of the two series of measurements, identified by the following "K series" and "L series" symbols, as well as their respective experimental conditions, are given in Tables II and III.

For the two boron solutions, neutron transmission always remained between 32 % and 76 %, values regarded as satisfactory for optimizing the effect of the counting statistics on the error of the result.

The effective number of pulses per channel varies considerably with the position of the channel over the energy range, assuming that for reasons of efficiency the works is carried out at around the peak of the Maxwellian spectrum, the total counts obtained being of the order of 10^3 to 10^5 pulses/channel.

Even the background neutrons were counted in order to be subtracted individually from each channel, their contribution passes from 4 % to 40 % in the case of channels subjected to a less intense flux.

4.2 — Experimental data processing.

As already pointed out in 4.1, each measurement given in Table II is the total of a number of cycles repeated a certain number of times varying from approximately 60 to 80. For each cycle, the time of flight analyser provides the following data:

- a) pulses counted by the fission monitor for the whole energy spectrum of the beam impinging on the chopper;
- b) pulses counted by the BF₃ detector and received by the analyser during the analysis time cycle.
- c) pulses analysed in each channel of the time of flight analyser.

After a first selection intended to eliminate either incomplete or obviously unreliable results, a second selection was made among the (b) type of counts normalized with respect to the monitor (a), the only ones accepted being those for which the deviation from the average

 $|x_i - \overline{x}|$, (i being equal to 1, 2... n), has in a normal Gaussian distribution

$$\varphi(\mathbf{x}) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\mathbf{x}-\mathbf{x}}{\sigma}\right)^2\right]$$
[6]

a probability $\tau > 1/2$ n, in which n is te number of times the measurement was repeated (Chauvenet's criterion).

It is only then that the data derived from the time-of-flight analysis (c) were accepted and applied according to formulae [1] and [5] of 4.1 to obtain the macroscopic cross-section of B_2O_3 .

The microscopic cross-section of the B_2O_3 molecule is subsequently derived from the relation

$$\sigma_{T} (B_{2}O_{3}) = \frac{M}{N_{A_{V}} c \rho} \Sigma_{T} (B_{2}O_{3})$$
[7]

M = molecular mass of B_2O_3

 N_{AV} = Avogradro number (6.02486.10²³ molecules/g mole).

The total cross-section [7] is, according to 1.3, assumed to be a function of the energy, or of the time-of-flight, and expressed as follows:

$$\sigma_{\tau} (B_2 O_3) = (2 K \tau + 21)$$
 barn, [8]

a justifiable hypothesis if the quantity

$$K = \frac{\sigma_{\tau} (B_2 O_3) - 21}{2 \tau}$$
 [9]

proves to be independent of τ .

The values of σ_T (B₂O₃) as a function of the time-of-flight τ are shown in Table IV for 25 channels of each of the two "K" and "L" series of measurements.

The error $\delta \sigma_{T}$, should be expected to represent the standard deviation of σ_{T} (B₂O₃) composed of the corresponding deviations of the directly measures quantities.

According to [5] and [7] of this chapter, σ_T (B₂O₃) is dependent upon the following quantities: c, ρ_o , ρ , d_0 , d, N, N₀ and \overline{N} , but only N₀ and N contribute to more than 0.1 % of the error involved in σ_T (B₂O₃); for this reason, they were taken into account for the calculation of $\delta\sigma_T$.

When we undertook to derive the quantity K from [9], the error contained in the 21 barn and in time of flight τ was also introduced. The first factor cannot be estimated, but as the 21 barn figure constitutes < 2.5 % of the total σ_{τ} (B₂O₃), the likely error cannot be regarded as affecting δ K to any appreciable extent. The time of flight being determined by.

$$\tau = \frac{t_A + t_0}{L}$$
[10]

 t_A = time measured by the analyser t_o = zero-time calibration

L = flight path.

The error contains either that resulting from the calibration of t_0 given in Table III or that of δL on the flight path.

It should be made clear, however, that δL is mainly determined by the finite dimensions of the neutron detector in the direction of the flight path, Appendix I.

Once the uniformity of K on the 2 x 25 channels measured (Fig. 8), has been determined, the weighted mean value can be derived from it,

$$\overline{K} = \left(\sum_{i=1}^{50} K_i w_i\right) / \sum_{i=1}^{50} w_i$$
[11]

with regard of the statistical weights,

$$\mathbf{w}_{i} = 1 / \delta \mathbf{K}_{i}^{2}$$
[12]

The corresponding standard deviation is derived from

$$\delta \ \overline{\mathbf{K}} = \mathbf{V} \sum_{i=1}^{50} (\mathbf{K}_i - \overline{\mathbf{K}})^2 \ \mathbf{w}_i / (50 - 1) \ \sum_{i=1}^{50} \ \mathbf{w}_i$$
[13]



4.3 — Results and Conclusions.

The development of the above mathematical operations in line with the method expounded in 4.2 leads individually, for the two boron samples with different B^{10} enrichments and in the aggregate for the two series "K" and "L" of measurements, to the following final results :

					$\overline{K}\pm\delta\overline{K}$
Natural boron	•				1.674 ± 0.0041 barn/µs m ⁻¹
Enriched boron	•		•		$8.15 \pm 0.019 $ barn/µs m ⁻¹

The data of paramont significance, however, is the value of σ_A (B) for neutrons travelling at the conventional velocity $v_o = 2200$ m/s and obtainable from the relation

$$\sigma_{A} (B)_{2200} \equiv \overline{K}/v_{o}$$
[11]

which after computation gives the following results :

					a				% B ¹⁰	σ_A (B) 2200 barn
Natural boron				•	•	•		•	19.81 ± 0.02	760.8±1.9
Enriched boron	•	•	•	•	•	•	•	•	96.515 ± 0.013	3703 ± 8.9

Bearing in mind that the two boron samples differ as to their characteristics, it would be interesting to compare the absorption cross-sections of the B^{10} , σ_A (B^{10}), derived from them, to confirm both the measurement accuracy and the validity of the method involved.

De	educ	ed j	from	e		$\sigma_{\text{A}}~(B^{10})$ at 2200 m/s		
Natural boron .								3840 ± 11 barn
Enriched boron						•		3837 ± 9 barn

The agreement between the two results can be regarded as very satisfactory, even when compared, as was the case in Table I, with the best and most recent determinations made.

The weighted mean of both results gives the value for the B^{10} absorption cross-section of (3838 \pm 10) barn.

APPENDIX I - FLIGHT PATH DEFINITION (*)

The two extremities of the nominal flight path L_o are located as follows: one on the rotor axis of the chopper provided with rectilinear slits and therefore symmetrical, the other on the geometrical centre of the detector. In view of the fact that the detectors consists of cylinder-shaped BF₃ counters, 2R in diameter, the axis of which is perpendicular to the neutron beam, neutron detection can be carried out at any point of their volume with the probability (see Fig. 9),

$$dp = \Phi (x, y) \Sigma dx dy$$
 [1]

per unit of length of the counter, Σ being the macroscopic cross-section of the filling gas. The neutron flux Φ , owing to its absorption by the counter, varies according to the following law :

$$\Phi (x, y) = \Phi_0 \exp \left[\Sigma \left(\sqrt{R^2 - y^2 - x} \right) \right]$$
[2]

so that, owing to the asymmetry with respect to x, the centre of the moments of the first order of the reactions will be found on an abscissa X which can be calculated in terms of [1] and [2]:

$$\frac{X = \int_{A} x \, dp / \int_{A} dp = \iint_{A} x \exp \left[\sum \left(\sqrt{R^2 - y^2} - x \right) \right] dx \, dy}$$

$$/ \iint_{A} \exp \left[\sum \sqrt{R^2 - y^2} - x \right] dx \, dy$$
[3]

The value of X, which depends upon the incident neutron energy through Σ , is calculated for each time of flight channel, and serves for correcting the flight path L₀ which thus becomes

$$L(\tau) = L_0 - X(\tau).$$
 [4]



Owing to the finite size of detectors, the flight path still contains a factor of uncertainty δL , account of which was taken in the time flight error $\delta \tau$ through 4.2 [10].

In order to assimilate this uncertainty factor to an equivalent standard deviation, we assumed, in the simplified case of a uniform flux $\Phi(x, y) = \Phi_0$, that the error on the flight path was the quantity δx , the distance from the axis of the counter limiting the interval within which occur 0.683 of the reactions of the entire counter.

$$\int_{0}^{\delta x} 4 \sqrt{R^{2} - y^{2}} \, dx = 0.683 \ \pi R^{2}$$
[5]

In the specific case of 12 E B 70/G counters, the correction of the flight path fell within the interval 0.020 - 0.075 cm.

The error δx which affects the estimation of δL , and indirectly, that of $\delta \tau$, is $\delta x = 0.55$ cm for a cylinder for which 2R = 2.54 cm.

APPENDIX II - CORRECTION FOR MULTIPLE SCATTERING (*)

Among the incident neutrons removed from the neutron beam as a result of absorption or scattering reactions, some are scattered towards the time-of-flight detector as a result of repeated collisions within the volume of the sample and are analysed together with those transmitted. In order to estimate the probability of such phenomena we used a simplified model, the scope of which was confined to the study of monokinetic neutrons which penetrate into the



29

sample along its axis and, once inside, undergo a process of isotropic scattering not involving any energy exchange. The probability P_1 that an incident neutron, after a first collision at point 1 (fig. 10) is scattered and transmitted into the solid angle Ω is

$$P_{1} = \frac{\Omega}{4 \pi} \int_{0}^{d} \exp\left(-\Sigma x\right) \cdot \Sigma_{s} \cdot \exp\left[-\Sigma (d-x)\right] dx =$$
$$= \frac{\Omega}{4 \pi} \Sigma_{s} d \exp\left(-\Sigma d\right)$$
[1]

The probability P_2 that the above process will occur after a second collision at point 2 subsequent to the first one is made up of the various probabilities which must be integrated within the entire volume V of the test specimen :

$$P_{2} = \frac{\Omega}{4 \pi} \int_{v} \left[\exp \left(-\Sigma x \right) \Sigma_{s} \cdot dx \right] \cdot \left[\frac{1}{2} \sin \theta d\theta \right]$$
$$\cdot \left[\exp \left(-\Sigma r \right) \Sigma_{s} dr \right] \cdot \left[\exp \left[-\Sigma \left(d \cdot x \cdot r \cos \theta \right) \right] \right] =$$

$$= \frac{\Omega}{8 \pi} \sum_{s}^{2} \exp(-\Sigma d) \int_{0}^{d} dx \int_{0}^{\pi} \sin\theta d\theta \int_{0}^{R(x,\theta)} \exp[-\Sigma r (1-\cos\theta)] dr \qquad [2]$$

The calculation of the probability for scattering of the order > 2 proves to be extremely complicated; accordingly, we shall confine ourselves to the first two terms of the total probability $P = P_1 + P_2 + ... + P_i + ...$, particularly since higher order probabilities grow more and more remote.

If we pass on to the numerical treatment of the special cases in which two samples are filled with heavy water and a B_2O_3 solution respectively, at a neutron energy of 0.0253 eV, and having the following characteristics :

we can derive directly from [1], and by means of numerical integration from [2].

									D ₂ O	$D_2O + B_2O_3$
P ₁ P ₂	•	•	•	•	•	•	•	•	0.00037 0.00039	0.00011

Results of the same order of magnitude would be found if more stringent theoretical treatments were applied; thus, the probability P for an infinite diffusing flat layer would be, according to ref. 28, 29 and 30 :

	_	•					D ₂ O	$D_2O + B_2O_3$
P.	•	• •	•	•	•	•	0.00082	0.00021

Whatever results are deemed to be more acceptable, values of P are small enough to be disregarded as compared with the other errors affecting the present measurements, particularly since the effect which they have in transmission experiments is determined by the difference between their values for heavy water and the boron solution respectively.

APPENDIX III - SYSTEMATIC ERRORS (*)

1. The start signal t_o :

- a) Phase shift with respect to the crystal pulses : The timing pulses are generated by a 2 MHz quartz oscillator. With this high frequency, the error due to the randomness of phase of the start signal and the 2 MHz pulses is reduced to a maximum of 0.5 µs.
- b) Sensitivity to the PM pulse width. The relatively broad PH pulse passes through a differentiating circuit yielding a sharp start pulse at the maximum of the PM pulse. But with different rotor speeds, ranging from 50 to 250 revolutions per second, the width of the PH pulse will change and the differentiating circuit is somewhat sensitive to this width. In this way shifts of 0.5 µs may occur.
- c) Parallelism of both mirrors and adjustment of the coincidence between PM pulse and burst.

For reasons of intensity we wanted two bursts per revolution; so two parallel mirrors were necessary. To minimize this correction, an end-gauge was fixed in the rotor-axis, with thickness variations over the whole surface smaller than $\pm 0.02 \,\mu$, yielding a parallelism between the two planes better than 1 second of arc. By rotating the rotor over 180° no corrections were necessary to the light source PM adjustment. The adjustment of the coincidence between burst and maximum of the PM pulse is limited by the mechanical finish of the positioning mechanism of the lamp with respect to the PM, and by the accuracy of the determination of the maximum intensity in the detectors. In this way the precision of the t₀-determination is smaller than 2 minutes of arc, a smaller than 0.5 μ s/m with a rotor speed of 100 revolutions per second. The mechanism we used could certainly be improved, and the error reduced.

- 2. Length of the flight-path : length measuring technique and shift of the mean detection point.
 - a) The end point of the flight-path is the detection point of the neutron. 12 EB 70/BF₃counters with a 1 inch diameter are used. The mean detection point does not coincide with the central wire, but is shifted towards the coming neutrons. This shift is a function of the neutron energy and typical figures are $E_n = 0.025$ eV, shift = 0.04 cm.

^(*) A. J. Deruytter, CEN-SCK.

b) To measure the length of the path a telescopic system was used, provided with a disk at one side and a threaded head at the other. The disk, perpendicular to the rod, served as a tangent plane to the rotor. At the other side a metal plate with accurately known thickness was put tangent to the counters. By the threaded head the system length was adjusted to the distance from rotor to counters and measured afterwards in the laboratory to \pm 0,1 mm. From the knowledge of the rotor diameter and the shift of the mean detection point the mean length can be determined for each neutron velocity with a precision of about 0.2 mm. For the 2 m flight path this measuring technique yields an error of about 10⁻⁴.

Due to the finite thickness of the BF₃-detectors there is a distribution of individual pathlengths about the used mean length. This distribution influence the resolution, but does not have a great effect on the calibration of the wavelength-scale, as there is a good agreement between experimental and theoretical resolution widths. For the cut-off of the (110)-planes in α Fe (See Fig. 4) the experimental resolution width is 52 µs. The theoretical width is 50 µs calculated from burst duration, channel width, mean thickness of the counters, etc., in the usual way. Both are in very good agreement.

3. Time of arrival.

The pulse from the BF₃-counter allows the measurement of the elapsed time. By means of a pulse-shifter this pulse is shifted towards the following clock-pulse. The BF₃-pulse only passes through this circuit upon arrival of the following 10 μ s pulse. The full width at half heighth of the clock pulse is 0.25 μ s. The rise time of the BF₃-pulse after amplification and discrimination is about 0.8 μ s. The pulse-shifter is adjusted so as to give an outputsignal when the BF₃-pulse reaches half its height when coincident with the clock pulse. So the BF₃-pulse should arrive at least 0.4 μ s before the end of the clock pulse in order to be counted in the right channel and not be shifted towards the following clock pulse. This means a 0.15 μ s-shift towards long wavelengths. An improvement would be possible by the use of fast transistorized equipment, or by using the 0.1 μ s fast scintillation time in a ZnS-B¹⁰ counter.

4. Elapsed time.

The frequency of the crystal, used as a master oscillator for the timing circuits, was established as 20° C as 2,000,203.6 \pm 0.1 Hz, and the temperature dependence of the frequency was determined as -0.4 Hz/ $^{\circ}$ C by C.C.R.M. (Centre de Contrôle des Radiocommunications des Services Mobiles, Uccle, Belgium). So the time frequency equals 2 MHz within 10^{-4} , an order of magnitude smaller than the final error in the flight time.

In the following table a list of the systematic errors is given with their absolute value and their relative error in the time-of-flight. The calculations are made for 500 μ s/m neutrons, 100 rps rotor speed and a 2m flight-path.

· .

Source of Error	Absolute Value	Relative error in the time-of-flight
1) Start Signal.		
a) Phase shift with respect to the crystal pulses	0.5 µs	5 x 10 ⁻⁴
b) Sensitivity of differentiation circuit to the PM pulse width	0.5 μs	5 x 10-4
c) Parallelism of both mirrors	1 second of arc	10-5
	(0.01 μs) 2 minutes of arc (1 μs)	10-3
 Length of the flight path. Shift of the mean detection point and the length measuring technique. 	0.2 mm	10
3) Time of arrival. Rise time of the detector pulses and width of the clock pulses	0.15 µs	1.5 x 10 ⁴
 4) Elapsed time. Deviation of the crystal frequency from 2 MHz 	$\begin{array}{c} 203.6 \pm 0.1 \text{ Hz} \\ (0.1 \mu \text{s}) \end{array}$	10+
Temperature dependence of the frequency .	—0.4 Hz/°C (2 x 10 ⁻³ µs)	2 x 10 ⁻⁶
Total error due to systematic errors		1.3 x 10− ³

-

REFERENCES

- 1 EGELSTAFF P. A. AERE Memorandum N/M 62 (1953).
- 2 HAMERMESH B., RINGO G. R., WEXLER S. Phys. Rev. 90, 603 (1953), mentioning unpublished results by Kimball, Ringo, Robillard and Wexler.
- 3 CARTER R. S., PALEVSKY H., MYERS V. W., HUGHES D. J. Phys. Rev. 92, 716 (1953).
- 4 GREEN A, LITTLER D. J., LOCKETT E. E., SMALL V. G., SPURWAY A. H., BOWELL E. – Jour. Nucl. Ener. 1, 144 (1954).
- 5 SCOTT F. R., THOMSON D. B., WRIGHT W. Phys. Rev. 95, 582 (1954).
- 6 COLLIE C. H., MEADS R. E., LOCKETT E. E. Proc. Phys. Soc. A 69, 464 (1956).
- 7 EGELSTAFF P. A. Jour. Nucl. Ener. I 5, 41 (1957).
- 8 HAVENS W. W., MELKONIAN E., RUSTAD B. Report CU 160 (1957).
- 9 --- von DARDEL G., SJOESTRAND N. G. --- Phys. Rev. 96, 1566 (1954).
 - von DARDEL G., SJOESTRAND N. G. Progr. Nucl. Energ., vol. 2, 216 (1958).
- IGNATEV, KIRPICHIKOV, SUKHORUCHKIN Proc. Int. Conf. on Peaceful Uses of Atomic Energy 1958, vol. 16, 209.
- 11 SCHMITT H. W., BLOCK R. C., BAILEY R. L. Nucl. Phys. 17, 109 (1960).
- 12 SAFFORD G. J., TAYLOR T. I., RUSTAD B. M, HAVENS W. W. Jr Phys. Rev. 119, 1291 (1960).
- 13 GOULD F. T., TAYLOR T. I., HAVENS W. W. Jr., RUSTAD B. M., MELKONIAN E. — Nucl. Sci. Eng. 8, 453 (1960).
- 14 DEBUS G., DERUYTTER A., LAUER K., MORET H., PROSDOCIMI A., this report.
- 15 SAFFORD G. J., TAYLOR T. I., RUSTAD B. M., HAVENS W. W. Jr. Nucl. Sci. Eng. 9, 99 (1961).
- 16 HUGHES D. J. Proc. Int. Conf. on Peaceful Uses of Atomic Energy 1958, vol. 16, 8.
- 17 HUGHES D. J., SCHWARTZ R. B. Neutron Cross Sections BNL-325, 2nd ed. and Suppl. (1958).
- 18 TITTMAN J., SHEER C. Phys. Rev. 83, 746 (1951).
- 19 LAUER K. F., LE DUIGOU Y. Z. Anal. Chem., 184, 4 (1961).
- 20 FINCK C., KLEINBERGER R. et al. J. Nuclear Energy, 3, 25 (1956).
- 21 SPAEPEN J. Meded. Kon. Vlaam. Akad. Wetensch. Belg. 19, no. 5 (1957).
- 22 MORET H. Rev. Sci. Instr., to be published.
- 23 MORET H. Nucl. Instr. and Meth., 14, 2 (1962).
- 24 CEULEMANS H., DERUYTTER A., MORET H., NEVE DE MEVERGNIES M. Symposium on Time-of-flight Methods, Paris 1961, p. 275.
- 25 CEULEMANS H., DEKEYSER A., MIES E. L'Electronique Nucléaire, vol. 2, 297, Paris (1959).
- 26 CEULEMANS H., DERUYTTER A. J. Symposium on Pile Neutron Research, IAEA, Vienna (1960); and CEN-SCK Report R-1900.
- 27 DERUYTTER A. J. Nucl. Instr. and Meth., 7, 145 (1960).
- 28 VINEYARD G. H. Phys. Rev., 96, 93 (1954).
- 29 CHANDRASEKHAR S. Radiative Transfer, Oxford (1950).
- 30 CHANDRASEKHAR S., ELBERT and FRANKLIN Astrophys. J. 115, 244 (1952) and 115, 269 (1952).

Tab. I — STATUS OF MEASUREMENTS

Ref.	Year	Laboratory	Method	Energy, eV
1	1953	AERE-Harwell	Transmission by slow chopper	0.01 - 0.1
2	1953	ANL - Argonne	Transmission	
3	1953	BNL - Brookhaven	Transmission by slow chopper	0.0003 - 0.04
3	1953	BNL - Brookhaven	Transmission by slow chopper	0.0008 - 0.04
4	1954	AERE-Harwell	Pile oscillator with reference to gold	thermal spectrum
5	1954	LASL-Los Alamos	Neutron lifetime	
6	1956	Oxf. UnOxford	Neutron lifetime	
6	1956	Oxf. UnOxford	Pile oscillator with reference to measured boron	
7	1957	AERE-Harwell	Transmission by slow chopper	0.01 - 0.1
7	1957	AERE-Harwell	Transmission by slow chopper	0.01 - 0.1
8-13	1957	Col. UnNew York	a) Chry:tal spectrometer b) Pulsed cyclotron	0.0006 - 1 0.012 - 0.12
9	1958	AE-Stockholm	Neutron lifetime	
9	1958	AE-Stockholm	Neutron lifetime	
10	1958	USSR	Transmission by pulsed cyclotron	
11	1960	ORNL-Oak Ridge	Transmission by fast chopper	0.018 - 0.4
11	1960	ORNL-Oak Ridge	Transmission by fast chopper	0.022 - 0.042
12	1960	Col. UnNew York	Transmission by crystal spectrometer	0.00291 - 0.1
12	1960	Col. UnNew York	Transmission by crystal spectrometer	
14	1961	CEN-BCMN - Mol	Transmission by slow chopper	0.006 - 0.08
14	1961	CEN-BCMN - Mol	Transmission by slow chopper	0.006 - 0.08

ON BORON THERMAL CROSS-SECTION

· .

Sample	Boron source - Isotopic ratio	2200 m/s cross-section barn	Value normalized to B ¹⁰ barn
Quartz cell containing D ₂ O+NaOD solution of B ₂ O ₃	natural	781 ± 10	3898± 63
	ANL standard boron	755 ± 3	3813±40
Aluminium cell containing D_2O solution of B_2O_3	California boron by ANL	749 ± 4	3782 ± 42
Boron glass plates manufactured by Corning Glass Co.	California boron	752 ±13	3797 ± 75
Heavy water solution of B_2O_3	Harwell standard boron	785 ± 8	3914± 57
	California boron	744 ± 20	3757 ± 107
Water solution of borax	natural	761 ± 2	_
	Harwell standard boron	760 ± 3	3792 ± 42
Quartz cell containing $D_2O+NaOD$ solution of B_2O_3	Harwell standard boron	771 ± 5	3847±47
Boron glass powder	Harwell standard boron	769 ± 5	3837 ± 47
oron glass plates manufactured by Corning Glass Co.	California boron	756 ± 6	3818 ± 48
Water solution of borax	natural	758 ± 4	_
Water solution of borax	ANL-BNL standard boron	760 ± 4	3838 ± 20
	natural	757.5 ± 3.5	
Aluminium cell containing D_2O solution of B_2O_3	enriched boron (result reduced to B ¹⁰)	_	3848 ± 38
	ANL-BNL standard boron 19.8 % B ¹⁰	753 ± 6	3803 ± 48
Quartz cell containing D ₂ O solution of NaBO ₂	enriched boron (result reduced to B ¹⁰)		3838 ± 11
Quartz cell containing D ₂ O solution of NaBO ₂	ANL-BNL standard boron	764 ± 3	3858 ± 25
Quartz cell containing D ² O solution of deuterated boric acid	natural, 19.81 % B ¹⁰	760.8 ± 1.9	3840 ± 11
Quartz cell containing D2O solution of deuterated boric acid	enriched, 96.51 % B ¹⁰	3703 ± 9	3837 ± 9
		1	

.

Tab. II — COUNTING IN THE TIME-OF

D	Changel	Natural bo	con sample	Enriched
Kun	Channel	А	A'	A
"K"	Monitor 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	$\begin{array}{c} 28858090\\ 53430\\ 48561\\ 41863\\ 34078\\ 27158\\ 22550\\ 18527\\ 16733\\ 16752\\ 15581\\ 13225\\ 11133\\ 9097\\ 7238\\ 6807\\ 6394\\ 5693\\ 5130\\ 4607\\ 4164\\ 3660\\ 3190\\ 2705\\ 2373\\ 2063\\ \end{array}$	9610550 967 924 882 971 833 806 800 770 761 678 678 674 616 649 613 549 550 467 445 442 437 369 347 303 285	$\begin{array}{c} 36961710\\ 64008\\ 58256\\ 50280\\ 40839\\ 32284\\ 26880\\ 22045\\ 20434\\ 19524\\ 18072\\ 15644\\ 12998\\ 10664\\ 8521\\ 8020\\ 7271\\ 6412\\ 6038\\ 5485\\ 4847\\ 4212\\ 3602\\ 3235\\ 2788\\ 2368\\ \end{array}$
"L"	Monitor 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	$\begin{array}{r} 44330570\\ 113373\\ 119811\\ 118211\\ 118211\\ 114379\\ 108051\\ 101096\\ 96380\\ 89209\\ 82633\\ 78700\\ 74052\\ 66846\\ 59549\\ 53909\\ 49512\\ 43725\\ 36661\\ 30965\\ 25993\\ 22720\\ 19422\\ 16760\\ 17247\\ 16209\\ 15307\\ \end{array}$	9144485 945 993 992 963 977 976 945 976 888 918 876 887 796 753 781 722 675 670 623 589 548 519 510 448 411	$\begin{array}{r} 47039965\\ 115000\\ 121426\\ 119980\\ 115519\\ 109589\\ 102235\\ 96619\\ 89917\\ 81914\\ 79134\\ 73683\\ 67417\\ 58875\\ 53192\\ 49244\\ 43329\\ 36480\\ 30726\\ 225506\\ 22351\\ 19153\\ 16674\\ 16807\\ 15926\\ 15034\\ \end{array}$

-FLIGHT ANALYSER AND MONITOR

· .

boron sample	Heavy w	ater blank	Helium	blank
A'	A	A'	A	A'
$\begin{array}{c} 10367630\\ 1013\\ 972\\ 976\\ 893\\ 888\\ 825\\ 840\\ 813\\ 841\\ 822\\ 744\\ 674\\ 645\\ 610\\ 620\\ 600\\ 537\\ 490\\ 516\\ 452\\ 421\\ 398\\ 386\\ 326\\ 297\\ \end{array}$	$\begin{array}{c} 29934400\\ 87940\\ 81371\\ 71592\\ 59013\\ 47948\\ 39996\\ 33320\\ 31290\\ 31578\\ 29627\\ 25588\\ 21595\\ 17505\\ 14173\\ 13471\\ 12764\\ 11469\\ 10334\\ 9483\\ 8574\\ 7586\\ 6500\\ 5788\\ 4738\\ 4188\\ \end{array}$	$\begin{array}{r} 8641810\\ 924\\ 914\\ 833\\ 813\\ 808\\ 752\\ 742\\ 716\\ 651\\ 717\\ 669\\ 624\\ 606\\ 576\\ 535\\ 504\\ 493\\ 435\\ 443\\ 394\\ 353\\ 315\\ 318\\ 275\\ 253\\ \end{array}$	$\begin{array}{c} 22772360\\ 171302\\ 156970\\ 136860\\ 117159\\ 97305\\ 81545\\ 68886\\ 65666\\ 67435\\ 64718\\ 57047\\ 48293\\ 40039\\ 32742\\ 31349\\ 29917\\ 27243\\ 25685\\ 23551\\ 21150\\ 19040\\ 16583\\ 14346\\ 12466\\ 10630\\ \end{array}$	$\begin{array}{c} 8658310\\ 1635\\ 1628\\ 1458\\ 1470\\ 1442\\ 1355\\ 1313\\ 1164\\ 1189\\ 1124\\ 1105\\ 1009\\ 1007\\ 915\\ 922\\ 837\\ 828\\ 731\\ 711\\ 657\\ 590\\ 560\\ 493\\ 435\\ 425\\ \end{array}$
$\begin{array}{c} 8523420\\ 849\\ 884\\ 890\\ 907\\ 895\\ 917\\ 908\\ 918\\ 864\\ 848\\ 765\\ 753\\ 693\\ 720\\ 680\\ 649\\ 620\\ 560\\ 562\\ 511\\ 482\\ 461\\ 466\\ 425\\ 379\\ \end{array}$	41203860 137485 147776 147883 145088 139765 133055 128319 121512 113615 111027 104683 96309 87138 79781 75426 66846 57424 48390 40936 35694 30993 27474 28350 27335 25993	8464320 923 978 976 936 980 975 992 950 826 909 836 760 751 777 748 704 678 629 609 558 511 468 477 417 402	35802625 273899 295795 297024 294146 286229 278824 266937 253876 239393 234390 223598 207072 188084 174417 163830 148055 127558 110091 93745 83538 72451 64822 67754 66546 64090	$\begin{array}{c} 7682195\\ 1334\\ 1441\\ 1452\\ 1543\\ 1524\\ 1556\\ 1522\\ 1474\\ 1419\\ 1375\\ 1380\\ 1279\\ 1222\\ 1112\\ 1185\\ 1076\\ 989\\ 942\\ 907\\ 793\\ 804\\ 794\\ 701\\ 655\\ 633\\ \end{array}$

	Run « K »	Run « L »	
Rotor speed	75.18±0.09 rps	100.24±0.22 rps	
Neutron burst length (full width)	71 µs	53 µs	
Delay time, t _A	1000 µs	1000 µs	
Channel width	40 µs	30 µs	
Zero-time calibration	(165 \pm 4) μ s	- (123 ± 3) µs	
Counting time	1000 s/cycle	1000 s/cycle	
Useful cycles	58 - 66	78 - 83	
Flight path, L_0	194.70 ± 0.02 cm		

Tab. III -- EXPERIMENTAL CONDITIONS FOR THE SLOW CHOPPER

·· .

Tab. IV. - BORON TOTAL CROSS-SECTION VERSUS TIME-OF-FLIGHT

· .

.

<u></u>	Chan-		Natura	al boron	Enriche	ed boron
Run	nel nº	τ µs/m	σ _T (B ₂ O ₃) barn	$\delta \sigma_T (B_2O_3)$ barn	σ _T (B ₂ O ₃) barn	δσ _τ (B ₂ O ₃) barn
"L"	$ \begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16\\17\\18\\19\\20\\21\\22\\23\\24\\25\end{array} $	252,47 267,88 283,29 298,70 314,11 329,53 344,94 360,35 375,77 391,18 406,59 422,00 437,42 452,83 468,25 483,66 499,08 514,49 529,91 545,32 560,74 576,15 591,57 606,98 622,40	860,10 913,15 958,78 1004,18 1065,70 1124,05 1155,65 1242,57 1277,21 1356,44 1368,62 1451,57 1489,88 1506,81 1622,47 1645,70 1731,44 1759,06 1792,22 1806,28 1879,48 2001,54 1989,32 2062,24 2065,16	$\begin{array}{c} 4,40\\ 4,25\\ 4,27\\ 4,32\\ 4,47\\ 4,64\\ 4,76\\ 4,97\\ 5,13\\ 5,31\\ 5,47\\ 5,80\\ 6,16\\ 6,52\\ 6,89\\ 7,41\\ 8,25\\ 9,29\\ 10.44\\ 11,00\\ 12,60\\ 13,86\\ 13,48\\ 13,52\\ 13,81\\ \end{array}$	4200,74 4434,04 4613,28 4925,88 5092,70 5396,58 5673,72 5958,44 6324,45 6467,11 6634,83 6759,40 7224,74 7462,13 7749,32 7893,66 8233,67 8257,88 8690,09 8640,06 8947,87 9359,12 9689,45 9944,91 9893,67	55,99 57,90 58,24 59,33 61,15 63,70 65,71 68,51 71,43 73,04 75,20 79,10 85,00 91,45 95,10 102,8 114,9 127,4 146,9 153,8 175,8 193,4 192,0 193,3 195,9
"К"	$ \begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 20\\ 21\\ 22\\ 23\\ 24\\ 25\\ \end{array} $	439,27 459,82 480,37 500,93 521,48 542,03 562,59 583,14 603,70 624,25 644,80 665,36 685,92 706,47 727,03 747,59 768,14 788,70 809,26 829,82 850,38 870,93 891,50 912,05 932,61	1498,16 1554,33 1632,37 1721,84 1769,48 1813,67 1895,03 2038,64 2049,08 2163,76 2242,47 2231,99 2478,31 2527,20 2533,87 2626,35 2566,31 2646,62 2784,81 2982,47 2852,47 3096,02 2840,97 3020,52	5,992 6,320 6,844 7,883 8,969 10,12 11,65 12,36 12,26 12,46 14,30 16,40 19,03 24,00 24,73 25,09 28,08 28,40 31,34 32,12 39,67 41,84 49,08 52,76 60,23	7215,86 7442,99 7812,54 8028,82 8533,05 8629,10 9062,32 9294,08 10308,93 10443,57 10430,98 10744,56 10866,00 11337,62 11949,37 12819,44 12961,20 12253,40 13021,22 13309,04 13989,64 14701,93 14905,06 13883,37 14976,65	77,93 82,00 89,43 101,1 117,8 132,2 153,4 160,6 165,0 172,5 191,6 216,6 255,4 307,6 328,3 354,0 381,8 387,7 445,3 469,5 525,7 614,0 645,7 744,2 858,6

