Validity and limits of applied exchange rate models: a brief survey of some recent contributions

by Giuseppe Tullio

Internal Paper
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The Directorate-General for Economic and Financial Affairs,
Commission of the European Communities,
200, rue de la Loi
1049 Brussels, Belgium
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by Giuseppe Tullio *

Internal Paper

* Economic adviser, Commission of the European Communities, Brussels. This paper was presented at the Annual Meeting of Italian Foreign Exchange Dealers held in Milan on October 28-29, 1985.

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The sharp appreciation of the dollar since 1980, both in nominal and real terms took both market operators and academic economists by surprise. The subsequent further upsurge between November 1984 and February/March 1985 was even more surprising and unexpected.

The fact that no model has been able to forecast the swings of the dollar in the late seventies and the eighties does not necessarily call them into question, for to forecast future exchange rate developments a "good" model is not enough, accurate forecasts of the future development of the model's explanatory variables are also required. In other words, it is necessary to have a "good" exchange rate model, but that alone is not sufficient. Therefore, exchange rate models cannot be justifiably rejected outright solely on the ground that a sufficiently reliable forecast of the exogenous variables is very difficult, because under some circumstances to be discussed below they may still be effective both for interpreting the past development of exchange rates and as a guide, although not a sure guide, to their possible future development.

The basic question that will be addressed in this paper is: are there exchange rate models that are sufficiently robust to withstand institutional changes, like the increase in the degree of integration of international financial markets, the change in the central banks' policies
on foreign exchange market intervention, for example, that announced on 22 September 1985 by the Finance Ministers of the world's five most industrialized countries, and the changes in monetary policy's operating procedures, for example, those announced in October 1979 by the US Federal Reserve. Although as things stand, doubts may still be entertained about the existence of satisfactory models, I will maintain in this paper that the experience acquired so far from the inter-war years and during the seventies, the econometric tests carried out up to now on the basis of various models and the very development of econometrics have supplied us with the information we require on what elements are crucial to make up a sufficiently sound exchange rate model.

This paper is in two parts: the first part briefly assesses the validity of some of the exchange rate models developed in the seventies, starting with two recent studies carried out in the United States by Meese and Rogoff (1983) and Bilson and Hsieh (1984). This is followed by an examination of the reasons why exchange rate models have not so far supplied the results hoped for. This examination is at the same time intended to provide suggestions for future research aimed at improving recent models. The reasons why exchange rate models have not so far come up with the results hoped for include one factor that I consider particularly important: applied economists have not taken sufficient account of institutional changes, such as the changes in the policy on intervention in exchange rate markets and the participation of various countries in exchange rate agreements such as the snake and the EMS. On the one hand, these agreements make the testing of the models for the bilateral rates of the member countries' currencies more difficult on account of the discontinuity in their fluctuations and on the other, they affect the determination of bilateral exchange rates of member countries with third countries, as demonstrated below in regard to the mark-dollar exchange rate after entry into force of the EMS in March 1979. Most applied researchers have reacted to the poor tracking ability of the simpler versions of exchange rate models by developing theoretically more complicated models, rather than by paying more attention to the institutional details.
In the second part I will illustrate the as yet unpublished results of an econometric testing of the monetary theory of exchange rate determination performed on the mark/dollar and ECU/dollar exchange rates in the period following the announcement of the establishment of the European Monetary System (July 1978 to December 1984). This test shows how the monetary theory must be rejected if no account is taken of the institutional change involved in setting up the European Monetary System (EMS), while it is borne out by the data if account is taken of the change in arrangements (Tullio and Brillembourg, 1985). I will also describe the results of previous tests of the monetary theory of exchange rate determination which I carried out alone or in cooperation with others, one for the lira Swiss franc exchange rate from March 1973 to December 1978 (Tullio, 1979, 1981) and the other for the mark-dollar rate during the German hyperinflation between 1920 and 1923 (Sommariva and Tullio, 1987). I will conclude that a sound exchange rate model must necessarily include as an explanatory variable the degree of restrictiveness in monetary policy in the two monetary areas concerned. However, I believe it is necessary at the same time to find a way of including the current account balance and its lags as an additional explanatory variable. In other words, the monetary theory must be combined with the portfolio balance theory. Finally Appendix 1 contains the model of the DM-Dollar estimated for the hyperinflation period of the 1920's by Sommariva and myself.

I will complete this introduction with a brief discussion of the principal exchange rate models developed during the seventies. The first distinction to be made is between random walk and autoregressive models, and models which have a basis in economic theory. The main feature of the former, which for the sake of simplicity I will call statistical models, is the fact that the past and current exchange rate are used to explain the future exchange rate. They use only exchange rate information to forecast the future trend, as if the exchange rate were isolated from the economic context. "Charting techniques" belong to the same family of models. Models which are based on economic theory, which for the sake of simplicity I will call "economic" models, may be classified in the chronological order of their theoretical development on the basis of the main explanatory variables of the exchange rate:
The purchasing power parity theory regards the inflation differential between the countries concerned as the major determinant of exchange rate developments. It was devised by the Swedish economist, Gustav Cassel (1922), in the inter-war years and applied by him to explain the long-term development of exchange rates in Europe from 1914 onwards. The idea at the basis of the theory is that the price of the same goods should tend towards uniformity in countries linked by international trade. Perfect uniformity is precluded by the existence of transport costs and the difference in national taxes. Cassel regarded the purchasing power parity theory as a long-term theory valid for an approximate ten-year period. Recent tests of the theory for the thirties and the seventies carried out on the basis of monthly data show that it did not hold (Frenkel, 1978). However the studies in question misinterpreted Cassel's thinking by assuming that exchange rates should converge towards purchasing power parity even for periods as short as a month.

Keynesian theory maintains that the exchange rate is determined by the intersection of the demand curve for foreign currency for the import of goods and services and the supply curve of foreign currency derived from the export of goods and services. This theory was very popular in the fifties, when capital movements were inexisten or extremely restricted in almost all countries by exchange controls. On the basis of a partial interpretation, this theory regards the current account balance as the proximate determinant of the exchange rate, but within a more general framework, monetary and fiscal policy may be incorporated into the model in so far as they influence the foreign currency demand and supply curves.

The interest rate parity theory regards the nominal interest rate differentials and the forward exchange rate as the main explanatory variables of the exchange rate. Against a background of high inflation, or where the forward market does not exist or is not very developed, it is the differential of real interest rates, i.e. nominal rates corrected for inflation, which is crucial. This theory has had the merit of diverting attention from the current account balance to international capital movements, which increased considerably following
the convertibility of currencies in 1958 and the development of the Eurodollar market in the sixties and seventies. However, while the Keynesian theory and, all things considered, also the purchasing power parity theory disregard capital movements completely, the interest rate parity theory completely disregards flows of goods and the real sector of the economy.

- The portfolio balance model, associated with Branson (1979), Kouri (1976), Richard (1980) and Armington (1985), incorporates the interest rate parity theory into a wider scheme of analysis, which takes account of the international flows of goods and services and the interactions between the current account balance and the accumulation of net foreign wealth. Assuming that a country's investors prefer to hold national currency at unchanged interest rate differentials, a current account surplus will be associated with an appreciation of the national currency and a deficit with a depreciation. In the case of the United States in the eighties up to March 1985 and to a lesser extent Italy, such a connection has not been observed, probably because the interest rate differentials or the restrictiveness of monetary policies have not remained unchanged while the current account deficit was increasing.

- Under the monetary theory of exchange rate determination relative supplies of money and the relative gross domestic products are the fundamental determinants of the exchange rate. According to this theory the money/income relationship is a more complete measure of the restrictiveness of monetary policy than the nominal or real interest rate. This theory presupposes the existence of a stable demand function of money. In its dynamic version it in no way presupposes that purchasing power parity is valid at any point in time. It is sufficient that the latter be verified in the state of final equilibrium. An example of an empirically verified dynamic monetary model which does not presuppose purchasing power parity and indeed explains real exchange rate fluctuations is that of Sommariva and Tullio (1987) estimated for the mark/dollar exchange rate between 1920 and 1923. This model is presented in the Appendix to this paper.
Each of the above theories may be interpreted in two different ways: firstly, regarding the exchange rate as a variable possessing a certain inertia and adapts to changes in the explanatory variables after some time and secondly assuming that the exchange rate reacts above all to the expected evolution of the explanatory variables so that the exchange rate jumps when "announcements" or "news" concerning the future development of the fundamental variables come to the attention of the market. Since clear, credible "announcements" are not so frequent, the exchange rate ends up being influenced systematically, in my opinion, above all by lagged variables; nevertheless, expectations of the future cannot be disregarded. An interesting case in which expectations played an important role was that of the dollar in 1981. In the spring of 1981, when the electoral promises of the newly-elected President Reagan became more clearly defined, the long-term interest rate increased in anticipation of the forthcoming reduction of tax rates and the increase in the federal deficit and the dollar exchange rate strengthened. Monetary policy was predetermined, the Federal Reserve having declared its unwillingness to finance the federal deficit by monetary means, otherwise the real interest rates would not have increased. The variations in the degree of importance of past and future variables cause problems in the formulation and estimation of econometric exchange rate models.

1. Validity and limits of applied exchange rate models

The view prevailing today among U.S. academics in regard to the forecasting accuracy of exchange rate models is strongly influenced by the study of Meese and Rogoff (1983). They compare statistical models with extremely simplified versions of some of the economic models outlined in the introduction, making forecasts with models estimated on the basis of monthly data from March 1973 to June 1981 for the dollar/mark, dollar/yen, dollar/pound and the trade weighted dollar exchange rate. For the forecasts with economic models, they use the actual (ex-post) values of the exogenous variables, rather than their predicted or expected (ex-ante) values. The forecasts are made for one month, three months, six months and 12 months. The principal conclusion they reach is that the forecasts based on statistical models are better than those based on economic models for all the horizons considered. However, the economic models used by Meese and
Rogoff are extremely simplified. For instance, no lags of the exogenous variables are considered whereas the study I carried out with Brillembourg on the mark/dollar and ECU/dollar exchange rates, which I will outline subsequently, suggests that for the relative money supply average lags are longer than one year and for the relative gross domestic product longer than six months. In addition the slow reaction of the US dollar to the worsening current account suggests that for the current account they are probably even longer. The estimates of Meese and Rogoff cover a period greatly disturbed by the two oil crises of the seventies and the subsequent adjustment of western economies and which, at least for the mark/dollar exchange rate, is characterized by major institutional changes, i.e. the passage from the unstable exchange rate agreements of the snake to the more stable and binding agreements of the EMS. It is not right to estimate one and the same model for a period characterized by different institutional set-ups without taking into account the effects of these changes on exchange rate determinants.

A second study recently completed by Bilson and Hsieh (1984) shows that the exchange rate expected by operators on the Chicago stock exchange is a weighted average of the past exchange rate and the exchange rate forecasts obtained by using various economic models. They find that the weight of the past exchange rate (statistical models) is generally 95%, while the weight of the economic models is only 5%.

The two above mentioned studies have two different objectives: the first is to test the soundness of economic models as forecasting instruments, without regard to the conduct of the market, the second is to record the importance the market attaches to economic models. It may just be that there are more sophisticated versions of the economic models used by Meese and Rogoff giving highly satisfactory results of which neither academic economists nor the market are aware, since studies similar to those of Meese and Rogoff with models paying more attention to institutional changes, the oil shocks and lags have not yet been carried out. As a result I have far fewer doubts about the conclusions in the study by Bilson and Hsieh than about those set out by Meese and Rogoff.
Theoretical developments in the course of the last ten to fifteen years and the empirical evidence that has built up on the operation and determinants of exchange rates as well as the development of econometric techniques enable us nowadays—I feel—to determine more successfully than in the past the future avenues of research which will provide us with more satisfactory exchange rate models. In the remaining part of this section I will discuss the conditions to which these models will have to adhere. The list of these conditions is also in part a critique of the models developed so far and in part a justification for the reason why, given the intrinsic difficulties, no sufficiently reliable economic models have yet been built some twelve years since generalized fluctuating exchange rates came into being.

(1) In his treatise "Della Moneta" published in Naples in 1750, Fernando Galiani wrote that the exchange rate is "the thermometer of States" (Galiani, 1750, vol. 11, p. 85). In a system of perfectly flexible exchange rates, the exchange rate reflects a country's general economic situation and therefore reflects all the economic variables which play a part in determining it. A comprehensive exchange rate model would therefore require a comprehensive model of the economic system to clarify the important interactions (for exchange rate purposes) between them. But it is extremely difficult to formulate such a model and even more difficult to estimate it. This is why applied economists have worked up to now generally with reduced form equations, i.e. simple equations which express the exchange rate as a function of very few fundamental variables.

(2) The exchange rate is not a perfectly endogenous variable; even in a system of flexible exchange rates it is an economic policy variable in all countries, excluding perhaps the United States. The policy of central bank intervention can be very important in explaining and forecasting the trend of exchange rates, particularly on weekly and monthly data, above all if the intervention is coordinated and not sterilized. Beyond one month probably only non-sterilized intervention can have lasting effects. When a country participates in exchange rate
agreements like the snake or the EMS, the bilateral exchange rate for the currencies of the member countries becomes again to a very large extent exogenous at least between realignments, its changes become discontinuous and their timing and size is influenced significantly by political, as well as economic considerations. In these circumstances it becomes impossible to test economic theories rigorously. In conclusion, models that do not take adequate account of intervention policy are defective, particularly in the case of models dealing with the dynamics of the exchange rate in the very short run.

(3) In classifying the economic exchange rate models I have pointed out that they may be interpreted in two ways: in the first case, past exogenous variables are important; in the second case, expected variables are. If expectations of the future are determined in turn by the past development of the same variables (adaptive expectations), then the past variables are still of importance in this second version also and it is easier to formulate expectations. But cases in which expectations are formulated with reference to the future are becoming increasingly frequent. A first example, already mentioned earlier, is the increase in the real long term interest rate and the restrictiveness of US monetary policy in the first quarter of 1981, following the announcement of President Reagan's new tax policy. On that occasion the dollar started to rise even before the new tax policy had been put into effect. Another example in which expectations regarding future interest rates are of importance to the exchange rate, but difficult to formulate, is the following: according to the monetary approach to exchange rate determination, an increase in the US money supply, real income remaining the same, weakens the dollar; however, in recent years an increase in the US money supply has strengthened the dollar in the very short term (one to two months) because, in the face of pre-established monetary objectives, its increase gives rise to expectations of higher interest rates aimed at bringing the money supply back within the band. Only when and if it becomes clear that the Federal Reserve has decided not to increase interest rates and to accept an
overshooting of the monetary target does the dollar weaken in accordance with the monetary theory. Accordingly, market expectations as to the future development of the money stock in the face of pre-established monetary objectives can lead to "perverse" affects of monetary growth on the exchange rate in the short term. Formalizing expectations remains, however, a largely unresolved problem.

Having carefully examined the contribution of the rational expectations theory to the understanding of German hyperinflation and the devaluation of the mark between 1920 and 1923, Sommariva and Tullio (1987) conclude that rational expectations, which are still very much in vogue among US academics, have made no contribution at all. Adaptive expectations, as formulated by Cagan (1956), work very well, also in the dynamic exchange rate model of Sommariva and Tullio. As Cagan's evidence that expectations were formed adaptively was so overwhelming, Sargent and Wallace (1973) made a major effort to find conditions under which adaptive expectations were also rational. The condition under which adaptive expectations are also rational is that prices determine money, rather than that money determines prices, as predicted by the quantity theory of money. It follows that in Sargent and Wallace's model prices become the driving force of the system and there is no longer a theory of inflation in their model, as correctly pointed out by B. Friedman (1978).

The major shocks to industrial economies in the seventies were the two oil shocks of 1973/74 and 1979/80. Few economic models estimated for this period take account of the differing effects of the oil crises on the exchange rates of the various countries. As regards the trade-weighted pound exchange rate, it has been shown that the present value of expected future revenues from North Sea oil has had a very significant effect (Bond and Knöbl, 1982). In theoretical terms, Giavazzi and Giovannini (1985) have shown that for two countries with different
structures which coordinate their monetary and exchange rate policies (like the countries belonging to the EMS), the best policy in the event of an exogenous shock is to realign exchange rates. It is logical to expect that the bilateral exchange rates most sensitive to changes in oil prices include the pound/yen exchange rate. But the US dollar also seems to react differently than the yen and the DM to changes in the price of oil, especially in the short run. Foreign exchange dealers argue that the dollar reacts favourably in the short run to increases in the price of oil because its demand goes up, since it is used to carry out oil transactions. Failure to take account of past and expected oil prices in the models for the seventies can therefore easily give rise to unsatisfactory estimates and unreliable forecasts. The study by Meese and Rogoff is no exception.

(5) In the exchange rate models based on the monetary theory, the demand function of money plays a central role. An incomplete specification of the demand of money may lead to an unjustified rejection of the monetary theory or to values for the sum of the current and lagged coefficients of money or income nowhere near one, the expected long-term value. Most of the models so far estimated do not take sufficient account of possible currency substitution. On the demand side currency substitution occurs when the demand for money in a country is influenced by foreign interest rates or by changes or expected changes in exchange rates. Indirect substitution occurs when, on account of negative exchange rate expectations, investors shift from national to foreign securities, thus provoking an increase in the domestic interest rate and a fall in the foreign one. In so far as the demand for money is affected by the interest rate, the demand for national money decreases as that for foreign money increases (McKinnon, 1982). Direct currency substitution occurs when the demand for money is directly influenced by actual or expected exchange rate changes. From an econometric point of view, however, it is very difficult to isolate the effect of exchange rate expectations on the demand for money, partly on account of
the difficulties in measuring expectations. In principle, if it were possible to measure the latter satisfactorily, it would also be possible to test separately the relevance of indirect substitution and of direct substitution in so far as the former assumes that exchange rate expectations have a significant impact on the interest rate. Tests of the significance of bilateral currency substitution in the seventies were carried out for the main currencies by Brittain (1981) and for the US dollar and the Canadian dollar by Miles (1979). Their studies suggest that currency substitution was significant. Sommariva and Tullio (1987) have shown that during the German hyperinflation of 1920 to 1923, the direct substitutability between the mark and the dollar was significant. However Brillembourg and Schadler (1979) include third currency effects in a portfolio balance model composed of the major currencies and do not find strong evidence in favour of currency substitution.

A practical example of the importance that currency substitution may have on the dynamics of the exchange rate is the following. Let us suppose that the main conclusion of the monetary theory is valid, i.e. that the restrictiveness of monetary policy in the two areas is fundamental to the determination of exchange rates. Let us also suppose, as is happening now in Germany, and at least until some time ago in the United States, that the monetary authorities proclaim and pursue with determination monetary objectives. Finally, let us suppose that there is a revaluation of the mark in prospect. The demand for the German currency would tend to increase on account of the possible substitution in demand from dollars to marks and German monetary policy would become more restrictive with respect to the initial central bank intentions, given predetermined money supply targets and real GDP growth. If the central bank rigidly pursues the pre-established monetary objective, a virtuous, but perverse circle may develop in which the revaluation expectations increase the demand for German money and the monetary policy becomes more restrictive than expected. This in turn strengthens the mark and so on.
Sooner or later the circle will break, but in the meantime the exchange rate dynamics is strongly influenced by currency substitution. According to McKinnon (1984), this is why US monetary policy has been much more expansionary than planned during the period when the dollar was weak (1977/78) and much more restrictive than planned during the period when the dollar strengthened (1980/81 and 1983/85). He concludes that the US monetary authorities too should take greater account of the level and changes in the exchange rate in formulating monetary policy and rely less on monetary objectives. He also suggests — given his belief that currency substitution is considerable — the fixing of target zones for the main exchange rates.

(6) The above discussion on the importance of currency substitution on the demand side already demonstrates that applied models cannot disregard the dynamics of the exchange rate. Future models will have to devote greater attention to this. Out of the applied models estimated so far only the portfolio balance model of Richard (1980) and Armington (1986) and the model of Sommariva and Tullio (1987) give explicit consideration to the exchange rate dynamics. Some applied models simply include lagged explanatory variables, such as Tullio's monetary models for the lira/Swiss franc (1978, 1979) and that of Tullio and Brillembourg (1985) for the mark/dollar and ECU/dollar exchange rate. However, many studies completely disregard these lagged effects even when they use monthly data for the estimates, as if the exchange rate always adjusted to fundamentals within one month. This last category includes Frenkel's monetary model for the mark/dollar exchange rate during the German hyper-inflation (1976) and Frenkel's (1978) tests of the purchasing-power-parity theory and the previously-mentioned models estimated by Meese and Rogoff (1983).

(7) To perform reliable econometric analyses it is necessary to have an adequate number of observations. For studies carried out with quarterly data in particular, the period covered by the data must
cover at least four or five years. The need to have a sufficient amount of observations often compels the researcher to make wild assumptions about the institutional uniformity of the sample from which the data originates. Mention has already been made (point 2) of the importance for the purposes of analysing exchange rates, of changes in intervention policy and in the accession of countries to exchange rate agreements. Other institutional factors which change over time and cannot be disregarded concern changes in the extent of trade and above all financial integration among countries and changes in the rules on the conduct of monetary policy.

Changes in the degree of trade and financial integration are usually continuous, but in countries which impose restrictions on capital movements there are numerous cases of discontinuous changes in the degree of financial integration which create major problems in the econometric analysis of exchange rates. However, even continuous changes can represent a considerable problem, particularly if they are relatively rapid, in so far as the relevance of the various exchange rate models varies along with the variation in the degree of financial integration. At a very low degree of financial integration, the Keynesian model is the most reliable. The portfolio balance model presumes imperfect substitutability between financial assets expressed in national and foreign currencies. Finally, the monetary model, at least in its most rudimentary version, assumes perfect substitutability between national and foreign financial assets.

Changes in the rules governing monetary policy (for instance, the decision to attach greater weight to monetary aggregates with respect to interest rates in October 1979 in the United States or the so-called "divorce" between the Banca d'Italia and the Italian Treasury in July 1981) can alter both the importance of the exchange rates' explanatory variables and the time lags between changes in these variables and the exchange rate. For example, greater emphasis in the conduct of monetary policy on monetary aggregates rather than on interest rates and a greater
variability of the latter could reduce the importance of interest rates in the determination of the exchange rate and enhance the role of the money stock.

2. Considerations on the most promising applied exchange rate models

In the previous section I described the reasons why economic models have not so far come up with the results originally hoped for and at the same time given certain hints as to possible future improvements, but I did not go into detail on which of the economic exchange rate theories referred to in the introduction I believe to be most promising. I will conclude that the two most promising theories are the monetary theory and the portfolio balance theory integrated in a dynamic context in which neither the market for goods nor the financial market adjust immediately in response to exogenous disturbances. At the same time the purchasing power parity theory, which tends to be valid only in the long run, must be imposed on this future "integrated" model as a condition of long-term equilibrium.

There is overwhelming empirical evidence to support the hypothesis that monetary market developments are crucial to exchange rate analysis. It comes from studies carried out in the twenties, in the seventies and early eighties. I will merely briefly summarize three empirical tests of the monetary theory which have, I believe, given excellent results, particularly because attention has been given to a certain number of factors mentioned in the previous section: relative institutional uniformity during the period considered for the estimates, explicit considerations of lags or explicit consideration of disequilibrium, selection of the bilateral exchange rate of two countries whose dependence on oil imports is fairly similar and consideration of substitution as between national and foreign currencies. The three studies are, in order of publication:

Lira was influenced by interventions in the exchange rate market for a fair proportion of the time, but this was taken into account in the estimates both because the monetary aggregates reflect non-sterilized interventions and because the updating of the estimates to December 1978 included among the exogenous variables a series of interventions in the exchange rate market by the Banca d'Italia. The estimates use as dependent variable the difference between the exchange rate for Italian banknotes in Zurich, a very flexible rate, and the exchange rate for cable transfers in Zurich which adjusts more slowly. These tests have the advantage of being free from reverse causation between the exchange rate and the stock of money. In addition, the relative monetary aggregates for Italy and Switzerland were lagged for up to 19 months. The model explains more than 80 percent of the variability of the spread between the two exchange rates. Some out of sample forecasts made for the Banca d'Italia in the first quarter of 1977 gave a very good forecast of the recovery of the Swiss franc with respect to the lira during 1977 and identified the weakness of the franc in the first quarter as a factor exogenous to the model, i.e. the crisis caused by lack of confidence in the franc on account of concern about the stability of the Swiss banking system.

b. The second example which confirms the importance of monetary factors in determining exchange rates is provided by the study of Sommariva and Tullio (1986) on the German mark - US dollar exchange rate during the German hyper-inflation of 1920 to 1923. As regards the specification and the method used for estimating the parameters, this is the most advanced of the three monetary models discussed here. It is made up of three differential equations of the first order which explain the dynamics of the price level in Germany, of the mark/dollar exchange rate and of the supply of money. Purchasing power parity is imposed as a condition for long-term equilibrium. The dynamics of the price level and of the exchange rate are each explained by the difference between their partial equilibrium level and their actual levels. The partial equilibrium level of prices is determined according to the quantity theory of money by the ratio between the money supply and the real demand for money. Currency is the monetary aggregate used, since it is the only one available on a monthly basis. The demand for German currency depends on real income, on inflation, which is assumed to reflect the substitution between currency and domestic goods,
the expected rate of inflation, which reflects the substitution with domestic financial assets and the rate of change of the exchange rate, which reflects the degree of substitution with foreign financial assets and foreign currency. The partial equilibrium level of the exchange rate is determined by the purchasing power parity condition. The model is estimated simultaneously with a maximum likelihood estimator developed by Clifford Wymer (1972, 1976). The estimates indicate that the substitution as between national and foreign currencies was important, that the exchange rate reacted far more rapidly to monetary imbalances than the price level. This explains the considerable real depreciation of the mark with respect to 1913 during the whole period of the hyper-inflation. The exchange rate adjusted on average in less than one day, while the price level required an average of 19 days. A fuller description of the empirical estimates and the stability analyses of this model is contained in Appendix 1.

c. The third example is a study involving tests of the monetary approach to exchange rate determination for the mark/dollar and ECU/dollar exchange rates since the creation of the European Monetary System (Tullio and Brillembourg 1985). The estimates cover the period between July 1978 (Bremen agreement) and December 1984, which was the last month available when the study was started. The aim of this applied research is not only to test the validity of the monetary theory, but also to demonstrate the importance for the mark/dollar exchange rate of institutional factors such as Germany's participation in the EMS. The model is highly simplified because account has not been taken of substitution between domestic and foreign currencies, nor of the dynamics of adjustment.

It would be hard to imagine that by participating to the EMS and partially subordinating its monetary policy to the aim of adhering to the agreements, the factors determining the level of the mark/dollar exchange rate would not be altered to some extent. It may be assumed that the monetary policies of the other member countries and the real growth of their economies has also had some effect on the mark/dollar exchange rate. To test this hypothesis the money stocks of the countries taking part in the exchange rate agreements were aggregated. At the same time, real
incomes were aggregated. The estimates show that the monetary theory is confirmed in this extended version. Vice versa, in the traditional version in which only the German and US variables are included, the monetary theory is rejected. If this combined hypothesis on the mark/dollar exchange rate is verified by the data, there is all the more reason for the monetary theory to be verified for the ECU/dollar exchange rate, where the ECU has to be redefined to exclude the pound since it is not part of the exchange rate agreements. The study also shows that indeed the monetary theory is verified for the ECU/dollar exchange rate.

Both for the mark/dollar exchange rate and for the ECU/dollar exchange rate, the time lags for the relative money stock are almost double the time lags of the relative real GDP, more than a year and a half for the first variable and six to eight months for the second. This confirms the impression gained from daily observation of the dollar-DM exchange rate in the course of 1985 that announcements concerning US GDP have a more rapid effect on the exchange rate than announcements concerning the US money supply. In the first quarter of the year the dollar appreciated considerably following the announcement of sustained growth in the US in the last quarter of 1984 and the announcement that the growth of GDP had been very high throughout 1984 (6.8% in real terms). When preliminary information on real growth was announced for the first quarter of 1985 (2.9% in relation to the previous quarter), the dollar did not depreciate much, but when as a result of subsequent revisions the figure for the first quarter was brought down to 0.3%, suggesting considerable slackening in growth, the dollar started to slide. These effects of announcements about US GDP on the dollar exchange rate therefore suggest relatively short lags for relative GDP. Vice versa, weekly announcements about US monetary growth did not have the same regular effect on the exchange rate, both because the market has now understood that the weekly figures fluctuate wildly and because of the perverse effects in the short run mentioned earlier (point 3 of section 1). In fact McKinnon (1985) argues that the overshooting of the dollar in late 1984/early 1985 was due at least in part to a conflict in the U.S. Federal Reserve System's immediate monetary objectives. To quote from McKinnon: "In November and again in December 1984, the FED cut the discount rate and embarked on much faster money
growth; it correctly noted that such expansion was warranted because (among other factors) the dollar at 3.0 marks was grossly overvalued even then. And for November, December and January, growth in U.S. M1 spurted to more than 11 percent measured on an annual basis. However, in January the FED then published - as required by the U.S. Congress - its money growth targets for all of 1985. A normal 5 to 7 percent growth range for M1 during 1985 was announced. Unfortunately, this published money growth target now conflicted with the higher money growth actually taking place in early 1985. In February and March, actual M1 was far above the cone of "permissible" levels officially published. The market came to expect that the FED would have to contract to get M1 back on its "normal" path. In anticipation, U.S. interest rates rose sharply in February 1985 and drove the dollar up further in the foreign exchanges. This surge into dollar assets assumed panic proportions when FED Chairman Volcker, testifying before Congress on February 20, suggested that the FED would end the progressively easier credit policy adopted in late 1984. Clearly the FED should have made clear that monetary ease would continue indefinitely, and that lower long-term growth in M1 would not be resumed until the dollar had fallen into its target zone.

Empirical tests of the portfolio models carried out by Branson and others (Branson, Halttunen and Masson, 1977) have not come up with satisfactory results. However, tests performed by Richard (1980) and Armington (1986) with much more sophisticated models both as regards the specification and as regards the empirical estimate have given much more satisfactory results. Richard and Armington specified the portfolio model in continuous time as a system of first order differential equations thereby introducing explicitly disequilibrium. They have estimated simultaneously the bilateral exchange rates of the main industrialized countries with respect to the dollar by using Wymer’s programs. Although the forecasting accuracy of their models for the dollar exchange rate in the eighties has so far been limited, in that the US current account has begun to exert the depressing effect on the dollar forecast by the
theory only in 1985, they must be given due consideration. The explicit consideration in the model of dynamic factors, the simultaneousness of the estimates for several exchange rates which makes it possible to incorporate easily substitution as between financial assets expressed in various currencies and the advanced econometric techniques they used, represent a considerable step forward.

It is still not clear how relative monetary restrictiveness can be reliably incorporated into a model like that of Richard and Armington in a treatable way. Their models must be simplified among other things by reducing the number of currencies, to make room for the restrictiveness of monetary policy without betraying the main thrust of Branson's model. In other words, I believe an attempt should be made to combine the model of Richard and Armington with that of Sommariva and Tullio presented in the Appendix, which, moreover, uses the same dynamic specification for the equations and the same simultaneous techniques for estimating parameters.
A disequilibrium model of the Exchange Rate during the German hyper-inflation: 1920-1923

This appendix presents a three equation model of inflation, exchange rate depreciation and monetary growth. The model is reprinted from Sommariva and Tullio (1987). After a review of the empirical literature on the German hyperinflation and a description of the main economic developments from 1919 to 1923 the authors conclude that a realistic model of the exchange rate for the German hyperinflation should possess the following characteristics:

a) the nominal exchange rate is allowed to diverge from purchasing power parity in the short run; b) currency substitution between domestic currency and foreign assets or goods is considered explicitly; c) the monetary approach to exchange rate determination and the quantity theory of money are imposed as long term properties on a disequilibrium model of inflation and exchange rate determination; d) the dynamic adjustment of prices and the exchange rate to monetary disturbances is made explicit.

Table 1 describes a three-equation model for prices, the exchange rate and money. The model is specified in continuous time according to the belief that macroeconomic phenomena, being the result of the behaviour of a large number of economic agents, are better described by continuous functions, although decisions taken by single economic agents may be discrete. Moreover, in a model specified in continuous time the estimates of the speeds of adjustment of the various markets have the advantage of being unbiased and independent of the length of the observation period to which the discrete data refer, and it is possible to obtain consistent estimates of the mean adjustment lags which may be much shorter than the observation period. The model is specified as a system of first order linear differential equations. For economy of notation the error terms are omitted in Table 1.
Table 1
A disequilibrium model of the exchange rate and prices

1. Exchange rate.

\[ \Delta \ln S = \alpha_1 \ln \left( \frac{S}{S_0} \right) \]  
(1)

where the partial equilibrium level of the exchange rate is given by:

\[ \hat{S} = \frac{\hat{P}}{\hat{P}_0} \]  
(1a)

where \( \hat{P} \) is the partial equilibrium level of prices:

\[ \hat{P} = \frac{B_u}{\hat{b}_u} \]  
(1b)

and \( \hat{b}_u \) is the partial equilibrium level of the demand for real cash balances:

\[ \hat{b}_u = \gamma e^{-\beta_1 \pi^e} e^{-\beta_2 \Delta \ln P} e^{-\beta_3 \Delta \ln S} \gamma \beta_4 \]  
(1c)

2. Prices.

\[ \Delta \ln P = \alpha_2 \ln \left( \frac{B_u}{P_{us} \hat{b}_u} \right) + \beta \ln \left( \frac{\gamma}{\hat{\gamma}} \right) \]  
(2)

\( \pi^e \) is expected inflation and is formed according to

\[ \Delta \pi^e = \lambda \left( \Delta \ln P - \pi^e \right) \]  
(2a)

3. Money supply.

\[ \Delta^2 \ln BU = \alpha_3 \left( \frac{\gamma - P}{BU} \right) + \alpha_4 \Delta \left( \frac{\gamma - P}{BU} \right) + \beta_2 \Delta \ln \left( \frac{\gamma}{\hat{\gamma}} \right) + \beta_3 \Delta \ln \left( \frac{\gamma}{\hat{\gamma}} \right) + \beta_3 \Delta \ln \left( \frac{\gamma}{\hat{\gamma}} \right) \]  
(3)

where \( \gamma \) is potential output

\[ \gamma = \gamma_0 e^{\lambda t} \]  
(3a)

List of variables

a) Endogeneous:

\( S \) = Mark/US dollar exchange rate
\( P \) = wholesale price index
\( BU \) = currency in circulation

b) Exogeneous:

\( P_{us} \) = US wholesale price index
\( y \) = real output in Germany
\( t \) = time trend

1/ Adjustment parameters are denoted by \( \alpha_i \) and \( \lambda \), elasticities by \( \beta_i \) and other parameters by \( \delta_i \).
Equation (1) assumes that changes in the nominal exchange rate are determined by discrepancies between the partial equilibrium level of the exchange rate and its actual level. The partial equilibrium level of the exchange rate (equation (1a)) is determined in the money market as predicted by the monetary approach to exchange rate determination. Equation (1c) is a demand function for real cash balances. The demand for real cash balances is assumed to depend on the expected and actual rate of inflation reflecting respectively substitution with domestic financial assets and goods,\(^2\) on the rate of change of the exchange rate reflecting foreign currency substitution, and on real income. The influence of currency substitution has been represented by the rate of change of the actual exchange rate, instead of the expected rate of change of the exchange rate principally in order to keep the model as simple as possible; the forward premium or discount which could have been used as a proxy for the expected rate of change of the exchange rate is available only from mid-1921. Its use would have substantially limited the sample period.

Equation (2) states that the inflation rate adjusts to the discrepancy between the supply of nominal cash balances and the demand for them. This equation is equivalent to one in which inflation adjusts to the discrepancy between the partial equilibrium and the actual levels of prices; and the partial equilibrium level of prices is determined by the discrepancy between the supply of nominal cash balances and the demand for real cash balances. Thus the steady state solution of this model implies that prices are formed according to the quantity theory of money and that purchasing power parity holds. In the short run, however, the exchange rate does not move according to purchasing power parity. The dynamics of inflation and the exchange rate depend crucially on the adjustment parameters \(\alpha_1\) and \(\alpha_2\). If \(\alpha_1\) is larger than \(\alpha_2\), the exchange rate is reacting more rapidly to monetary disequilibrium than prices thus producing a real depreciation of the exchange rate in the short run; the opposite is true if \(\alpha_2\) is greater than \(\alpha_1\).
Equation (3) is a simple money supply process, with the acceleration of nominal cash balances depending on the ratio of the government budget deficit to nominal cash balances and its changes as well as on the position of the economy in the cycle. Since no reliable observations are available on nominal government expenditures and revenues on a monthly basis, the deficit has been approximated in equation (3) by assuming that government expenditures and revenues are proportional to nominal income \((G = \alpha y p \text{ and } T = \beta y p)\), where \(G\) and \(T\) are respectively nominal government expenditures and revenues). The last two terms in equation (3) represent the reaction function of the monetary authorities to deviations of actual from potential output. These two terms can be interpreted as a linearization of a nonlinear model, in which parameters \(\lambda_3\) and \(\lambda_4\) change in time according to the gap between actual and potential output.\(^3\)

Estimates of the parameters of this model are presented in Table 2. The parameters of the exact discrete model equivalent to the continuous time system have been estimated simultaneously by full information maximum likelihood.\(^4\) The exact discrete model was used in estimation because it gives more efficient parameter estimates, which is particularly important when using the present set of data which is characterised by extreme variations. Another advantage in using the exact discrete model in estimation is that it allows mean time lags to go to zero, which is not the case when the approximate discrete model is used in estimation. A full information maximum likelihood estimator is perhaps the most satisfactory since it enables non-linear within and across equations restrictions to be imposed on the system and is consistent and asymptotically efficient. The sample period is April 1920 to July 1923. All estimated parameters are significantly different from zero with the exception of the income elasticity of the demand for real cash balances. The \(\lambda_1\) parameter turned out to be very large, indicating that the adjustment was extremely rapid and thus that the exchange rate was maintained in partial equilibrium.\(^6\) The \(\lambda_2\) parameter implies a mean adjustment lag of about 19 days indicating that disequilibrium in the money market affected prices with a relatively long lag. The \(\lambda\) parameter turns out to be about 0.27, implying a mean adjustment lag of about 3.7 months. This is more plausible than Cagan’s estimate which implied a mean lag of about 5.5 months. These results
Table 2

Empirical estimates of the model
(April 1920 - July 1923).

<table>
<thead>
<tr>
<th>Estimated parameters</th>
<th>Asymptotic standard error</th>
<th>Mean adjusted lag (in days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_t$</td>
<td>200.0 $^1$</td>
<td></td>
</tr>
<tr>
<td>$\alpha_t$</td>
<td>1.604</td>
<td>0.423</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.059</td>
<td>0.019</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>0.511</td>
<td>0.061</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.166</td>
<td>0.124</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>3.941</td>
<td>1.124</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.403</td>
<td>0.078</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>5.654</td>
<td>0.324</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>0.672</td>
<td>0.188</td>
</tr>
<tr>
<td>$\rho_3$</td>
<td>0.078</td>
<td>0.031</td>
</tr>
<tr>
<td>$\rho_4$</td>
<td>0.067</td>
<td>0.102</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.268</td>
<td>0.026</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>-0.001</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Chi-square value of the log-likelihood ratio 75.707 $^2$

Carter-Nagar chi-square statistic for the over-identified model 603.934 $^3$

Carter-Nagar $R^2$ of overidentified model 0.834

$^1$ In preliminary estimates and using an approximate discrete version of the model, parameter $\alpha_t$ turned out to be equal to 5.2 and significantly different from zero. Using an exact discrete estimator, for any value of $\alpha_t$ greater than 50, all other parameters remain unchanged and the likelihood function is flat. The high value of this coefficient, which corresponds to an extremely low mean time lag, indicates that equation (1) should really be of zero order. For this reason this parameter was set to 200 in the final estimates of the model.

$^2$ The value at the 5 per cent level of the chi-square distribution with 63 degrees of freedom is 82.5, so that the hypothesis that the overidentifying restrictions are not consistent with the sample must be rejected.

$^3$ The critical value at the 5 per cent level of the chi-square distribution with 15 degrees of freedom is 25.0, so that hypothesis that this model is not consistent with the data must be rejected.
explain the very long deviations of the real exchange rate from purchasing power parity during the German hyperinflation.

The estimates show that the degree of foreign currency substitution was significant during the German hyperinflation, although it is much lower than the degree of substitution of cash balances with domestic goods and financial assets. The income elasticity of the demand for real cash balances is around 0.1 and not significantly different from zero. This may, however, be the result of the way in which the monthly output series was reconstructed. German output statistics are available only on a yearly basis (Graham), and the interpolation was made by using monthly data on real imports of goods. Furthermore, the precision of the estimate of the income elasticity could have been affected by the low variability of monthly output with respect to the extremely high variability of nominal variables. The parameters $\alpha_3$ and $\alpha_4$ turn out to be significantly different from zero and around 0.06 and 0.5 respectively. Parameter $\alpha_3$ corresponds to $\alpha_3'(a-b)$, where $a$ and $b$ are the shares of government expenditures and revenues in nominal income. The average ratio of the Central Government budget deficit to nominal income for the sample period was about 13 percent, so that the $\alpha_3'$ parameter is 45 percent. The average share of government debt held by the Reichsbank in total government debt during the sample period was 33 percent, which is within the confidence interval of the estimated $\alpha_3$ parameter.

The chi-square statistic of the likelihood ratio indicates that the hypothesis that the overidentifying restrictions are consistent with the data cannot be rejected. It has to be remarked that the chi-square statistic of the likelihood ratio is an asymptotic test and that it is biased towards rejection in small samples. The Carter-Nagar system $R^2$ and the Carter-Nagar chi-square statistic of the overidentified model indicate that the model is consistent with the data. The root mean square errors (RMSE) of the static (or one period ahead) and dynamic forecasts are presented in Table 3. Since all variables are in logarithms, the RMSE gives the average error as a proportion of the actual level of variables.
Table 3

Ex-post root mean square errors of forecast.
(in percent)

<table>
<thead>
<tr>
<th></th>
<th>Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>D ln S</td>
<td>23.1</td>
<td>27.1</td>
</tr>
<tr>
<td>D ln P</td>
<td>17.1</td>
<td>18.2</td>
</tr>
<tr>
<td>D ln BU</td>
<td>4.1</td>
<td>8.2</td>
</tr>
<tr>
<td>ln S</td>
<td>17.1</td>
<td>32.5</td>
</tr>
<tr>
<td>ln p</td>
<td>18.7</td>
<td>29.3</td>
</tr>
<tr>
<td>ln BU</td>
<td>3.8</td>
<td>27.4</td>
</tr>
</tbody>
</table>
The RMSE in the static forecast is about 23 percent for the rate of change of the exchange rate and 17 percent for inflation while the RMSE for changes in the nominal cash balances is about 4 percent. The RMSEs in the dynamic forecast are similar to those in the static forecast for the rate of change of the variables, while they are higher for the level of the variables, partly because of cumulation of errors in the dynamic forecast and partly because the reaction function of the monetary authorities does not fully capture the money supply process. Charts 1, 2 and 3 plot the actual and simulated levels of the endogenous variables. They show that, in the dynamic simulation, equation (3) overestimates the growth of money both in 1921 and 1922. This can be explained by the fact that policy measures adopted by the German government in early 1920, caused the budget deficit in relation to GDP to fall from 19 per cent in 1920 to 6.2 per cent in 1922. Thus the approximation of the government budget deficit by nominal income in equation (3) leads to an overestimation of the growth of nominal cash balances both in 1921 and 1922, especially in the dynamic simulation.

Table 4 reports the results of a test on the autocorrelation of the residuals (static simulation). The residuals of each behavioural equation have been regressed both on their lagged values and on the lagged residuals of the other equations. Neither the autocorrelation parameters nor the cross correlation ones were significantly different from zero, indicating absence of autocorrelation.

The system of differential equations reported in Table 1 can be represented as follows:

\[ Dy(t) = Ay(t) + BZ(t) + u(t) \]

where \( y(t) \) is a vector of endogenous variables, \( Z(t) \) a vector of exogenous variables, and \( u(t) \) a vector of white noise disturbances. The endogenous part of the system \( Dy(t) = Ay(t) \) can be characterized by a set of eigenvalues containing all the dynamic information about the system. The estimated model is dynamically stable as all the real parts of the eigenvalues are negative (Table 5). The existence of two eigenvalues with imaginary parts implies that the model converges to equilibrium in cycles which have a period of about 3 months. The stability of the estimated model confirms
CHART 1
Mark-Dollar Exchange Rate (1)
(April 1920 - July 1923)


(1) Logarithms of the exchange rate index, 1913=1
CHART 2
Prices (1)
(April 1920 - July 1923)


(1) Logarithms of the index of prices, 1913=1
Chart 3
Money (1)
(April 1920 - July 1923)


(1) Logarithms of the index of nominal cash balances, 1913=1
<table>
<thead>
<tr>
<th>Independent variables</th>
<th>Dependent variables</th>
<th>$u_{S,t-1}$</th>
<th>$u_{S,t-2}$</th>
<th>$u_{p,t-1}$</th>
<th>$u_{t,t-2}$</th>
<th>$u_{BU,t-1}$</th>
<th>$u_{BU,t-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_{S,t}$</td>
<td></td>
<td>-0.113</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.068)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_{S,t}$</td>
<td></td>
<td>-0.111</td>
<td>0.049</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.680)</td>
<td>(0.280)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$u_{S,t}$</td>
<td></td>
<td>-0.232</td>
<td>0.256</td>
<td>0.701</td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.140)</td>
<td>(1.030)</td>
<td>(0.770)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_{S,t}$</td>
<td></td>
<td>0.006</td>
<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.040)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_{P,t}$</td>
<td></td>
<td>0.001</td>
<td>0.129</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.001)</td>
<td>(0.780)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$u_{P,t}$</td>
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<td>0.034</td>
<td>0.078</td>
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<tr>
<td></td>
<td></td>
<td>(0.230)</td>
<td>(0.160)</td>
<td>(0.100)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_{BU,t}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.149</td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td>(0.870)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_{BU,t}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.143</td>
<td>0.029</td>
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</tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>(0.790)</td>
<td>(0.160)</td>
<td></td>
</tr>
<tr>
<td>$u_{BU,t}$</td>
<td></td>
<td>0.032</td>
<td>-0.029</td>
<td>-0.159</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.820)</td>
<td>(0.630)</td>
<td>(0.910)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1/ Number in parenthesis are t statistics.
Table 5

Stability analysis 1/

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Asymptotic standard errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real part</td>
<td>Imaginary part</td>
</tr>
<tr>
<td>-0.268</td>
<td>0.026</td>
</tr>
<tr>
<td>-0.285</td>
<td>0.177</td>
</tr>
<tr>
<td>-0.842</td>
<td>0.520</td>
</tr>
<tr>
<td>-1.117</td>
<td>±2.235</td>
</tr>
</tbody>
</table>

1/ Although a second order system of differential equations would usually have six non-zero distinct eigenvalues, in the present model each differential depends on the rate of change of the exchange rate, prices and cash balances as well as on the level of cash balances deflated by either prices or the exchange rate. Hence one eigenvalue is zero and can be neglected. This does not affect the estimation of the model. It may be noted, however, that if the restrictions implicit in the theory on which the model is based were not enforced during estimation this result would be unlikely to hold.
findings by other authors that the German inflationary process was not feeding upon itself but was caused by fiscal deficits and excessive monetary growth.

A crucial assumption in the monetary analysis of inflation is the stability of the demand for money. According to the theory, structural parameters of the demand for real cash balances should be stable, while it cannot be excluded a priori that adjustment parameters shift as inflation accelerates. The model presented in Table 1 has been re-estimated for the period April 1920 to March 1923. The period from April to July 1923 was excluded from the sample because of the substantial acceleration of inflation in that period. Estimates of the model are presented in Table 6. Estimates of the parameters of the demand for real cash balances lie within the confidence interval of the parameters estimated for the sample period April 1920 to July 1923, thereby confirming previous findings by other authors that the demand for real cash balances was stable during the hyperinflation. The results, however, are not so clear for the parameters \( \alpha_2 \) and \( \lambda_6 \).
### Table 6
Parameter estimates  
(April 1920 - March 1923)

<table>
<thead>
<tr>
<th>Estimated parameters</th>
<th>Asymptotic standard error</th>
<th>Mean adjustment (in days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td>200.01</td>
<td></td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>1.259</td>
<td>0.336</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>0.045</td>
<td>0.016</td>
</tr>
<tr>
<td>$\omega_4$</td>
<td>0.421</td>
<td>0.064</td>
</tr>
<tr>
<td>$i_1$</td>
<td>0.333</td>
<td>0.179</td>
</tr>
<tr>
<td>$i_2$</td>
<td>2.899</td>
<td>1.135</td>
</tr>
<tr>
<td>$i_3$</td>
<td>0.352</td>
<td>0.195</td>
</tr>
<tr>
<td>$i_4$</td>
<td>6.063</td>
<td>0.479</td>
</tr>
<tr>
<td>$p_1$</td>
<td>0.852</td>
<td>0.252</td>
</tr>
<tr>
<td>$p_2$</td>
<td>0.093</td>
<td>0.039</td>
</tr>
<tr>
<td>$p_3$</td>
<td>0.352</td>
<td>0.195</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.304</td>
<td>0.038</td>
</tr>
<tr>
<td>$i_4$</td>
<td>-0.001</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Chi-square value of the loglikelihood ratio: 67.755<sup>1</sup>

Carter-Nagar chi-square of the overidentified model: 1076.72<sup>3</sup>

Carter-Nagar $R^2$ of overidentified model: 0.906

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1/ In preliminary estimates and using an approximate discrete version of the model, parameter $\omega_4$ turned out to be equal to 4.9 and significantly different from zero. Using an exact discrete estimator, for any value of $\omega$, greater than 50 all other parameters remain unchanged and the likelihood function is flat. The high value of this coefficient, which corresponds to an extremely low mean time lag, indicates that equation (1) should really be of zero order. For this reason this parameter was set to 200 in the final estimates of the model.

2/ The critical value at the 5 per cent level of the chi-square distribution with 63 degrees of freedom is 82.5, so that the hypothesis that the overidentifying restrictions are not consistent with the sample must be rejected.

3/ The critical value at the 5 per cent level of the chi-square distribution with 15 degrees of freedom is 25.0, so that hypothesis that this model is not consistent with the data must be rejected.
(1) Models that include a monetary variable (money, the monetary base and to a lesser degree the interest rate) take account indirectly of non-sterilized intervention.

(2) An appropriate specification of a demand for real cash balances would include a nominal interest rate and expectations of inflation reflecting substitution with financial assets and goods. However, since no reliable data on interest rates are available, the nominal interest rate was proxied by the expected inflation rate, under the assumption that interest rates were perfectly flexible during the German hyperinflation. In this model, actual inflation reflects substitution of cash balances with goods.

(3) A second interpretation of equation (3) is that the acceleration of nominal cash balances is a function of the ratio of nominal income to cash balances and its changes, as well as of the position of the economy in the cycle. The fact that parameters a and b are not identified makes it difficult to interpret this equation.

(4) The iterative procedure is Newton - Raphson followed by quasi-Newton once the gradient is small or the likelihood is almost stationary.

(5) In preliminary estimates, using an approximate discrete version of the continuous model, parameter \( \alpha_1 \), turns out to be equal to 5.2, significantly different from zero and larger than parameter \( \alpha_4 \). Using the exact discrete estimator, for any value of \( \alpha_4 \), greater than 50 all other parameters remain unchanged and the likelihood function is flat. The high value of this coefficient, which is equivalent to an extremely small mean time lag, indicates that equation (1) should really be of zero order, that is \( \hat{\hat{S}} = \hat{S} \). For this reason this parameter was set to 200 in the final estimates of the model.

(6) The RMSE of this model are very similar to those presented in table 5. In addition the stability analysis indicates that the model is stable for the subperiod.
REFERENCES


BILSON J. and HSIEH D.: The profitability of currency speculation, manuscript, University of Chicago, 1984.


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