PROGRAMME OF RESEARCH AND ACTIONS ON THE DEVELOPMENT OF THE LABOUR MARKET

ANALYSIS OF THE RESULTS OF THE SAMPLE SURVEYS ON THE LABOUR FORCES


By:
Jacques A. ZIGHERA
Annick GUILLOUX
Raymond HARA
Centre d'Analyse Statistique des Structures et des Flux
Université de Paris X Nanterre

For:
The Commission of the European Communities

Study number 79/17
November 1979
PROGRAMME OF RESEARCH AND ACTIONS ON THE DEVELOPMENT OF THE LABOUR MARKET

ANALYSIS OF THE RESULTS OF THE SAMPLE SURVEYS ON THE LABOUR FORCES


By:
Jacques A. ZIGHERA
Annick GUILLOUX
Raymond HARA
Centre d'Analyse Statistique des Structures et des Flux
Université de Paris X Nanterre

For:
The Commission of the European Communities

Study number 79/17
November 1979
SUMMARY

On the basis of the results of the 1973, 1975 and 1977 Labour force sample surveys, the rate of employment is studied, generation by generation, in the nine member countries. From these data, concerning males and females separately, a comparison is drawn between national rates of employment at various ages; a study is also made country by country, of the effects, on the various generations, of difficulties in finding a job, of increasing female entry or reentry, of earlier retirement age.

RESUMÉ

A partir des résultats des Enquêtes par sondage sur les forces de travail de 1973, 1975 et 1977, on examine les taux d'emploi des diverses générations dans les neuf pays membres. Ceci permet, tant pour les hommes que pour les femmes, d'une part de comparer les taux d'emploi nationaux aux divers âges, d'autre part d'analyser comment au cours de la période les différentes générations ont été plus ou moins, suivant les pays, affectées par les difficultés à trouver un emploi, l'entrée ou la reentrée croissante des femmes sur le marché du travail, l'avancement de l'âge de la retraite.

This study was financed by the Commission of the European Communities as part of its « Programme of Research and Actions on the Development of the Labour Market ». The submitted analysis and proposals concerning the Labour market and other departments do not bind the Commission.

The report is available only for information. Neither quoting, nor allusion to this study in published works or in the newspapers are allowed without the authorization of the Commission. Requests concerning this study must be addressed to the Direction Générale de l'Emploi et des Affaires Sociales (à l'attention de la division V/B/3), Commission des Communautés Européennes, 200, rue de Loi, B-1049 Bruxelles.
INTRODUCTION

0.1. This report presents, on the basis of the results of the Labour force sample surveys of 1973, 1975, and 1977, the evolution of the employment rates of the different generations, for both men and women, in the several countries of the Community.

0.2. The surveys of 1973, 1975, and 1977 have been selected, since previous surveys are not strictly comparable; in addition, with the exception of Ireland and Denmark which did not participate in the 1973 survey, all the member countries are covered by these three surveys. The data which, according to our methods, must be itemized by year of age, have been furnished by the Statistical Office of the European Communities.

0.3. It will be immediately evident that this study concerns the employment rate and not the activity rate. The activity rate includes the unemployed; but, on the one hand, there remain institutional and economic differences which make it difficult to compare the various countries, and, on the other hand, the variations in unemployment and employment are related in a complex manner. The idea that is at the basis of the notion of activity rate, i.e., that there are persons available for activity, does not seem to correspond to the present situation, since there are, on the one hand, physically available people who, for example, give up seeking a job after many failures, and, on the other hand, people who are not available but who pretend to be so in order to obtain unemployment benefits. We shall therefore study here the percentage of persons having a job (occasional activity being excluded).

0.4. If one examines in graphic form the employment rate by age, recorded at the time of a given survey, one should not take the curve for a representation of the evolution of the employment rate according to age. Thus, with reference to the 1973 curve
of female employment in France, on page 14, one should not deduce that women, once they have reached the age of 24, retire progressively from the labour market until the age of 34. This sort of reasoning is absurd, because these are not the same women in both cases; this reasoning has long been responsible for the refusal of certain forms of aid, vocational training for example, to women under 35, on the pretext that this aid would be wasted, since it would be given in part to women who were going to quit the labour market.

0.5. If one really wants to follow this evolution, one must observe the evolution of the same group of persons, that is, for example women (or men) who were 23 years old in 1973, 25 in 1975, and 27 in 1977, therefore the generation born in 1950; this is what appears in the second series of graphs, which are drawn up by generation.

0.6. But the data on which we are working are the result of a sample survey and therefore show some irregularity, and their presentation in rough form would lead to graphs that would be illegible and difficult to interpret. We have therefore made the hypothesis that the 1973-1975-1977 evolution of a generation was regularly deformed when one proceeded to the following generations; we have therefore smoothed the curves by means of double orthogonal polynomials, some corresponding to deformation according to the year of birth, and others to the 1973-1975-1977 evolution. In order to make the graphs more regular and therefore more legible, we have intrapolated between the years by the same method. Accessorily, this method permits, in the case of the countries where one asks the age of the persons studied rather than the year of birth, to eliminate « waves » due to the rounding off of the answers.

0.7. In the following pages we shall therefore present, with a commentary, the representative graphs thus obtained, first for women, and then for men. In each of the two corresponding parts, we shall compare first the situations of the different countries by regrouping on the same graph the 1975 data smoothed in the way indicated above. Next, we shall study the evolution of each country in turn by presenting both the smoothed survey curves for 1973, 1975, and 1977 (even pages) and the smoothed evolutions of each generation (odd pages).
0.8. The following color codes are used in the graphs:

- when all countries are shown on the same graph:

<table>
<thead>
<tr>
<th>color</th>
<th>country</th>
</tr>
</thead>
<tbody>
<tr>
<td>blue</td>
<td>France</td>
</tr>
<tr>
<td>yellow</td>
<td>F.R. Germany</td>
</tr>
<tr>
<td>brown</td>
<td>Belgium, Luxemburg</td>
</tr>
<tr>
<td>red</td>
<td>Netherlands, Denmark</td>
</tr>
<tr>
<td>green</td>
<td>Italy, Ireland</td>
</tr>
<tr>
<td>violet</td>
<td>United Kingdom</td>
</tr>
</tbody>
</table>

- on the survey graphs for each country shown separately:

<table>
<thead>
<tr>
<th>color</th>
<th>year</th>
</tr>
</thead>
<tbody>
<tr>
<td>green</td>
<td>1973</td>
</tr>
<tr>
<td>brown</td>
<td>1975</td>
</tr>
<tr>
<td>violet</td>
<td>1977</td>
</tr>
</tbody>
</table>

- on the graphs that show the evolution of the generations for each country separately:

<table>
<thead>
<tr>
<th>color</th>
<th>generations born in 1900, 1905, 1910, etc...</th>
</tr>
</thead>
<tbody>
<tr>
<td>brown</td>
<td>generations born in 1901, 1906, 1911, etc...</td>
</tr>
<tr>
<td>blue</td>
<td>generations born in 1902, 1907, 1912, etc...</td>
</tr>
<tr>
<td>green</td>
<td>generations born in 1903, 1908, 1913, etc...</td>
</tr>
<tr>
<td>red</td>
<td>generations born in 1904, 1909, 1914, etc...</td>
</tr>
<tr>
<td>yellow</td>
<td>generations born in 1905, 1910, 1915, etc...</td>
</tr>
</tbody>
</table>

0.9. In reading all the following comments one should keep these points in mind:

- all the comments are founded on smoothed data; this means that when one speaks of generations born in 1930 it means the average of generations born between 1925 and 1935 or 1928 and 1932, and when one speaks of 20 year old people it means people from 19 to 21 or 18 to 22 years of age.

- all the comments concern the employment rate for a given generation; according to the country the relative weight of the various generations may be very different, and the overall consequences may therefore differ greatly according to the demographic structures of the various countries.
1. FEMALE EMPLOYMENT RATE

1.1. On the graph on the following page, the curves obtained for 1975 are arrived at by double smoothing, according to the year of birth and the year of the survey; this permits a significant comparison to be made among countries. This comparison will be made for each of the main age groups in turn.

1.2. If one first examines the profiles in their ascending parts, they fall into three groups:

- Luxemburg, F.R. Germany, the United Kingdom, Ireland, and the Netherlands: in these five countries the maximum employment rate occurs around the age of 21 and reaches 73% to 67% in that order for the first four countries and 63% for the Netherlands;

- France, Denmark, and Belgium: these countries differ from the preceding in that their maximum employment rate occurs at a later age, around 23 for the first two, with a rate of 66-67%, and 24 for the latter, with a rate of 63%;

- Italy: this country is a special case, since other studies show that regional differences are more marked here than elsewhere and, for this age group as for the others, the figures given here are an average of several very different regional situations; the maximum correspond to 22-23 years and is only 40%.

1.3. If one now examines the later age group up to 50, the groupings change greatly:

- Denmark, France, F.R. Germany, and the United Kingdom: in these four countries the average rates are relatively high, with the following differences: Denmark, France, and Germany have relatively flat profiles situated respectively around 60%, 55%, and a little under 50%; of course, being given the different values for the maximum rates attained, it is implied that on the curves there will be decreases of varying sizes.
between 20 and 30, slight in Denmark, average in France, and large in F. R. Germany; finally the United Kingdom remains the only country to keep a sizeable trough around 25-30 years, corresponding to women's quitting their work to devote themselves to their families; inversely, above 35-40 the employment rate is very high but it is well known that it is easier to declare part-time work in the United Kingdom because of differences in social contributions; it is to be wished that, when the necessary data are available, this analysis will be carried further by separating full-time and part-time work;

- Belgium: this country has a very particular profile, a nearly regular decline between 25 and 50 years of age; a closer analysis will show that this corresponds to a thorough evolution that is taking place;

- Luxemburg, Ireland, and the Netherlands: these countries from the age of 30 are at a low level, 20% for the two latter, and 30% for the former.

1.4. If one now looks at what is happening after the age of 50, one must be cautious in one's comments, for the populations concerned are increasingly small and the samples are therefore unreliable.

One notices however that in relation to the level previously reached, decreases are more or less sudden:

- in the United Kingdom, F. R. Germany, and Belgium, the decrease is already considerable at 60;
- in Denmark, France, and Italy, the decrease with age is less rapid;
- in the Netherlands and especially in Luxemburg and Ireland the decline is even slower.

It is possible— but here too, more thorough studies are needed— that these differences of profile for advanced ages are linked to the size of a residual aged agricultural population.

1.5. We shall return to this later, but the first scrutiny of these curves leads us to think that if one hypothesizes a future upward harmonization of employment rates, the changes will be of little amplitude in France, in F.R. Germany, and in the United Kingdom, and that in these countries a continuation of the increase in employment rates for the middle-aged groups will be in part compensated for by the prolongation of
education and the lowering of the retirement age. On the other hand, in Ireland, the Netherlands, Italy, and Luxemburg, one can expect in the coming years a great increase in the employment rate between 30 and 50 years of age, which will correspond to (and therefore be multiplied by) the arrival of large age groups, since these countries are all more dynamic demographically. Belgium occupies an intermediate position.

1.6. In the following paragraphs, the country-by-country evolution will be shown, on the one hand by comparing the profiles of the three surveys (two for Denmark and Ireland), and on the other hand by examining in more detail the changes in the curves representing the various generations. The curves that are commented upon are grouped together starting with page 14, the survey curves being on the lefthand pages and the generation curves on the righthand pages. Before beginning the commentary, one should stress that what is being commented upon is in fact the difference between entries into and exits from the labour market; for example, a stagnation of the employment rate does not mean that there are no entries but that the entries balance the exits. Each time therefore that one speaks of entry, net entry is meant.

1.7. In the Federal Republic of Germany, if one first examines the three survey curves, one notes that 1973 and 1975 seem very little different, but 1977 much more so; one sees first a definite decline in the maximum rate attained, which was moreover the highest in the Community and which is about 66%; on the other hand, around the age of 30-35 the activity rate is 8 or 9% higher than in 1973. If one now examines the generation curves beginning with the youngest generations one notes an increasingly tardy entry into employment and, it seems, a rather rapid decline in the maximum rate attained; however one should not exaggerate the importance of a phenomenon that seems to appear clearly for only two or three generations. The next important phenomenon is the entry or rapid re-entry into the labour market of women who were about 25-30 years of age in 1977; this evolution shows no sign of slowing down and can therefore be expected to continue. For older generations, transversal and longitudinal changes merge; it is only towards the age of 60 that retirements accelerate.

1.8. On the survey curves for France, one notes the disappearance of the trough that was formerly typical of the 30-35 year olds, and which appeared for the last time in 1973. Between 1973 and 1975 the differences seem more marked than between 1975
and 1977, as much in what concerns the employment of the middle-aged as for the lowering of the retirement age. If one then moves to the generation curves, there is for the youngest a slight delay in entry. For the generations born between 1950 and 1945 it seems that once found a job is not given up. Among the generations born around 1940, which were those most concerned by the net entry or re-entry into the labour market, there is a marked trend towards a slowing down of this phenomenon; for the generations born around 1930 the entry or re-entry into the employment market was weaker but not negligible. Finally, among the generations born in the first twenty years of the century, retirement occurs increasingly early, about one year every four or five generations.

1.9. In the case of Italy, our remarks concerning the regional non-homogeneity of the situations apply to the changes also. On the survey curves there is a considerable rise for the middle-aged and elderly; for example at 30 the increase, in relative percentages, is almost one-third in five years; inversely, the profile is more and more inclined for the young women and the maximum employment rate is reached increasingly late. If one now examines the generation curves, the young generations enter the labour market more and more slowly, the oldest who have jobs keep them (still with the understanding that this refers to a remainder), and all the generations born between 1950 and 1935 greatly increase their participation in the labour market. Finally, the trend seems to be more towards an extension of professional life, this being the only case in all nine countries.

1.10. In the Netherlands, one observes the progressive rise of the employment rate at all ages between 22 and 55 years and a rather marked progressive rise in the age of entry for the youngest generations. As for the generations, it seems that the high employment rate of the young at the same time increases and shifts towards older people. All the middle-aged generations, born between about 1940 and 1920, have seen their employment rate increase by about 5% in absolute value, that is on an average one-fourth in relative value. Finally, the lowering of the retirement age is important but concerns few people.

1.11. In Belgium, it is especially between 1973 and 1975 that the differences are marked, the profile declining at an increasingly regular pace between 25 and 55 years;
B.R. DEUTSCHLAND
WOMEN
SURVEYS
1973 1975 1977

FRANCE
WOMEN
SURVEYS
1973 1975 1977

ITALIA
WOMEN
SURVEYS
1973 1975 1977
B.R. DEUTSCHLAND
WOMEN
GENERATIONS

FRANCE
WOMEN
GENERATIONS

ITALIA
WOMEN
GENERATIONS
NEDERLAND
WOMEN
SURVEYS
1973 1975 1977

BELGIQUE/BELGIE
WOMEN
SURVEYS
1973 1975 1977

LUXEMBOURG
WOMEN
SURVEYS
1973 1975 1977
UNITED KINGDOM
WOMEN
SURVEYS
1973 1975 1977

IRELAND
WOMEN
SURVEYS
1975 1977

DANMARK
WOMEN
SURVEYS
1975 1977
UNITED KINGDOM
WOMEN
GENERATIONS

IRELAND
WOMEN
GENERATIONS

DANMARK
WOMEN
GENERATIONS
there is also a rise in the age for the maximum employment rate. If one now looks at
the generations, the decline in the entry into the labour market among the young
generations seems more marked than elsewhere. The slightly older generations, those
born between 1950 and 1945, seem to quit the labour market less than the youngest
and the sizeable entry or re-entry into the labour market seems rather narrowly concen­
trated in the generations born around 1940. The evolution does not seem very significant
for the older generations, except for a definite increase in retirements from the age of
55 on.

1.12. Concerning Luxembourg, the size of the sample leads one to be circumspect
when commenting on this evolution; what is significant is the decline in job-leaving
for the generations born around 1950 and a large entry or re-entry for the generations
born around 1940.

1.13. In the United Kingdom, the evolution of the profiles is marked especially
between 1973 and 1975, but it is recognized that the 1973 survey is perhaps not exactly
comparable to those of the following years. Between 1975 and 1977, the movement is
limited to a partial effacement of the still important trough situated just under 30
years. If one examines the generation curves, forgetting the reservations made concerning
the 1973 survey, the main movement that appears is first the trend towards the filling
in of the trough, whose minimum remains around 27 years. Moreover, the increase in
the employment rate seems important for the generations born between 1940 and
1920. There seems to be no definite movement in what concerns retirement.

1.14. The results concerning Ireland, limited to 1975 and 1977, seem to correspond
to a rather high stability since the two survey curves almost merge and since most of
the movements of the generations are parallel to these curves. At the most one can
detect a slight shift in the part of the curve concerning the 15-30 year olds and an
increase in retirements from 60 years on.

1.15. In Denmark, only the 1975 and 1977 surveys are available. The rise in the
profiles between 30 and 45 years is important at 5%, when one considers the level
already attained. It corresponds to an increase in the employment rate for the genera­
tions born between 1940 and 1925; one also observes for the generations born before
1910 a definite decrease in retirement age.
2. MALE EMPLOYMENT RATE

2.1. On the graph on the following page the male employment rate is depicted using the arrangement that was used previously for the female rate. The three age groups will be commented upon in turn, which correspond to the entry into the labour market, to the presence there in large numbers and to the exit from it. Concerning the middle-aged male group the viewpoint will be more that of the economy's capacity to furnish jobs than that of men's desire to find them, contrary to what was the case for women. Due to the simpler shapes of the curves, comments will be shorter.

2.2. There are sizeable differences in male employment rates at the youngest ages. Even if in all the countries there appear to be entries until around the age of 30, the differences at 20 and 25 are very marked. At 20, the employment rate remains around 50% only in Italy, Belgium, and the Netherlands; it is a little less than two-thirds in Denmark and in France, reaches almost three-fourths in Ireland, F.R. Germany, and Luxemburg, and even reaches 80% in the United Kingdom, where there is no military service. At 25 the differences are slighter; all the countries are situated between 85 and 90% except for Italy, Denmark (the curve of this country has a very curious shape, however, and a more detailed analysis is required), and F.R. Germany.

2.3. Among the middle-aged, i.e. between 30 and 50, the employment rates come very close together around 95%, with higher values however for Luxemburg, and definitely lower ones, around 90%, for Denmark and Ireland.

2.4. In the part corresponding to exit from work one distinguishes first the United Kingdom, Ireland, and Denmark, countries where over 50% of the 65 year old men have a job; for the other countries this rate is only about one-third and for Belgium and Italy one-fourth.
Employment rate

Age

0 10 20 30 40 50 60 70

50 60 70 80 90
2.5. If one now examines these profiles as a whole, one is struck by the great difference between the United Kingdom, which is relatively «square» (almost simultaneous entry and exit), and Italy, which is very «pointed» (very progressive entry and exit).

2.6. In the Federal Republic of Germany, one notes a general retraction of the profiles, especially between 20 and 30 years and above 60. If one examines the generations, the phenomenon is a little more complex. First, for the generations born after 1955, there seems to be a definite slowdown in entry into the labour market. Next, the generations born around 1950 seem to have been particularly touched by unemployment in 1975, more so than the older generations; inversely (but this may be giving too much importance to a slight movement) the generations born around 1930 seem to have benefitted less from the recovery of 1977. Finally, one should note the rapid decline in the retirement age which can be estimated at one year every two or three generations.

2.7. In France one also notes a lowering of the profiles, which was evident especially between 1973 and 1975 but is of much less magnitude. If one examines the generations, one sees a slight slowdown in entries into the labour market among the youngest generations. Unemployment through job loss seems to affect above all the generations born around 1930. Retirement occurs earlier but the evolution is slower than in F.R. Germany, with a lowering of age estimated at one year every four generations.

2.8. In Italy, the profile whose shape we have already very particularly stressed, seems relatively stable in relation to what can be observed for other countries. On the other hand, if one examines the generation curves, all the young generations born after 1945 seem to be affected by a decline in the already slow speed of entry into the labour market. For the older generations, born around 1930, contrary to what happens in other countries, the situation deteriorated somewhat between 1975 and 1977. Finally, one notes a very slight lowering of the retirement age.

2.9. In the Netherlands, the main overall lowering of profile occurred between 1973 and 1975. If one examines the generations one notes the rather marked increase in age
B.R. DEUTSCHLAND
MEN
GENERATIONS

FRANCE
MEN
GENERATIONS

ITALIA
MEN
GENERATIONS
NEDERLAND
MEN
GENERATIONS

BELGIQUE/BELGIE
MEN
GENERATIONS

LUXEMBOURG
MEN
GENERATIONS
UNITED KINGDOM
MEN
SURVEYS
1973 1975 1977

IRELAND
MEN
SURVEYS
1975 1977

DANMARK
MEN
SURVEYS
1975 1977
at entry for the very young generations; employment of the generations born between 1945 and 1935 is about stationary during the period covered; on the other hand, the generations born around 1930 seem to have been very preferentially affected by the slowdown in activity. Finally, the lowering of the retirement age is almost as rapid as in F.R. Germany, about one year every three generations.

2.10. In Belgium, the profile declined little at the median ages and slightly at the extreme ages. If one now examines the generations, one notes here also an increase in the age at entry. The decline in employment seems to have affected just about uniformly all the middle-aged generations during the period covered. Retirement is lowered by about one year every three generations.

2.11. In Luxembourg, the profile has stayed stable at the median ages and has declined, especially between 1975 and 1977, at the extreme ages. If one examines the generations, the entry of the very young generations seems to have occurred without change between 1973 and 1975, then to have reached a maximum suddenly. The generations born between 1930 and 1950 seem to have been affected, but the acceleration of retirements seems as rapid as in the rest of the Benelux, affecting however not only those over 60 but also those over 50.

2.12. In the United Kingdom, one notes between 1973 and 1977 a slight lowering of the profiles. If one examines the generations, one notes the delay at entry, but beyond that, all the generations seem about equally touched by the lowering of the employment rate. At the advanced ages, the lowering of the retirement age occurs but takes place rather slowly.

2.13. In Ireland, the profile is relatively stable except around 20-25 years. This is to be seen again in the profiles, but the movement based on only two years seems difficult to explain. For the higher ages, one notes a very great stability, with transversal and longitudinal profiles merging.

2.14. Concerning Denmark too, one should be circumspect in one's comments,
since only two years are available. One notes a rise in the profiles. As for the generations one observes what seems to be an increase in the employment rate for all generations under 45. There seems to be no evolution in the retirement age.
Annex: ORTHOGONAL POLYNOMIALS

1. Basis data and their transformation

- $s$: year of the survey
- $S$: number of surveys
- $t$: age
- $g$: generation ($g = s - t$), i.e. year of birth
- $G$: number of generations
- $\theta_{g,s}$: employment rate of the generation $g$ collected through the survey $s$

$\theta_{g,s}$ is an estimation of the real rate obtained on the basis of a sample of size $n_{g,s}$; the variance of this estimation depends on the value of $\theta_{g,s}$ (heteroscedasticity); to obtain homoscedasticity (supposing that $n_{g,s} = n$ is independent of $g$ and $s$, which is not perfectly right), one replaces $\theta_{g,s}$ by:

$$\tau_{g,s} = \text{Arc} \sin \sqrt{\theta_{g,s}}$$

which variance is constant and equal to $\frac{1}{4n}$

2. Orthonormal polynomials

- $P_i(g)$: value of the orthonormal polynomial of degree $i$ for the generation $g$
- $N_j(s)$: value of the orthonormal polynomial of degree $j$ for the survey $s$

To calculate the values of the orthonormal polynomials $P_i(g)$, one draws in the $G-1$ dimensional space the vectors $V_i$ of components $g^i$, then, by iterative process from the length 1-vector $P_0$ which all components are equal between themselves, one builds
vector $P_i$ of the same length as $P_0$ and projections of $V_i$ on the hyperplane perpendicular to all $V_k (k = 0, 1, ..., i-1)$; the components of these vectors $P_i$ become the successive values of $P_i (g)$.

To calculate the $N_j (s)$, one proceeds the same way in the $S-1$ dimensional space without forgetting that, contrary to the values of $g$, the values of $s$ are not necessarily regularly spaced.

3. Computation of the regression coefficients

$$a_{ij} \quad \text{single regression coefficient of } \tau_{g,s} \text{ compared with the double explanatory variable } P_i(g) N_j(s)$$

$$a_{ij} = \frac{1}{GS} \sum_i \sum_j P_i(g) N_j(s) \tau_{g,s}$$

The corresponding explanatory formula of $\tau_{g,s}$ is thus:

$$\tau_{g,s} = \sum_i \sum_j a_{ij} P_i(g) N_j(s)$$

Each $a_{ij}$ contributes to the total correlation $R^2$ (which is equal to 1 if and only if one takes all the values of $i$, and all the values of $j$), by a partial correlation $R_{ij}^2$ which is no other than $a_{ij}^2$. 