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Measuring the Term Structure of ECU Interest Rates

Johan Verhaeven and Werner R6ger*

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MEASURING THE TERM STRUCTURE OF ECU INTEREST RATES

(J. VERHAEVEN / W. RÖGER)

Part II : **ESTIMATING ECU RATES**

Summary

The paper develops a methodological basis for determining Ecu interest reference rates (EIRRs); these rates are destined to reflect the level of medium and long term Ecu interest rates observed in the secondary market for prime borrowers.

The first part gives a general overview of the various factors that influence bond yields. Some of these factors reflect the prevailing market conditions (e.g. term strcuture, tax regime, ...) while others are related to the characteristics of each particular security (e.g. liquidity, credit rating, coupon level, ...). The Ecu itself has a number of peculiarities due to its basket definition and the absence of a natural domestic market.

The second part concentrates on the Ecu bond market. Regression analysis is used to describe the important yield variations that are observed in the market for the currently outstanding Ecu securities issued by either sovereign or supranational borrowers. This analysis is then used to derive a number of criteria to select the most representative (benchmark) issues in the secondary market.

After comparing various methods, the report concentrates on the yield curve approach as the preferred method for determining interest reference rates.

ECU INTEREST REFERENCE RATES

Part I: GENERAL

A. INTRODUCTION

Medium and long term capital market interest rate levels condition bond prices as quoted in the secondary market. Inversely, the level of bond prices can be used to obtain fairly accurate indications on a daily basis of interest rates prevailing in the market.

One such measure consists in calculating the yield-to-maturity (or gross redemption yield) of existing bonds. This measure takes account of the size and payment dates of all future cash flows resulting from an investment in a particular security and provides a good initial approximation for the level of interest rates prevailing for investments with similar characteristics (term to maturity, risk, etc.). The rest of this report will mostly deal with various methodological as well as practical considerations that have to be taken into account if one wants to arrive at accurate estimates of prevailing medium and long term interest rate levels for the Ecu in order to determine reference rates for typical maturities (e.g. 3 yrs, 5 yrs, 7 yrs and 10 yrs).

The approach developed for the Ecu extends the information on interest rates which is already available for other currencies. For most currencies, representative bond yields are published on a daily basis by either stock exchanges, securities dealers or central banks. Several securities dealers publish cross-currency comparisons on a regular basis. Domestic government debt is generally taken as a benchmark for reasons related to both the volume of outstanding debt in the domestic currency (liquidity) and the sovereign (default-free) status of the issuer.

B. CHARACTERISTICS OF REFERENCE RATES

B.1 Requirements

Reference rates have a variety of applications and should therefore be publicly available so that any interested party can make use of them for either practical or academic purposes.

A number of requirements should be met in order to ensure that the rates are accurate and authoritative. Firstly, the rates should be representative and therefore reflect market levels as close as possible; they should for instance be unbiased as far as one's position (e.g. holding either assets or liabilities) in the Ecu market is concerned. In addition, transparency should be ensured by indicating both the origin of the data and the methodology to calculate the rates. Finally, rates should be made available on a regular and continuous basis.

B.2 Comparability

The reference rates should be representative for top quality borrowers (e.g. the EEC institutions, a number of EEC Member States) and ideally be "base rates". This will allow meaningful comparisons with similar figures reported for the national currencies, which are generally based on domestic (default-free) government bonds.

In this regard, the Ecu has some peculiar characteristics which have an influence on the level of interest rates. More specifically:

- it lacks a domestic market or lender of last resort;

- although Ecu interest rates are related in a complex way to the interest rates of the underlying component currencies, they are also conditioned by a number of additional elements, such as the possibility of future recompositions (adjustments of weightings, or inclusion of additional currencies in the basket). Market views and expectations on the future role of the Ecu are also reflected in secondary market yields.

C. THE DETERMINANTS OF INTEREST RATES

C.1 General considerations

The yields applying to fixed-income securities (alternatively, the secondary-market price) are determined by an important number of factors, of which the most relevant ones are the following:

- general level of interest rates;
- maturity (remaining life) of the security;
- credit quality of the issuer (default risk) and of the issue itself (guarantee provisions, ...);
- tax features related to the issue;
- liquidity of the security;
- the characteristics of the issue (e.g. provisions relative to calls and puts, sinking funds, reinvestment, coupon size, ...)

The relevance of these various elements in the context of the current exercise will be studied in greater depth in the next sections (see also the short bibliography annexed to this report for additional information).

The general interest rate level is conditioned by general macro-economic factors such as the supply of savings, the demand for credit (government borrowing, corporate investment, ...), the rate of inflation, etc. Moreover, in an open economy without capital movement restrictions, interest rates applying to various currencies are interrelated through the foreign exchange market as both savers and borrowers are not restricted to the domestic currency. Interest rate levels will therefore also integrate market perceptions on exchange rate evolutions for each particular currency.

C.2 The term structure of interest rates

Various hypotheses have been put forward to provide a theoretical explanation for the term structure of interest rates i.e. the influence of maturity on credit cost. The most customary theories are illustrated by the following example based on a two-period investment model.

We consider the various possibilities open to investors having an investment horizon of either one or two years:

- investor A with a horizon of a single year could either invest in a 1-year security (at spot rate r1), or otherwise invest in a 2-year security (at spot rate r2) and sell the security after 1 year (at an uncertain price, which will depend on the yet unknown 1-year spot rate "s" applying at the end of year 1);

- investor B with a 2-year horizon could directly invest at r2 for 2 years (certain outcome) or buy a 1-year security (at r1) and re-invest the proceeds at the end of year 1 at the then applying 1-year spot rate "s" .

According to the **expectations** hypothesis, the implicit forward rate "f" must equal the expected future spot rate "s" i.e. f = E(s), because:

- if f > E(s): both investors will prefer 2-year securities (r2 will therefore decrease)
- if f < E(s): both investors will prefer 1-year securities (r1 decreases).

The maturity structure of interest rates therefore reflects expectations about future interest rate levels e.g. an upward sloping curve (r2 > r1) indicates that interest rates are expected to rise. Once equilibrium is reached, investor A will prefer the 1-year security and investor B the two-year security (risk is minimized for both of them as the outcome is certain).

The **liquidity-preference** hypothesis (which is generally seen as a complement to the previous proposition) states that the difference between f and E(s) (the "liquidity premium") will generally be positive so as to compensate investors for the higher risk related to investments in securities with longer maturities. Advocates of the theory believe that for the most part, investors want to lend short and companies want to borrow long. The liquidity premium compensates investors with a shorter horizon (e.g. investor A) for the additional risk. If this hypothesis is right, the term structure should be upward sloping more often than not (the term structure could also be downward sloping, but the liquidity premium will then attenuate the negative slope).

The **segmentation** hypothesis is in contradiction with the two previous propositions as it denies the existence of a formal relationship between interest rate levels for various maturities. According to this theory, borrowers and lenders are constrained to particular segments of the maturity spectrum (e.g. pension funds and insurance companies prefer long-term instruments while commercial banks prefer short-term ones). As both borrowers and lenders have their own horizons, from which they will not depart, the market is made up of separate and unrelated "segments" where interest rate levels can differ substantially as they are conditioned by separate demand and supply factors. Changes in demand or supply for a particular maturity will affect the interest rate for that maturity only and have no impact on the other rates.

The **preferred habitat** theory is related to the two previous ones in that it takes account of risk aversion (investors do however not necessarily prefer the short maturities and borrowers the long ones) and also assumes that both borrowers and lenders have (less strict) maturity preferences. The theory thus allows for some substitution between maturities, mainly on the basis of relative differences in interest rate levels along the maturity spectrum.

D. ESTIMATING THE LEVEL OF INTEREST RATES

Before moving on to the formal description of the methodology used to construct a model determining Ecu reference rates, the main concepts and central issues are further elaborated in this section.

D.1. Redemption yield: definition and limitations

The redemption yield (RY), also called yield-to-maturity, is defined as the interest rate equalizing the bond's current price with the discounted value of all future cash-flows (payments of coupons and principal) stemming from the security.

Alternatively, the redemption yield can be defined as the internal rate of return associated with an investment in a particular security.

Annex 1 contains further details on the formulae and methods to be used for calculating yield figures.

Two important observations should however be made regarding this definition:

1. for a particular security, the same discount rate (i.e. $r1 = r2 = ... = rm = RY$) is used for discounting all future payments, irrespective of the period in which these payments take place; the method therefore implicitly assumes that the term structure of interest rates is perfectly flat;
2. when comparing two bonds with a different yield RY, it is apparent that identical cash flows with the same payment date stemming from these two securities will (all other things being equal) be discounted at different rates i.e. similar cash flows will be treated differently depending on the overall RY of each security.

These two shortcomings could in theory be overcome if the present value of all payments occurring in any given period would be calculated on a consistent basis. This implies the use of a single discount function (see fig. 1) for all securities from the same borrower.

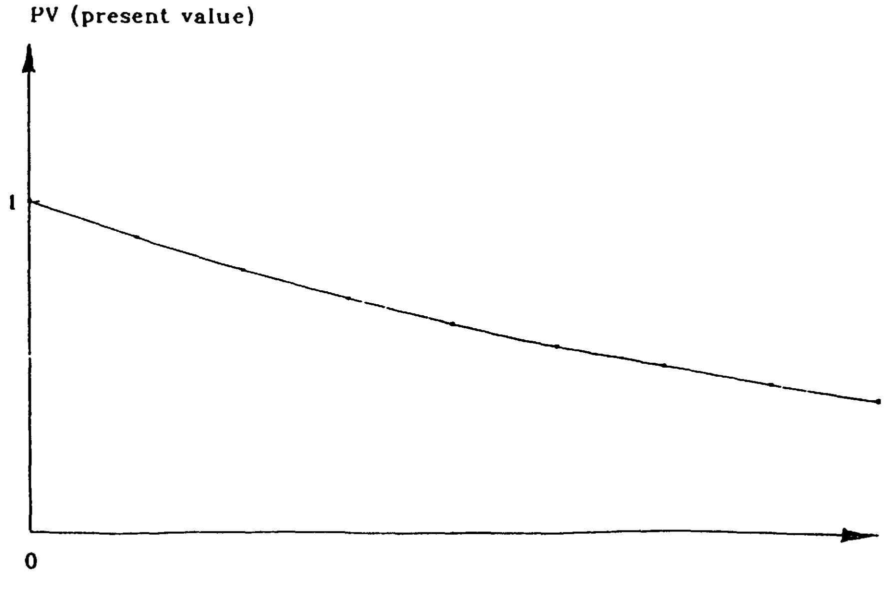


FIGURE 1

The discount function in turn allows the determination of a set of discount or spot rates (ri) to be applied for discounting payments occurring in period i (ri no longer a constant).

Zero-coupon (capitalization or pure discount) bonds can be used to derive the discount function (applying to a particular currency, risk category, ...) as they involve a (unique) single payment in period i. Moreover, the ri corresponding to period i can be estimated quite easily as it is equal to the RY of the bond.

D.2. <u>Par yield</u>

Apart from securities in which a single payment is involved, discount rates cannot be directly observed in the market and can generally only be derived in an indirect and cumbersome way. For bonds involving a series of payments (e.g. the traditional fixed-rate "bullet" bond), the problem has to be approached differently. We will therefore examine how yields are related to discount (spot) rates and how yields can be used to estimate the term structure of interest rates. The mathematical relationships between these variables will be further illustrated by a number of examples considering various hypotheses on the maturity structure of the interest rates.

We consider various term structures as proposed in fig. 2 (numbered 1 to 3) and examine the "behaviour" of a series of bullet bonds (i.e. regular coupon payments; all principal repaid at final maturity) with different coupon levels and different maturities. We provisionally exclude any other external factors (e.g. taxation, risk, etc.) that might influence bond prices.

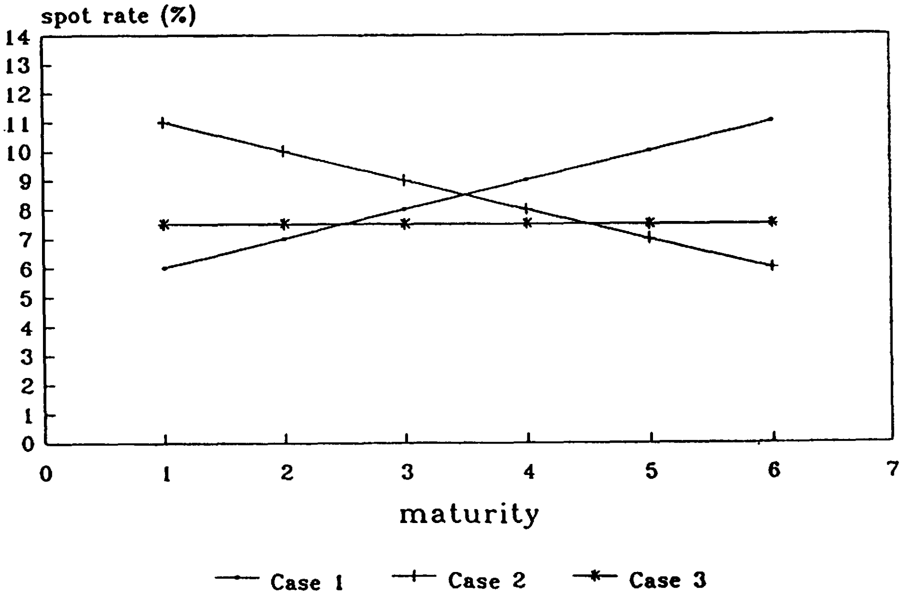


FIGURE 2

Case 1: Upward sloping term structure

We assume that the spot (discount) rates "ri" increase linearly (by 1 % per year and starting from a 6 % level) as displayed in fig. 2:

The price of a bond with maturity "m" can be obtained by calculating the net present value of all future cash flows (principal and coupon stream) using the discount rates applying for each particular period "t":

$$P = \frac{V}{(1 + r_m)^m} + \sum_{t=1}^{m} \frac{C}{(1 + r_t)^t}$$

(V = principal, C = Coupon)

Taking bonds with different coupon levels, each of them with equal maturity (6 years in this example) and deriving the RY according to the formula given in annex 1, we obtain the following results:

Coupon level (%)	Price (%)	RY (%)
0	53.46	11.00
5	75.84	10.65
8	89.26	10.50
10	98.21	10.42
13	111.63	10.30
10.40 (PAR)	100.00	10.40

As explained in the previous section, the RY of the zero-coupon bond equals the 6-year spot rate. However, as far as coupon bonds are concerned, we can draw the following conclusions on the basis of the above example:

1. Although each bond is correctly priced, the coupon level influences the outcome of the yield calculations as RY figures differ according to the coupon level of the bond; in the above example, low-coupon bonds seem to yield more than the high-coupon ones; (this is caused by the fact that the RY

computation assumes that discount rates are equal across all periods; if discount rates increase over time, this will lead to underestimation of the present value of early cash flows combined with overestimation of later cash flows; as high-coupon bonds are more affected by this, the RY method will compensate this by a lower yield figure)

2. Each yield figure is a complex average of the underlying discount rates ri; for a bullet bond, the discount rate corresponding to the final maturity (6 years in the above example) will have a proportionally larger influence (especially for bonds with short maturities) because the outstanding principal is discounted at this rate (in the price);

3. We can derive from the central part of the table that a coupon level exists (somewhere between 10 and 13 %) at which the bond will trade at par (100) i.e. the coupon level at which the present value of future coupons and principal equates to 100. This fictitious bond would bear a 10.40 % coupon (par yield).

4. The par yield has the further property that the coupon of this fictitious bond is equal to its yield RY (10.40 %).

The par yield avoids the effect of variation in coupon on the yield to maturity and therefore provides a smooth and logically consistent yield figure for each maturity (par yield curve). The par yield curve can be constructed on the basis of the discount (spot) rates, supposing these are known (see annex 2 explaining the methodology for deriving the par yield curve). Inversely, the underlying discount rates can be derived from the par yields (see also annex 2); the par yield curve can therefore be used as a starting point for determining all other interest rate or yield measures: spot rates, rolling yields (used by investors), RY for bonds bearing coupons different from the par level, etc.

The example described above has been generalized for all maturities (from 1 to 6 years) using the same spot rate curve. The upper part of figure 3 shows the spot rate curve as well as the par yield curve (derived for all maturities ranging from 1 to 6 years). We notice that in this particular case the par yield curve is systematically below the spot rate curve.

The lower part of figure 3 plots the difference (expressed in basis points) between the RY of bonds with a specific coupon level and the par yield. Computing this difference for all maturities provides us with a "constant coupon" curve for each of the 4 coupon levels considered in our example. It becomes apparent that low (high) coupon (e.g. 5 %) bonds have a higher (lower) RY than the par yield corresponding to a particular maturity. This difference is seen to increase significantly with the maturity of the bond. The intersections of the constant coupon curves with the par yield curve correspond to the maturities where the coupon level equals the par yield (e.g. between 5 and 6 years for 10 % bonds). Although this is

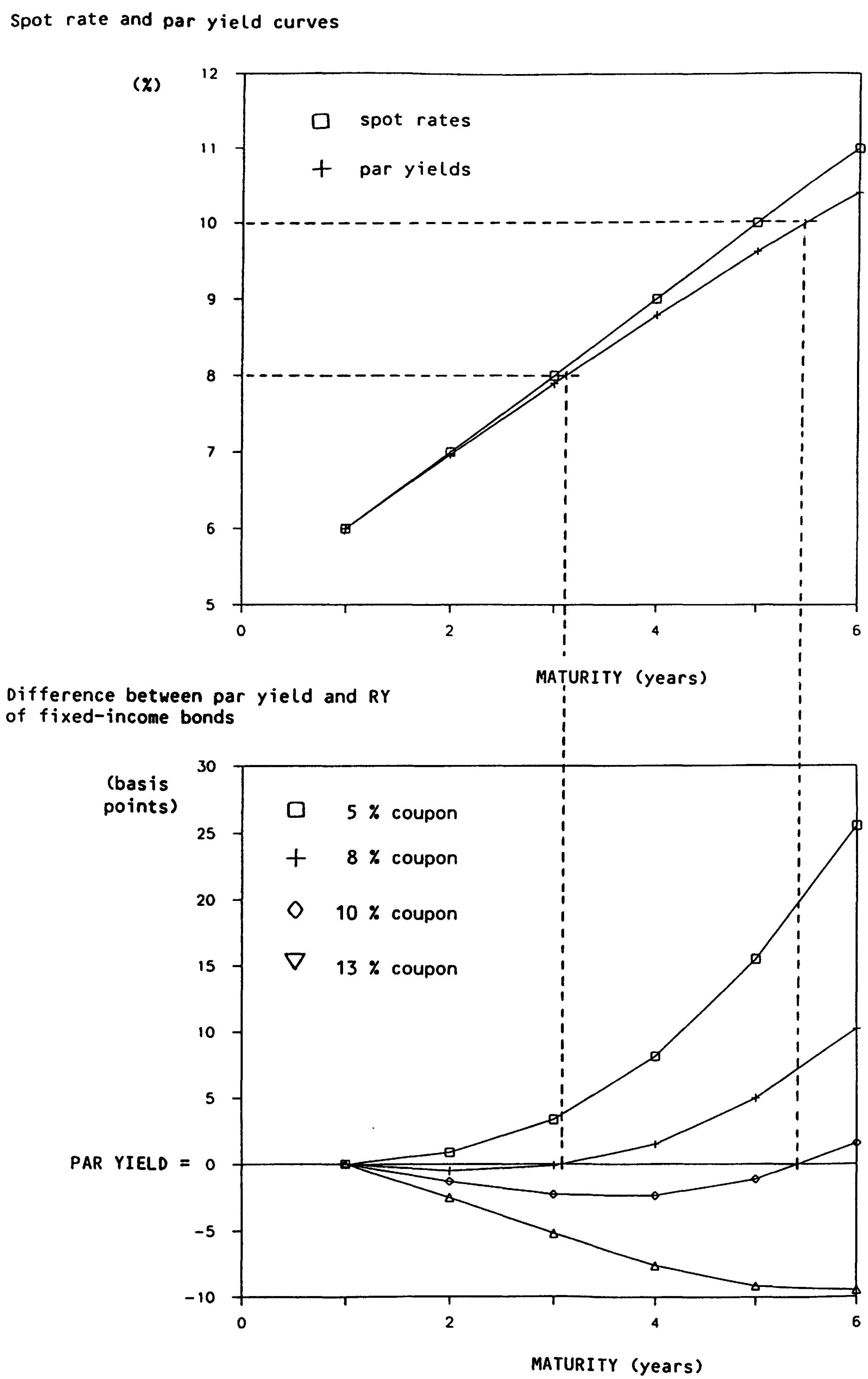

Spot rate and par yield curves
(%)
12
11
10
9
8
7
6
5
spot rates
par yields
0
2
4
6
MATURITY (years)
Difference between par yield and RY
of fixed-income bonds
(basis
points)
30
25
20
15
10
5
PAR YIELD = 0
-5
-10
5 % coupon
8 % coupon
10 % coupon
13 % coupon
0
2
4
6
MATURITY (years)

FIGURE 3

not apparent in fig. 3, the RY of coupon bonds is in all cases below the level of the spot rate curve (and equal to the spot rate for zero-coupons).

Case 2: Downward sloping curve

Figure 4 examines the consequences of a downward sloping spot rate curve (curve 2 on fig. 2) on the resultant yields of a similar set of securities. Here again, coupon levels will influence the yields, although in the opposite direction of the pattern observed in case 1 (small coupon bonds "yielding" less than high coupon ones). The par yield curve is located above the level of spot rates.

Case 3: Flat curve

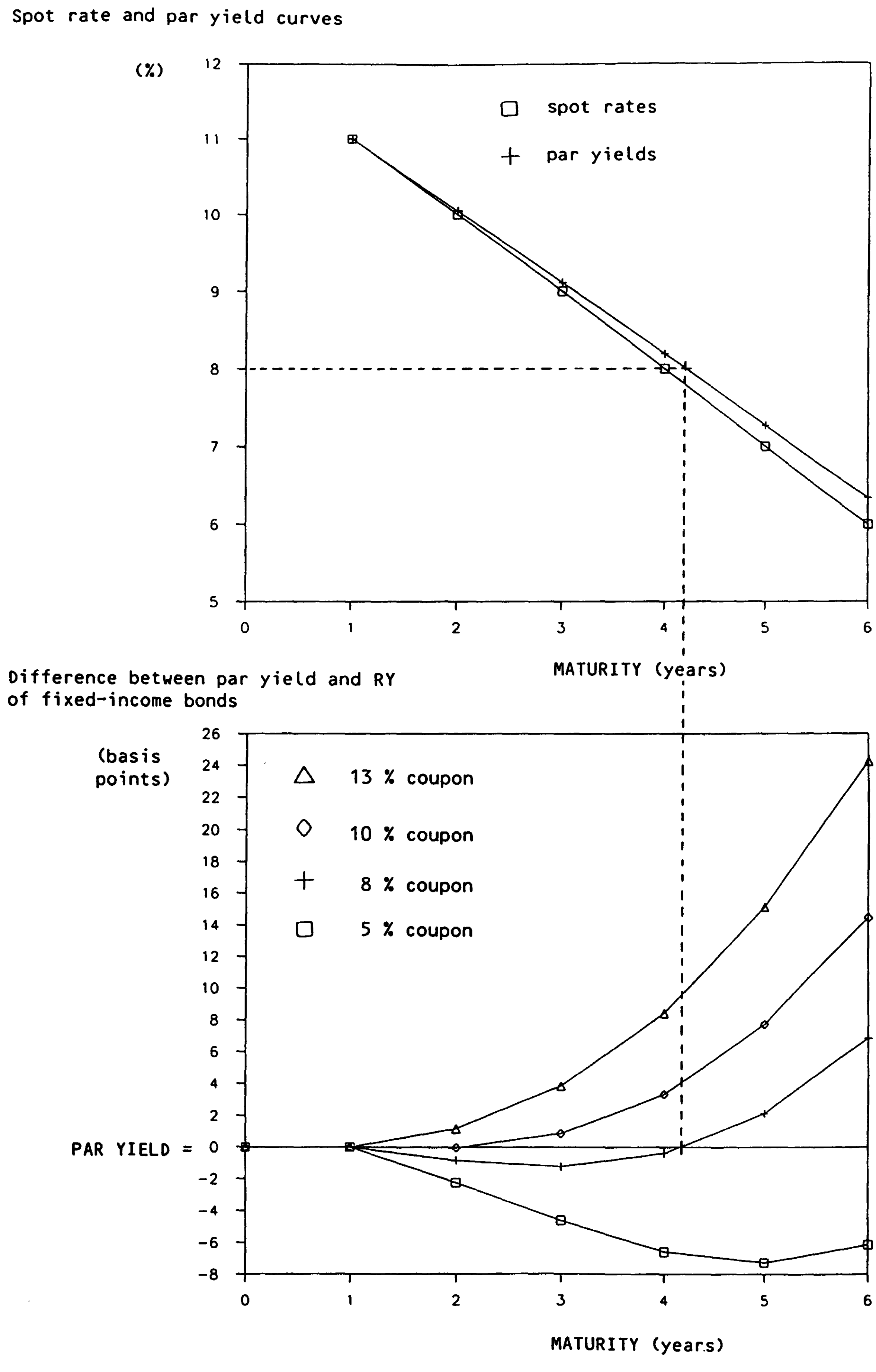
Case 3 (see fig. 2) assumes that the spot rate curve is flat i.e. discount rates to be applied to future payments are constant for all maturities (ri = R). As this is one of the implicit assumptions of the yield definition, we find that under these conditions:

- all bond yields are equal to the par yield, irrespective of their coupon level or maturity (RY = R);
- as a consequence, the par yield for each maturity is also equal to the level of the discount rates (PY = R);

Case 4: Other term structures

Real life situations can obviously differ from the few simple pattern that have been considered so far. In particular, yield curves can have bended shapes, involving a more complex relationships between spot rates, redemption yields and par yields. Two additional examples involving situations where the curves show a maximum (minimum) for a particular maturity are illustrated in annex 3; in both cases the par yield curve intersects the spot rate curve at a particular point. The effect of the coupon level on the RY becomes obviously less straightforward than in the simulations carried out before. The situation would become even more erratic if the segmentation hypothesis on the term structure of interest rates would hold i.e. if no formal term structure exists.

Prices and yields applying to fixed-income securities form generally the basis for determining interest rate levels as spot rates can usually not be observed in a direct manner. Coupons and principal can however be traded separately (e.g. after a "coupon strip") in which case a series of pure discount "bonds" is created. One such operation has already been performed on an Ecu bond (Italy 2011 issue) and is briefly commented in annex 4.



Spot rate and par yield curves
(%)
12
11
10
9
8
7
6
5
spot rates
par yields
0
1
2
3
4
5
6
MATURITY (years)
Difference between par yield and RY
of fixed-income bonds
(basis
points)
26
24
22
20
18
16
14
12
10
8
6
4
2
PAR YIELD = 0
-2
-4
-6
-8
13 % coupon
10 % coupon
8 % coupon
5 % coupon
0
1
2
3
4
5
6
MATURITY (years)

FIGURE 4

D.3 Provisional conclusions on the use of redemption yields

The above examples illustrate that the redemption yield is potentially confusing if one wants to determine a reference rate corresponding to a specific maturity as it depends on the coupon of a particular bond. Deviations can be significant if:

- spot rate curves are steeply sloped (downward or upward);
- residual maturities are long;
- coupon levels are either relatively low or relatively high compared to the current level of interest rates for a particular maturity.

The par yield concept provides a more coherent and representative basis for representing the interest rate levels applying for each maturity.

Bonds with current coupon levels (i.e. coupons relatively close to the prevailing level of interest rates) are generally to be preferred when deriving yield curves; the US Treasury only uses its most recent issues for determining the US $ reference rate.

Only in the case of a flat yield curve does the coupon size become irrelevant (as far as the RY concept is concerned).

D.4 Price volatility and duration

The coupon level of a bond and its residual maturity were used in the previous section to derive its current price and yield characteristics. The analysis was a purely static one, comparing yields applying to different securities in the same time period on the basis of a given term structure of interest rates. Interest rates do however change over time, thereby influencing current prices so as to bring yields in line again with the current market situation.

It appears that the price sensitivity of individual bonds to these interest changes differs substantially, e.g. bonds with long maturities are more exposed to price variations than bonds with shorter maturities. Similarly, zero-coupons (and more generally, bonds with low coupon levels) will be affected more than high-coupon ones, etc. Theoretical analysis has demonstrated that the relevant concept describing a bond's price volatility as a result of interest rate movements is its "modified duration" (a notion integrating maturity, coupon and other characteristics of a bond): see annex 5 for the mathematical definition of duration as well as the relation between duration and price volatility.

The duration of a bond describes one of the main risk components associated with an investment in a fixed-income security. For the same final maturity, bonds with high durations (e.g. low-coupons) should therefore have a lower price (and hence a higher yield) than similar instruments with low durations (all other things being equal).

A different source of price variability again related to duration is caused by market expectations on the direction of interest rate movements. In the case of assymmetrical expectations about the future evolution of interest rates, bonds with high (low) durations will command a higher (lower) yield if interest rates are expected to rise (diminish).

Pushing this type of analysis further, one could theoretically also consider the convexity features of the security, although the impact is likely to be minor within normal (fairly narrow) differences of yield levels.

E. CREDIT QUALITY

The analysis put forward in the previous section only holds to the extent that all cash flows and their time patterns are known with complete certainty (risk-free securities). If this were not the case (e.g. because of default risk), the present value of all future payments would decrease and hence push up security yields.

For a given currency, the risk characteristics of a security (and the size of the risk premium) are related to several factors, of which the main ones are:

- the creditworthiness of the borrower: which is itself related to his particular status: corporate borrower, sovereign borrower (capability to tax and to create money), supranational institution (callable capital), etc.;

- specific provisions applying to the issue itself: some of these provisions will decrease the risk (e.g. guarantees, sinking funds, ...) while others will increase it (e.g. call option, subordination clause,...);

- the currency in which the borrowing is expressed: a sovereign borrower provides high security when borrowing in his domestic currency.

These various risk elements are often combined by specialized agencies into a single indicator or credit rating (*), which reflects the overall relative risk level of a security.

As a result of the existence of risk differentials and their implication on yields, the interest rate levels measured across the maturity spectrum are only representative for securities with an equivalent degree of risk. Interest rates applying to (virtually) risk-free securities constitute the most natural reference base used for either cross-currency comparisons or term structure representations as they constitute "base rates" for interest rate levels applying to the whole risk spectrum and are not contaminated by underlying default probabilities.

(*) opinion expressed by an independent agency on the ability of issuers to honour punctually their debt obligations

F. TAXATION

F.1 General

In real life situations, both investors and borrowers will act so as to optimize net returns, given their particular tax situation. On the other hand, taxation will also exert influence on gross interest rates; this effect is unlikely to be equal for all kinds of securities and could help to explain a number of yield effects observed in the market.

The practical impact of taxation will depend on a variety of often interacting elements, related to the following factors:

a. the characteristics of the security:

- the specific status of the security: e.g. tax-exempted, level of withholding tax, ...;
- the relative importance of income derived from coupons vs. capital gains (e.g. zero-coupons);
- etc.

b. the status of the investor with regards to taxation:

- the country where the investor is taxed;
- the tax regime (e.g. tax-exempted or not, individual vs. institutional investor, etc.);
- possibilities for hedging against or evading tax;
- etc.

c. the market on which the security is traded

- withholding taxes (e.g. eurobond vs. domestic market);
- transaction costs and taxes;
- etc.

A detailed analysis of the impact of all these (interacting) elements being impossible in the context of the current report, we limit ourselves to a few general considerations and generally applying conclusions:

F.2 Impact of taxes on yield levels

As taxes have a negative impact on the value of present and future income deriving from a particular security (and hence lower its price), they will drive up the yields. This effect can be readily observed by comparing interest rate levels applying:

a. in different markets for the same currency (e.g. Eurobond vs. domestic markets in the absence of exchange controls and restrictions on capital movements in general);

b. to securities with differing tax-status traded in the same market (e.g. tax-exempt vs. non-exempted bonds);

c. after the introduction or disappearance of tax measures (e.g. withholding tax in Germany).

As investors focus on net returns, some of the taxes mentioned above (case b. in particular) will lead to a segmentation of the market between those liable to the tax and the others for whom gross and net returns are identical.

In other instances, investors not subject to the tax might derive additional income from its existence, in the form of higher interest rates, swap opportunities between different markets (tax arbitrage), etc. and thereby exert a "stabilizing" influence on prices until equilibrium levels are attained (between markets, types of investors, instruments, ...). Even investors subject to taxation could derive some benefits from the existence of the tax under some particular circumstances (e.g. if capital losses are deductible).

The overall effect on yields is therefore complex and difficult to quantify.

F.3 Levy base of the tax

Most taxes will be related to the revenues derived from the investment in a particular security, either in the form of coupon income or capital gains. Many tax systems include a distinction between either source of income (for certain categories of investors); for practical reasons, capital gains are often treated more favourably (provided some re-investment provisions are complied with), thereby introducing a bias (and hence a yield reduction) in favour of securities generating a larger part of their income in the form of capital appreciation (e.g. low-coupon investments). In a dynamic context, where investors buy and sell securities, this might also affect the time-pattern of yields (especially around coupon payment dates).

The influence of taxes and duties levied on securities transactions is discussed more extensively in section E as its effect is quite comparable to dealing spreads.

G. COUPON EFFECTS

The previous sections illustrated several possible reasons for expecting a relationship between the redemption yield of a security and the size of the annual coupon payments. These various effects are summarized in the table below, which indicates the relative impact, either positive (+) or negative (-), of each individual factor.

	LOW COUPON (*)	HIGH COUPON (*)
Rising yield curve	+	-
Inverted yield curve	-	+
Duration effect (price volatility)	+	-
Expected increase in interest rates	+	-
Expected decrease in interest rates	-	+
Capital gains tax	-	+
Coupon tax, withholding tax, ...	+	-

Some effects are more or less permanent over time (tax structure, duration of a particular security, ...), contrary to other factors (interest rate expectations, shape of yield curve, ...).

As the various effects might either compensate or reinforce each other, it is difficult to develop ex ante views on the final outcome. It is however possible to verify to what extent and in which direction (positive or negative) the coupon level influences the yield. A correct estimate of this "coupon effect" will allow the determination of the par yield curve on the basis of the redemption yields observed for bonds with various coupon levels.

(*) "+" indicates that the yield will be relatively higher
"-" indicates that the yield will be relatively lower

H. TRANSACTION AND INTERMEDIATION COSTS

H.1. General

Secondary market yields are derived from price quotations. The method by which these prices are fixed or quoted can therefore affect yield measurements as illustrated in the two following examples of market organization:

a. market operation on a centralized "bourse" basis: all trade is combined in a single system or location where all demand meets all supply; a stock exchange is a typical example of this market type. Supply for a particular security is regularly confronted with demand (e.g. once or several times a day, continuously, ...). At one particular point in time, the system produces a single price for each security, although this price can evolve during the course of the day. This reference price (e.g. as published by newspapers) will be based on a particular moment during the day (e.g. fixing price) and seems a valid starting point for deducing interest rates as it applies to both buyers and sellers. The price (and hence the yield) applying to the final customer (buyer/seller) will be adjusted for all transaction costs: taxes and duties, brokerage fees, etc., which are negotiated separate from the transaction.

b. over-the-counter trade operates in a decentralized manner, whereby buyers and sellers are either brought together by brokers or whereby each dealer more or less operates as a market on his own, acting as a counterparty for each transaction on either side of the market (market making) and hence taking positions in the security (e.g. most Eurobond trades). Two prices (bid and offer) are generally quoted; the spread between these two prices will a.o. depend on:

   - the liquidity of the security (conditioned by: the total issue size, the amount of paper in active circulation, the residual maturity, ...): illiquid issues will command a higher spread because of factors such as the difficulty to undo a certain position (find counterparties), the impact of any supply or demand on the price level of the security, valuation problems, etc.;

   - the average transaction size for the security.

   - etc.

   In the eurobond market, spreads (on the price, not on the yield) applying to Ecu securities will generally range between 10 basis points or less for very actively traded issues (benchmarks) and up to a 1.5 % for illiquids.

H.2. <u>The impact of spreads (and transaction costs) on yields</u>

The relationship between yield variations on bond price volatility was explained in section C.2, where the duration concept was introduced. The relationship also holds in the opposite direction and therefore also describes the influence of price movements on yields. The impact of a given level of spreads (or transaction costs in general) will therefore be greatest for securities with relatively small durations (e.g. short maturities) or relatively low prices (e.g. below par quotations).

Figure 5 provides a theoretical illustration of the yield differentials resulting from dealing spreads (set at 50 and 100 bp; par yield set at 7.5 % for all maturities; 7.5 % coupon).

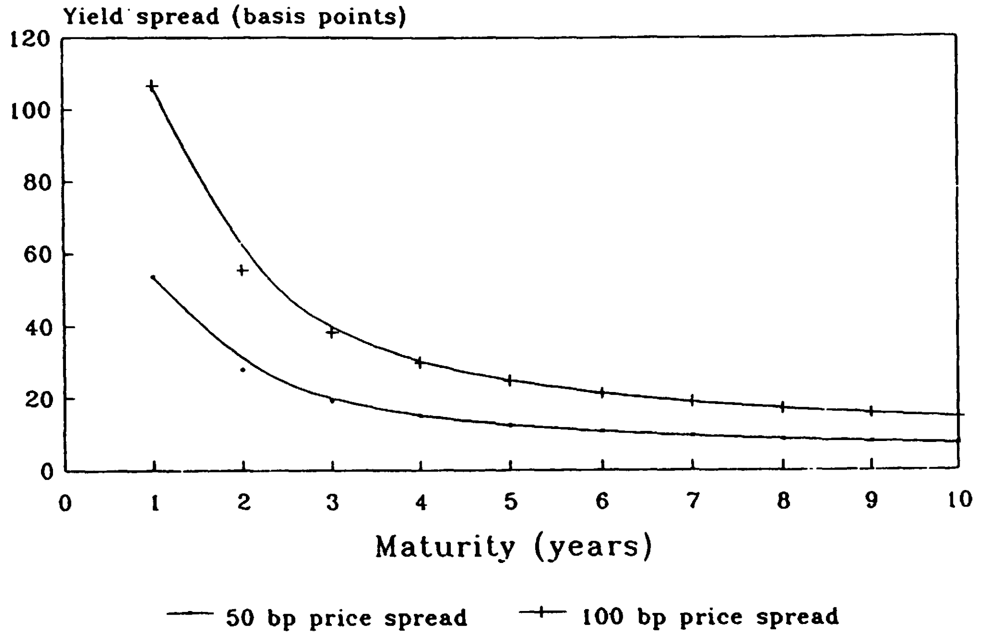


Figure 5

The figure illustrates that:

- bonds with short maturities are particularly affected by spread levels: for securities with residual maturities of 1 year or less, the yield spread will exceed the price spread; this effect contributes to the illiquidity of bonds with short maturities as transactions become expensive in yield terms;

- for longer maturities, the transaction spread will be distributed over the remaining life of the bond but never totally disappear: for a bond with infinite maturity (perpetual) the yield spread in the above example would amount to around 7.5 bp;

- the yield differential is roughly proportional to the size of the spread; the influence of the coupon level is marginal compared to the impact of the maturity factor.

Fig. 6 illustrates the actual yield differentials for 20 selected Ecu securities considered highly representative and liquid in the Ecu bond market. Yields were computed on the basis of actual bid and offer prices (31 May 1991; ISMA data). High differentials appear for either short-term securities or smaller issues.

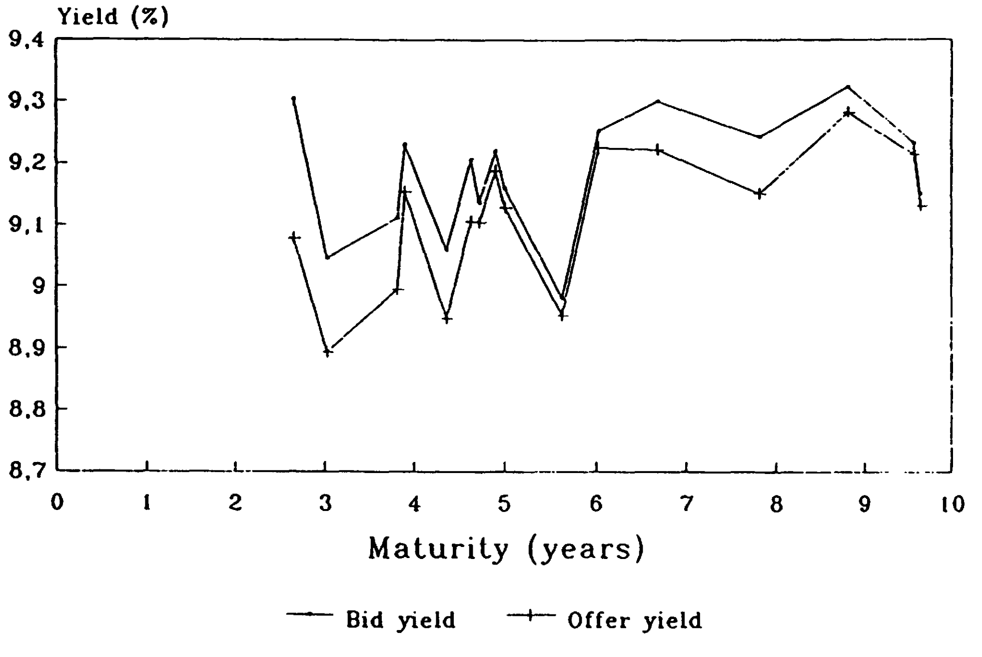


Figure 6

H.3. <u>Choosing the appropriate reference base</u>

The choice of the (bid or offer) price for calculating the yields will obviously affect the overall level of yields (highest if bid prices are taken). Furthermore, because of the higher variability of short-term bonds, the slope of the yield curve will also be influenced: if a yield curve based on mid-prices would be perfectly flat, it can be seen that the "bid" yield curve would be downward sloping, whereas the "offer" yields would be upward sloping (see fig. 7):

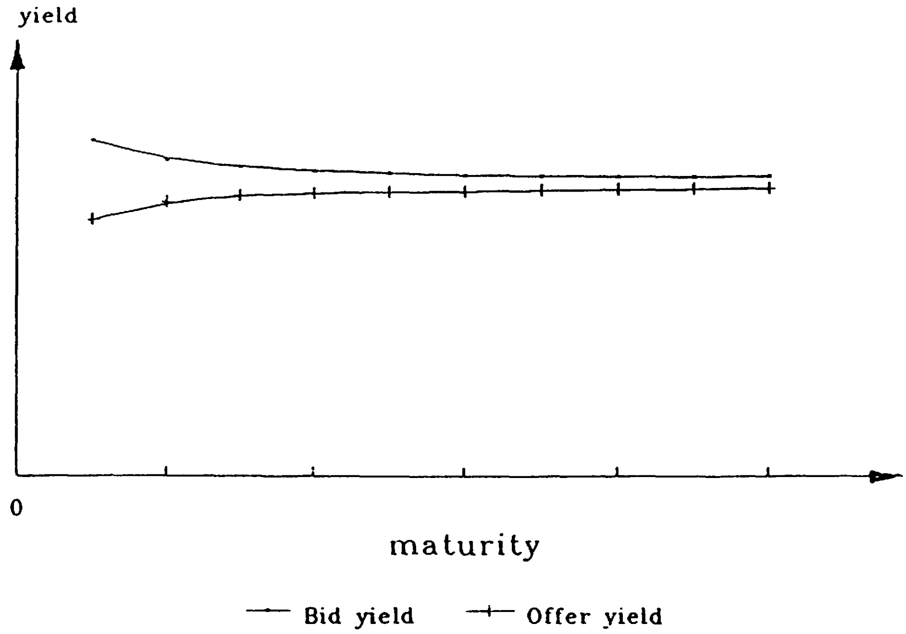


Figure 7

It is likely that the various actors in the market view prices and yields from different perspectives:

- yields are relevant for those holding securities over longer periods (typically the investors)

  - offer yield: when acquiring a new security

  - bid yield: opportunity cost for giving up (selling) a security (e.g. when using the proceeds to acquire another one)

- a borrower considering a new bond issue will probably concentrate on offer yields (minimum interest rate to be offered);

- prices matter as well for investors when they valuate their portfolio on a particular point in time (static perspective): the bid price seems the most relevant one (value of the bond when sold in the market);

- dealers and market makers might concentrate more on prices and spreads (as they derive their income from them), unless they consider long-term investments in a particular security;

The mid-price between bid and offer therefore seems an acceptable compromise for constructing reference rates as it takes into account the views and interests of the various market participants. Furthermore, the mid-price is comparable with the single price quotations provided by stock exchanges, assuming that market makers charge an equal amount of costs to sellers and buyers (price equilibrium in the middle of bid and offer).

Although this conclusion is presumably valid for the market as a whole, it does not necessarily apply to each individual market maker or security dealer. A particular intermediary might want to revert out of an unwanted - long or short - position (or move into a new position) for a particular security and set his prices accordingly. An average of the mid-prices quoted by a range of representative intermediaries will therefore provide a more accurate reflection of the overall position of the market.

Part II: ESTIMATING ECU RATES

I. THE ECU BOND MARKET

I.1. Development of the market

The first Ecu bond was issued in April 1981, only two years after the introduction of the EMS (European Monetary System). For many years, the Ecu bond market remained a relatively small, mainly retail-driven although steadily developing market, characterized by a variety of relatively small issues. The evolution of total issue volumes (see annex 6) showed a pattern of regular increases, which was however interrupted in the middle of 1992 (cf. section I.2).

Despite the current slowdown in new issues, the nature of the Ecu bond market changed profoundly in the early nineties. Governments and institutions from the EC have played an important role in this as they became the main players in the primary market as opposed to the business sector which was traditionally dominating the primary market. All but three of the Community's Member States (Germany, Luxembourg, Netherlands) now have Ecu borrowings outstanding. Several non-Community sovereign borrowers or supranationals joined in, often with large issue volumes. The main consequences of these developments are the following:

a. appearance of true benchmark issues: while the market was formerly characterized by a variety of small and often illiquid issues trading at substantial spreads, "jumbo" issues (500 Mecus or more) now represent the major part of market activity. First introduced in July 1988 (Jean Monnet 500 Mecu issue), jumbos have become increasingly customary, some of them attaining sizes in excess of Ecu 3 bn. All of these are characterized by a high issue quality.

b. increase of secondary market activity: largely as a result of the existence of highly liquid benchmark issues traded at small spreads and displaying high credit quality, institutional investors came on the forefront replacing the former retail investor. Besides increasing the absorption capacity of the primary Ecu bond market, institutional trade vastly increased secondary market activity as illustrated by clearing house statistics: at some stage, both Euroclear and Cedel figures indicated that about half of the ten most heavily traded Eurobonds were Ecu bonds. The number of market makers in Ecu bonds rose significantly as a result.

c. existence of hedging vehicles: bond future contracts provide institutional investors with new means for hedging against Ecu interest rate risks and therefore contribute to the development of the bond market itself;

d. appearance of long maturities: while some domestic currency bond markets in the Community do not exceed 10 (or even 5) years maturity, the Italian 20-year issue and the French 30-year issue confirmed the Ecu market's ability to sustain long-term borrowing instruments.

I.2. Impact of mid-1992 crisis

The growth path of the Ecu bond market was suddenly interrupted in the middle of 1992, mainly as a result of political developments related to the ratification process of the Treaty on European Union, of which Economic and Monetary Union is an integral part.

The outcome of the referendum in Denmark on 2 June 1992 was the first element affecting the Ecu market. Market uncertainty kept rising with the organization of a referendum in France on 20 September 1992. In the meanwhile, pressures started to build up on the monetary front, successively leading to the devaluation of the lira (by 3.5 %; other currencies were revalued by 3.5 %) on 14 September, the withdrawal of both the pound sterling and the lira from the ERM on 17 September, and the devaluation by 5 % of the peseta on the same date. On 22 November, the peseta and the escudo were both devalued by 6 %.

Realignments continued in 1993, with a 10 % devaluation of the Irish pound on 1 February, followed on 14 May by a devaluation of both the peseta (8 %) and the escudo (6.5 %). On 1 August, EC finance ministers decided to widen the fluctuation bands around the central rates in the ERM from 2.25 % to 15 %.

These developments evidently had a significant impact on the Ecu bond market. Activity on the primary market slowed down considerably from mid-1992 onwards (see annex 6b); domestic Ecu issues by EC Member States (French OATs, Italian CTEs, UK notes, ...), together with EC institutions represented the bulk of the issue activity. This situation has continued in the course of 1993 (see annex 6c): total issue volumes in the first semester of 1993 roughly stand at 60 % of 1991 figures (and at 40 % of the total issue size in the first semester of 1992).

Prices in the secondary market started to drop significantly in mid-1992 and bid-offer spreads widened. The market virtually came to a stand-still at the end of July 1992 when professional dealing obligations between market makers were suspended. The situation improved in the weeks thereafter, although the liquidity in the Ecu secondary market remained seriously impaired. Secondary market turnover of fixed-income Ecu bonds, as measured through Cedel and Euroclear statistics, dropped from as high as $ 95 bn per month (first quarter 92) to somewhat less than $ 40 bn per month (first semester 93). The relative share of Ecu bond trading in the secondary market dropped from close to 30 % (first quarter 1992) to around 11 % (first semester 1993).

Despite the significant downturn in both the primary and the secondary market, the Ecu bond market has stabilized again, albeit at a lower level of activity.

J. APPROACHES FOR DERIVING REFERENCE RATES

A variety of methods exist to derive medium and long term interest rate figures, some of which have already been applied to the Ecu. An overview is provided below.

J.1. Synthetic measures

During the period in which the Ecu bond market was retail-driven and few benchmark issues existed, Ecu reference rates for various maturities were generally computed as "theoretical" interest rates by recomposing the interest rates or yields applying to government borrowings in the twelve component currencies of the Ecu basket. It is intuitively clear that the evolution of the Ecu rate is conditioned to a considerable extent by the fluctuations of the underlying components; the possible deviation between "synthetic" rates and actual ones was assumed to be restricted (arbitrage constraint). Several problems and limitations associated with the synthetic approach became however apparent:

- as long as the definition of the Ecu basket is not irrevocably fixed, actual yields will also reflect market expectations on the future composition of the basket;

- the level of domestic taxes (e.g. level of withholding tax) differs between countries; as these taxes influence gross interest rates, this induces some amount of incoherence in the calculation of the Ecu rates; a similar problem results from the differing credit quality of the various sovereign issuers;

- as a result of the inexistence of long maturity bonds for a few currencies, long term Ecu rates have to be approximated;

- dealing expenses and transaction costs in general make it more costly to arbitrage so weakening the link between the theoretical and actual Ecu.

With the advent of the liquid Ecu issues, the deviation between theoretical and actual Ecu rates could be estimated more precisely and appeared to be quite substantial at times (cf. annex 7). Moreover, both the absolute size of this difference (sometimes as much as one percent) and its sign (negative or positive) varied over time. The theoretical approach however remains valid to obtain further insights on the underlying structure of Ecu interest rates and on their relative level.

J.2. Index or portfolio method

This approach is based on actual issues and is widely used for various currencies. The liquid issues traded in a particular bond market are divided into homogeneous subsets (e.g. classification of issues according to maturity range or according to issuer quality). Each subset constitutes a hypothetical portfolio which is used as a basis for deriving yield figures which are considered representative for this particular segment of the market; the issues in each portfolio are generally weighted according to the total amount in circulation.

The portfolios can also be used to calculate other indicators, such as yield, average maturity, duration, convexity, etc. In combination with predefined re-investment rules, the portfolio also forms the basis for total return measurements (ignoring transaction costs). Each investor can therefore compare the characteristics and performance of his individual portfolio with the characteristics and the performance of the index.

As far as the Ecu is concerned, two market makers (Paribas and J.P. Morgan) have developed an index methodology so as to assist and advise their clients on Ecu investments. In both cases, yield and return calculations are based on internal price quotations.

The main advantage of the index or portfolio method is that it provides performance measures (yield, total return, etc.) which are directly usable by portfolio managers to evaluate the performance of their Ecu investments. These measures are not based on theoretical models or other assumptions: investors can replicate the composition of an "index" portfolio and know that, apart from transaction costs, their investment performance will be accurately reflected by the evolution of the index.

The main disadvantages of the method are the following:

- portfolios are selected according to one or at most two criteria (e.g. maturity, credit rating, etc.); within each portfolio, issues will however vary widely according to other relevant criteria (e.g. coupon, issue size, etc.), which are ignored in the subsequent computations;

- compared to a yield curve, the method only provides discontinuous yield estimates over the maturity spectrum (e.g. estimates for 3 years, 5 years, etc.); moreover, each particular portfolio will not provide estimates for a particular maturity, but for a specific range (e.g. from 2 to 4 years);

- specific problems arise when the total number of representative securities in the market is limited:

    - some portfolios will contain very few (or even no) securities (or otherwise the number of market segments needs to be severely reduced);

    - over their remaining life, bonds will periodically drop out of a maturity range and be incorporated in the previous one; this gives rise to discontinuities if the number of securities portfolio is small.

J.3. <u>Benchmark method</u>

This method is comparable to the previous one, except that individual securities, instead of portfolios, are taken as a reference base. The selected securities are considered the most representative ones for a given maturity and are either the most liquid ones or the ones that were most recently issued.

Simplicity is the main advantage as no further computations are required and results can be easily verified. On the other hand, a large part of the market is ignored when determining reference rates; it should therefore preferably be applied in very large and liquid markets with regular new issues (e.g. US Treasury bonds).

J.4. Swap market rates

For the major currencies, swap market rates can be readily obtained for a wide range of maturities (2 to 10 years). The method is therefore relatively straightforward and easily verifiable; cross-currency comparisons on the same basis are also convenient.

Bond market yields for top quality borrowers are lower than swap rates for comparable maturities. This has mainly to do with credit risk factors as swap rates represent fixed-rate equivalents for floating rates at Libor levels. High quality borrowers have access to short term funds at rates below Libor, which represents the average interbank rate (the banking system having an average rating below triple-A). Swap rates should therefore be adjusted to take account of this difference in credit quality.

The data collection process to compute an average swap market rate is rather cumbersome as, contrary to the Eurobond market, no central source of swap market data is available (ISDA only collects information on volumes, not on rates). Furthermore, the liquidity in the swap market fluctuates considerably over time.

J.5. Yield curve

Yield curves display the term structure of interest rates and are generally based on regression models, whereby secondary market yields are estimated as a function of maturity and sometimes other variables on the basis of a prespecified functional form. This method is used for many currencies (cf. annex 8 illustrating yield curves for the £ and FF as provided by the Bank of England and the Banque de France), although the underlying theoretical base which is used to determine the functional forms of the yield curve can vary widely from one model to another.

The main advantage of the method lies in its flexibility as a wide range of explanatory variables can be integrated in the computations. Moreover, the yield curve allows to produce continuous estimates over the whole maturity spectrum, even if no bonds are traded in a specific maturity range. Similarly, estimates for spot and forward rates can be derived.

On the other hand, the estimates are conditioned by the definition of the model: different models will produce different outcomes. As some models are highly complex, results are less verifiable. The method is also of less interest to professional investors who prefer measures based on actual bond portfolios and who are also interested in a number of complementary indicators (total return measures, etc.).

J.6. Choice of methodology

After a comparison of the various methods listed above in the light of the current characteristics of the Ecu bond market (limited number of benchmark issues, heterogeneity of issuers, ...), the yield curve approach seems the most appropriate.

Despite the difficulties mentioned above, the method is capable of producing reliable measures along the maturity range and hence to provide a good basis for deriving reference rates at predefined maturities (e.g. 3, 5, 7 and 10 years). Moreover, the consistency of the results produced by diverse yield curve models (with different underlying assumptions) provides further evidence in favour of this choice (cf. section M).

K. FACTORS DETERMINING BOND YIELDS

Before moving on to a detailed description of the various yield curve models which were tested, this section explores which criteria are relevant for selecting representative issues; these issues are the only ones that are to be considered when determining reference rates.

The pertinence of the bond selection criteria which are presented in detail in section L will be illustrated by the statistical analysis of Ecu bond yields as observed in the secondary market. Figure 8 displays the yields of a all outstanding fixed-rate Ecu Eurobonds issued by either sovereign borrowers or supranational institutions. All securities with specific provisions (calls, ...) are removed from the set; yields are based on mid-prices (average of bid and offer price; closing quotations) as reported by market makers to the ISMA on 28 June 1991. The resulting yield levels vary between 8.7 and 10.9 %.

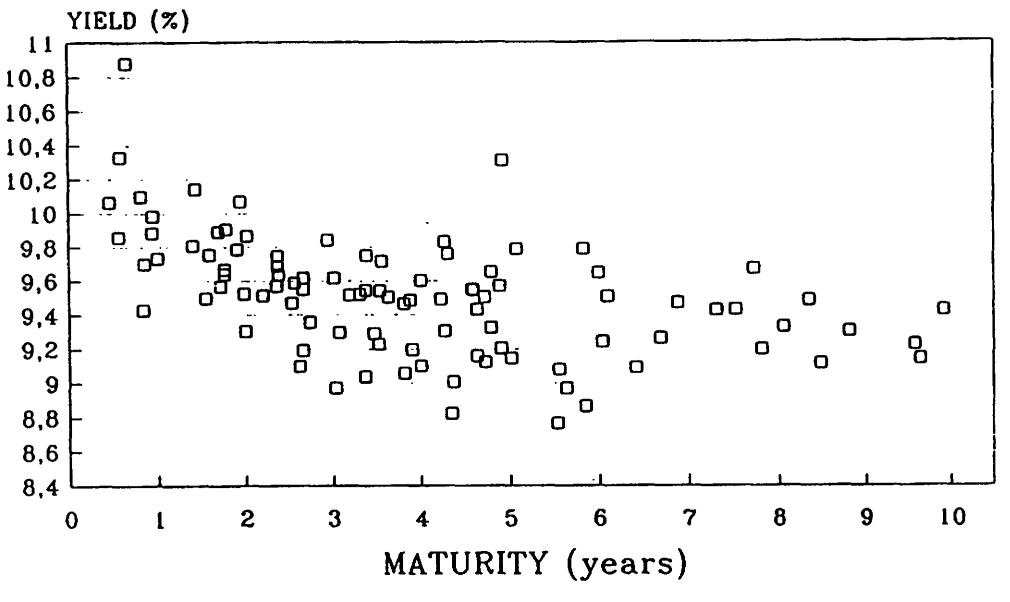


Figure 8

Annex 9 brings some additional perspective into the above graph as it illustrates the influence of the following factors on the yield level of this set of issues:

- the credit quality of the issuer (as measured by the rating figures of Moody's and Standard & Poor's: see also annex 11)
- the size of the issue (estimated outstanding; in Mecus);
- the price spread (bid-offer spread in basis points);
- the number of market makers for the issue.

Further relationships between these and other variables (coupon, yield spread, ...) are displayed in annex 10. Both annexes 9 and 10 clearly demonstrate the paramount importance of liquidity factors (as measured by dealing spreads, issue size, number of dealers) as well as the issuer's credit quality on the yield levels; the influence of coupon levels appears to be weak.

A more quantative analysis was carried out by regressing the secondary market yield of the various securities represented in figure 8 against the list of variables discussed above. The table below compares the results of these regressions on yield data for 28 June 1991 and 31 December 1990 with the following explanatory variables:

Y = yield (calculated on the basis of the mid-price);

M = residual maturity (in years);
Q = credit quality (see annex 11);
C = coupon of the issue (in %);
S = issue size (in Mecus; estimated amount outstanding);
P = price spread;
D = number of market makers (reporting to ISMA);

	DATE = 28/06/91		DATE = 31/12/90	
VARIABLE	Coefficient	t-statistic	Coefficient	t-statistic
(constant)	13.15	10.7	13.88	9.9
M	-0.49	-2.4	-0.49	-1.5
M^2	0.11	1.3	0.13	1.2
M^3	-0.01	-0.8	-0.01	-1.0
M^4	0.00	0.5	0.00	0.8
Q	0.04	1.2	0.06	1.6
Q^2	0.01	1.7	0.01	1.2
C	-0.59	-2.2	-0.63	-2.1
C^2	0.03	2.0	0.03	2.0
S	0.00	0.1	0.00	0.6
S^2	-0.00	-0.1	0.00	0.1
P	-0.00	-0.1	0.01	1.3
P^2	-0.00	-0.2	-0.00	-1.5
D	-0.03	-2.2	-0.06	-3.1
D^2	0.00	0.9	0.00	1.4
	R2 = 0.77		R2 = 0.83	

A comparison of the coefficients and their level of significance (t-values) on these two dates indicates a high degree of stability of the model over time. The regressions are however unsatisfactory from a statistical point of view because of the strong multicollinearity in the model. This mainly stems from the presence in the equation of several proxy variables for liquidity (trading spread, issue size, number of market makers), which are obviously heavily correlated (see also annexes 10G, 10H, 10I). A simplified and statistically more meaningful model is presented below.

	DATE = 28/06/91		DATE = 31/12/90	
VARIABLE	Coefficient	t-statistic	Coefficient	t-statistic
(constant)	11.33	30.9	12.49	26.0
M	-0.54	-2.7	-0.69	-2.1
M^2	0.11	1.5	0.19	1.7
M^3	-0.01	-0.9	-0.02	-1.4
M^4	0.00	0.5	0.00	1.2
Q	0.04	1.4	0.11	3.1
Q^2	0.01	3.3	-0.00	-0.9
log (C)	-0.45	-2.7	-0.31	-1.7
log (D+1)	-0.09	-4.8	-0.34	-12.3
	R2 = 0.73		R2 = 0.76	

The main conclusions to be drawn from the graphical analysis and from the regressions are the following:

- liquidity has an undeniable influence on secondary market yields (cf. annexes 9C, 9D); several variables can be used to obtain indications on liquidity characteristics;

- the relationship between credit quality (measured by the credit rating) and yields is also highly significant (cf. annex 10A) and to some extent non-linear; further tests on the relationship between the size of the risk spread and the maturity of the security remained unconclusive;

- the evidence on a possible influence of the coupon term remains statistically unconclusive; the negative coefficients obtained in the table above suggests that bonds with high coupons might have slightly lower yields on average.

L. SELECTING REPRESENTATIVE ISSUES

L.1. Representative market segment

The Ecu bond market is composed of several segments:

- the Eurobond market, where most Ecu bonds are issued and traded;

- the domestic markets: in some cases (e.g. French Ecu OATs), bonds were issued on the same terms as international issues and can therefore be assimilated with the former market segment (because of absence of withholding taxes, similar clearing and settlement procedures, etc.). This is however not the case for "pure" domestic issues (e.g. Italian CTEs, Greek Ecu-linked bonds), as each of these is characterized by specific attributes.

In view of this, the part of the market composed of both Eurobond issues and "assimilated" domestic ones seems the most representative for deriving interest reference figures. Bond prices are quoted continuously by a variety of market makers and can be consulted via Reuters or Telerate, in particular for the benchmark issues. Closing quotations are reported daily to the ISMA (International Securities Markets Association) by the associated dealers and published daily (average prices per issue). Daily price information from a variety of sources is furthermore available via other information providers (Bloomberg, Datastream, etc.).

L.2. Selection criteria

In view of the objectives described in section B and of the empirical tests described in the previous section, the following criteria are to be retained when selecting the issues to be used for determining Ecu yields:

a) **criteria pertaining to the issuer:**

- only sovereign and supranational issuers (acting as borrowers and not as guarantors for the issue) are to be considered (both EEC and non-EEC issuers);

- the issuer's Ecu borrowings should enjoy a high credit quality as measured by the major rating agencies e.g. be rated AAA-AA (Standard and Poor's) or Aaa-Aa1 (Moody's); unrated issues are assumed to have the same rating as other rated issues of the same borrower, if they include a "pari passu" clause;

b) criteria pertaining to the issue:

- the issue should be highly liquid and therefore generally meet the following conditions:

  - issue size: at least 500 Mecu (preferably more than Ecu 1 billion);

  - trading spread (difference between bid and offer price) less than 50 bp;

  - number of dealers reporting to the ISMA: at least 15;

  Liquidity for each issue needs furthermore to be regularly verified on the basis of Cedel and Euroclear turnover statistics;

- only fixed-rate bonds are to be considered; the issue should furthermore be exempted from special features influencing yield levels (e.g. call or put options, zero-coupons, etc.);

- domestic Ecu issues are only to be considered to the extent that they can be assimilated with Eurobond issues; they should more specifically:
  - be free of withholding tax to non-residents;
  - be subject to similar clearing and settlement procedures (7-day settlement through Cedel or Euroclear);

- only capital market (and no money market) instruments are to be considered; residual maturities should exceed one year;

- no upper limit is imposed on maturities; issues with maturities beyond ten years are however only to be taken into account if the maturity "gap" with the previous issue is smaller than two years.

Although these selection criteria should remain fixed over a relatively long period, adjustments could be envisaged as a result of evolving market conditions.

The representative set of issues corresponding to the above criteria needs to be regularly reviewed as new issues enter the market or existing ones become illiquid. The yield curve approach allows for a high degree of flexibility in this respect as the requirement of a stable reference portfolio (as required by the index method) is less urgent.

The application of the selection criteria to the list of Ecu issues which are currently outstanding in the secondary market produces a small, but highly representative set (see example in annex 12). Only issues with high credit ratings are considered as indicated above.

Although the model intends to measure Ecu rates applying to prime quality issues, benchmark issues with a slightly lower credit rating have not been discarded as they can contribute to the reliability of the estimates, particularly for those maturities with few triple-A issues outstanding. About 10 issuers are represented in the set, while the majority of issues enjoy a triple-A rating and exceed an issue size of Ecu 1 billion.

M. DEFINING THE ECU YIELD CURVE

M.1. Methodologies

A variety of methodological approaches to construct yield curves have been presented in the literature. Broadly speaking, most methods can be classified in two categories. One approach, illustrated in section M.2., is based on the redemption yields of the individual issues. Alternatively, the secondary market price of each of the same issues can be used to derive the Ecu discount function, which is then used as a basis to construct the yield curve (cf. section M.3.).

M.2. Yield curve based on redemption yields

This method is the most straightforward one and uses the yields to maturity observed in the secondary market. The yield curve itself is estimated via a traditional OLS regression after specification of the desired functional form; all data are processed in unweighted form i.e. the issue size is not considered. A polynomial function is both simple and flexible as illustrated in the equation below:

$$Y = a_0 + a_1M + a_2M^2 + a_3M^3 + b_1Q \qquad \text{(model A1)}$$

with: Y = yield
M = residual maturity
Q = quality of the issue
(equal to 0 if triple-A and equal to 1 otherwise)
a_i, b_i = coefficients

The last term of the equation takes account of the fact that some of the issues have a credit quality (as measured by the credit rating) which is just below the triple-A category; the b_1 coefficient therefore estimates the average spread between the two categories of bonds. This functional form however assumes a constant spread between both types of issues; a more convenient form, allowing for a maturity-dependent spread, is given below:

$$Y = a_0 + a_1M + a_2M^2 + a_3M^3 + b_1Q + b_2QM \qquad \text{(model A2)}$$

with: QM = product of Q and M

Figure 9 illustrates estimations with model A2, the lower curve representing the yield curve for default-free securities. Annex 13 provides monthly test results, including estimated reference rates for high-quality bonds (i.e. Q = 0), for the year 1992.

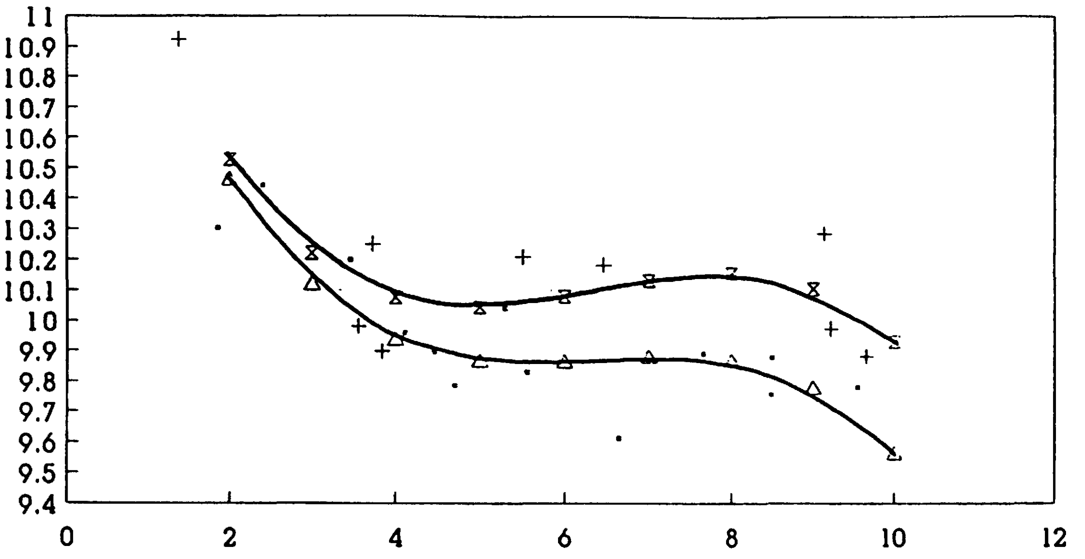


Figure 9

A third order polynomial sometimes proves insufficient to provide an adequate fit over the whole maturity range. One possibility consists in introducing polynomials of a higher order (4th order and beyond) in order to improve the flexibility and fitting properties of the regression. A more appropriate alternative consists in making use of spline regressions, whereby the maturity range is segmented in several sections and a separate (third order) polynomial fitted in each section (cf. annex 14 for a discussion of spline functions). The yield curve equation is then adjusted as follows:

$$Y = a_0 + a_1M + a_2M^2 + a_3M^3 + b_1Q + b_2QM + \sum_{i=1}^{n} X_i$$

with n = number of knotpoints
$X_i = (M - V_i)^3$ if $M > V_i$
$= 0$ for $M < V_i$
V_i = value of knotpoint i

Test results using the above model with a single knotpoint at 5 years are also included in annex 13 (model A3); the improvement of the regression results is however limited as the sample size is small and the maturity range limited to 10 years.

M.3. <u>Yield curve based on the discount function</u>

The approach presented under M.2 is both simple and straightforward. The method is however debatable from a theoretical point of view because of the questionable properties of redemption yields (see section D). It is for instance assumed in the yield formula that discount rates are constant over time i.e. identical for all maturities, which is basically in contradiction with the concept of a yield curve. Moreover, the actual discount rates are different for each security as they are calculated independently.

The estimation of the Ecu discount function (EDF) allows to relieve these two incongruities as spot rates are allowed to vary over the maturity range and as the same spot rates are consistently applied to all bonds. The EDF reflects the interest rate conditions in the market (spot rates and implied forward rates) and allows to estimate a market value for any security as long as both the periods and sizes of all future payments are known with complete certainty. In the absence of default risk, the DF is unique at any particular moment in time. For securities where the market perceives an underlying credit risk, the EDF will be below the default-free curve. As the set of representative issues includes two categories of bonds, the estimation procedure takes account of the quality factor, as was the case for the methods described in section M.2.

Figure 10 compares the EDFs on 28 May and 28 August 1992, estimated on the basis of the selected set of representative issues; only the EDFs for triple-A securities are represented.

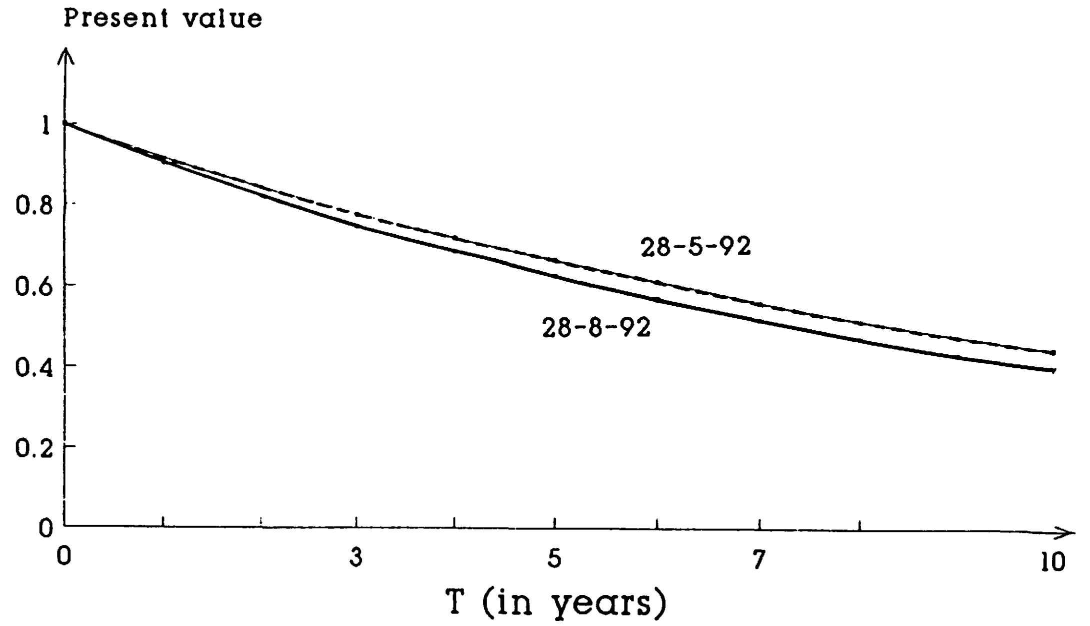


Figure 10

The estimation procedure of the discount function is somewhat complex as it cannot be observed as such in the market. Zero coupon (pure discount) bonds are the only ones providing a direct estimate of the function at a specific maturity since they produce a single future payment. The situation is less transparent as far as the traditional bullet bonds are concerned: the price of these securities represents the value of a series of future payments (coupons and principal) while the individual value or price of each individual payment remains unknown. The EDF estimation procedure via linear regression is further detailed in annex 15.

Once the EDF is estimated, it can be used to compute an estimated price for any fixed-income bond by adding up the present value of all its future cash flows. Inversely, the EDF can be used to calculate the estimated coupon of a bond with a particular maturity once its price is known. We could for instance consider a hypothetical 3-year fixed-income bond which is traded at par (price equal to 100) and use the discount function to derive its estimated coupon. The estimated coupon value not only represents the par yield for the 3-year maturity, but also the redemption yield of this hypothetical bond (property of par bonds cf. section D.2). This value could then be considered as the 3-year reference rate.

In order to obtain a par yield curve, the same computations have to be repeated (point by point) for the whole maturity range. Figure 11 displays the par yield curves for 28 May and 28 August 1992 computed on the basis of the EDFs represented in figure 10.

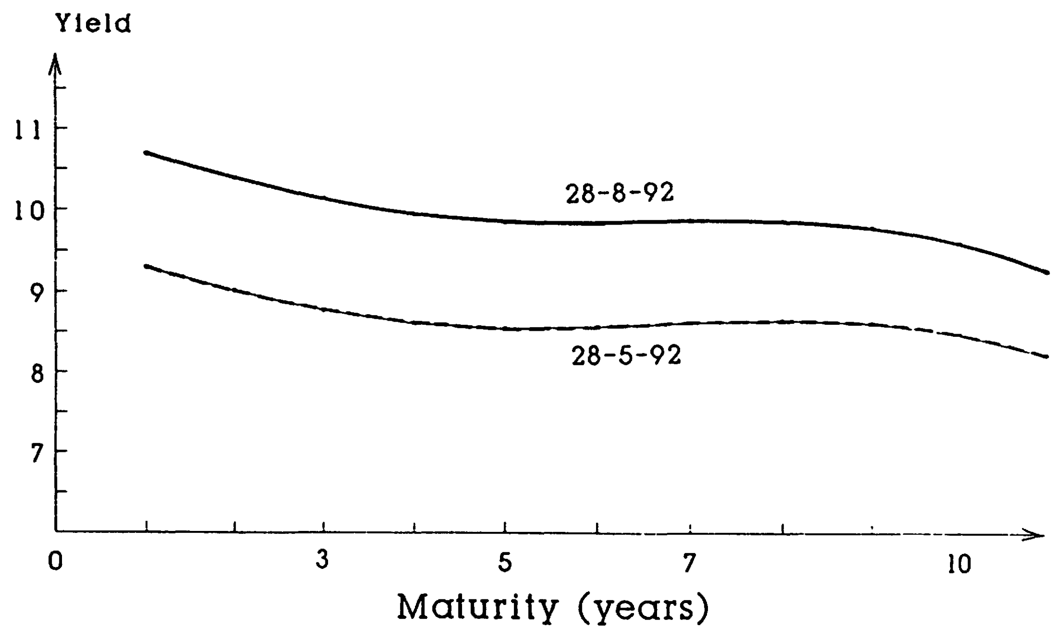


Figure 11

Monthly estimates of Ecu interest reference rates (EIRRs) for 1992, using the same sample as for the earlier models, are also provided in annex 13 (model B). R^2 values (stemming from the EDF estimates, as the par yield curve results from a mathematical transformation of this) are usually much higher, which is explained by the fact that price data show a much larger variation than yield data.

M.4. Comparison of results

A comparison of the results produced by the four models described so far shows striking similarities between the various EIRR estimates, which are generally within a very small range of a few basis points. More specifically, the coherence of the results produced via the estimated EDF with those calculated via the more conventional approach presented in section M.2., provides strong evidence of the overall reliability of the EIRRs produced by the various models. The model A3 deserves however to be favoured because of its inherent simplicity compared to the more cumbersome EDF approach. Moreover, financial markets are more familiar with approaches based on redemption yields.

Model A4 constitutes a further extension of model A3 in that bonds with maturities extending beyond 10 years are also taken into account in order to provide an "anchor" to the curve at the high end of the maturity range and thus to improve the 10 year EIRR estimates. The impact of these additional observations (e.g. OAT 2022, Italy 2011, Finland 2007) on the shorter maturities is restricted by the introduction of an additional knotpoint at 10 years. These observations are however not being used in order to produce EIRR values beyond 10 years, but are only destined to secure continuity and stability of the yield curve around the 10 year maturity.

Results for model A4 are also listed in annex 13; they are however not directly comparable with the results for the other models because of the higher number of observations.

M.5. Possible future improvements

The current model relies on a rather restrictive set of issues, resulting from the application of strict selection criteria listed above. In view of the limited sample size, scope for further refinement of model A4 is limited and even undesirable in order to avoid excessive complexity.

A number of improvements could however be considered if a larger number of Ecu benchmarks were available. Most importantly, the estimated yield curve could then be exclusively based on the highest quality issues (triple-A only).

A N N E X E S

ANNEX 1: Computing redemption yields RY (*)

a. The price (P) of a fixed coupon bond at m years of its final maturity will be equal to the sum of the present values of:

- its redemption price V (generally the principal or face value of the bond);

- the stream of coupon (C) payments (i.e. an annuity).

$$P = \frac{V}{(1 + y)^{m}} + \frac{C}{y} \left(1 - \frac{1}{(1 + y)^{m}}\right) \qquad \text{(eq. 1.1)}$$

with y = yield
P, V and C: expressed in % or as fractions.

b. Between two coupon payments, the above equation adjusts as follows (ISMA rule 803):

$$P + C\,(1 - f) = \frac{V}{(1 + y)^{n+f}} + \frac{C}{y\,(1 + y)^{f-1}} \left(1 - \frac{1}{(1 + y)^{n+1}}\right)$$

(eq. 1.2)

with P = "clean" price (excluding accrued interest);
n = number of full years before maturity;
f = fractional part of residual maturity.

c. The equation can only be solved by iteration (e.g. Newton-Raphson, binary search, ...).

d. The equation also applies for zero-coupons (C = 0).

e. Yields are not additive: the yield of a bond portfolio is not equal to the weighted average of the individual yields.

(*) equivalent to yield-to-maturity

ANNEX 2: Computing par yields PY

a. ri represents the spot rates (i.e. r1 in period 1, r2 in period 2, etc.) A bond maturing in m years and trading at par will bear a coupon (c) for which the following equation holds:

$$\frac{1}{(1 + r_m)^m} + c \cdot \sum_{i=1}^{m} \frac{1}{(1 + r_i)^i} = 1 \qquad \text{(eq. 2.1)}$$

(c and r are expressed as fractions)

b. As the coupon in the above equation will be equal to the par yield, the latter can be defined as:

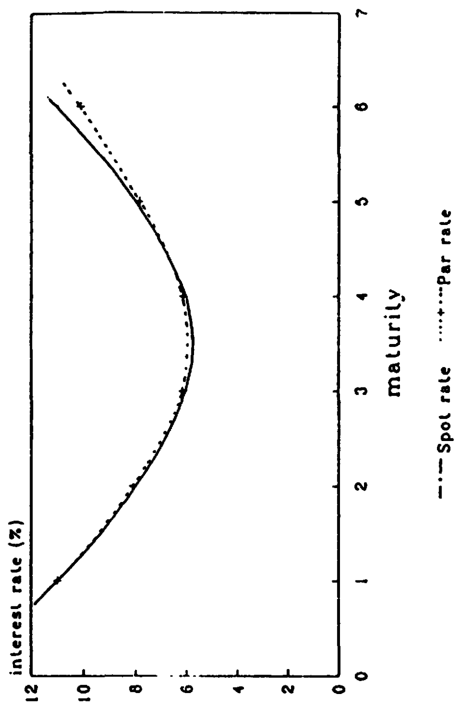
$$PY = \frac{1 - \dfrac{1}{(1 + r_m)^m}}{\displaystyle\sum_{i=1}^{m} \frac{1}{(1 + r_i)^i}} \qquad \text{(eq. 2.2)}$$

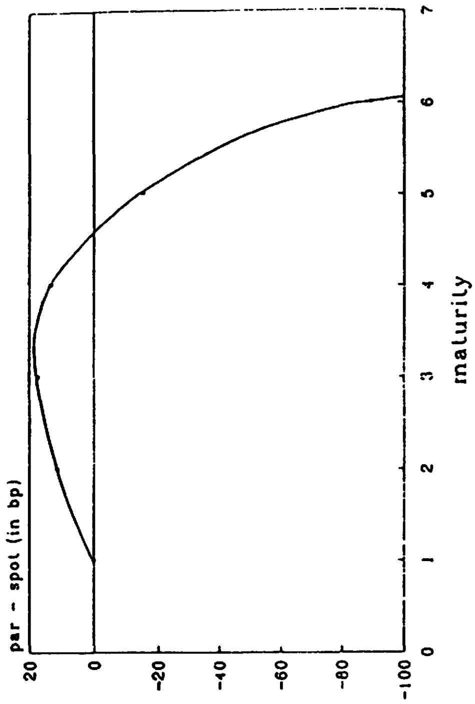
c. Inversely, we can use equation (2.1) to derive the discount rates for each period if we assume the yields are known, i.e.:

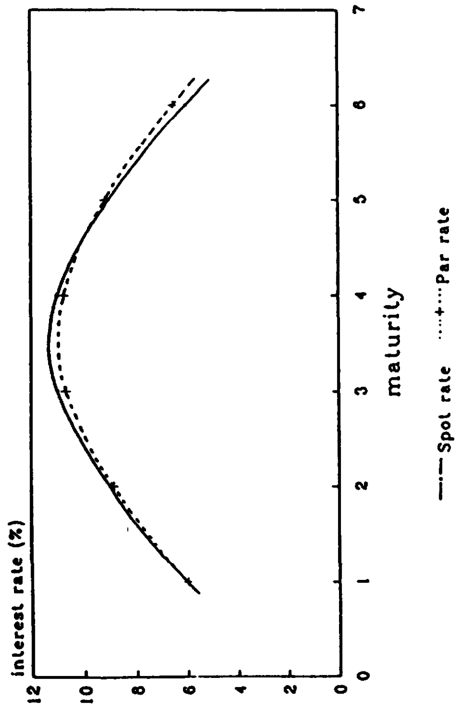
$$r_t = \left(\frac{c_t + 1}{1 - c_t \displaystyle\sum_{i=1}^{t-1} \frac{1}{(1 + r_i)^i}} \right)^{\frac{1}{n}} - 1 \qquad \text{(eq. 2.3)}$$

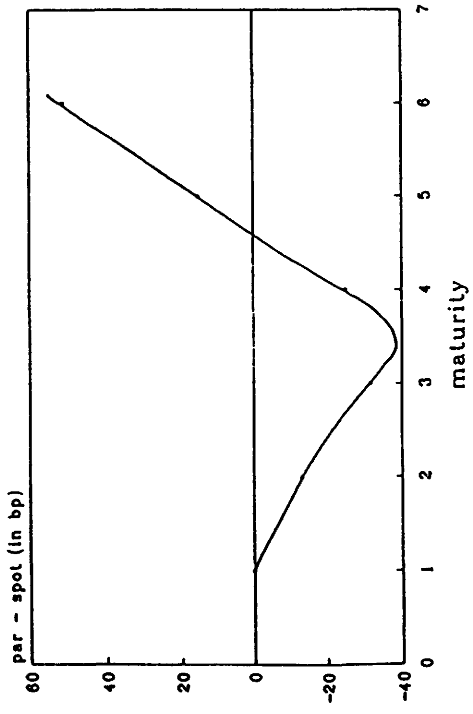
Solving this equation iteratively for t = 1, 2, ... we obtain the various discount rates.

ANNEX 3


interest rate (%)
maturity
Spot rate
Par rate


par - spot (in bp)
maturity


interest rate (%)
maturity
Spot rate
Par rate


par - spot (in bp)
maturity

ANNEX 4: Ecu strips

The first non-US dollar denominated Eurobond to be stripped was the Italian "2011" issue. Ecu 400 mn of Italy's issue were transformed in March 1991 by Goldman Sachs into 21 series of zero-coupon bonds, each of the first 20 tranches comprising the 9.25 % yearly coupon payments, the last tranche furthermore containing the reimbursement of the principal. The zero-coupon bonds are issued at their present discount value and guaranteed by the payments resulting from the underlying Italian issue.

As each of the tranches is being traded and priced separately, the resulting yields could be seen as a spot rate curve, albeit only applying to a single issuer and of limited practical value because of the small size of each individual issue

ANNEX 5: The Duration concept

a. Duration is mainly used as a measure for the volatility or sensitivity of the market value of a bond to changes in interest rates. Contrary to the maturity concept, duration takes all interim cash flows (and their payment dates) into account and is defined as (Macaulay duration):

D = the time-weighted average of the present value of all future cash flows (CF), divided by the price of the bond, i.e.:

$$= \frac{\sum_{t=1}^{m} \frac{t \times CF_t}{(1 + r)^t}}{\sum_{t=1}^{m} \frac{CF_t}{(1 + r)^t}} \qquad \text{(eq. 3.1)}$$

Note: in equation 3.1 all CFs are discounted at the same rate (a flat yield curve is assumed).

b. The main properties of duration are the following:

- for coupon bonds, duration will be smaller than maturity; for zero-coupons, duration is equal to maturity;

- for perpetuals, the duration equals:

  $D = (1 + r) / r$

- for the same coupon level, duration will generally increase with maturity;

- all other things equal, low-coupon bonds will have higher durations than high-coupon ones (because a larger proportion of the cash-flows is paid out at final maturity); similarly, discount bonds have higher durations than premium bonds;

- duration decreases when market yields increase;

- durations are additive: the duration of a portfolio is equal to the weighted average of the durations of the individual securities.

c. relationship between duration and price of a bond:

$$\frac{\Delta P}{P} = - \frac{D}{(1 + y)} \cdot (\Delta y) \qquad \text{(Eq. 3.2)}$$

with P = full price (including accrued interest);
D = duration;
y = yield.

The relative price change (price sensitivity or volatility) of a bond as a result of a yield change will be proportional to the duration (which is therefore related to interest rate elasticity). All bonds bearing characteristics that increase duration (e.g. long maturity, low coupon, ...: see b.) will therefore be more volatile.

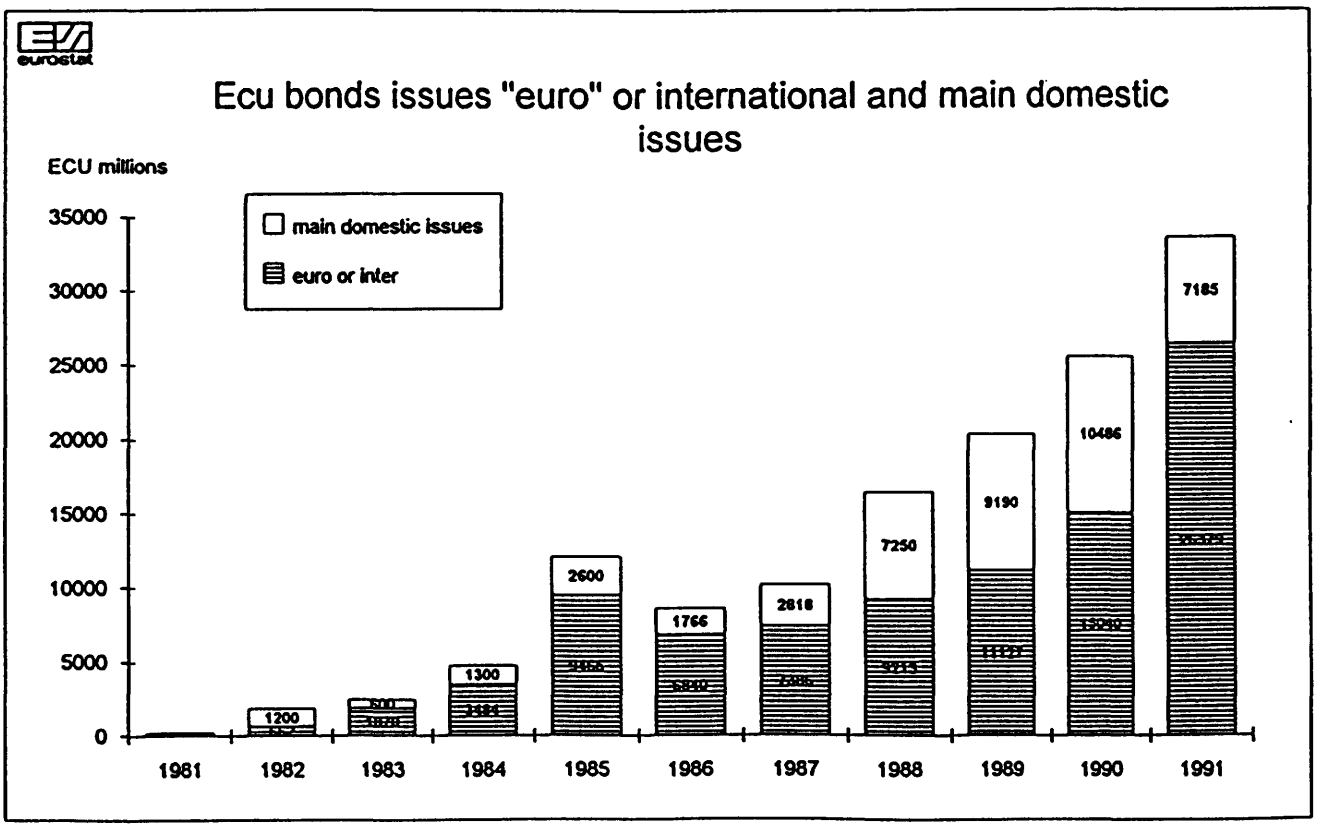
ANNEX 6a: Evolution of Ecu bond issue volumes (1981 - 1991)

"Euro" or international ecu bond issues and main domestic issues recorded at the date of payment, in millions of ecus.

Summary table:

	euro or internat.	main domestic issues					Total
		OAT France	CTE Italy	ELB Greece	3YN United-Kingdom	other	
1981	232	0	0	0	0	0	232
1982	662	0	1200	0	0	0	1862
1983	1870	0	600	0	0	0	2470
1984	3484	0	1300	0	0	0	4784
1985	9465	0	2500	0	0	100	12065
1986	6840	0	1600	166	0	0	8606
1987	7386	0	1500	218	0	100	9204
1988	9213	0	7250	0	0	0	16463
1989	11127	1652	6000	1538	0	0	20317
1990	15040	2357	5250	2379	0	500	25526
1991	26379	3134	2400	1651	0	0	33564

Source:Eurostat



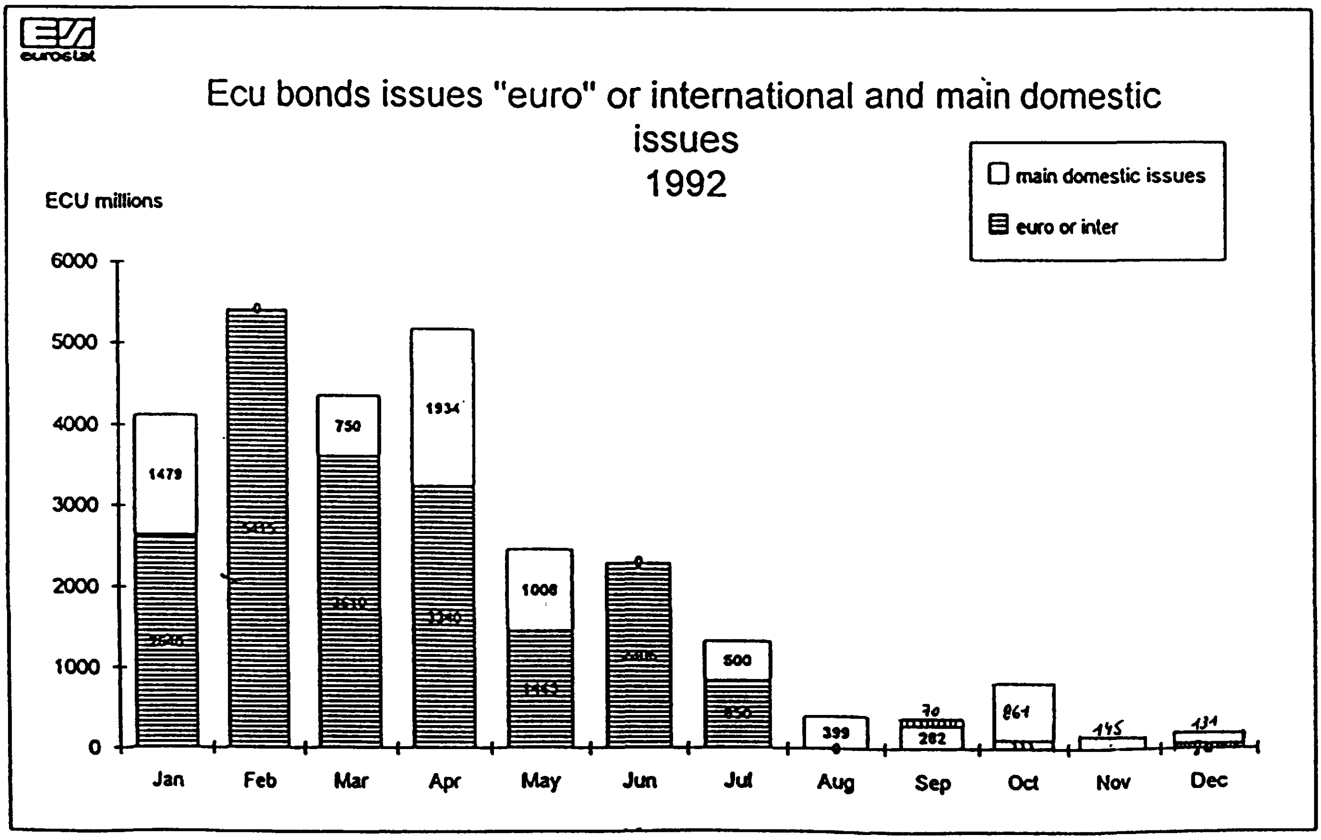
ANNEX 6b: Ecu bond issues in 1992

1992	euro or internat.	main domestic issues					Total
		OAT France	CTE Italy	ELB Greece	3YN United-Kingdom	other	
January	2640	0	0	479	1000	0	4119
February	5415	0 (1)	0	0	0	0	5415
March	3610	0 (2)	750	0	0	0	4360
April	3240	526	750	158	500	0	5174
May	1465	0	750	258	0	0	2473
June	2305	0	750	0	0	0	3055
July	850	0	0	0	500	0	1350
August	0	0	0	399	0	0	399
September	70	0	0	282	0	0	352
October	0	0	750	111	0	0	861
November	0	0	0	145	0	0	145
December	80	0	0	131	0	0	211
1st quarter	11665	0	750	479	1000	0	13894
2nd quarter	7010	526	2250	416	500	0	10702
3rd quarter	920	0	0	681	500	0	2101
4th quarter	80	0	750	387	0	0	1217
year	19675	526	3750	1963	2000	0	27914

source: Eurostat *last update: 10/02/1993*

(1) The ecu 1500 Mio OAT 8.25% payable on April 25, 2022 (issued in January with a payment date in February) was issued in the same way as Eurobonds, and is thus taken into account for February in the column "Euro" in this table.

(2) The ecu 125 Mio tranche of the OAT 8.5% payable on May 12, 1997 was issued in the same way as Eurobonds, and is thus taken into account for May in the column "Euro" in this table.

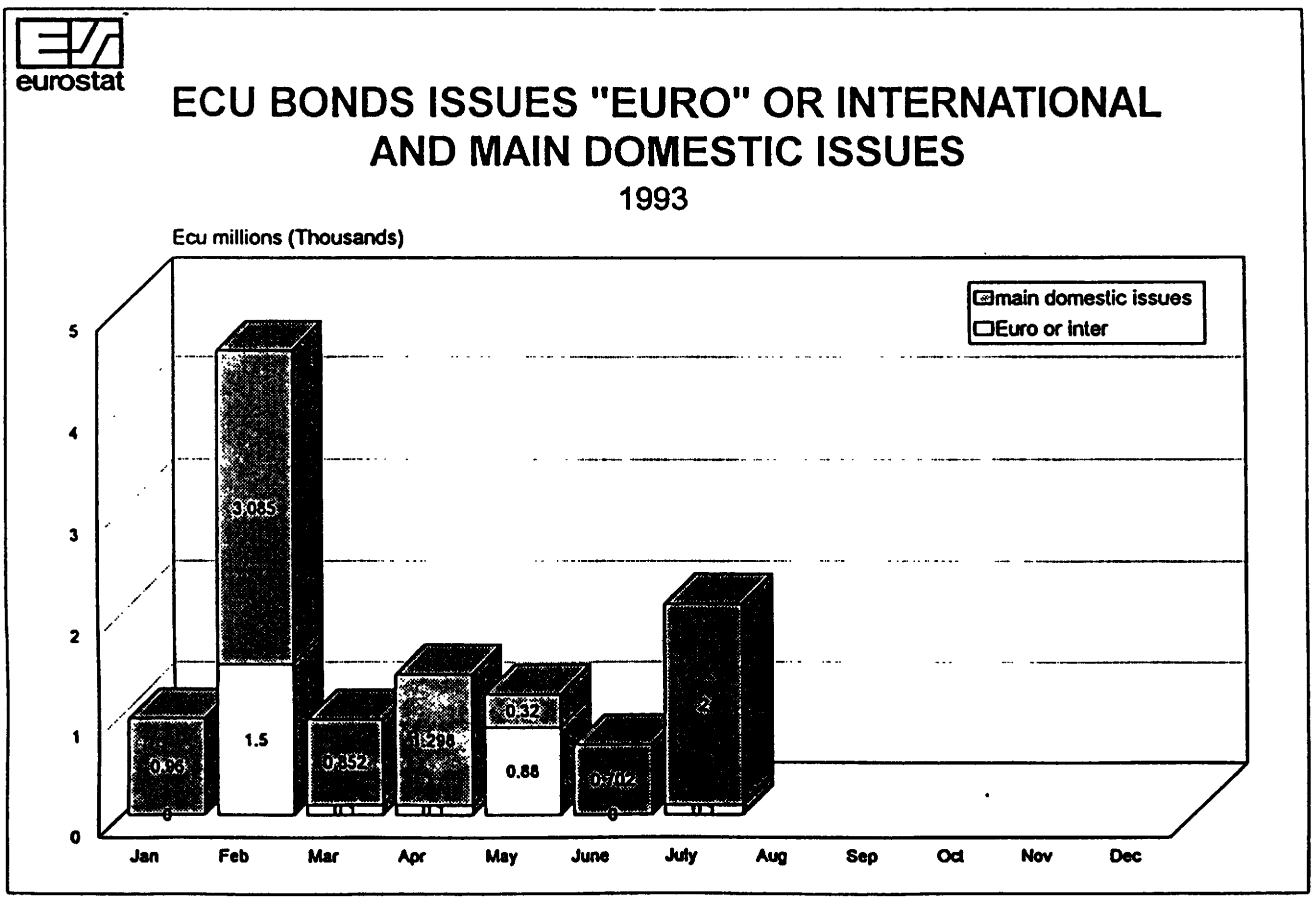


ANNEX 6c: Ecu bond issues in 1993

"Euro" or international ecu bond issues and main domestic issues recorded at the date of issue, in millions of ecus.

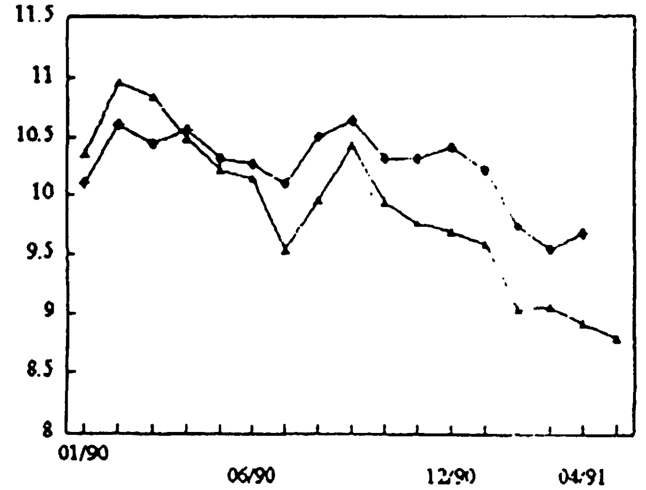
1993	euro or internat.	main domestic issues					Total
		OAT France	CTE Italy	ELB Greece	3YN United-Kingdom	other	
January	0	710	0	250	0	0	960
February	1500	0	500	85	500	2000 (1)	4585
March	100	0	750	102	0	0	952
April	100	0	750	48	500	0	1398
May	880	0	0	320	0	0	1200
June	0	0	600	102	0	0	702
July	100	500	1000	0	500	0	2100
August							
September							
October							
November							
December							
1st quarter	1600	710	1250	437	500	2000	6497
2nd quarter	980	0	1350	470	500	0	3300
3rd quarter							
4th quarter							
year							

(1) French BTAN

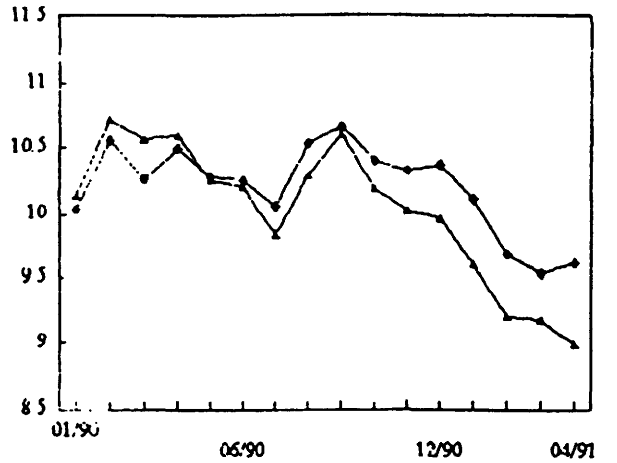


ANNEX 7 : comparison between Ecu theoretical (synthetic) bond yields and actual yields.

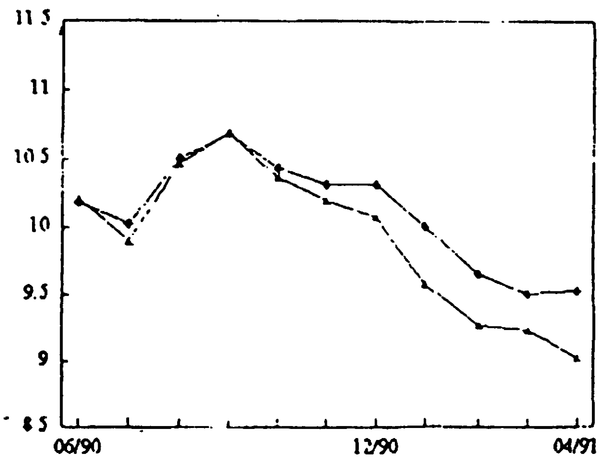
3-year Ecu bond yields



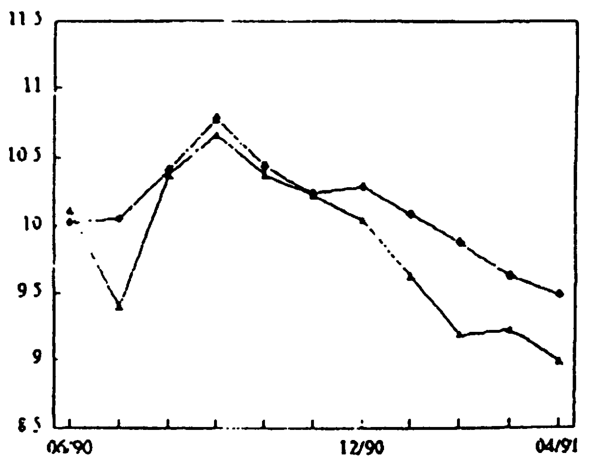
5-year Ecu bond yields



7-year Ecu bond yields



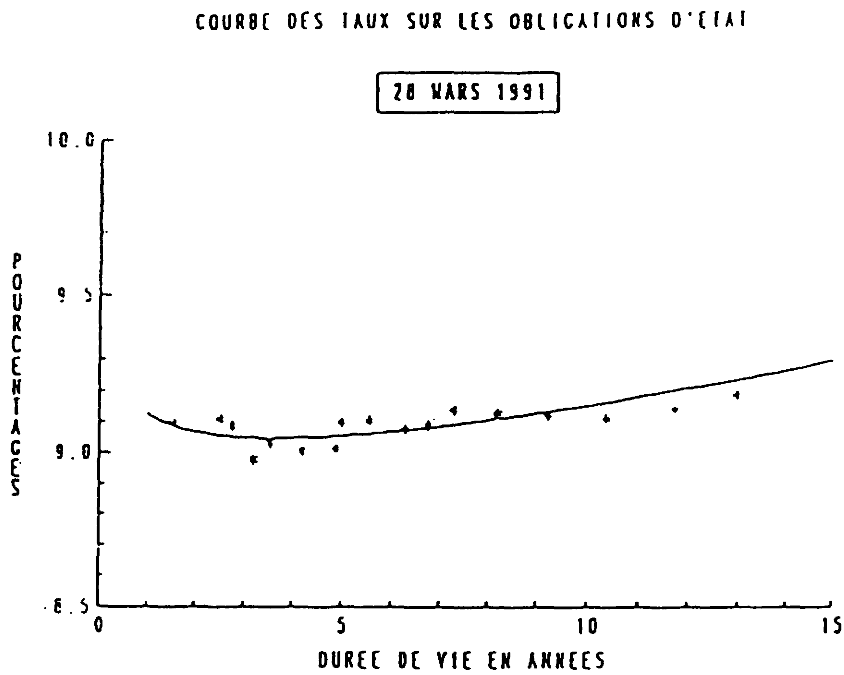
10-year Ecu bond yields



Synthetic — Actual

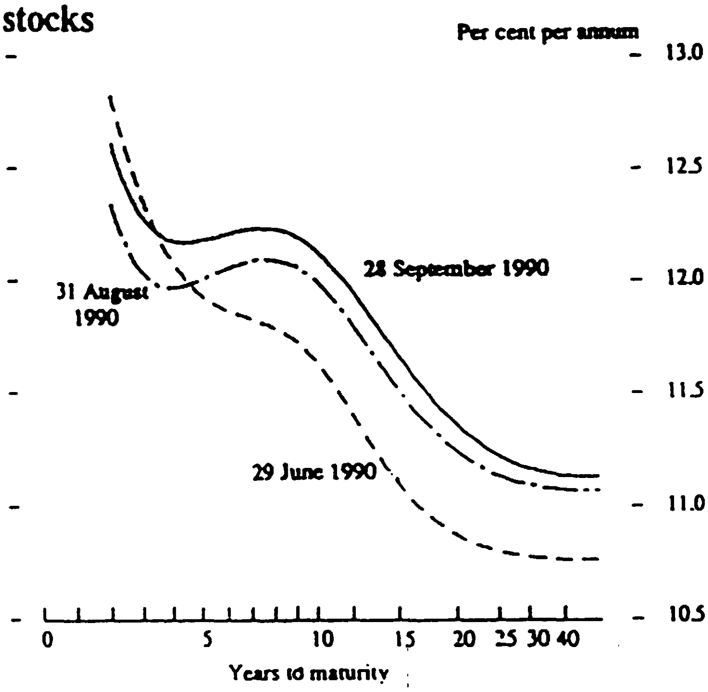
Source: EBA Newsletter, Special issue 6, June 1991.

ANNEX 8



Source: Banque de France, Bulletin mensuel

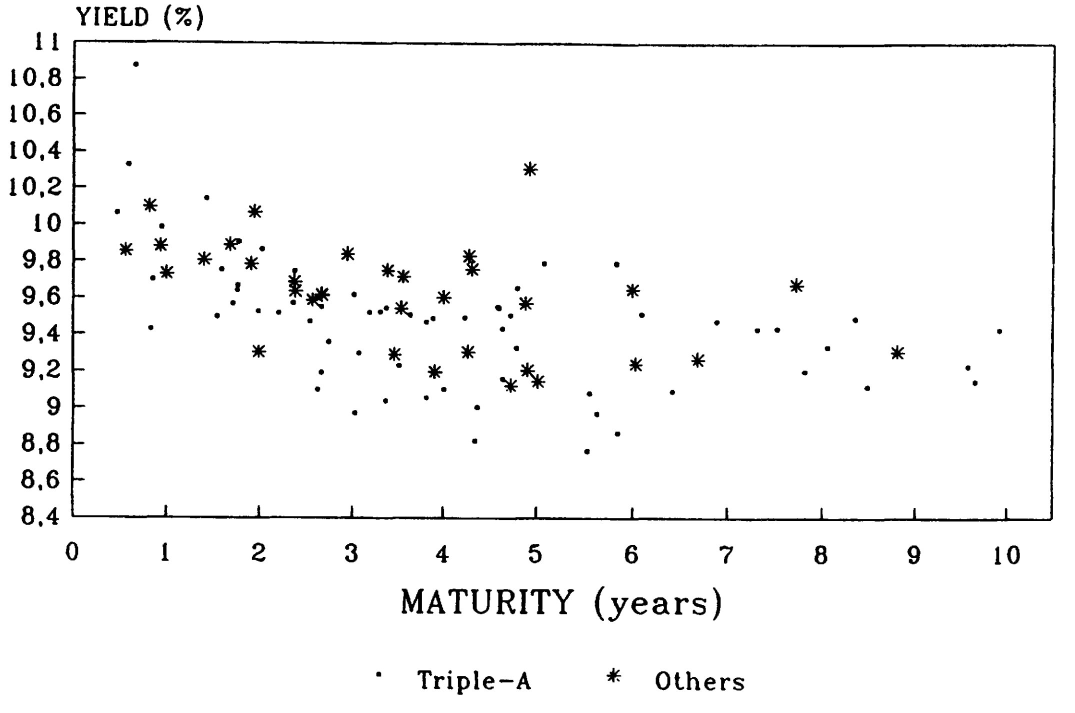
Time/yield curves of British government stocks



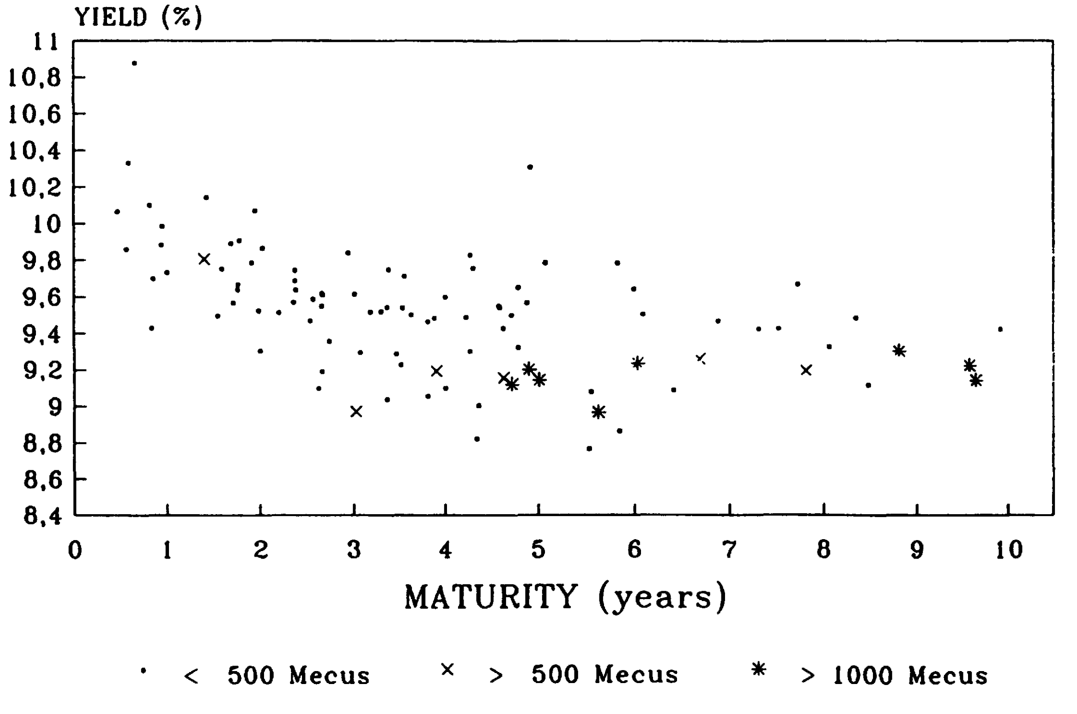
Source: Bank of England Quarterly Bulletin

ANNEX 9

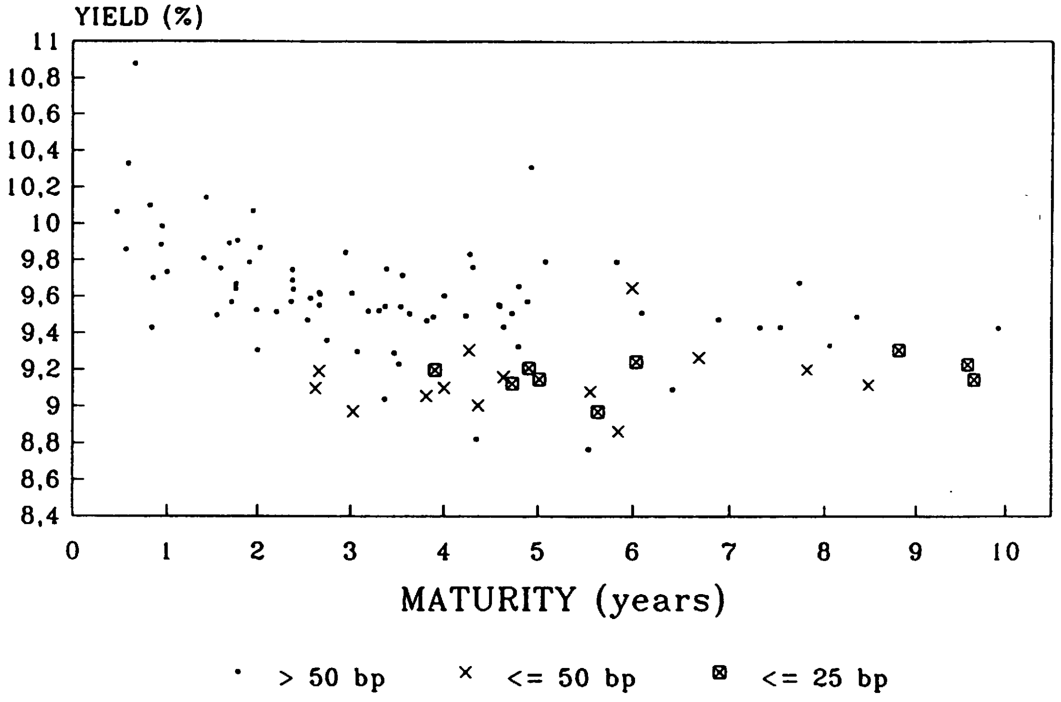
Annex 9A : CREDIT RATING



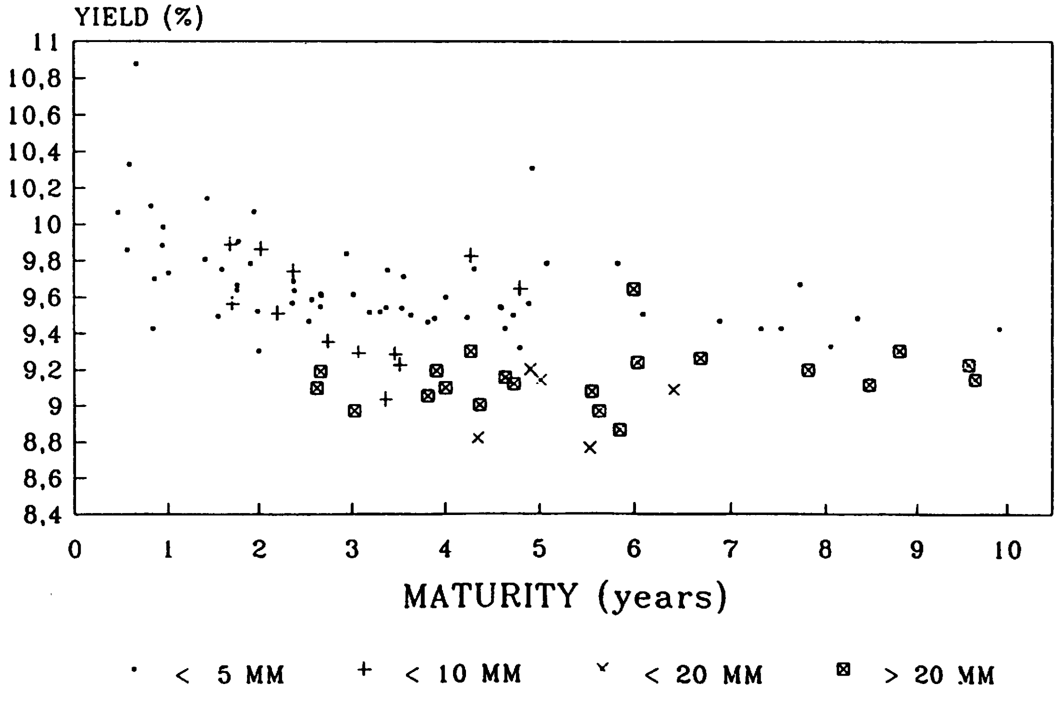
Annex 9B : ISSUE SIZE



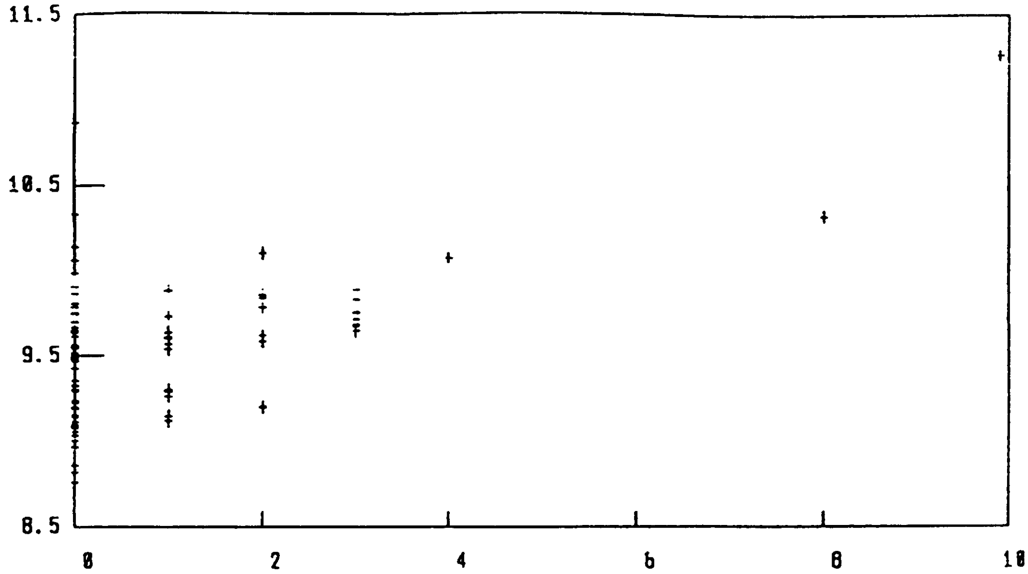
Annex 9C : PRICE SPREAD



Annex 9D : NUMBER OF MARKET MAKERS

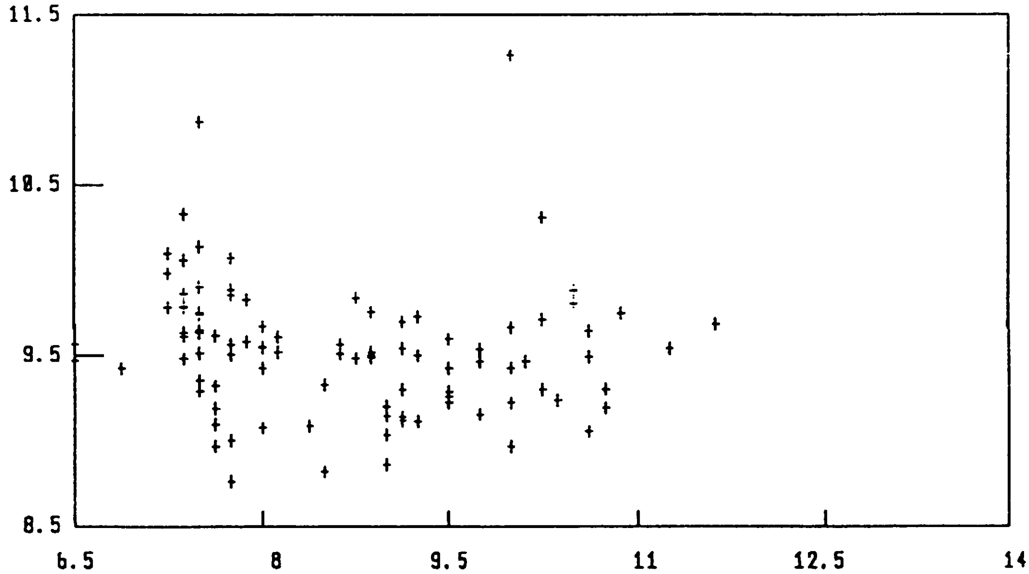


ANNEX 10



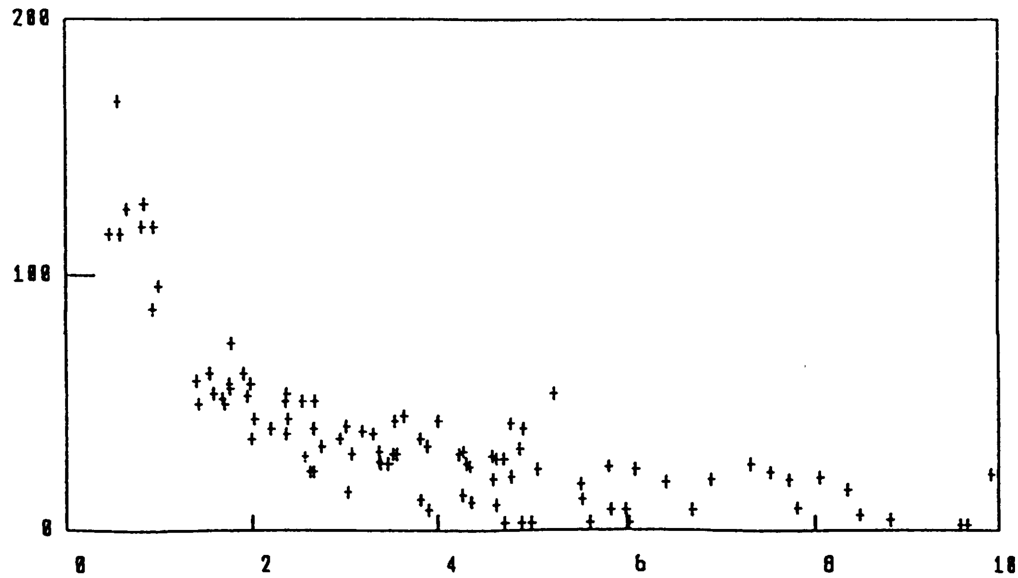
10 A

Y : yield
X : credit rating



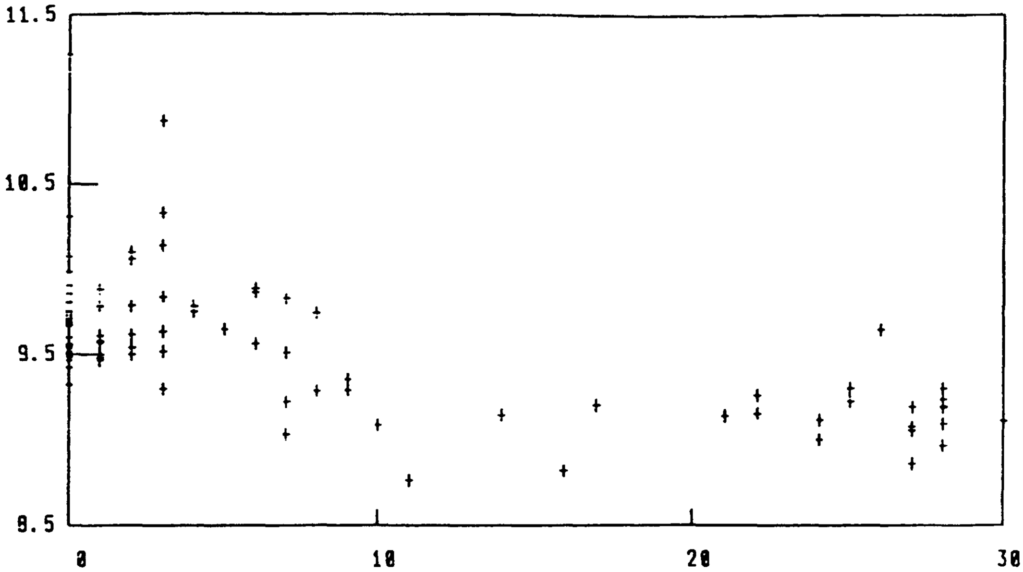
10 B

Y : yield
X : coupon



10 C

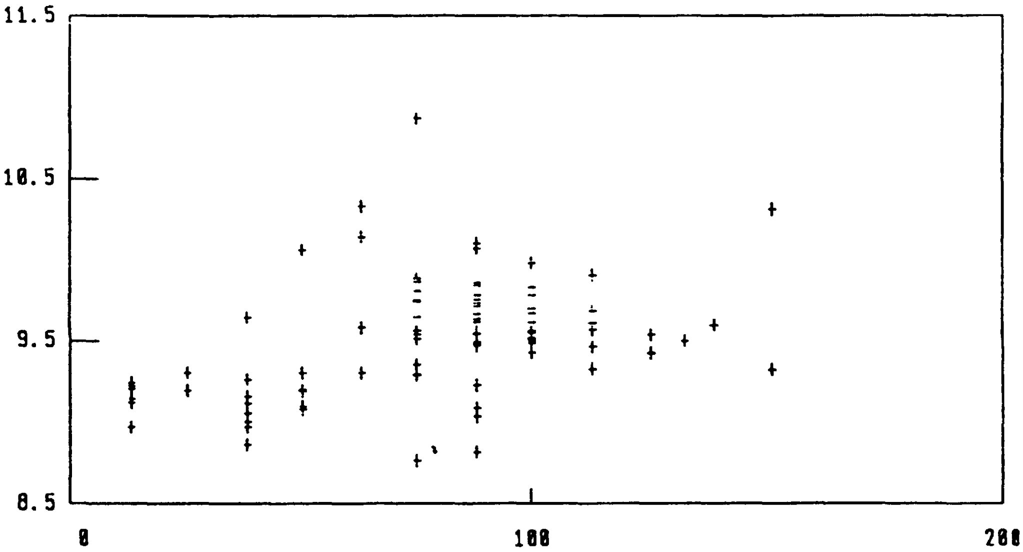
Y : yield spread
X : maturity



10 D

Y : yield

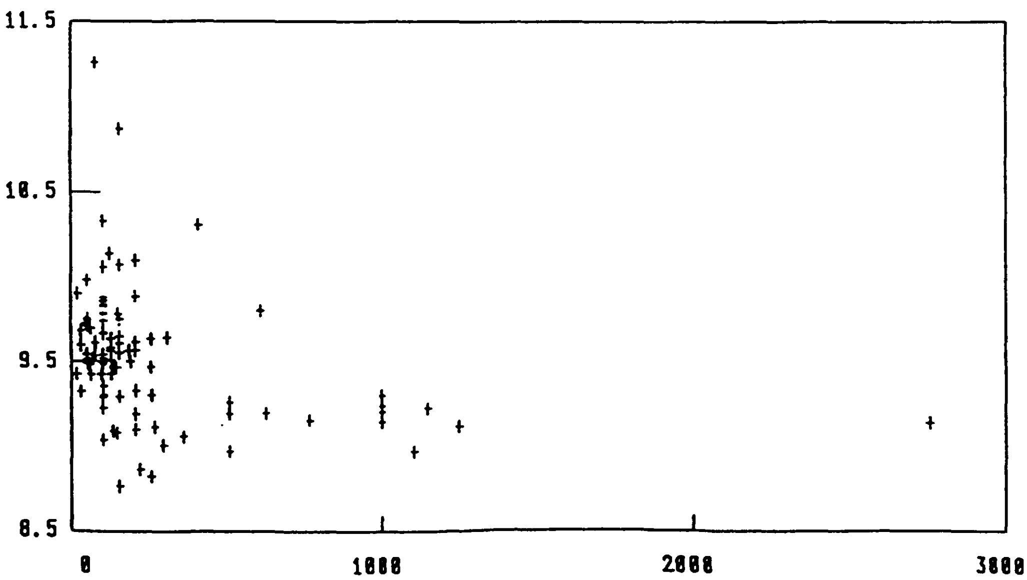
X : number of market makers



10 E

Y : yield

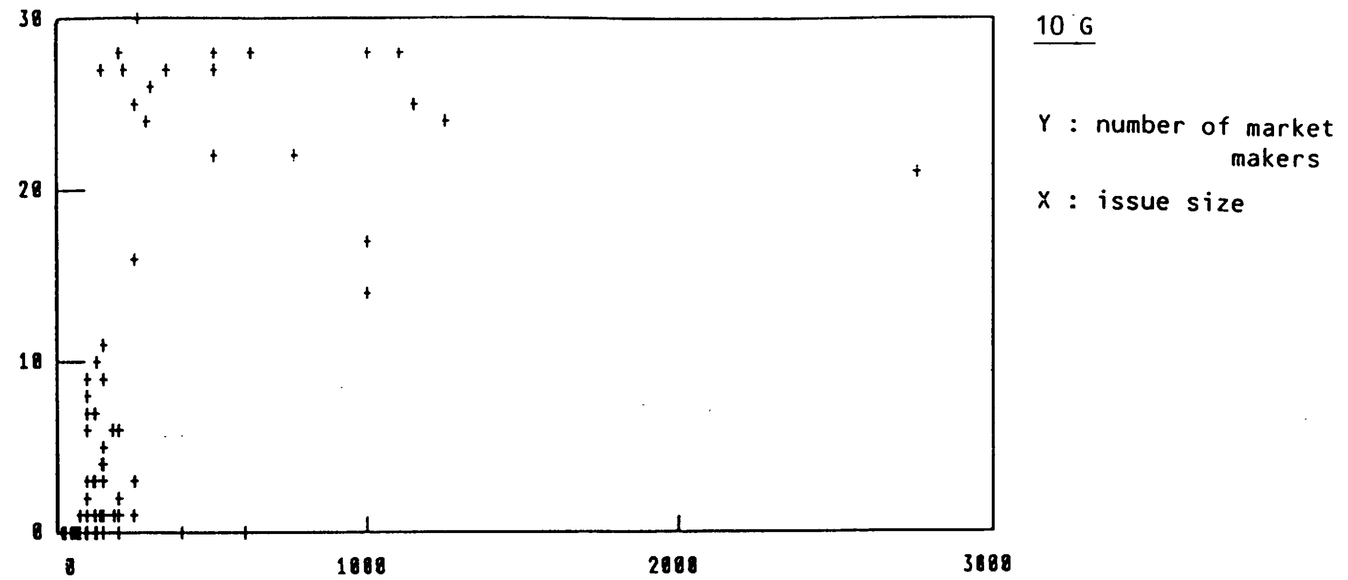
X : price spread

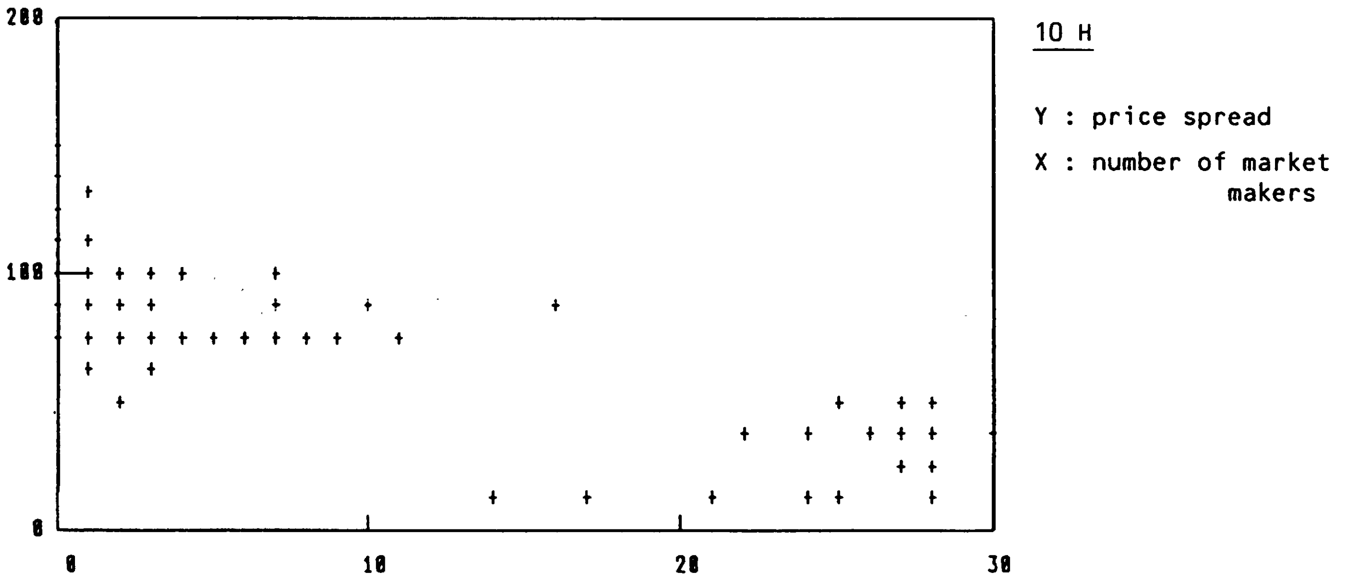


10 F

Y : yield

X : issue size


30
20
10
0
0
1000
2000
3000
10 G
Y : number of market makers
X : issue size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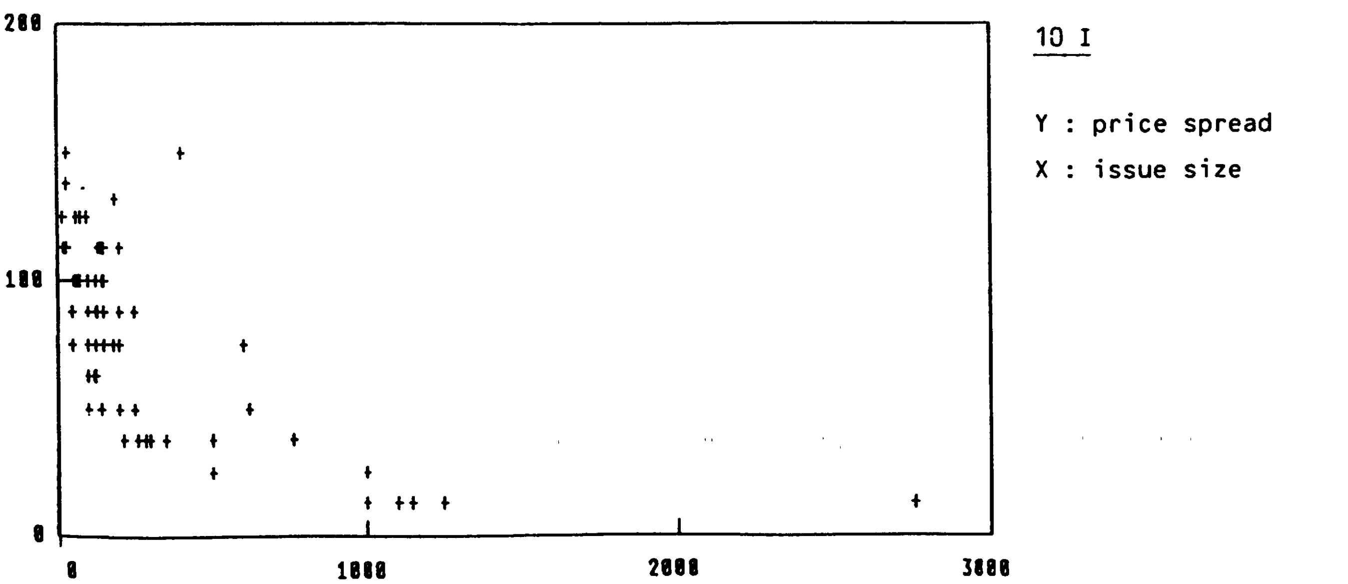
10
20
30
10 H
Y : price spread
X : number of market makers


200
100
0
0
1000
2000
3000
10 I
Y : price spread
X : issue size

ANNEX 11: Credit quality of Ecu issuers (sovereigns and supranationals); situation at 31.12.92.

ISSUER	S & P's rating	Moody's rating	Regression value (*)
AUSTRIA	AAA	Aaa	0
BELGIUM	-	Aa1	1
DENMARK	AA	Aa1	2
FINLAND	AAA	Aa1	1
GREECE	BBB	Baa1	8
HUNGARY	-	Ba1	10
IRELAND	AA-	Aa3	3
ITALY	-	Aa3	3
NEW-ZEALAND	AA-	Aa3	3
NORWAY	AAA	Aa1	1
PORTUGAL	-	A1	4
SPAIN	AA	Aa2	2
SWEDEN	AAA	Aa1	1
UNITED KINGDOM	AAA	Aaa	0
COUNCIL of EUROPE	AA+	Aa1	1
EURATOM	AAA	Aaa	0
ECSC	AAA	Aaa	0
EEC	AAA	Aaa	0
EIB	AAA	Aaa	0
INTER-AM. DEV. BK.	AAA	Aaa	0
IBRD	AAA	Aaa	0

(*) value based on the table below; highest value is taken in case of difference.

S & P's	Moody's	Value
AAA	Aaa	0
AA+	Aa1	1
AA	Aa2	2
AA-	Aa3	3
A+	A1	4
A	A2	5
A-	A3	6
BBB+	Baa1	7
BBB	Baa2	8
BBB-	Baa3	9
BB+	Ba1	10

Annex 12: List of selected representative issues (date: 31/12/92)

MATURITY	ISSUER	SIZE (Mecus)	COUPON (%)
24/01/95	U.K.	1.000	8.250
14/02/96	EEC	760	9.750
18/03/96	Belgium	1.250	9.125
22/05/96	Spain	1.000	9.000
01/07/96	Norway	1.000	9.000
10/10/96	EBRD	500	8.875
14/02/97	EIB	1.104	10.000
12/05/97	France	873	8.500
15/12/97	EEC	740	8.625
04/03/98	Finland	500	9.500
18/03/98	EEC	935	9.250
20/04/99	EIB	616	9.000
25/04/00	France	3.371	9.500
24/01/01	EIB	1.150	10.000
21/02/01	U.K.	2.750	9.125
26/02/01	France	1.500	10.000
17/10/01	Finland	500	8.750
14/11/01	Council of Europe	1.000	9.000
15/03/02	France	1.004	8.500
24/04/02	Denmark	1.000	8.500
-------------- beyond 10 years --------------			
13/02/07	Finland	750	8.500
25/04/22	France	1.500	8.250

ANNEX 13: Comparison of test results (period: JAN - DEC 1992)

REF. DATE (*)	MODEL	ESTIMATED REFERENCE RATES				R2	R2 Adj	Nr. of obs.
		3 Yr	5 Yr	7 Yr	10 Yr			
03-01-92	A1	8.92	8.77	8.70	8.63	0.82	0.76	18
	A2	8.94	8.79	8.69	8.61	0.87	0.82	18
	A3	8.96	8.79	8.69	8.60	0.88	0.82	18
	B	8.95	8.78	8.69	8.60	0.99	0.99	18
	A4	8.96	8.79	8.69	8.60	0.88	0.81	19
31-01-92	A1	8.57	8.44	8.39	8.46	0.51	0.38	20
	A2	8.59	8.46	8.39	8.44	0.59	0.44	20
	A3	8.63	8.45	8.37	8.40	0.69	0.54	20
	B	8.64	8.44	8.39	8.40	0.99	0.99	20
	A4	8.62	8.46	8.37	8.43	0.74	0.62	23
28-02-92	A1	8.58	8.41	8.43	8.43	0.57	0.46	21
	A2	8.60	8.43	8.42	8.41	0.66	0.55	21
	A3	8.62	8.43	8.41	8.39	0.67	0.53	21
	B	8.61	8.42	8.42	8.40	0.99	0.99	21
	A4	8.60	8.44	8.40	8.43	0.74	0.62	24
03-04-92	A1	8.89	8.73	8.78	8.69	0.49	0.37	21
	A2	8.91	8.75	8.77	8.67	0.53	0.38	21
	A3	8.97	8.74	8.76	8.61	0.59	0.41	21
	B	8.93	8.72	8.78	8.65	0.99	0.99	21
	A4	8.92	8.76	8.73	8.74	0.58	0.40	24
30-04-92	A1	8.92	8.76	8.81	8.75	0.50	0.37	21
	A2	8.96	8.79	8.80	8.72	0.64	0.52	21
	A3	9.00	8.78	8.79	8.68	0.67	0.52	21
	B	8.97	8.77	8.80	8.69	0.99	0.99	21
	A4	8.98	8.80	8.76	8.78	0.71	0.59	24
28-05-92	A1	8.73	8.55	8.62	8.55	0.73	0.68	25
	A2	8.77	8.56	8.62	8.47	0.77	0.70	25
	A3	8.77	8.56	8.62	8.47	0.77	0.69	25
	B	8.76	8.53	8.61	8.50	0.99	0.99	25
	A4	8.75	8.59	8.58	8.60	0.74	0.65	28
03-07-92	A1	9.10	8.92	9.03	8.86	0.79	0.75	25
	A2	9.13	8.92	9.03	8.79	0.81	0.76	25
	A3	9.13	8.92	9.03	8.79	0.81	0.75	25
	B	9.14	8.89	9.02	8.81	0.99	0.99	25
	A4	9.10	8.96	8.98	9.00	0.77	0.69	28
31-07-92	A1	9.73	9.59	9.74	9.44	0.62	0.54	25
	A2	9.78	9.61	9.73	9.30	0.66	0.57	25
	A3	9.74	9.64	9.73	9.36	0.67	0.56	25
	B	9.76	9.58	9.72	9.39	0.99	0.99	25
	A4	9.71	9.67	9.69	9.58	0.71	0.61	28

(*) Fridays closest to each end of month were chosen as reference dates.

REF. DATE (*)	MODEL	ESTIMATED REFERENCE RATES				R2	R2 Adj	Nr. of obs.
		3 Yr	5 Yr	7 Yr	10 Yr			
28-08-92	A1	10.10	9.86	9.87	9.70	0.75	0.69	23
	A2	10.12	9.87	9.87	9.58	0.77	0.70	23
	A3	10.11	9.88	9.87	9.61	0.77	0.68	23
	B	10.13	9.85	9.86	9.61	0.99	0.99	23
	A4	10.09	9.92	9.84	9.79	0.81	0.73	25
02-10-92	A1	9.75	9.53	9.49	9.37	0.61	0.50	20
	A2	9.80	9.57	9.50	9.17	0.68	0.56	20
	A3	9.81	9.56	9.50	9.14	0.68	0.53	20
	B	9.75	9.54	9.50	9.22	0.99	0.99	20
	A4	9.77	9.60	9.47	9.38	0.81	0.72	22
30-10-92	A1	8.56	8.62	8.84	8.60	0.64	0.55	20
	A2	8.58	8.64	8.84	8.51	0.66	0.54	20
	A3	8.57	8.64	8.84	8.54	0.66	0.50	20
	B	8.52	8.61	8.83	8.62	0.99	0.99	20
	A4	8.53	8.69	8.80	8.87	0.85	0.78	22
27-11-92	A1	8.57	8.64	8.82	8.64	0.66	0.57	20
	A2	8.60	8.66	8.82	8.52	0.69	0.58	20
	A3	8.58	8.67	8.82	8.56	0.70	0.55	20
	B	8.54	8.64	8.81	8.62	0.99	0.99	20
	A4	8.55	8.70	8.79	8.85	0.90	0.84	22
31-12-92	A1	8.56	8.66	8.85	8.63	0.68	0.60	20
	A2	8.59	8.69	8.85	8.50	0.71	0.61	20
	A3	8.56	8.70	8.85	8.60	0.72	0.59	20
	B	8.53	8.67	8.84	8.65	0.99	0.99	20
	A4	8.54	8.73	8.82	8.86	0.90	0.84	22

ANNEX 14: The use of spline functions

A single polynomial does often not allow for sufficient precision, particularly if the maturity range is wide. Instead of increasing the order of the polynomial, it is often preferable to segment the maturity range in a number of intervals (e.g. from 0 to 2 years, from 2 to 5 years, etc.) and to fit a separate polynomial in each segment. A number of constraints have to be brought in so as to ensure continuity (i.e. adjacent polynomials should have identical values in the knot points) as well as a smooth transition in the knot points (first and second derivatives should be equal). These constraints also ensure smoothness in both the term structure of interest rates and in the forward rates.

All parameters are estimated in a single regression; each additional interval used to fit the function involves an additional variable in the regression equation and the loss of a degree of freedom in the residual.

The general functional form of a succession of cubic polynomials, satisfying the constraints indicated above, can be written as:

$$F(t) = a_1 + a_2t + a_3t^2 + a_4t^3 + \sum_{i=1}^{k} b_{i+1}\,(t - t_i)^3 D_i(t)$$

with k = number of knot points
t_i = value of knot point i
$D_i(t)$ = dummy variables equal to:
  0 for $t < t_i$
  1 for $t >= t_i$
a_i, b_i = parameters

ANNEX 15: Estimation of the Ecu discount function (EDF)

The estimation procedure is based on the principle that the value (i.e. the price of a bond in the secondary market) is equal to the present value of all future payments. For a bond with a full number of years to maturity, this can be represented as follows:

$$P = \frac{V}{(1 + r_m)^m} + \sum_{t=1}^{m} \frac{C}{(1 + r_t)^t} = V.D(m) + \sum_{t=1}^{m} C.D(t)$$

with P = price
V = principal
C = coupon
r_t = spot rate for period t
m = maturity of the bond
D(t) = value of the DF for period t

The desired functional form for the DF (e.g. a polynomial) is first substituted in the price equation, which is then used to estimate tha value of the parameters via linera regression. The high degree of precision which is required for the estimation of the DF makes the use of spline functions highly recommendable.

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