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Monetary and Fiscal Rules for  
Public Debt Sustainability (\*)

Marco Buti

Internal paper



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The Directorate-General for Economic and Financial Affairs,  
Commission of the European Communities,  
200, rue de la Loi  
1049 Brussels, Belgium

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# MONETARY AND FISCAL RULES FOR PUBLIC DEBT SUSTAINABILITY

## A Note on the Arithmetic of the Government Budget Constraint

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## SUMMARY

The question of sustainability of public sector debt in the presence of high budget deficits and a positive interest rate-growth rate differential has received widespread attention in recent years. The risks of "explosion" of the public debt and the eventual monetization which could result in high inflation have often been at the roots of several medium-term programmes of budgetary consolidation.

The paper, after presenting the classical results of the debt sustainability analysis (section I), introduces gradual budgetary consolidation in a model à la Sargent and Wallace (exogenous interest rate-growth rate differential and quantitative money demand function) so as to derive the adjustment in the "primary" balance necessary in order to stabilize the debt-income ratio by a certain time in the future (section II).

The consequences of different policy regimes - cooperation between monetary and fiscal authorities, leadership by fiscal authorities or by monetary authorities - for the budgetary adjustment, the rate of inflation and the level of the debt-GDP ratio are explored in section III. If the fiscal authorities are not able (or willing) to carry out the whole adjustment, debt stabilization requires an increase in the "seigniorate tax" which could imply a substantial acceleration in the rate of monetary growth. If the monetary authorities act as leaders and the fiscal authorities pursue an adjustment lower than the equilibrium one, the stabilization of the debt-GDP ratio could require a significant "jump" in the primary balance at the end of the period.

A simple application of the model to Italy's debt-stabilization problem is made in section IV. The main conclusion of the analysis is that the Government programme of budgetary consolidation appears to require a considerable increase in seigniorage which is likely to contrast with the objectives of monetary stability.

In the last section, after summarizing the main results of the analysis, a preliminary attempt is made to go beyond the arithmetic of the debt sustainability issue and to consider its economics.

## 1. PUBLIC DEBT (UN)SUSTAINABILITY

During the past few years the so-called "fixed-parameters" projections of public debt have been largely used in the analysis of the effects of sustained fiscal deficits (1).

The basic equation used in these exercises represents the constraint faced by the government in financing public deficits :

$$(1) \quad \dot{M} + \dot{B} = -F + iB + i_m M$$

where,  $M$  : money base,  $B$  : market holdings of government bonds,  $F$  : deficit (-) or surplus (+) net of interest payments ("primary" balance),  $i$  : nominal interest rate paid on market securities,  $i_m$  : institutionally fixed interest rate paid on  $M$ .

By taking ratios to GDP (lower case letters) and re-arranging, (1) becomes :

$$(2) \quad \dot{b} = (r - y)b - f - (g_m - i_m)m$$

where  $r$  and  $y$  are the (ex-post) real interest rate paid on market securities and real growth rate of the economy, respectively;  $g_m$  is the rate of growth of the money base (we assume that deficit financing is the only channel of creation of money base).

The term  $(g_m - i_m)m$  is the seigniorage tax extracted through monetary financing, i.e. the value of real resources appropriated by the government through net issues of high-powered money.

Let us assume :

1. the interest rate-growth rate differential ,  $r - y$ , is fixed;

2. the demand for money is "quantitative", i.e.  $M/Y = m$  fixed, thus  $G_m = y + \pi$ , where  $\pi$  is the inflation rate;
3. the money base grows at a steady rate, i.e.  $G_m$  is given;
4. the primary balance  $F$  is kept in a fixed proportion to GDP, i.e.  $f$  is given.
5. private sector willingness to hold government debt is assumed to be limited, i.e.  $b$  is constrained by  $b \leq \bar{b}$ ;

The hypothesis of exogeneity of  $(r - y)$  is highly restrictive.

The real rate of interest is exogenous if there exists perfect substitutability between government debt and real assets or, in open economies, between government debt and assets denominated in foreign currencies. The real growth of the economy is exogenous if the so-called Ricardian equivalence holds, i.e. economic agents discount fully future taxes to re-pay the debt which implies that government bonds are not considered net wealth (2).

The assumption of a quantitative demand for money ( $m = \bar{m}$  fixed) implies that the value of the seigniorage tax revenue rises proportionally with increases in  $G_m$ .

By following Bulter's definitions (see Bulter, 1987), the seigniorage revenue can be decomposed into two components : the seigniorage tax rate,  $G_m - i_m$ , and the seigniorage tax base,  $m$ . The assumption of a quantitative money demand implies a constant value of the tax base. However, if real money balances are a decreasing function of inflation (e.g. Cagan's money demand function), and higher money growth rates sooner or later imply higher inflation, the velocity of circulation,  $1/m$ , will rise, i.e. the seigniorage tax base will fall. Therefore, a

higher value of  $G_m$  will only rise seigniorage if the elasticity of velocity with respect to inflation is less than unity : this is likely to be the case, at least for relatively moderate rates of inflation.

If a higher value of the seigniorage is needed in order to stabilise the debt, the quantitative money demand assumption "minimises" the rise in  $G_m$  required to achieve it : in the real world a higher rise in  $G_m$  would be probably needed in order to compensate for the fall in  $m$ .

Assumption 3 and 4 define the fiscal rule and the monetary rule, respectively (3).

Assumption 5 asserts that, like all private borrowers, the government is faced by an upper limit to the amount of debt it can issue : "In August 1982, (b) in Mexico appeared to have hit  $(\bar{b})$ . In France, between 1924 and 1926, (b) appeared to have been close to  $(\bar{b})$ , precipitating a continuing financial crisis and the "waltz of portfolios" of the finance ministers of France" (Sargent, 1986, p. 25).

Given assumptions 1 - 4, the differential equation (2) can be solved for the level of the debt-income ratio,  $b(t)$ .

The condition for convergence to a finite value is  $y - r > 0$ , i.e. the real rate of growth of output is higher than the (ex post) real rate of interest. Under such a condition, the long run equilibrium value of  $b(t)$  is :

$$(3) \quad b^* = \frac{f + (G_m - i_m)m}{r - y}$$

If the government is running a "seigniorage-corrected" primary surplus (i.e.  $f + (G_m - i_m)m > 0$ ),  $b(t)$  will converge to a negative value : the debt is progressively reduced and, eventually, is transformed into a net asset position.



If  $y < r$ , the debt-income ratio is generally unstable. Under this assumption, either  $b(t)$  is on an explosive path or, if  $b(0) = \bar{b}$ , is constant. Therefore, if  $y < r$ , the only "sustainable" position (which is the one of constant  $b(t)$ ) requires a seigniorage-corrected primary surplus :

$$(4) \quad f + (a_m - i_m)m = b(0)(r-y) > 0$$

If the value of the seigniorage is relatively low, a sufficiently large primary surplus is needed (i.e.  $f > 0$ ).

The case of a positive interest rate-growth rate differential is pictured in Figure 1.

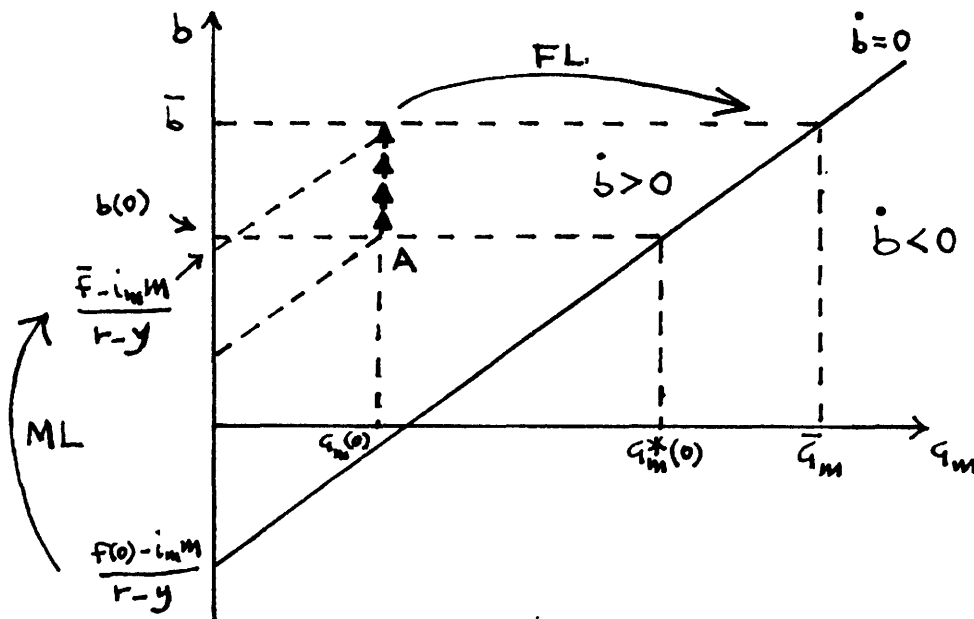


Fig. 1

By setting  $\dot{b} = 0$  in eq. 2, and solving for  $b$  we obtain :

$$(5) \quad b = \frac{f - i_m m}{r - y} + \frac{m}{r - y} G_m$$

which is represented by the line ( $\dot{b} = 0$ ) in Fig. 1. Line ( $\dot{b} = 0$ ) is the locus of all the combinations of  $f$  and  $G_m$  which give rise to a

stable debt-GDP ratio. Since  $r - y > 0$ , the line is upward sloping. Any combination above (below) the line corresponds to a point of rising (decreasing) debt.

Let us assume that  $A (b(0), G_m(0))$  is the starting point.  $A$  is a point of non-equilibrium : the stability of the debt-GDP ratio at the level  $b(0)$  would require either a higher money base growth,  $G_m^*(0)$ , which would produce a higher seigniorage revenue, or a sufficiently large primary surplus,  $f^*(0)$ . In the latter case, the line ( $\dot{b} = 0$ ) would be shifted upwards so as to pass through  $A$ .

In the absence of either the debt rises. If no adjustment is implemented, the debt eventually reaches the upper threshold  $\bar{b}$ . At that moment, since the private sector refuses to increase its holdings of government securities, the debt has to be stabilised. This implies either a faster expansion of money base from  $G_m(0)$  to  $\bar{G}_m$ , which is the case referred to as fiscal leadership (FL), or a sudden jump into primary surplus (from  $f(0) < 0$  to  $\bar{f} > 0$ ) which we define as monetary leadership (ML). The picture shows that in both cases, because of the debt accumulation which has taken place in the meanwhile, the extent of the forced eventual adjustment is higher than that required at the beginning of the period.

In particular, under the FL regime, "with the government budget persistently in deficit and real rates of interest exceeding the economy's growth rate, the (central bank) must choose between fighting present inflation with "tight" monetary policy now or fighting future inflation with "easy" monetary policy now" (Sargent-Wallace, 1981, p. 159).

The condition of stable debt  $\bar{b} = b(0)$ , implies that, in a situation of seigniorage-corrected primary deficit, if the interest rate-growth rate differential becomes positive, a sudden jump into surplus is needed : since plans for fiscal adjustment are usually characterized by a

phasing out of the deficit over several years, the implications of gradualness have to be explored.

The question of gradualness was first raised by Blanchard (1984).

Let us define, with Blanchard,  $f^M$  as the maximum politically feasible primary surplus,  $f^M > 0$ , given by the difference between the minimum socially acceptable amount of (net of interest) government spending and the maximum amount of taxes which can be collected by the government.

The long-run equilibrium value of the debt corresponding to  $f^M$  is :

$$(6) \quad b^M = \frac{f^M + (G_m - i_m)m}{r - y}$$

If the debt exceeds  $b^M$ , it can be stabilized only through a rise in  $G_m$ . If we define  $G^M$  as the maximum accepted monetary expansion, the maximum sustained level of debt becomes :

$$(7) \quad b^{MM} = \frac{f^M + (G^M - i_m)m}{r - y}$$

If the debt ever exceeds  $b^{MM}$ , it will be forever increasing :

If  $b(t)$  exceeds  $b^{MM}$  (e.g. because the Government is running a primary deficit and the central bank is committed to monetary stability), a primary surplus or a monetary expansion larger than  $f^M$  and  $G^M$ , respectively, would be required to stabilize the debt (this is prevented by the definition of  $f^M$  and  $G^M$ ).

If  $b(0) < b^{MM}$  and the adjustment is phased out over several years, the reduction of the primary deficit has to be fast enough in order to prevent  $b(t)$  exceeding  $b^{MM}$  during the adjustment period.

Blanchard's model is pictured in Figure 2.

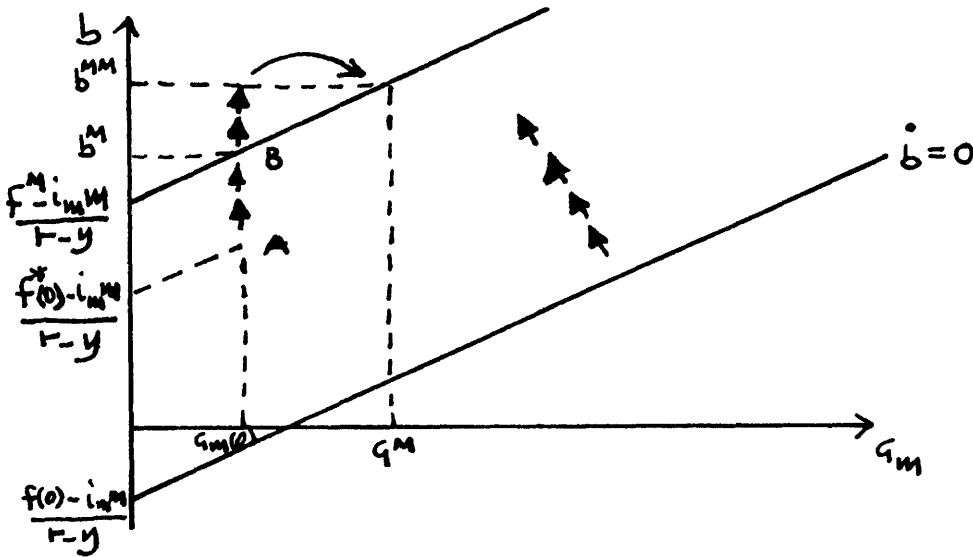


Fig. 2

If the adjustment in the primary surplus is fast enough the debt can be stabilized at a level  $b \leq b^M$ . If the budgetary consolidation is not carried out sufficiently quickly, monetary authorities have to step in. If the level of debt ever exceeds  $b^{MM}$ , it will grow boundlessly in that any adjustment programme would require  $f > f^M$  and/or  $G_M > G^M$ .

In section II, a simple model of gradual fiscal adjustment is presented: the model will be solved for the linear equilibrium adjustment in the primary balance to be carried out over a period of  $\bar{t}$  years, which allows the stabilization of  $b(t)$  at the level attained at time  $\bar{t}$ .

## II. LINEAR ADJUSTMENT AND DEBT STABILIZATION

The present section deals with the problem of debt stabilization over a period of  $\bar{t}$  years (from  $t = 0$  to  $t = \bar{t}$ ).

We are interested in deriving the annual adjustment in the primary balance which allows the stabilization of the debt-GDP ratio at time  $\bar{t}$ .

Monetary authorities can contribute to the debt stabilization programme by increasing the rate of growth of money base,  $G_m$ , which gives rise to a higher seigniorage tax revenue.

In the present section, we retain assumptions 1 and 2 (fixed interest rate-growth rate differential and quantitative money demand function) and explore the consequences of adjusting  $f$  and  $G_m$  so as to stabilize the debt-GDP ratio over a period of  $\bar{t}$  years.

The actual form of the monetary and fiscal rules depends on the leadership position of monetary and fiscal authorities.

In a non-cooperative setting, fiscal authorities' leadership (FL) is described by equations (8a) and (8b) :

$$(8a) \quad \begin{aligned} f(t) &= f(0) + \alpha t && \text{for } t < \bar{t} \\ f(t) &= f(0) + \alpha \bar{t} && \text{for } t > \bar{t} \end{aligned}$$

$$(8b) \quad \begin{aligned} G_m(t) &= G_m(0) && \text{for } t \leq \bar{t} \\ G_m(t) &= G_m(0) + Z^F && \text{for } t > \bar{t} \end{aligned}$$

Equations (8a) state that the primary deficit (surplus) is reduced (increased) linearly by  $\alpha$  % of GDP per year over a period of  $\bar{t}$  years. The monetary authorities stick to the initial money base growth,  $G_m(0)$ , until the final year; then, given the hypothesis of FL, they give up and raise  $G_m$  at once by  $Z^F$ .

In Sargent and Wallace's seminal work (see Sargent-Wallace (1981)), the behaviour of policy makers is described by equations (8a) and (8b) under the assumption  $\alpha = 0$ .

The case of monetary authority leadership (ML) is described by equations (9a) and (9b) :

$$(9a) \quad \begin{aligned} f(t) &= f(0) + \alpha t && \text{for } t \leq \bar{t} \\ f(t) &= f(0) + \alpha \bar{t} + K && \text{for } t > \bar{t} \end{aligned}$$

$$(9b) \quad G_m(t) = G_m(0) \quad \text{for } t > 0$$

In such a case, monetary authorities stick to the initial money base growth,  $G_m(0)$ , while fiscal authorities, if the adjustment is not completed by time  $\bar{t}$ , have to make a further correction in the primary balance by  $K$  % of GDP in the final year.

In a set up of cooperation (C) both monetary and fiscal authorities are willing, from the beginning, to contribute to the adjustment.

Such a scenario is described by equations (10a) and (10b) :

$$(10a) \quad \begin{aligned} f(t) &= f(0) + \alpha t && \text{for } t \leq \bar{t} \\ f(t) &= f(0) + \alpha \bar{t} && \text{for } t > \bar{t} \end{aligned}$$

$$(10b) \quad G_m(t) = G_m(0) + z^C \quad \text{for } t > 0$$

In such a case  $\alpha$  is jointly determined and the monetary authorities step in at the beginning of the period and raise  $G_m$  by  $Z^C$ .

In policy regimes C and FL,  $G_m$  "jumps" either at the beginning (equation 10b) or at the end of the period (equation 8b). An interesting intermediate case is that in which  $G_m$  is increased linearly over the period :

$$(11) \quad G_m(t) = G_m(0) + \beta t \quad \text{for } t \leq \bar{t}$$

$$G_m(t) = G_m(0) + \beta \bar{t} \quad \text{for } t > \bar{t}$$

Under this assumption the "seigniorage-corrected" primary balance is adjusted linearly over the period so as to stabilize  $b(t)$  at  $t = \bar{t}$ .

Policy regimes described by equations (8) - (10) can be summarized by equations (12a) and (12b) :

$$(12a) \quad f(t) = f(0) + \alpha t \quad \text{for } t \leq \bar{t}$$

$$f(t) = f(0) + \alpha \bar{t} + k \quad \text{for } t > \bar{t}$$

$$(12b) \quad G_m(t) = G_m(0) + Z^C \quad \text{for } t \leq \bar{t}$$

$$G_m(t) = G_m(0) + Z^C + Z^f \quad \text{for } t > \bar{t}$$

by setting  $K$ ,  $Z^C$ ,  $Z^f$ , two by two, equal to zero, we obtain the three scenarios previously described.

A useful benchmark case is obtained by setting  $Z^C = Z^f = K = 0$ , thus fiscal authorities pursue an annual equilibrium adjustment in the

primary balance which allows the stabilization of  $b(t)$  at time  $\bar{t}$ , given the initial rate of growth of money base.

By retaining the assumption of a positive interest rate-growth rate differential, and substituting equations (12a) and (12b) in equation (2), we obtain the following differential equations :

$$(13) \quad \dot{b} = (r - y)b - f(0) - [G_m(0) + Z^C - I_m]m - \alpha t \text{ for } t \leq \bar{t}$$

$$(14) \quad \dot{b} = (r - y)b - f(0) - k - [G_m(0) + Z^C + Z^F - I_m]m - \alpha \bar{t} \\ \text{for } t > \bar{t}$$

Equations (13) and (14) can be solved to find  $b(t)$ . The equilibrium adjustment in the primary balance,  $\alpha$ , can be found by imposing the condition  $\dot{b} = 0$  for  $t > \bar{t}$  (see Appendix ) :

$$(15) \quad \alpha = \gamma - \theta Z^F - \varepsilon Z^C - \mu K$$

where  $\theta$ ,  $\varepsilon$ , and  $\mu$  are all positive.

The equilibrium adjustment  $\alpha$  is a function of the initial conditions and the policy variables :

$$(16) \quad \alpha = \alpha(b(0), f(0), G_m(0), I_m, m, r-y, \bar{t}, Z^F, Z^C, K)$$

where signs above the arguments are those of partial derivatives.

It is important to note that a longer adjustment period allows a lower effort to be made each year ( $\alpha(t_1) > \alpha(t_2)$  if  $t_1 < t_2$ ), but requires a larger overall adjustment ( $f(t_1) < f(t_2)$ ).



It is easy to show (see Appendix ) that under the assumption of a linear rise in  $G_m$  (equation 11), the expression for  $\alpha$  is the following :

$$(17) \quad \alpha = \gamma - m\beta$$

If the fiscal authorities carry the whole burden of the adjustment, the annual correction in the primary balance becomes :

$$(18) \quad \alpha = \gamma$$

If the monetary authorities pursue a stricter anti-inflationary policy, by decreasing of  $G_m$  either at the beginning or gradually over the period, the adjustment to be carried out by fiscal authorities would have to compensate for the reduced value of the seigniorage tax revenue ( $\alpha < \gamma$ ).

If the fiscal authorities are not able (or willing) to adjust the primary balance by  $\gamma$  % of GDP per year, and set  $\alpha = \bar{\alpha} < \gamma$ , the stabilization of  $b(t)$  requires a rise in the seigniorage revenue or a further correction by  $K$  % of GDP in the primary balance in the final year. Therefore, given  $\alpha = \bar{\alpha}$ , we obtain :

$$(C) \quad z^C = \frac{\gamma - \bar{\alpha}}{\varepsilon}$$

$$(FL) \quad z^F = \frac{\gamma - \bar{\alpha}}{\theta}$$

$$(ML) \quad K = \frac{\gamma - \bar{\alpha}}{\mu}$$

If  $G_m$  is increased linearly over the period, the equilibrium  $\beta$  is

$$\beta = \frac{\gamma - \bar{\alpha}}{m} .$$

III. STEADY-STATE RESULTS UNDER DIFFERENT POLICY REGIMES

The present section deals with the effects on  $b(t)$ ,  $f(t)$  and  $\Pi(t)$  of budgetary consolidation under different policy regimes, under the assumption  $r-y > 0$ . We assume that  $\alpha$  is set at  $\bar{\alpha} \leq \gamma$  and  $z^C$ ,  $z^F$  and  $K$  are endogenously determined.

A. Total adjustment in the primary balance

Under C and FL regimes, if  $\alpha$  is set at  $\bar{\alpha} < \gamma$ , the rate of growth of money base,  $G_m$ , adjusts by  $z^C$  and  $z^F$ , respectively, so as to stabilize  $b(t)$  at  $\bar{t}$ . Therefore, the total adjustment in the primary balance is the same under the two regimes.

The adjustment is higher under ML : In such a case the seigniorage tax revenue is given, thus the primary surplus eventually has to "jump" by  $K$  % of GDP.

It is easy to show that, under ML, the total primary adjustment is minimized if fiscal authorities choose  $\bar{\alpha} = \gamma$ , which corresponds to our benchmark case : In such a case the correction in the primary balance is "just enough" to stabilize the debt at  $\bar{t}$ , without the need for a further rise in the surplus ( $K=0$ ):

$$f(\bar{\alpha} = \gamma)^{ML} = f(0) + \gamma \bar{t}$$

$$f(\bar{\alpha} < \gamma)^{ML} = f(0) + \bar{\alpha} \bar{t} + K$$

where  $K = \frac{\gamma - \bar{\alpha}}{\mu}$

Therefore by substituting from the Appendix (4) :

$$(19) \quad f(\bar{\alpha} = \gamma)^{ML} - f(\bar{\alpha} < \gamma)^{ML} = \frac{\gamma - \bar{\alpha}}{r-y} \left[ \bar{t} (r-y) + 1 - e^{-(r-y)\bar{t}} \right] < 0$$

Since debt rises faster under  $\bar{\alpha} < \gamma$  as a consequence of a lower adjustment, the interest burden rises, thus the fiscal jump  $K$  required to stabilize the debt leads to a larger overall adjustment.

### B. Effects on the Inflation rate

Given the assumption of a quantitative money demand function, the rate of inflation fully reflects the rate of growth of money supply,  $G_m$ :  
 $\pi(t) = G_m(t) - y$ .

The lowest inflation rate is clearly achieved under monetary authority leadership (ML) : in such a case  $G_m$  does not rise, thus the inflation rate is constant over the period ( $\pi^{ML} = \pi(0)$ ).

Under C,  $G_m$  rises by  $z^C$  at the beginning of the period, while, under FL,  $G_m$  is raised eventually by  $z^F$ .

We can show that, for a given  $\bar{\alpha} < \gamma$  we obtain,  $z^F > z^C$  :

$$z^F = \frac{\delta - \bar{\alpha}}{\vartheta} \quad \text{and} \quad z^C = \frac{\delta - \bar{\alpha}}{\varepsilon} .$$

By substituting from  $\vartheta$  and  $\varepsilon$ , in Appendix , we obtain :

$$(20) \quad z^F - z^C = \frac{(\delta - \bar{\alpha}) [e^{(r-y)\bar{E}} - 1]}{m(r-y) e^{(r-y)\bar{E}}} > 0$$

Equation (20) re-states the classical Sargent and Wallace result in a context of gradual adjustment in the primary balance : under inconsistent monetary and fiscal objectives, if fiscal authorities act as leaders, a lower rate of inflation in the short run gives rise to a higher rate of inflation in the long run.

As we can see from equation (20) the inflation differential is inversely correlated with  $\bar{\alpha}$ : the lower the annual adjustment in the primary balance, the higher the eventual rise in inflation.

It is easy to show that a linear increase in  $G_m$  over the period gives rise to a long run inflation rate which is intermediate between those observed under co-operation and fiscal leadership.

C. Equilibrium level of the debt

The lowest level at which the debt is stabilized is that achieved under co-operation. At the limit, if the whole adjustment is carried out by the monetary authorities (i.e.  $\bar{\alpha} = 0$ ,  $z^C = \delta/E$ ), the debt is stabilized at its starting level :  $\bar{b} = b(0)$ .

For a given annual adjustment in the primary balance, the level of the debt is the same under FL ( $\alpha = \bar{\alpha}$ ,  $z^F > 0$ ) and ML ( $\alpha = \bar{\alpha}$ ,  $K > 0$ ).

$\bar{b}^{FL} = \bar{b}^{ML}$  because over the period, the seigniorage revenue and the correction in the primary balance are the same in the two regimes (the final jump in the primary surplus or in the money growth are needed in order to stabilize the debt at the level attained in  $\bar{t}$ ).

As is implicit in the above reasoning, the effect on  $b(t)$  of a shift in the burden of the adjustment between monetary and fiscal authorities depends on the policy regime. Under C, a rise in  $\bar{\alpha}$ , which implies a fall in monetary financing, raises the equilibrium level of debt (5) :

$$(21) \quad \frac{\partial \bar{b}^C}{\partial \bar{\alpha}} = \frac{1 - e^{(r-y)\bar{t}} [1 - \bar{t}(r-y)]}{(r-y)^2 e^{(r-y)\bar{t}}} > 0$$

On the contrary, under ML and FL, a rise in  $\bar{\alpha}$  results in a fall of the equilibrium level debt (6) :

$$(22) \quad \frac{\partial \bar{b}^{ML,FL}}{\partial \bar{\alpha}} = \frac{1 + \bar{t}(r-y) - e^{(r-y)\bar{t}}}{(r-y)^2} < 0$$

Under both, ML and FL, the lowest level of  $\bar{b}$  is achieved by setting  $\bar{\alpha} = \delta$ .

It is interesting to note that in the case of linear increase in  $G_m$  (i.e.  $\beta = \frac{\gamma - \bar{\alpha}}{m}$ ), the equilibrium level of debt is independent of the distribution of the adjustment between monetary and fiscal authorities:

$$\bar{b}(\bar{\alpha} = \gamma)^\beta = \bar{b}^* = \bar{b}(\bar{\alpha} < \gamma)^\beta, \quad \frac{\partial \bar{b}^\beta}{\partial \bar{\alpha}} = 0$$

This result is due to the fact that, unlike the other policy regimes, the adjustment in the "seigniorage-corrected" primary balance is the same under  $\bar{\alpha} = \gamma$  and  $\bar{\alpha} < \gamma$ ,  $\beta = \frac{\gamma - \bar{\alpha}}{m}$ . Since the debt accumulation depends on the adjustment in the seigniorage-corrected primary balance, the level of debt attained at  $t = \bar{t}$  is the same under the two policy options.

Results established in sections A-C can be summarized as follows :

$$\bar{f}^{FL} = \bar{f}^C = \bar{f}^\beta < \bar{f}^* < \bar{f}^{ML}$$

$$\pi^*(t) = \pi^{ML}(t) = \pi^{FL}(t) < \pi^C(t), \quad \text{for } t \leq \bar{t}$$

$$\pi^*(t) = \pi^{ML}(t) < \pi^C(t) < \pi^\beta(t) < \pi^{FL}(t), \quad \text{for } t > \bar{t}$$

$$\bar{b}^C < \bar{b}^* = \bar{b}^\beta < \bar{b}^{FL} = \bar{b}^{ML}$$

where "\*" indicates the benchmark case  $\bar{\alpha} = \gamma$  and " $\beta$ " that in which  $G_m$  is increased linearly over the period.

#### IV. APPLICATION TO A HIGH DEBT ECONOMY : THE CASE OF ITALY

Italy's public finance situation is a "natural" case study for an illustration of the "arithmetic model" developed in the previous section (7).

The behaviour of the relevant parameters for the period 1980-88 is summarized in Table 1.

The total public sector debt has risen from 59% of GDP in 1980 to 96% in 1988. The rise has almost entirely been in market held debt while the debt held by the central bank has remained flat at around 13-14% of GDP.

The primary deficit, after rising until 1985, declined in the subsequent two years to attain 3,1% of GDP in 1988.

The "self-perpetuating" element of debt growth, the interest rate-growth rate differential, was clearly negative in 1980-81, virtually zero in 1982-84 and positive in the final four years. Therefore, in the period 1985-88, the debt has been growing on a path of dynamic unsustainability.

The Treasury seigniorage has shown an erratic behaviour : since the seigniorage tax base,  $m$ , and  $l_m$  have been virtually constant over the period, the whole variability can be attributed to the money growth element,  $G_m$ , of the seigniorage tax rate.

At the end of May 1989, the Government presented an updated version of the medium-term plan of fiscal adjustment (<sup>8</sup>) which aims at stabilizing the debt-GDP ratio by 1992 (see Table 2). The objective is to be met through an adjustment of the primary balance of 0,9% of GDP per year over three years, to reach a small primary surplus of 0,6% of GDP in 1992. The interest rate-growth rate differential is expected to remain positive between 2 and 3%.

As a result, the public sector borrowing requirement falls from 11,4% of GDP in 1989 to 7,6% of GDP in the final year. The public sector debt is stabilized at around 106% of GDP.

No explicit assumptions are made on monetary policy. However, the programme points out that a rise in money financing is endangered by the increasing European monetary integration and by the forthcoming liberalization of short-term capital movements.

In the application of the model developed in section II, to Italy's debt stabilization problem, we assume as a starting point the situation of 1989 as it was expected by the Government in May 1989.

The initial rate of growth of money base,  $G_m(0)$ , which in the model corresponds to the rate of growth of nominal income, and the money-base-GDP ratio,  $m$ , are set at 9,0% and 13% respectively.

Therefore, given the assumption of a net interest rate on central bank holdings of Government debt,  $i_m$ , of 5,5%, these assumptions imply an initial seigniorage tax revenue of 0,5% of GDP.

Tables 3-6 present the results of the application of the linear adjustment model to Italy's debt stabilization under different assumptions about the interest rate-growth rate differential (1,0%, 2,0%, 3,0%) and the adjustment period (3, 7, 10 years). After deriving the linear equilibrium adjustment in the benchmark case,  $\bar{\alpha} = \delta$  (table 3), we examine the consequences of setting  $\bar{\alpha} = \delta/2$  on the monetary expansion under regimes C and FL (table 4), the primary surplus under regime ML (table 5), and the equilibrium level of debt under different regimes (table 6).

A summary of the results under  $\bar{t} = 3$ ,  $\bar{\alpha} = 0,9\%$ , as in the government adjustment plan, is presented in table 7.

Such a degree of adjustment would be sufficient to stabilize the debt-GDP ratio, under constant  $G_m$ , if the interest rate-growth rate differential does not exceed 1,0%.

Under higher (and, probably, more realistic) values of the differential, 2,0% and 3,0%,  $\delta$  becomes 1,2% and 1,5% respectively. If the fiscal authorities are willing to reduce the primary deficit by only 0,9% of GDP per year, debt stabilization requires a rise in the

money base growth (either at the beginning (C) or at the end of the period (FL)) or a final additional jump in the primary surplus which has to be increased by a further  $K = 0,9\%$  of GDP at the end of the period.

The lowest equilibrium level of the debt is attained under C : under  $r - y = 2,0\%$ , the "Jump" in the money base from  $9,0\%$  to  $15,6\%$  at the beginning of the period allows a "gain" in  $\bar{b}$  of  $1,3\%$  relative to the FL regime ( $93,3\% - 90,7\%$ ).

The results of the calculations under the assumption  $r - y = 3\%$  are shown in Figure 3.

Three interesting phenomena are highlighted by table 7 :

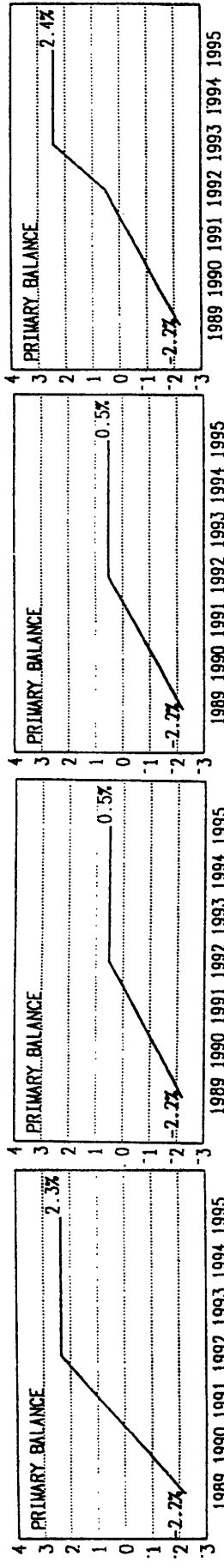
- I) Given a seigniorage tax base,  $m$ , of  $13\%$  of GDP, a non-negligible contribution to monetary authorities to the adjustment process, implies a substantial acceleration in the money base growth; the required higher  $G_m$  is very sensitive to the value of the interest rate-growth rate differential : under C and FL, a differential of  $3,0\%$  requires a jump in  $G_m$  which is  $13,6\%$  and  $14,8\%$ , respectively, higher than when the differential is  $1,0\%$ .
- II) Given the short adjustment period ( $\bar{t} = 3$ ), the steady state level of  $G_m$  is not very different under C and FL : if the adjustment is spread over a longer period, the difference between  $G^C$  and  $G^{FL}$  may become important (see table 4). Therefore, if 1992 remains the deadline for the stabilization of the debt-GDP ratio, the rise "as soon as possible" in  $G_m$  does not allow any relevant gain in terms of inflation in the final period.
- III) Although the overall adjustment in the primary balance is only slightly higher under ML, than under  $\bar{\alpha} = \delta$  ( $0,1\%$  of GDP under  $r - y = 3,0\%$ ), the eventual correction  $K$  could be substantial :



Fig. 3.

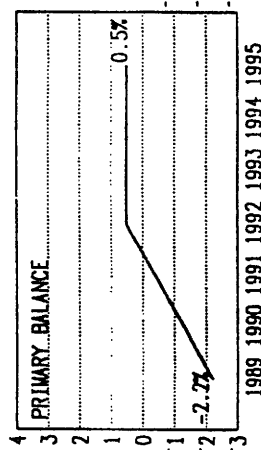
MONETARY LEADERSHIP

$\bar{\alpha} = 0.9\%, K > 0$



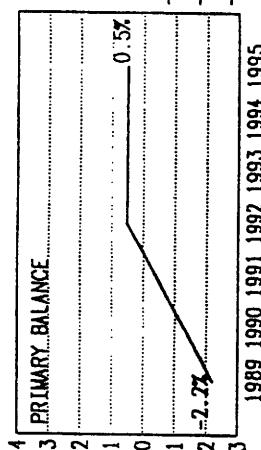
FISCAL LEADERSHIP

$\bar{\alpha} = 0.9\%, z^F > 0$



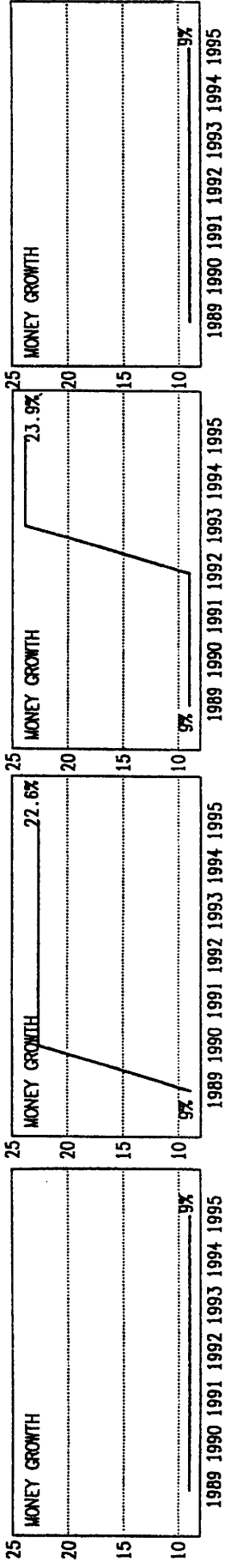
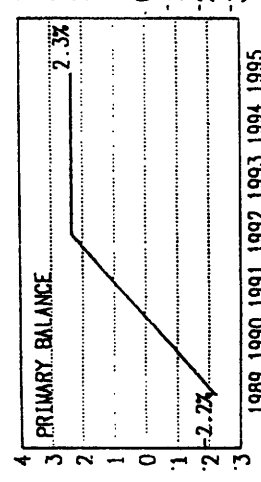
COOPERATION

$\bar{\alpha} = 0.9\%, z^C > 0$



BENCHMARK CASE

$\bar{\alpha} = 1.5\%$



given  $\pi = 0,9\%$ , the eventual adjustment in the primary balance is  $1,9\%$  of GDP under  $r - y = 3,0\%$ . Therefore, relatively higher fiscal adjustment year by year could allow the government to avoid a "cold turkey" correction in the primary balance at the end of the period (9).

#### V. GOING BEYOND THE ARITHMETIC

The paper develops a simple method for calculating the annual adjustment in the primary balance necessary in order to stabilize the debt-income ratio at a certain time in the future, given the interest rate-growth rate differential and a quantitative money demand function.

Three "extreme" policy regimes are singled out : co-operation between monetary and fiscal authorities, in which the distribution of the adjustment is jointly determined and the monetary authorities raise the money base growth at the beginning of the period; fiscal authority leadership, in which the adjustment in the primary balance is lower than that needed to stabilize the debt-GDP ratio and monetary policy eventually gives in by making the remaining part of the adjustment; monetary authority leadership, in which the primary balance, if the adjustment is not completed, has to undergo a further correction.

The analysis in section II is based on the assumption of a fixed interest rate-growth differential. Such an over-simplification has to be kept in mind in assessing the numerical computations presented in section III.

Some of the consequences of relaxing the assumption of exogeneity of the differential have been explored by Galili (1985) and Masson (1985).

In a full employment model with consumption as a function of real wealth and quantitative money demand, Gallí (1985) shows that, if (i) the Ricardian equivalence does not hold, i.e. real wealth includes government debt, and (ii) government debt and real assets are imperfect substitutes, the interest rate-growth rate differential is a positive function of the stock of government debt: a higher debt implies a higher risk premium; moreover, the rise in consumption, brought about by the higher real wealth, decreases the rate of growth of the economy.

Analogous results are obtained by Masson (1985) in an overlapping-generations model with uncertainty about government deficit financing. In such a model, a rising debt, by affecting the subjective probability of future monetization, leads to a rise in government's real borrowing costs which feed into the deficit and bring forward the prospects of debt unsustainability.

Both models point to the conclusion that the simple requirement of a negative differential at a point in time is not enough to assess the sustainability of a set of government policies.

If, e.g., we assume that the interest rate-growth rate differential is a linear function of the level of public debt :  
 $r - y = -a + cb$ , the non-linear differential equation (2) has two stationary equilibria if  $a^2 + 4c[f + (g_m - i_m)m] > 0$ . Therefore, the stable case corresponds to "small" seigniorage-corrected primary deficits  $-f - (g_m - i_m)m < a^2/4c$ , while in the case of "large" deficits, the system is unstable.

In the case of the implementation of a credible fiscal adjustment plan, the fall in the risk premium in interest rates has to be weighted against the negative Keynesian demand impact produced by the lower budget deficit.

In the medium run, the fiscal consolidation is likely to entail a permanent reduction of the differential. In the short run, however, the net effect depends on many factors, e.g. the availability of consumption-smoothing instruments, the expectations of consumers on the "inevitability" of the fiscal adjustment, the confidence effect on investment decisions, etc.

The short run effect is important because a possible unfavourable impact on the differential may undermine the political sustainability of the fiscal adjustment.

In the model developed in Section II, the effectiveness of monetary authorities in contributing to the adjustment depends on the level of the seigniorage tax base, i.e. the money base-income ratio: the lower the value of the seigniorage tax base, the higher the rise in the growth rate of money base to extract a given level of seigniorage.

Given the current level of the money base - income ratio in the main industrial countries, if the central bank retains a broad target of monetary stability, any plan of debt stabilization is bound to be (almost) entirely carried out by the fiscal authorities.

Even in the case of Italy, where the money base-GDP ratio is considerably higher than that in the other industrial countries, the rise in the growth rate of the money base, either at the beginning or at the end of the period, necessary in order to increase the level of seigniorage non-negligibly is likely to conflict with the requirement of monetary stability. Furthermore, it is doubtful that, in Italy's case, even the present level of seigniorage could be maintained in the near future (10):

- 1) the abolition of capital controls in a context of (almost) fixed exchange rates implies a loss of autonomy of monetary

policy : the need for a further convergence of the inflation rate toward the average of the EC could require stricter monetary discipline;

- ii) the liberalization of capital movements in 1990 and the single market in 1992 will stimulate competitiveness and financial innovation in Italy's repressed financial system and require a lower reserve coefficient; these effects are likely to cause a drop in the money base-GDP ratio toward a level typical of other industrial economies.

Therefore, if the debt stabilization target is to be achieved, Italy's fiscal authorities would need to pursue a stronger adjustment in the primary balance so as to offset the lower level of the seigniorage revenue.

In the above model, the only contribution that monetary authorities can give to fiscal consolidation is through rising the seigniorage tax revenue. However, it has been shown <sup>(11)</sup> that in a model of an open economy with free capital mobility, monetary authorities can, through a once-for-all devaluation, reduce the interest rate on government debt. Such a strategy will result in a partial implicit repudiation of public debt.

A devaluation-induced inflation can be effective in the stabilization of public debt provided that :

- a) no further devaluation is expected, and
- b) a virtuous fiscal behaviour is credibly implemented.

The favourable effect devaluation on the debt dynamics is twofold:

- i) a once-for-all devaluation, by restoring the trust in the currency, entails a reduction in nominal interest rates;

- ii) the induced inflation allows a further temporary reduction in the interest rate-growth rate differential. Moreover, if prices are not perfectly flexible, the devaluation gives rise to a temporary real depreciation which may offset some of the demand effects of the adjustment.

However, if after the devaluation the "permanent" differential remains positive, the primary balance must eventually show a surplus.

The above considerations bring the issue of credibility of fiscal authorities. An essential pre-condition to minimise possible output losses and to guarantee, in the case of a fiscal package-cum-exchange rate devaluation, a permanent reduction in the differential, is a fiscal policy fully committed to public finance consolidation. Without such a commitment, no cooperation (vs. conflict) scenario can prevent the system to run into the "debt unsustainability trap".

#### APPENDIX

##### The Determination of the Equilibrium Adjustment

The solutions of equation (13) and (14) are, respectively :

$$(A1) \quad b(t) = \frac{f(0) + [G_m(0) + Z^c - i_m]m}{r-y} + \frac{\alpha}{(r-y)^2} + \frac{\alpha}{r-y} t +$$

$$+ \left\{ b(0) - \frac{f(0) + [G_m(0) + Z^c - i_m]m}{r-y} - \frac{\alpha}{(r-y)^2} \right\} e^{(r-y)t}$$

for  $t < \bar{t}$

$$(A2) \quad b(t) = \frac{f(0) + \alpha \bar{t} + k + [G_m(0) + Z^C + Z^F - \dot{L}_m]m}{r-y} +$$

$$+ \left[ b(\bar{t}) - \frac{f(0) + \alpha \bar{t} + k + [G_m(0) + Z^C + Z^F - \dot{L}_m]m}{r-y} \right] e^{(r-y)(t-\bar{t})}$$

for  $t \geq \bar{t}$

The stabilization of  $b(t)$  at the level attained at  $t = \bar{t}$  implies :

$$b(\bar{t}) = \frac{f(0) + \alpha \bar{t} + k + [G_m(0) + Z^C + Z^F - \dot{L}_m]m}{r-y}$$

(A3)

By substituting  $b(\bar{t})$  from (A1) and solving for  $\alpha$ , we obtain :

$$(A4) \quad \alpha = \gamma - \theta Z^F - \varepsilon Z^C - \mu k$$

where :

$$\gamma = \frac{(r-y) \{ f(0) + m[G_m(0) - \dot{L}_m] - b(0)(r-y) \} e^{(r-y)\bar{t}}}{1 - e^{(r-y)\bar{t}}} > 0$$

$$\theta = \frac{m(r-y)}{e^{(r-y)\bar{t}} - 1} > 0$$

$$\mu = \frac{r-y}{e^{(r-y)\bar{t}} - 1} > 0$$

$$\varepsilon = \frac{(r-y)m e^{(r-y)\bar{t}}}{e^{(r-y)\bar{t}} - 1} > 0$$

If fiscal authorities carry out an annual adjustment  $\bar{\alpha} < \gamma$ , then :

$$z^F = \frac{\gamma - \bar{\alpha}}{\theta} \quad \text{under Fiscal Leadership}$$

$$z^C = \frac{\gamma - \bar{\alpha}}{\varepsilon} \quad \text{under Cooperation}$$

$$K = \frac{\gamma - \bar{\alpha}}{\mu} \quad \text{under Monetary Leadership.}$$

If monetary authorities follow the rule  $G(t) = G_m(0) + \beta t$ , equations (A1) and (A2) become, respectively :

$$(A5) \quad b(t) = \frac{f(0) + [G_m(0) - i_m]m}{r-y} + \frac{\alpha + \beta m}{(r-y)^2} + \frac{\alpha + \beta m}{r-y} t + \\ + \left\{ b(0) - \frac{f(0) + [G_m(0) - i_m]m}{r-y} - \frac{\alpha + \beta m}{(r-y)^2} \right\} e^{(r-y)t} \\ \text{for } t < \bar{t}$$

$$(A6) \quad b(t) = \frac{f(\bar{t}) + [G_m(\bar{t}) - i_m]m}{r-y} + \left\{ b(\bar{t}) - \frac{f(\bar{t}) + [G_m(\bar{t}) - i_m]m}{r-y} \right\} e^{(r-y)(t-\bar{t})} \\ \text{for } t \geq \bar{t}$$

By following the above procedure, we obtain the linear equilibrium adjustment :

$$(A7) \quad \alpha = \gamma - m\beta$$

If monetary authorities are willing to raise  $G_m$  by  $\beta$  each year, the equilibrium  $\beta$  corresponding to  $\alpha = \bar{\alpha}$  is  $\beta = \frac{\gamma - \bar{\alpha}}{m}$ .



SYMBOLS

- b : market held public debt (% of GDP)  
f : primary deficit (-) or surplus (+) (% of GDP)  
m : money base (% of GDP)  
r-y : Interest rate-growth rate differential  
 $l_m$ : Institutionally fixed nominal interest rate on central bank  
holdings of public debt  
 $G_m$ : rate of growth of money base  
x :  $\partial x / \partial t$   
t : adjustment period  
: annual adjustment in f  
: annual adjustment in f which allows the stabilization of b at  $\bar{t}$ , given  $G_m(t) = G_m(0)$ .  
 $z^C$ : rise in  $G_m$  at the beginning of the period  
 $z^F$ : rise in  $G_m$  at the end of the period  
K : rise in the primary surplus at the end of the period  
(C : cooperation ; FL : fiscal leadership; ML : monetary leadership)  
Values of the parameters used for the calculation of Tables 3-7 :  
b(0) = 86,7%; f(0) : -2,2%; m = 13,0%;  $G_m(0) = 9,0%$ ;  $l_m = 5,5%$ ;  
 $\bar{t} = 3, 7, 10$ ; r-y = 1,0%, 2,0%, 3,0%.

Table 1

## The growth of public debt in Italy : relevant parameters

	1980	1981	1982	1983	1984	1985	1986	1987	1988
Public sector debt (% of GDP)	59,0	61,1	66,4	72,0	77,2	84,0	88,5	92,9	96,1
- of which market held debt (b)	45,3	46,8	52,0	59,4	64,4	69,2	73,9	78,9	83,1
Interest rate - growth rate differential ( $r - y$ )	-12,8	-4,9	-1,0	-0,4	-0,2	1,3	2,4	1,7	0,7
Primary balance (f)	-4,4	-5,4	-6,9	-6,8	-5,9	-6,5	-3,6	-3,5	-3,1
Growth rate of Treasury money base ( $G_m$ )	22,4	25,6	18,2	1,2	16,6	29,5	8,9	5,4	1,7
Nominal interest rate on Bank of Italy Government debt ( $i_m$ )	5,6	5,2	5,3	4,3	5,6	5,3	5,3	5,5	5,5
Money base-GDP ratio (m)	13,7	14,3	14,4	12,6	12,8	14,8	14,6	14,0	13,0

Table 2

Targets of the medium-term public finance programme presented by the Italian Government in May 1989

	1988	1989	1990	1991	1992
Primary deficit (-) of surplus (+)	-3,1	-2,2	-1,2	-0,3	0,6
Interest burden	8,4	9,2	9,4	8,8	8,2
Public sector borrowing requirement	11,5	11,4	10,6	9,1	7,6
Total public sector debt (b + m)	96,1	99,7	103,0	105,5	106,2
Interest rate-growth rate differential <sup>(1)</sup>	0,7	2,1	2,8	2,8	2,7

(<sup>1</sup>) Calculated from the official programme; the interest rate is the average cost of market-held debt, calculated by assuming  $m = 13,0\%$  and  $i_m = 5,5\%$ .

Source : Documento di programmazione economico-finanziaria, May 1989

Table 3 :

Linear adjustment in the primary balance to be carried out from 0 to  $\bar{t}$ , to stabilize  $b(t)$  at  $\bar{t}$ , given  $G_m = G_m(0) : \gamma, (f(\bar{t}))$

$\bar{t}$	3	7	10
$r - y$			
1,6	0,9 (0,45)	0,4 (0,50)	0,3 (0,54)
2,0	1,2 (1,38)	0,5 (1,53)	0,4 (1,64)
3,0	1,5 (2,34)	0,7 (2,62)	0,5 (2,83)

Table 4 :

Rise in the money base growth to stabilize  $b(t)$  at  $t = \bar{t}$ , when the annual adjustment carried out by fiscal authorities is half of the equilibrium one :  $(\bar{\alpha} = \gamma/2), z^C, z^F$

$\bar{t}$	3			7			10		
$r - y$									
1,0	(0,44)	10,0	10,4	(0,19)	10,0	10,8	(0,14)	10,0	11,1
2,0	(0,60)	13,4	14,2	(0,27)	13,4	15,4	(0,19)	13,4	16,3
3,0	(0,76)	16,7	18,3	(0,34)	16,7	20,6	(0,25)	16,7	22,6

Table 5 :

"Jump" in the primary balance at  $t = \bar{t}$ , needed to stabilize  $b(t)$ , when  $\bar{\alpha}$  is half of the equilibrium adjustment and  $G_m = G_m(0) :$

$(\bar{\alpha} = \gamma/2), K, f^K(\bar{t}), (f^*(\bar{t}))$

$\bar{t}$	3			7			10		
$r - y$									
1,0	(0,44)	1,3	0,5 (0,5)	(0,19)	1,4	0,6 (0,5)	(0,14)	1,4	0,6 (0,5)
2,0	(0,60)	1,8	1,4 (1,4)	(0,27)	2,0	1,7 (1,5)	(0,19)	2,1	1,8 (1,6)
3,0	(0,76)	2,4	2,4 (2,3)	(0,34)	2,7	2,9 (2,6)	(0,25)	2,9	3,2 (2,8)

Table 6:

Equilibrium level of the debt-GDP ratio, under equilibrium adjustment,  
 ( $b^* \mid \bar{\alpha} = \delta$ ) cooperation ( $bC \mid \bar{\alpha} = \delta/2$ ) fiscal or monetary leadership,  
 ( $bFL = bML \mid \bar{\alpha} = \delta/2$ )

$\bar{t}$	3			7			10		
$r - y$									
1,0	90,6	88,7	92,6	95,9	91,3	100,8	100,0	93,3	107,1
2,0	92,0	89,3	94,7	99,2	92,9	106,0	104,7	95,7	114,9
3,0	93,3	90,0	96,8	102,4	94,6	111,5	109,5	98,1	123,4

Table 7

Public debt stabilization under alternative policy regimes ( $\bar{t} = 3$ )

	Benchmark case	Cooperation	Fiscal leadership	Monetary leadership
	$\bar{\alpha} = \gamma, G_m = G_m(0)$	$\bar{\alpha} = 0.9, Z^C \geq 0$	$\bar{\alpha} = 0.9, Z^F \geq 0$	$\bar{\alpha} = 0.9, K \geq 0$
r - y	f(3) $G_m(3)$ b(3)	f(3) $G_m(3)$ b(3)	f(3) $G_m(3)$ b(3)	f(3) $G_m(3)$ b(3) K
1,0	0,5 9,0 90,6	0,5 9,0 90,6	0,5 9,0 90,6	0,5 9,0 90,6 0,0
2,0	1,4 9,0 92,0	0,5 15,6 90,7	0,5 16,0 93,3	1,4 9,0 93,3 0,9
3,0	2,3 9,0 93,3	0,5 22,6 90,7	0,5 23,8 96,2	2,4 9,0 96,2 1,9

Footnotes

- (1) See, e.g., Bulter (1985), Cividini-Galli-Masera (1987), Galli-Masera (1988), Masera (1987), Spaventa (1987).
- (2) Some of the implications for debt sustainability of relaxing the assumption of a fixed interest rate-growth rate differential will be explored in the final section.
- (3) Other monetary rules, e.g. a constant share of the overall deficit is money financed, have been explored in the literature. For a detailed discussion, see Cividini-Galli-Masera (1987).
- (4) The term  $1 + \bar{t}(r - y) - e^{(r - y)\bar{t}}$  is negative.

Lets call  $\bar{t}(r - y) = x$  and rearrange :  $1 + x < e^x$ . By taking the logarithm of both sides we have :  $\ln(1 + x) < x$  which is always true, in that  $\ln(1 + x)$  lies below the 45 degree line for any value of  $x$ .

- (5) The term  $1 - e^{(r - y)\bar{t}}(1 - \bar{t}(r - y))$  is positive. As in footnote (4) lets call  $\bar{t}(r - y) = x$ , rearrange and take the logarithm :  
 $x < -\ln(1 - x)$  which is true for any value of  $x$ .
- (6) See footnote (4).
- (7) Several authors have used "fixed-parameter" analysis to investigate the growth of public debt in Italy. See, e.g., Cividini-Galli-Masera (1987), Masera (1987), Rossi-Salvemini (1987), Spaventa (1984, 1987). For a comprehensive set of papers on the consequences of high public debt in the Italian experience, see Giavazzi-Spaventa (1988).

- (8) See Documento di programmazione economico-finanziaria, May 1989.
- (9) Here we implicitly assume that the expected lifetime of the Government is longer than  $\bar{t} = 3$ . Since the average lifetime of Government in Italy is only a few months, the difficulty in pursuing a stronger adjustment could be easily explained.
- (10) For a recent analysis of the behaviour of the seigniorage tax in Italy during the period 1976-1987, see Bruni-Penati-Porta (1988). Analyses of the seigniorage tax revenue in several countries can be found in Giavazzi (1989), Drazen (1989), and Grilli (1989).
- (11) See Buti (1990), Calvo (1989).



## References

1. Blanchard, O.J. (1984), Current and Anticipated Deficits, Interest Rates and Economic Activity, *European Economic Review*, 25.
2. Bruni, F. (1987), *Debito Pubblico e Politica Economica In Italia*, SIPI.
3. Bruni, F., Penati, A., Porta, A. (1988), Financial Regulation, Implicit Taxes and Fiscal Adjustment in Italy, presented at the Conference on Fiscal Policy, Economic Adjustment and Financial Markets, IMF-Università Bocconi, January.
4. Bulters, W.H. (1985), A Guide to Public Sector Debt and Deficits, *Economic Policy*, 1.
5. Bulters, W.H. (1985), The Current Global Economic Situation, Outlook and Policy Options, with Special Emphasis on Fiscal Policy Issues, *CEPR Discussion Paper n. 210*, November.
6. Buti, M. (1990), Implicit Repudiation of Public Debt, Brussels, March, mimeo.
7. Calvo, G.A. (1989), Is Inflation Effective for Liquidating Short-term Nominal Debt?, *IMF Staff Papers*, 36.
8. Cividini, A., Galli, G., Masera, R.S. (1987), Vincolo di bilancio e sostenibilità del debito : analisi e prospettive, in F. Bruni (1987).
9. De Cecco, M., Giovannini, A. (1989), A European Central Bank?, CUP.
10. Drazen, A. (1989), Monetary Policy, Capital Controls and Seigniorage in an Open Economy, in M. de Cecco, A. Giovannini (1989).
11. Eitis, W., Sinclair, P. (1988), *Keynes and Economic Policy*, Macmillan.
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