CAPITAL/LABOUR SUBSTITUTION
AND ITS IMPACT ON EMPLOYMENT

Fabienne Ilzkovitz*
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AND ITS IMPACT ON EMPLOYMENT

Fabienne Ilzkovitz*

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  Financial Affairs, Commission of the European Communities.

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I. INTRODUCTION

In 1986, the average unemployment rate in the European Community was 11.9%, considerably higher than the 6.9% observed in the United States and the 2.8% in Japan. Despite the improved economic climate, unemployment has remained at an unacceptably high level in the Community, and everything must be done to reduce it. Dissatisfaction with or constraints concerning traditional policies (demand expansion, wage moderation) have led some economists to propose more structural policy measures. In order to obtain growth with a greater employment content, they have proposed slowing down the rate at which capital is substituted for labour. Many factors may be responsible for this substitution process: accelerating technical progress, changes in the organization of work, specialization in new activities and the cost of capital relative to that of labour. Among these factors, the link between substitution and factor costs is frequently highlighted, since it is easier to identify. It is therefore suggested that the substitution process can be altered by influencing factor costs. Is such a policy appropriate and feasible? What are the likely effects on employment? Which method of influencing relative cost should be used in order to obtain the most positive effect on employment? These are questions which are tackled in this paper.

To this end, it is necessary to verify the existence of a substitution effect. We must therefore first examine the scope for substitution between capital and labour. Then we have to determine whether the empirical analysis confirms the hypothesis that relative factor costs affect the rate of substitution of capital for labour. Finally, it is necessary to establish whether a consensus is emerging as to the importance of this effect and of its impact on employment.

Before reviewing the empirical work carried out, it is necessary to define a theoretical framework for studying the substitution process and its dependence on factor costs. This framework and the main concepts linked to the substitution process are examined in Part I. Then, in Part II, an attempt is made, on the basis of empirical investigations, to determine whether there is a substitution effect and how it is linked with factor costs.
II. THEORETICAL APPROACH

The aim of Part I is to present theoretical concepts linked to the substitution between factors of production, which will make it easier to interpret the empirical work. First of all, the problem of measuring substitution will be discussed. Secondly, we shall define the substitution effect. We shall also show that the impact of factor costs - and therefore the substitution effect - varies according to economic situations and production technology. Finally, as substitution is studied here in relation to factor costs, the difficulties of finding a correct formulation for the cost of capital will be tackled.

1. Measurement of capital/labour substitution

Where a firm can achieve the same output with different combinations of factors of production, it may substitute one factor for one or more others. Economists have been particularly concerned with the substitution of capital for labour. However, other types of substitution, involving energy or raw materials, are also possible.

The substitution of capital for labour therefore corresponds to an increase in the relative importance of capital compared with labour in the course of the production process. It is measured by the variation in the capital stock per unit of employment. This method of measurement raises a number of highly controversial questions. Those questions are commented on briefly here.

First of all, there is the problem of valuing the capital stock. This valuation is generally based on the perpetual inventory method, which consists of estimating the stock in a base year from past investment and increasing that by net investment. While the flows of investment in new plant and machinery are well known, this is not the case with plant and machinery which is scrapped. In order to determine the latter, the perpetual inventory method assumes that the retirements of capital assets are stable over time. This assumption is open to criticism, since
retirements may accelerate as a result of technological progress, increased bankruptcies or higher energy prices, which reduce the profitability of plant and machinery with a high consumption of energy. However, the reverse may also occur where there is a slowdown in the rate of growth of real wages or energy prices. In that case, old plant and machinery requiring more labour and/or energy than their new counterparts may remain profitable.

Secondly, it is necessary to examine the need to adjust the substitution measure to take account of the variation in working time of capital and labour. Such an adjustment would make it possible to identify the substitution not connected with changes in the organization of work. A reduction in working hours may encourage firms to take on additional staff without that involving greater use of the factor labour in terms of the number of hours worked. The net effect on the capital-labour ratio adjusted in this way therefore depends on whether the working time of capital\(^1\) is reduced like the working time of labour or whether the reduction in the latter is accompanied by a reorganization which increases shiftwork in such a way that the working time of plant and equipment does not vary.

2. **Substitution effect**

The substitution effect indicates the extent to which the variation in the relative cost of two factors affects the demand for one of these factors: for example, the extent to which a variation in the relative cost of labour and capital influences demand for labour (substitution effect in an employment function) or demand for capital (substitution effect in an investment function).

The link thus established between demand for a factor and its cost relative to that of another factor is derived from the hypothesis that firms which seek to maximize their profits vary their production techniques

\(^1\) The working time of capital is determined by the working time of labour and by the level of shiftwork /17/. 
it being necessary for the factors to be substitutable — according to relative factor costs. For example, if the cost of labour increases by 1% in relation to the cost of capital and if the elasticity of demand for labour with respect to relative labour/capital cost is -0.2, employment falls by 0.2%, with firms turning towards more capital-intensive production techniques.

The substitution effect is studied by estimating equations of demand for factors involving cost variables. For there to be a substitution effect, it is necessary for the elasticity of demand for a factor with respect to its cost to be negative and for the elasticity of demand for a factor with respect to the cost of the other factor to be positive.

The following sub-sections show that the substitution effect depends on:
- hypotheses concerning the production technology;
- constraints perceived by firms on different markets;
- the measurement of relative factor costs (and in particular of the user cost of capital).

2.1 Substitution effect and production technology

The substitution effect is linked to the characteristics of production technology. It increases with the possibilities of substitution between factors of production. Such possibilities are measured by their elasticity of substitution, a concept which most directly reflects the technical constraints inherent in production processes.

The substitution effect will also vary depending on whether the factors of production are substitutable only at the time when equipment is purchased (ex ante substitution and putty-clay technology) or at the time of purchase and throughout the working life of the equipment (ex post

2) A more technical description of the elasticity of substitution is provided in Annex I.
substitution and putty-putty technology). In the first case, a change in relative prices will affect only the choice of production technique for the new equipment, whereas in the second case it may have an impact on the entire capital. It follows that weak elasticity in the demand for a factor with respect to relative costs (a weak substitution effect) may either reflect limited scope for substitution between factors (a weak elasticity of substitution) or result from a lack of flexibility of capital combined with a high degree of substitution relating only to new equipment (putty-clay hypothesis).

Lastly, even if the production factors are complementary (clay-clay technology), capital-labour substitution may take place through the rate at which equipment is retired, where this rate varies in line with the real wage. In this case, an increase in the real wage has the effect of accelerating the retirement of equipment by making the oldest vintages unprofitable, and if the new equipment is more productive than the equipment withdrawn, employment decreases. It is thus possible to ensure the same level of production with newer equipment and fewer jobs. Measuring this substitution means that the rate of retirement (or the actual life of the equipment) must be endogenized as a function of the real wage. This endogenization raises difficulties due in particular to the lack of statistical data, and the results so far obtained must be treated with caution.

2.2. Substitution effect under different economic regimes

Strictly speaking, for there to be a substitution effect, it is necessary that an increase in the relative cost of capital, for example, leads the firm to substitute labour for capital. Such an effect does not appear in all the regimes\(^3\) in which a firm may find itself. Thus in a repressed inflation situation, this effect cannot emerge since employment is constraint by the supply of labour.

\(^3\) The terminology used is that of the disequilibrium models. The three situations are as follows:
- Classical unemployment: excess supply of labour and demand for goods;
- Keynesian unemployment: excess supply of labour and goods;
- Repressed inflation: excess demand for goods and labour (and therefore full employment).
In a classical unemployment situation\(^4\), the substitution effect is more than offset by the "variation in the volume of production" effect. The latter is due to the fact that firms without a sales constraint may choose the level of production which maximizes their profit. In this case, an increase in the cost of one factor has the effect of reducing profitable production capacity, which depresses the demand for the two factors (and therefore also the demand for the factors whose relative cost has diminished) unless the substitution possibilities are very great (elasticity of substitution \(> 1\)). Consequently, in such a regime, it is necessary to bring down the global costs of production rather than the cost of labour relative to capital in order to stimulate employment.

Finally, only the Keynesian unemployment regime\(^4\) is always compatible with a substitution effect in the strict sense: in this regime firms with a sales constraint vary their production technology and hence their demand for factors in line with relative factor cost. In such a situation, factor demand also grows in line with anticipated sales.

2.3 **Substitution effect and relative factor costs**

The substitution effect is based on the hypothesis that the choice of production techniques is influenced by the relative factor costs, i.e. in the case of two factors, by the ratio of the user cost of capital to the cost of labour.

The capacity of models for assessing this substitution effect, or more precisely for assessing the impact of measures designed to change relative capital/labour costs, is thus linked to the accuracy with which factor costs are measured. The cost of labour is generally defined as the direct wage cost plus social security contributions and other taxes linked to labour. The theoretical definition of the user cost of capital is more complex. Let us define this concept.

\(^4\) See Annex II for the precise expressions of factor demand.
The user cost of capital is the cost of holding a unit of capital per unit of time\(^5\). It depends on the purchase price of equipment and on its expected inflation rate, which allows account to be taken of capital gains, of the financial cost of the capital, of depreciation cost, and of the tax legislation applicable to investment. Such tax legislation includes in particular the rate of tax on profits (corporation tax rate), investment allowances or credits, the tax deductibility of physical depreciation, and the tax deductibility of interest charges.

Problems arise when appropriate empirical measurements have to be found for the financial cost of the capital and the capital gains expected and in taking account of the tax system specific to investment. Very often, it is assumed for the sake of simplicity that the capital market is perfect and expectations certain. The financial cost is then taken to be the long-term market interest rate and capital gains to be the observed growth rate in the price of equipment goods or indeed the observed inflation rate.

Some analyses do not apply these assumptions and present more sophisticated formulations of the user cost of capital. Thus, two indicators of the financial cost are proposed where the perfect financial market hypothesis is abandoned. The first corresponds to the discount rate at which the present value of expected future earnings during the life-time of an investment project (approached by current profits or a distributed lag function for present and past profits) equals the market value of securities. The second is a weighted average of the marginal cost of three sources of financing, namely debt, new equity and retained earnings. An illustration of these two measurements is given in /24/. Furthermore, if one removes the hypothesis of the certainty of expectations, the expectation function of firms must be stipulated. This is often represented by a distributed lag model.

\[ c = P_k (f + \delta - \frac{P_k}{\frac{P_k}{P_k}}) \]

where \( P_k \) = the post-tax price of new investment;
\( f \) = the post-tax cost of finance
\( \delta \) = the rate of physical depreciation of capital.

\(^5\) The following is a possible theoretical formulation of the user cost of capital (c) /21/.
Measurement of the influence of taxation on the user cost of capital is also highly simplified in empirical works. Often a synthetic measurement is applied which does not allow a distinction between the different effects which taxation may have on the cost of capital. However, the formulation of the tax system within the cost of capital must be particularly precise if the impact of tax changes on the relative factor costs is to be measured.

III. REVIEW OF EMPIRICAL WORK

Empirical work regarding substitution can be of several kinds. First, a distinction has to be made between studies, depending on whether or not they are based on a production function. Only those studies which stipulate assumptions relating to the production function make it possible to measure the elasticity of substitution. In general, the latter is derived from estimated investment or employment functions, or from the simultaneous estimation of these two functions. However, even when the substitution effect is the main point of interest, it is desirable, in empirical work, for a link with a production function to be established. This link, even if only implicit, makes the interpretation of the coefficients easier, and also enables their economic likelihood to be tested.

A second distinction can be made, depending on whether the substitution effect is estimated from a single equation (employment or investment) or from a complete macro-economic model. The results obtained from each approach are not comparable. In the first case, the coefficient of the relative cost variable measures its direct effect on employment or investment, with the other factors being maintained constant. In the second case, simulations with models do not give results which can be so easily interpreted, as they describe the direct and indirect effects of a change in relative costs. A reduction in wages and salaries, for example, will have effects on demand, profits, competitiveness and prices, depending on the particular structure of the model considered. In addition, the resulting change in employment is due to the interaction of all these effects, and not the result of the substitution effect alone. This also explains why the results from a model will vary according to which component of the relative cost variable is altered: thus a fall in
the real wage will not have the same impact on employment (and investment) as an increase in the user cost of capital, even if the relative factor cost variable changes by the same amount in both cases.

The critical analysis of Bureau and Norotte /14/ may be quoted by way of illustration. This study measures the effects on employment of a sustained 10% shock on the cost of capital relative to labour, first, solely by means of factor demand equations (non-linked model), and second, by means of a small model which enables account to be taken of the effects of Keynesian linkage, i.e. the expansionary effect induced by the increase in consumption and the depressive effect resulting from the fall in investment. In particular it shows that in the non-linked model, the effects on employment are greater in the short term than in the linked model, and that the converse is true in the long term.

The analysis of empirical work will group together, on the one hand, the estimates of the elasticity of substitution, and on the other hand estimates of the substitution effect. For the reasons given above, the results given will be those drawn from equations. Nonetheless certain results from models will also be commented upon.

1. Elasticity of substitution

The studies /4/, /7/, /8/ and /10/ consider a production function with two factors: capital and labour. They allow only capital-labour substitution to be examined. But in the past few years, a number of studies have been published which consider production functions comprising three or even four factors of production (see /15/ and /21/). The additional factors generally included are energy or raw materials, and energy and other inputs in the case of a production function comprising four factors.

The impact of the increase in raw material prices has been studied by Bruno and Sachs (see /12/ and /13/). The two authors conclude that this increase is responsible for the slowdown in the growth of labour productivity which occurred in the manufacturing industry of several industrialized countries in the 1970s. They show in particular that this adverse effect of raw material prices is connected with the substitution which took place between raw materials and capital and labour.
But it is energy which is usually included as the third factor of production. For the past ten years, energy prices have followed specific trends compared to the prices of the other factors, and it seems useful to be able to measure the impact of this. This is the case in the studies /3/, /8/, /9/, and /21/ which consider a production function with three factors of production, capital, labour and energy, of the following type

\[ Q = f(g(K,E), L) \]

with \( g \) a CES function and \( f \) a CES or Cobb-Douglas (CD) function.

This particular formulation implies an initial choice combining labour with the grouping capital-energy, then the combination of capital with energy. It is then possible to estimate the elasticity of substitution between labour and the capital-energy bloc and between capital and energy. It is also possible to define partial elasticities of substitution (Allen elasticities) which describe the influence of the cost of energy on investment (demand for capital) taking account not only of the substitution between capital and energy, but also of the substitution between capital-energy and labour. These Allen partial elasticities of substitution make it possible to determine whether capital and energy are, in total, substitutable or complementary.

1.1. Substitution between labour and the other factors of production

Table I shows that the elasticity of substitution between labour and the grouping capital-energy (\( \sigma_{L/KE} \)) is high, but in general is not as great as that required by the Cobb-Douglas function, and that it differs from country to country. Recent estimations by the OECD /21/ and the IMF /3/ (fourth and fifth columns of Table I) show that this elasticity lies between 0.6 and 1. However, the country values differ significantly as between the two studies. This is particularly the case for the United States where the elasticity of substitution is unity in the OECD study, but 0.55 in the study published by the IMF. It seems therefore that the estimates of the elasticity of substitution are not robust and are very
sensitive to the specification chosen. This is also the conclusion of the OECD: "The derived equations for production and factor demands revealed very little power to choose a value for the (...) elasticity of substitution. The evidence suggests that in most countries it could easily be as low as 0.65 or as high as 1.0, and that the choice has virtually no effects on the fit or the parameter values of the estimated production and factor demand equations" /21/. In these circumstances it is very risky to try and estimate the consequences of the flexibility or rigidity of wages and salaries on the demand for labour, since such consequences are very sensitive to different values of the elasticity of substitution.

A comparison of the elasticities of substitution obtained when using a putty-putty model or a putty-clay model is provided by a number of French studies (/4/, /8/, and /10/, seventh and eighth columns of Table I). In the putty-putty hypothesis, the change in technology concerns the entire capital stock but the elasticity of substitution is low (between 0.03 and 0.09). By contrast, in the putty-clay hypothesis, the change in technology affects only new equipment, but the elasticity of substitution is much greater (between 0.6 and 1). Despite these differences, the results lead to similar short term effects concerning the impact of a change in relative costs on investment. This shows that the elasticity of substitution alone does not permit a judgement of the short-term effect of relative cost changes. It is equally necessary to take into account the degree of flexibility of the capital stock.

1.2. Substitution between capital and energy

The results obtained by the OECD /21/ and the IMF /3/ (fourth and sixth columns of Table II) also differ with regard to the substitution between capital and energy (\( \sigma_{KJ} \)). Thus according to the OECD this elasticity lies between 0.3 and 0.9, depending on the country concerned, whereas according to the IMF the variation is between 0.2 and 0.4. The types of models used in these two studies may explain these differences.
Table I - Elasticities of substitution between labour and the other factors of production

<table>
<thead>
<tr>
<th>Country</th>
<th>/8/ and /9/</th>
<th>/9/</th>
<th>/21/</th>
<th>/3/</th>
<th>/7/</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>0,5</td>
<td>0,8</td>
<td>0,99</td>
<td>0,73*</td>
<td>0,7</td>
</tr>
<tr>
<td>France</td>
<td>1 *</td>
<td>1 *</td>
<td>0,80 *</td>
<td>0,63</td>
<td>0,7</td>
</tr>
<tr>
<td>Italy</td>
<td></td>
<td></td>
<td>0,80</td>
<td>0,79</td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>0,2</td>
<td>0,15</td>
<td>0,60 *</td>
<td>0,79</td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>1 *</td>
<td></td>
<td>1,01</td>
<td>0,85</td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td>1 *</td>
<td>1 *</td>
<td>1,01</td>
<td>0,55</td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>0,5</td>
<td>0,8</td>
<td>0,70 **</td>
<td>0,80</td>
<td></td>
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</table>

Elasticities between labour and capital-energy

<table>
<thead>
<tr>
<th>Country</th>
<th>putty-putty clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>putty-putty clay</td>
</tr>
<tr>
<td>France</td>
<td>putty-putty clay</td>
</tr>
<tr>
<td>Italy</td>
<td>putty-putty clay</td>
</tr>
<tr>
<td>UK</td>
<td>putty-putty clay</td>
</tr>
<tr>
<td>Canada</td>
<td>putty-putty clay</td>
</tr>
<tr>
<td>United States</td>
<td>putty-putty clay</td>
</tr>
<tr>
<td>Japan</td>
<td>putty-putty clay</td>
</tr>
</tbody>
</table>

Estimation period

|-----------|-----------|-----------|-----------|-----------|-----------|

Production functions

<table>
<thead>
<tr>
<th>putty-putty clay CES</th>
<th>putty-putty clay CES</th>
<th>putty CES</th>
<th>putty CES</th>
<th>clay CES</th>
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<tbody>
<tr>
<td>except*CD</td>
<td>except*CD</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In effect the IMF model is of the putty-putty type whereas the production function used by the OECD is of the putty/semi-putty type. Such a function allows for complete flexibility with regard to energy consumption for new equipment and a partial flexibility for equipment already installed. The underlying hypothesis is that part of the capital stock can be reconverted to benefit from the most up-to-date energy-saving techniques. In this case the production function includes a reconversion parameter (R) which defines the proportion of the capital stock which can be reconverted\(^6\)). This proportion (fifth column of Table II) is highest in Japan (0.68) and the United States (0.45) and lowest in Canada (0.05) and the United Kingdom (0.05).

According to the French studies analysed in the preceding section, greater capital flexibility is associated with a lower elasticity of substitution. It is therefore logical that the elasticities of substitution obtained from a putty-putty model are smaller than those obtained from a putty/semi-putty model. But, the elasticities estimated from the latter model should also be smaller than those obtained from putty-clay models. Yet, if the first two columns of Table II are compared with the third, we find that this is not the case. The greater values for the elasticity of substitution given in /21/ could be due to the different period for which they have been estimated, and they perhaps reflect an increase in the possibilities of substitution between capital and energy in more recent years. The perfecting of new of energy conservation techniques could be one of the cause of this phenomenon.

\(\text{KE} = (\text{KE}_{-1}) (1-\delta - R) + (\text{IB} + R\text{K}_{-1})(\text{KE}) \)

with \(\text{KE}\) : regrouping of capital stock by vintage and energy type
\(\delta\) : scrapping rate
\(R\) : reconversion parameter
\(K\) : capital stock
\(\*\) : optimal or desired
This is equivalent to the putty-clay model if \(R = 0\) and to the putty-putty model if \(R = 1-\delta\).

\(^6\) Thus:
Table II - Elasticities of substitution between capital and energy

<table>
<thead>
<tr>
<th>Country</th>
<th>$\sigma_{K/E}$</th>
<th>$R$</th>
<th>Partial Elasticsities (Allen)</th>
<th>Capital and energy in total are</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>0.60</td>
<td>0.63</td>
<td>0.5</td>
<td>0.29* - 0.07 complementary</td>
</tr>
<tr>
<td>France</td>
<td>0.37</td>
<td>0.16</td>
<td>0.8*</td>
<td>0.16 0.29 - 0.95 complementary</td>
</tr>
<tr>
<td>Italy</td>
<td></td>
<td></td>
<td>0.5</td>
<td>0.37 0.22</td>
</tr>
<tr>
<td>UK</td>
<td>0.41</td>
<td>0.20</td>
<td>0.3*</td>
<td>0.05 0.36 0.13 substitutable</td>
</tr>
<tr>
<td>Canada</td>
<td>0.66</td>
<td>0.9</td>
<td>0.9</td>
<td>0.05 0.24</td>
</tr>
<tr>
<td>United States</td>
<td>0.26</td>
<td>0.25</td>
<td>0.5</td>
<td>0.45 0.26 - 0.70 complementary</td>
</tr>
<tr>
<td>Japan</td>
<td>0.28</td>
<td>0.07</td>
<td>0.8**</td>
<td>0.68 0.29</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Production function</th>
<th>putty- clay CES</th>
<th>putty- clay CES</th>
<th>putty/semi- putty CES</th>
<th>putty- putty CES</th>
</tr>
</thead>
</table>


Finally, to determine whether capital and energy are substitutable or complementary it is possible either to consider Allen's partial substitution elasticities, or to compare \( \sigma_{L/E} \) with \( \sigma_{K/E} \). According to /8/ and /9/, \( \sigma_{K/E} \) has to be greater than \( \sigma_{L/E} \) if capital and energy are to be, in total, substitutable. From this point of view, the different studies seem to show that capital and energy are complementary in the majority of countries7). Dramais' study /18/ based on a production function comprising four factors (capital, labour, energy and non-energy intermediate inputs) and concerning manufacturing industry confirms these results for four European countries, France, Germany, Italy and the United Kingdom. A fall in the price of energy involves an increase in the demand for capital and energy to the detriment of labour, and this effect should be greater than the substitution effect between capital and energy itself so that, overall, a fall in the price of energy stimulates investment.

2. Substitution effect

The results regarding the substitution effect must also be examined with caution. The validity of these results depends chiefly on the accuracy with which the user cost of capital is defined and, in certain studies (/32/ for example), the indicators used for this concept are highly simplified. Moreover, empirical work sometimes establishes no more than a very vague link with the production function, so that it is difficult to interpret the estimated coefficients. Finally, the influence of factor costs can vary with the specification of the equation. Thus, in /30/ it is found that in an employment function which takes account only of cost variables (a real neo-classical model), the coefficient of these variables measures both the substitution effect and the effect of the change in production brought about by the change in costs. On the other hand, in a Keynesian employment function where demand plays a part, the elasticity of employment with respect to relative cost represents a "pure" substitution effect. This supports the conclusions in section 2.2 of the theoretical part.

7) Following the results obtained in /8/ and /9/ for UK and in /21/ for Japan, capital and energy would be substitutable in those countries.
There are problems in measuring the cost of capital, in the link with the production function, in isolating the substitution effect; clearly, anyone who wishes to study the impact of a change in relative costs on factor demand faces many obstacles. Nevertheless below we shall attempt to outline the main features which emerge from empirical work. To this end, we shall first analyse the substitution effect in investment functions, and secondly in employment functions.

2.1 **In investment functions**

Empirical studies show the existence of a substitution effect in investment functions. But when the relative cost variable is significant, its influence is small, especially in the short term, and is always less than the influence of expected demand. Moreover expected demand emerges as the determining factor in firms' investment decisions. Recent studies also point to the conclusion that investment is sensitive to profits which themselves reflect either the influence of expected profitability or a liquidity restraint. French studies /8/ and /19/ and Belgian studies /22/ indicate that response lags of investment may be influenced by profits. Moreover, estimates made in a British Treasury study /24/ indicate that the relative cost effect becomes insignificant if the investment function does not include a variable measuring firms' liquidity: "This result appears to stem from the 1974-75 episode when inflation tended to sharply reduce the cost of capital, but investment fell back quite a lot while company liquidity deteriorated".

The results of Table III show that it is not possible to assign a precise value to the elasticity of demand for capital with respect to relative costs. Elasticity varies particularly in keeping with the indicators used for the user cost of capital. Thus, the derivation of low - or even insignificant - relative cost elasticities may be due to errors in measuring the cost of capital. This is the conclusion of B. Dormont /20/ who, by the use of an estimation technique which eliminates the bias due to these measurement errors, shows a higher relative cost effect on factor demand in France and Germany. However, the large variation in elasticity with regard to relative cost in the two estimations can also be explained by the fact that the cost of capital proxy used by Dormont is not particularly precise (total financial costs related to long and medium term debt).
### Table III - Substitution effect in investment functions

<table>
<thead>
<tr>
<th>Studies</th>
<th>Country</th>
<th>Estimation period</th>
<th>Estimated relationship</th>
<th>Long term elasticity with respect to $\frac{w}{c}$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>/20/</td>
<td>Germany</td>
<td>1967-1977</td>
<td>Investment in industry</td>
<td>+ 0,01</td>
<td>+ 0,56</td>
</tr>
<tr>
<td></td>
<td>France</td>
<td>1967-1975</td>
<td></td>
<td>+ 0,02</td>
<td>+ 0,63</td>
</tr>
<tr>
<td>/4/</td>
<td>France</td>
<td>1965-1978</td>
<td>Total Investment</td>
<td>+0,17 + 0,43</td>
<td>These are results arising from various studies quoted in /4/</td>
</tr>
<tr>
<td>/32/</td>
<td>Germany</td>
<td>1960-1980</td>
<td>Investment in industry</td>
<td>No substitution effect</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Belgium</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>France</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Italy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Netherlands</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>UK</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>/24/</td>
<td>UK</td>
<td>1968-1984</td>
<td>Investment in manufacturing industry</td>
<td>+ 0,18</td>
<td></td>
</tr>
<tr>
<td>/22/</td>
<td>Belgium</td>
<td>1953-1979</td>
<td>Total Investment</td>
<td>+ 0,09*</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1963-1982</td>
<td></td>
<td>+ 0,86</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1953-1979</td>
<td>Investment in manufacturing industry</td>
<td>+ 0,60*</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1954-1982</td>
<td></td>
<td>+ 0,24*</td>
<td></td>
</tr>
</tbody>
</table>
The study by D. Weiserbs /32/ shows the advantage of estimating a single investment function for six Community countries. He also concludes that demand expectations are the main determinant of investment in most of the countries considered. The relative price effect plays a part in only three countries, France, the UK and Belgium. This result is surprising since other studies have shown the existence of such an effect in Italy /23/ and Germany /20/. In Germany, firms' substitution behaviour is even found to be equivalent to that in France. Here too, one may ask whether the relative cost variable used is valid. Weiserbs uses the ratio of investment prices to value-added prices as a relative cost indicator. His argument is that "When the distribution of value-added remains constant, an increase in this index of relative prices expresses a rise of the cost of equipment with respect to effective labour cost" /32/.

2.2 In employment functions

Most of the studies which attempt to show a substitution effect in employment functions have been carried out quite recently. Until the start of the 1970s, employment was mainly explained by demand, and empirical investigations failed to find any macro-economic relationship between employment and factor costs. Opinions are more divided today. Nevertheless, the substitution effect seems even weaker - and harder to identify - for employment than for investment.

Employment functions in which cost variables occur can be placed in two categories according to whether or not output appears in the equation. Models based upon an output constraint appear to give better results than those which take account only of cost variables. But Symons and Layard /29/, by introducing raw material prices, succeed in estimating a true neo-classical labour demand for six large OECD countries.

8) The functions considered here are labour demand functions. In other words, it is assumed that labour is never in short supply.

9) This result is consistent with theory. In fact, if firms are constrained by their outlets, the elasticity of the demand for labour with respect to relative capital/labour costs is equal to the product of the elasticity of substitution and the share of the cost of capital in value-added. On the other hand, the elasticity of the demand for capital with respect to relative capital/labour costs is equal to the product of the same elasticity of substitution and the share of wage costs in value-added.
Table IV - Substitution effect in employment functions

<table>
<thead>
<tr>
<th>Studies</th>
<th>Country</th>
<th>Estimation</th>
<th>Estimated relationship</th>
<th>Long term elasticity with respect to</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>period</td>
<td></td>
<td>w</td>
<td>p</td>
</tr>
<tr>
<td>/20/</td>
<td>Germany</td>
<td>1967-1977</td>
<td>Employment in industry</td>
<td>-0,02</td>
<td>0,15</td>
</tr>
<tr>
<td></td>
<td>France</td>
<td>1967-1975</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>/11/</td>
<td>France</td>
<td></td>
<td>Employment in the private sector</td>
<td>-0,1</td>
<td>0,1</td>
</tr>
<tr>
<td>/29/</td>
<td>US</td>
<td>1956-1980</td>
<td>-0,6*</td>
<td>-3,4</td>
<td>* Insignificant variables</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Japan</td>
<td>-2,4</td>
<td>2,6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Canada</td>
<td>-2,6</td>
<td>1,8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Germany</td>
<td>-1,8</td>
<td>1,8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>France</td>
<td>-0,3</td>
<td>0,1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>UK</td>
<td>-1,8</td>
<td>0,4</td>
</tr>
<tr>
<td>/30/</td>
<td>UK</td>
<td></td>
<td>Employment</td>
<td>-0,5</td>
<td>&lt;0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Japan</td>
<td>-0,2</td>
<td>0,3</td>
</tr>
<tr>
<td>/24/</td>
<td>UK</td>
<td>1967-1984</td>
<td>Employment in manufacturing industry</td>
<td>-0,33</td>
<td>+0,06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1971-1983</td>
<td>Employment in the non-manufacturing sector</td>
<td>-0,05</td>
<td></td>
</tr>
<tr>
<td>/1/</td>
<td>10 major</td>
<td></td>
<td>OECD countries</td>
<td>-0,2</td>
<td></td>
</tr>
</tbody>
</table>
In line with the theoretical analysis set out in Section 2.2, the negative elasticity of employment with respect to real wages is higher in neo-classical models. Thus, a survey /30/ of studies of the relationship between employment and wages in the UK finds elasticities of -0.5 to -1 in neo-classical models compared with -0.2 to -0.3 in models which take account of output. But this elasticity measures a "pure" substitution effect only in the latter models. In neo-classical models, output is allowed to vary, so that changes in real wages affect employment through both factor substitution and induced changes in output. Furthermore, in these models, the elasticity of employment with respect to raw material prices is negative (see /29/ and /30/). This is also the case in the hybrid classical-Keynesian equation estimated by P. Artus /11/. These results indicate that labour and raw materials are insufficiently substitutable to compensate for the effect of changes in output10).

In most studies, the elasticity of employment with respect to factor costs is small and less than demand effects. On the other hand, Symons and Layard /29/ find higher elasticities and reject the idea of demand having an effect on employment, except for France and the USA. But the tests carried out to give this conclusion are unconvincing. The comparison of elasticities in six large OECD countries which this study permits, however, is more interesting. This shows that the influence of real wages is weaker in France and insignificant in the USA, and that the influence of real raw material prices is minimal in the UK and insignificant in Japan and France. This last result is not confirmed by P. Artus /11/.

Furthermore, one may ask whether the differences in country results reflect behavioural differences or simply the rather approximate character of the relations tested. Indeed, according to one study, quoted in /1/, the elasticity of labour demand with respect to the relative price of labour and capital averages -0.2 for the 10 main OECD countries. But the reduction in the demand for labour induced by the substitution effect varies, according to the country, with the rate of growth of the relative

10) An increase in raw material prices reduces profitable productive capacity. The substitution effect and the effect of the change in profitable capacity thus work in opposite directions on labour demand. As the latter effect is generally the greater, employment declines.
cost of labour and capital encountered. Table V shows that this varies between 0.8% (USA) and 13.2% (UK) over the period 1973 to 1981.

Table V Change in the relative cost of labour and capital, and substitution effect

<table>
<thead>
<tr>
<th>Country</th>
<th>( \frac{w}{c} ) in 1981</th>
<th>( \frac{w}{c} ) in 1973 = 100</th>
<th>Fall between 1973 and 1981 in the demand for labour due to the growth of ( \frac{w}{c} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>104,1</td>
<td>100</td>
<td>0.8</td>
</tr>
<tr>
<td>Japan</td>
<td>119,8</td>
<td>1</td>
<td>3.9</td>
</tr>
<tr>
<td>Germany</td>
<td>117,7</td>
<td>1</td>
<td>3.5</td>
</tr>
<tr>
<td>France</td>
<td>123,9</td>
<td>1</td>
<td>4.8</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>166,1</td>
<td>1</td>
<td>13.2</td>
</tr>
</tbody>
</table>

Source: /1/.

Finally, certain conclusions can be drawn from the estimates put forward in a British Treasury study /24/. The estimates, which are derived from employment equations in which demand plays a part, confirm the existence of a substitution effect between capital and labour, but less clearly in the non-manufacturing sector. Moreover, in manufacturing industry employment is affected more by the relative cost of labour and raw materials than by that of labour and capital.

3. Reconciliation of results concerning substitution possibilities and substitution effects

At this stage, it may appear contradictory to observe relatively high substitution possibilities between capital and labour at the same time as an altogether marginal influence of relative costs on the demand for capital and labour. This can be explained by the following reasons:

- The existence of substitution possibilities between capital and labour does not necessarily mean that substitution of capital for labour can be slowed down by reducing the relative cost of labour. There are other factors - more difficult, or even impossible to manipulate - which affect the rate of substitution, such as technical innovations specialization in more capital-intensive activities etc.
- Even if the substitution elasticities are high, the impact of relative costs can be weak if capital is not very flexible. Empirical results seem to confirm the putty-clay technology hypothesis in which factors of production are substitutable only at the time that equipment is purchased. In this case a change in relative prices affects only new equipment and not the whole of the capital stock.
- Relative costs are not the chief determinant of firms' demand for labour and capital. Other factors, such as demand and profitability, have greater influence.

4. Substitution effect and wage moderation policies

The effects of wage moderation should not be confused with substitution effects. Firstly, a fall in the relative cost of labour may be achieved by means other than reducing wages. Secondly, the substitution between labour and capital is only one of the channels through which a policy of wage moderation may have an impact on the economy. Other effects are produced mainly by means of prices, competitiveness, growth in income and demand and they depend on the other economic policy instruments which are manipulated, notably money supply and the public sector borrowing requirements. Dynamic macroeconomic models must be used to measure these different effects. Below, we comment briefly on simulations of a wage moderation policy carried out on three different models: the French Metric model /6/, the British Treasury model /30/ and the Commission's Compact model for the Community. It will thus be possible to evaluate differences in the results obtained for reasons intrinsic to the mechanisms of each model.

Within Metric the relative cost of capital and labour influences, in industry, the choice of production techniques for new equipment. A simulation which allows for an initial reduction of 5% in the real hourly wage rate shows that the substitution effect is weak relative to the consequences of a fall in demand resulting from the loss of wage earners' purchasing power, so that, in the first six years, the impact on employment is negative. Another scenario tested a fall of 5 points in the rate of
employers' social security contributions (i.e. a 2.8% reduction in the wage costs born by firms) and the application of an 18.5% tax on investment. These measures permit a 21.3% reduction in the cost of the labour relative to capital without reducing household demand or causing a deterioration in public finance. They lead to a 2% increase in employment after 6 years, though a quarter of this increase results from a prolongation of the life time of capital equipment, estimated at six months on the basis of calculations made outside the model.

In the Treasury model /30/ the relative cost of capital and labour is included in the investment and employment functions. In this model, a fall in real wages has an impact on both the supply of and the demand for goods. With regard to supply, the immediate effect of a reduction in real wage costs is to increase firms' rate of profit and therefore their production capacity and their demand for labour. With regard to demand, the influence of the fall in real wages is exerted through three channels:
- it redistributes incomes from wage earners to entrepreneurs, hence reducing private consumption and investment in housing, and increasing company investment;
- it improves the economy's competitiveness and therefore stimulates exports;
- it permits a reduction in inflation and, given the concomitant fall in the demand for money, a fall in interest rates, which lessens the reduction in households' expenditure (real wealth effect).

According to this model a 2% reduction in real wages over four years should increase employment by between 0.7% and 1.4% after four years, depending on the price effects and the monetary and fiscal policies followed. But three quarters of the variation in employment can be explained by the variation in output and only a quarter by the substitution effect. These results for employment more favourable than those of the first simulation by the Metric model are obtained, on the one hand, by means of the fall in prices which limits, in time and scale, the reduction in household expenditure, and on the other, because the reduction in household expenditure is offset by an increase in other categories of
expenditure. These simulations also show that the stance of fiscal and monetary policy has a great impact on the response of employment to a fall of wages: "A policy that keeps the financial framework (especially PSBR/GDP ratio and money supply) unchanged in nominal terms permits a larger rise in output and hence employment than a policy in which tax rates and interest rates are unchanged, and hence the money supply and the PSBR/GDP ratio fall" /30/. In this case, the reduction in interest rates and the lighter burden of taxation which can be afforded as a result, have the effect of increasing the growth of companies' investment and of limiting the reduction in private consumers' expenditure.

Finally, in the Compact model, relative costs influence the level of potential employment which corresponds to the full utilization of profitable production capacity. Effective employment is lower than potential employment when capacity is underutilized as a result of insufficient demand. The results of this model show that a policy of wage moderation which slows down the growth of wage earners' purchasing power by 3.2 points over 6 years allows employment to increase by 1.4%. These positive results for employment can chiefly be explained by an improvement in profitability which encourages firms to invest. The reduction in the relative cost of capital and labour certainly increases potential employment. But effective employment does not vary greatly if demand does not follow. Employment does not improve until the third year when additional demand emerges as a result of the increase in companies' investment. Thus, according to the Compact model, in order to attain the most favourable results for employment, it is necessary to implement a policy of reducing wage costs without causing too much harm to demand (for example, by achieving wage moderation partly through reducing social security contributions) and to combine supply-side policy with a policy of demand support. Thus, a policy of supporting demand which consists in increasing the proportion of public investment in GDP by 0.5 points between 1986 and 1991, and reducing real wage costs by 4.6% (75% of the reduction coming from wage moderation and 25% from cutting employers' social security contributions), would permit employment to grow by 3.2% after six years. Such a policy increases companies' profitable supply at the same time as demand, and leads to a more rapid improvement (from the first year) in effective employment.
CONCLUSION

This study has shown the difficulties which arise in evaluating the scope for substitution between factors of production (i.e., their elasticity of substitution) or in measuring the impact of a change in relative costs on factor demand (i.e., the substitution effect). For one thing, it is not possible to attribute a precise value to the elasticities of substitution because of the lack of robustness in estimating them. For another, the sensitivity of employment or investment to the relative cost of capital and labour may vary with the definition of the user cost of capital and with the specification of the equation. Thus, in a true neo-classical model of demand for labour or capital, it is not possible to isolate the substitution effect from the effect of the variation in profitable capacity. Lastly, it is difficult to compare the results derived from different studies given the diversity of methods and models used. Consequently, great caution is required in drawing economic policy conclusions from empirical studies, especially as regards the advisability of undertaking a wage moderation policy. Nevertheless, we shall attempt to draw together the main lessons from these studies.

At the macroeconomic level, there seems to be significant scope for substitution between labour and the other factors of production. However, empirical studies do not agree on the value of the elasticity of substitution; depending on the studies and the countries involved it varies from 0.6 to 1. Nevertheless, these studies show that if capital and labour are substitutable, capital and energy would seem to be complementary. It follows that a reduction in energy prices should stimulate the utilization of capital and hence investment.

Since, at the macroeconomic level, there is technical scope for substitution between capital and labour, the question arises of whether the relative cost of these factors can influence firms' technological choices, and consequently, their demand for capital and labour. Empirical studies show that this is the case. But the substitution effects are somewhat weak and slow to appear. This apparent contradiction between substantial scope
for substitution and the small influence of relative costs can be explained by the fact that the technologies utilized are of the putty-clay type. In this case, a change in relative costs will only affect new equipment (and the employment which can be associated with it) and not the entire capital stock.

Despite the existence of significant scope for substitution between labour and capital, it therefore seems difficult to significantly reduce unemployment solely by endeavouring to influence the technological choices of companies by reducing the relative cost of labour. First, since the substitution effects are weak and slow to appear, the change in the relative cost would have to be substantial and lasting in order to produce a positive effect on employment. Consequently, a policy aiming solely to reduce the relative cost of labour would not be a short-term instrument for combating unemployment. It could, however, make a contribution to a medium/long term policy which, by gradually modifying relative costs, would lead to a productive structure which is more employment intensive.

Secondly, in this area as in others, the preferred economic policy depends on whether unemployment is predominantly classical or Keynesian. In the first case, a fall in relative costs may encourage employment but it must be complemented by an adequate policy of demand support. In the second case, rather than controlling relative costs, it would be better to try to reduce overall production costs in order to increase profitable capacity. But since classical and Keynesian unemployment situations generally coexist, any policy of combating unemployment must combine supply-side measures which improve company profitability with measures to support demand.

Lastly, any policy which slows the rate of substitution between capital and labour is not without risk and could be difficult to implement. First, such a policy may jeopardize the future production capacity of European economies if it has too great an effect in discouraging investment. This is especially detrimental at the present time since existing capacity in the European Community is no longer (since 1978) sufficient to ensure full employment. Second, such a policy is
non-selective and it could impede the development of the industries of the future ("the new technologies") which, because of the rapid evolution of technical progress, require investment in order to remain competitive.

To conclude, it seems appropriate to indicate how to reduce the relative cost of labour in order to maximize the positive effect on employment while eliminating the disadvantages of such a policy as much as possible. In this case, it is necessary to consider not only the direct effects but also the induced macroeconomic effects of a change in relative costs. These effects will clearly vary, depending on the method used to achieve the change. For example, the same variation in relative costs will have different macroeconomic effects depending on whether it is achieved by a fall in real wages or an increase in the cost of capital because of an increase in the rate at which profits are taxed. On this subject, model simulations show that measures which have a particularly deflationary impact should be avoided. It follows that the type of measure to be advocated is one which allows a reduction in companies' wage costs without damaging the net income of households, for example a reduction in employers' social security contributions. However, the budgetary difficulties which affect some European economies may severely restrict the room for manoeuvre in this area.
ANNEX 1  DEFINITION OF THE ELASTICITY OF SUBSTITUTION

The elasticity of substitution (\( \sigma \)) describes the substitution possibilities which exist between factors of production. Where factors of production may be combined in variable proportions to achieve the same output, they are substitutable and the elasticity of substitution is positive. The greater the possibilities of substitution, the greater the elasticity of substitution. On the other hand, where factors of production have to be used in fixed proportions, they are complementary and not substitutable.

The common definition of the elasticity of substitution is based on a production function which comprises only two factors of production. Consider a production function with two factors (labour and capital) possessing the usual properties\(^{11}\).

\[
Q = F(K,L)
\]

At a given level of production (\( Q \)) the different combinations of factor inputs are described by an isoquant which is convex to the origin.

Graph 1

\(^{11}\) The production function is homogeneous, continuous, differentiable at least twice, with marginal products which are positive and decreasing.
The slope of the tangent at each point on the isoquant gives the **marginal rate of substitution** between the two factors (R).

\[ R = -\frac{\partial Q}{\partial L} = \frac{F_L}{F_K} \]

with \( \frac{\partial F}{\partial K} = F_K \) and \( \frac{\partial F}{\partial L} = F_L \)

For any given combination of factors the marginal rate of substitution defines the change in capital which is necessary to maintain constant output when there is a small change in labour input. As the isoquant is convex to the origin, the marginal rate of substitution is increasing as one factor is substituted for the other.

**The elasticity of substitution** (\( \sigma \)) measures the ease with which two factors can be substituted for each other. It is defined as the elasticity of relative factor proportions with respect to their marginal rate of substitution.

\[ \sigma = \frac{\frac{\partial \ln (K)}{\partial \ln R}}{\frac{\partial \ln (L)}{\partial \ln R}} \]

Therefore a change in labour input from \( L_1 \) to \( L_2 \) (see Graph 1) involves a change in relative factor proportions from \( K_1 \) to \( K_2 \), (these ratios are \( \frac{L_1}{L_2} \) measured respectively by the slopes of OA and of OB in Graph 1). The latter change is a function of the change in the slope of the tangent from point A to point B, i.e. the change in the marginal rate of substitution.

If firms minimize their costs of production,

\[ R = \frac{F_L}{F_K} = \frac{P_L}{P_K} \]

with \( P_L \): cost of labour

\( P_K \): cost of capital
The elasticity of substitution may be written:

\[ \sigma = \frac{d \ln \left( \frac{K}{L} \right)}{d \ln \left( \frac{R}{R_K} \right)} \]

This elasticity of substitution defines the sensitivity of factor proportions to their relative prices. For example, if the price of labour increases by 1% in relation to the price of capital and the elasticity of substitution is 0.5, the capital/labour ratio increases by 0.5%.

The elasticity of substitution has the following characteristics:
- it is symmetrical relative to the two factors;
- it is positive;
- it lies between zero and infinity.

When \( \sigma \) equals zero, the two factors are complementary (the isoquant is L-shaped) and when it equals infinity, the two factors are perfectly substitutable (the isoquant is a straight line).

Two production functions have interesting properties in terms of their elasticity of substitution:
- in the case of the CES production function,
  \[ Q = A \left[ \delta K^{-\frac{\ell}{\ell}} + \left( 1 - \delta \right) L^{-\delta} \right]^{-\frac{1}{\ell}} \]
  the elasticity of substitution is equal to \( \frac{1}{1+\ell} \)
- in the case of the Cobb-Douglas production function (which is a CES function with \( \ell = 0 \)),
  \[ Q = A K^\alpha L^\beta M^\gamma \]
  the elasticity of substitution equals 1.

Where the production function comprises more than two factors, the concept of elasticity of substitution becomes more complex. The elasticities of substitution which may be defined depend on the assumed behaviour of firms (minimization of costs, maximization of profit) (see /23/) and on the way in which the factors are grouped together. For example, in the case of a production function which distinguishes four factors - capital (K), labour (L), energy (E) and other inputs (M) - and which groups together, on the one hand, capital and energy and, on the other, labour and other inputs, it is possible to define:
the elasticity of substitution within each group (i.e. between K and E on the one hand and between L and M on the other);
- the elasticity of substitution between the group K-E and the group L-M;
- partial elasticities of substitution.

If it is assumed that firms seek to minimize their production costs, it is possible to derive Allen's partial elasticities of substitution. These describe the elasticity between two factors taking into account both the substitution between those two factors and the inter-group substitution if the factors belong to different groups. Allen's elasticities of substitution make it possible to obtain cross price elasticities which describe the sensitivity of demand for a factor at the price of another factor. Unlike the elasticity of substitution of a production function involving two factors, the Allen elasticities (and therefore the cross price-elasticities) may be negative where the factors are complementary.
ANNEX 2  THE DEMAND FOR CAPITAL AND LABOUR IN SITUATIONS OF CLASSICAL AND KEYNESIAN UNEMPLOYMENT

The equations for the demand for labour and capital are derived assuming that firms maximize expected profits ($\Pi^e$) taking account of production technology.

\[ \text{i.e. Max } \Pi^e = p^eQ - w^eL - c^eK \]  \hspace{1cm} (1)

Maximising (1) implies that marginal products of factors equal respective factor incomes.

1. Model of notional demand (Classical unemployment)

Assumptions: - competitive markets
- no constraints for firms in goods or labour markets (excess supply of labour and demand for goods)

Under conditions of perfect competition and in the absence of constraints for firms on the two markets, notional demand for capital ($K^*$) and for labour ($L^*$) from firms depends only on the anticipated costs and prices of output ($p^e$) and labour ($w^e$).

\[ K^* = csr(e \frac{c^e}{p^e}) a_1 \frac{w^e}{p^e} a_2 \frac{a_3 t}{a} \]  \hspace{1cm} (2)

\[ a_1 = -[\beta \sigma + \frac{1-\beta}{1-\nu}] , \hspace{1cm} a_2 = -\beta \left[ \frac{1}{1-\nu} - \sigma \right] \hspace{1cm} \]

\[ L^* = csr(e \frac{c^e}{p^e}) a_4 \frac{w^e}{p^e} a_5 \frac{a_3 t}{a} \]  \hspace{1cm} (3)

\[ a_4 = (1-\beta) \left[ \sigma - \frac{1}{1-\nu} \right] \hspace{1cm} a_5 = \left[ (1-\beta) \sigma + \frac{\beta}{1-\nu} \right] \]

\( \beta \) is the share of wages in total cost ($\beta = \frac{\omega^L}{\omega^L + c^eK} $), \( \nu \) is returns to scale, \( \sigma \) is the elasticity of substitution and \( \delta \) is the rate of technical progress.
Equations (2) and (3) show that:

- the demand for capital (labour) is defined only on the assumption of a production function with diminishing returns to scale;
- the demand for capital (labour) is always a decreasing function of the real cost of capital (labour), \((a_1 < 0 \text{ and } a_5 < 0)\);
- the demand for capital (labour) may be an increasing or diminishing function of the real cost of labour (capital) (increasing if \(\sigma > \frac{1}{1-\nu} > 1\), which is generally not the case according to Artus and Muet /9/).

2. Model of effective demand

Assumption: constraint for firms in goods market (excess supply of labour and goods)

Where firms perceive a constraint on the goods market, i.e. where they are unable to produce as much as they would wish, demand for capital (labour) is an increasing function of the anticipated demand for goods \((Q^e)\) and a diminishing (increasing) function of the relative factor costs \((c^e)\). This gives

\[
K^* = c^{r^e} (Q^e)^{b_1} \left(\frac{c^e}{w^e}\right)^{b_2} \varepsilon^{b_3} \nu^{b_4} \quad (6)
\]

\[
b_1 = \frac{4}{\nu}, \quad b_2 = -\beta \sigma, \quad b_3 = -\frac{\xi}{\nu}
\]

\[
L^* = c^{r^e} (Q^e)^{b_1} \left(\frac{c^e}{w^e}\right)^{b_4} \varepsilon^{b_3} \nu^{b_4} \quad (7)
\]

\[
b_4 = (\lambda - \beta) \sigma
\]

It should be noted that:

- this model is less restrictive than the previous one, since there is no need for any assumption concerning returns to scale;
- the elasticity of demand for capital and labour with respect to production is positive and equal to the inverse of the returns to scale;
- the elasticity of demand for capital (labour) with respect to relative capital/labour costs is negative (positive) and equal to the product of the share of wages (cost of capital) in the total cost and the elasticity of substitution.


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