IS THERE A STABLE MONEY DEMAND EQUATION AT THE COMMUNITY LEVEL?

-EVIDENCE, USING A COINTEGRATION ANALYSIS APPROACH, FOR THE EURO-ZONE COUNTRIES AND FOR THE COMMUNITY AS A WHOLE-

INTRODUCTORY REMARKS

A stable and predictable demand for money function is a pre-requisite for the use of targets for monetary aggregates as a suitable intermediate objective of monetary policy. A strong and sustained empirical interest in the stability of such functions and in the estimates for the income and interest elasticity of the demand for money is, of course, justified given the crucial role exercised by this relationship, allied to its parameter estimates, in terms of influencing the relative effectiveness of monetary and fiscal policies.

Up until the 1970’s the money demand function was regarded as one of the best understood and most highly stable equations in macro-economic models. Unfortunately, since then, due to some spectacular “breakdowns” or shifts in the money demand relationship in both virtually all of the major economies and for all of the various measures of money i.e. both the narrow and broad aggregates, it has come to be viewed as one of the weakest elements in the overall framework. Most analysts attributed this widespread instability, over the period in question, to the rapid pace of financial sector innovation in the economies concerned.

In spite of the clear difficulties which have manifested themselves in the estimation of such money functions, empirical interest has been renewed over the last decade driven by the belief that a stable long-run money demand relationship continues to exist, determined essentially by income and interest rates, and that any instability is merely a reflection that the short run adjustment processes underlying this linkage are substantially more complicated than earlier models had allowed for. This revival of activity in this area was aided and abetted by the coming on stream of new econometric techniques, especially in the area of cointegration analysis and error correction models. With the development of these new techniques for dealing with the problems associated with non-stationary time series, it has become customary to specify and estimate money demand functions in error-correction form since the latter type of models are ideal for reflecting the adjustment towards a long-run equilibrium state as a result of short-term disturbances.
Using these new econometric techniques, this paper looks again at the aggregate data for the Euro Zone countries and for the Community as a whole and tries to answer the question of whether a stable, cointegrating, money demand function exists at the Euro Zone and EC15 levels.

Research interest in this area of EC-wide money demand functions has been growing over the last two decades, driven initially by the interest generated by the Exchange Rate Mechanism (ERM) of the European Monetary System and, of course, more recently, by the single currency project. This research appeared to suggest that a Community-wide money demand equation had better stability properties than the national equivalent. This is also the conclusion reached by the present study.

It is important to highlight at the outset that despite this apparent stability any policy conclusions to be drawn from the paper must be very cautious indeed regarding the potential role of money targeting in guiding the monetary policy course of any future European Central Bank (ECB). With regard to this latter point the present paper does not attempt to draw any firm policy conclusions from the analysis undertaken since it is essentially a methodological exercise. Such cautiousness is justified given that any empirically supported ex post stability of a EU monetary aggregate does not suggest anything about ex ante stability given the substantial amount of evidence which suggests that EMU will result in important structural changes in monetary relations involving a permanent shift in the velocity of money.

Furthermore, even if the aggregate money demand function is stable, the regime change engendered by the creation of the EURO area would certainly call into question the usefulness of knowing that the aggregate money demand function was stable in the past. Policy makers would in fact do well to heed the warning contained in the concluding sentence of the 1993 paper on “A European Money Demand Function” by Artis, Blanden Hovell and Zhang “There must be a nagging doubt that underlying this apparently stable relationship are aspects of the Lucas critique and Goodharts’ law which are simply waiting to have a further laugh at the expense of monetary economists”.

**STRUCTURE OF STUDY**

**Section 1**: Rationale for the Modelling Strategy Adopted in the Study

**Section 2**: Specification Issues

**Section 3**: Establishing the Time Series Properties of the Variables

**Section 4**: Cointegration Analysis: The Multivariate Approach

**Section 5**: Single equation cointegration analysis : The Engle-Granger Procedure for Estimating the Error Correction Model

**Section 6**: Evaluation of the Robustness of the Error Correction Model: Diagnostic and Stability Tests

**Overview and Concluding Remarks**
SECTION 1: RATIONALE FOR THE MODELLING STRATEGY ADOPTED IN THE STUDY

In trying to provide answers to this question of stability in the money demand function a cointegration analysis approach is used. Cointegration analysis concerns itself with the relationships between variables that have random or stochastic trends, with the latter type of time trend being the hallmark of what are termed “non-stationary” variables. “Stationary” variables, on the other hand, are distinguishable by the possession of deterministic, i.e. fixed, time trends. If there is a stationary, linear, combination of a group of non-stationary time series then the latter series are said to be cointegrated. The latter stationary linear combination can be interpreted as a long-run equilibrium association and is often referred to as a cointegrating equation.

This distinction between stationary and non-stationary variables is an important one and depends on whether or not the variable contains a unit root. In this regard, it is essential to test for the presence of unit roots in the data before estimating a regression model containing non-stationary variables. Estimating such a model, without prior testing to establish what is termed the correct order of integration of the series, carries the danger of producing at worse “nonsense” regressions, which produce spuriously attractive results due to the random trends in the data, or at best ignores important information concerning the statistical and economic processes underlying the series.

Consequently, if unit roots are present in the data, and they usually are in time series, appropriate modelling procedures must be adopted. In the past researchers overcame this problem by removing the non-stationary (stochastic) trend in the data through simply differencing. This, however, provided only a partial solution since while it avoided the “nonsense” regression problem, it also removed important long-run information from the data concerning the natural equilibrating propensity in economic forces, as reflected in the co-movement of particular economic variables over time.

It is clear therefore that appropriately modelling the long run when the variables are non-stationary requires more than just differencing the data. It is also widely accepted that regressions involving the levels of non-stationary variables are only meaningful if the researcher has already shown that these variables are linked together in a long-run cointegrating relationship. If such a relationship can be established then an error correction model can be estimated using a sensible and statistically significant disequilibrium error term.

WHAT IS THE ATTRACTION OF USING ERROR CORRECTION MODELS: The use of such short-run Error Correction Models (ECM’s) constitutes an effective framework for establishing links between the short and long-run approaches to econometric modelling. Such dynamic type models are increasingly used for dealing with the problems associated with non-stationary series and spurious correlations. Simple first differencing, as mentioned earlier, does not provide a satisfactory solution to these problems. Among the main advantages of using an ECM approach is that:

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1 Note: If the mean, variance and covariances of a time series remain constant over time then that time series is said to be stationary.
1) no information on the levels of variables is excluded because of the inclusion of a disequilibrium term in ECM’s which picks up and reflects the extent of departure from the long-run equilibrium relationship;

2) provided that the variables in levels cointegrate, ECM’s avoid problems relating to spurious regressions because they are formulated in terms of first differences and consequently the variables are detrended;

3) the ECM incorporates both short-run and long-run effects, with a clear distinction made between the short-term dynamic effects (i.e. the speed of adjustment of the dependent variable in response to disequilibrium) and the long-run equilibrium parameters in the model.

**PROPOSED MODELLING STRATEGY:** The modelling strategy to be adopted in the paper is reasonably clear therefore. One must:

- firstly analyse the time series properties of the data in order to establish whether the variables are stationary or, more normally, non-stationary;
- secondly, if the series are shown to be non-stationary, then tests must be conducted to establish if the series concerned are cointegrated since it is only when non-stationary variables are cointegrated that it is acceptable to infer a causal long-run relationship(s) between those variables;
- finally, the dynamic model is estimated in error correction form.

**SECTION 2: SPECIFICATION ISSUES**

The demand for money is a demand for real balances. This implies that citizens do not suffer from money illusion over the long run in that their demand for nominal money balances is directly proportional to the prevailing price level. Consequently, it is theoretically appropriate to impose a unitary elasticity on the price level term in the money demand equation. Empirical testing also showed that this assumption of price homogeneity was indeed a valid restriction to impose. While some studies have shown a price coefficient which is significantly different from one, this can reasonably be explained as resulting from ongoing structural changes in both financial markets as well as in the money-holding behaviour of individuals.

Consequently, a common empirical specification of the demand for money function links the demand for transactionary and precautionary real money balances to have a positive relationship with some “scale” variable such as income, with some rate of interest variable varying inversely with a demand for speculative balances.

The data set used in the study involves annual data\(^2\) from 1970-1996 for the Euro Zone countries and for the Community as a whole. It contains series for real broad money (deflated by the GDP deflator and aggregated using PPS weights), real GDP and short-

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\(^2\) **Note:** While it would clearly have been more appropriate to use quarterly data, this was not possible as a reliable, sufficiently long, series for broad or narrow money was not available at the aggregate Euro Zone or EC15 levels.
term interest rates. All series except interest rates were expressed in logarithms\(^3\). This data set is used to model the demand for money.

As regards the choice of the most appropriate scale variable, while income, non-human wealth and permanent income were all possible candidates, pragmatism was the deciding factor in favour of income since reliable data for the other variables was not available for all countries and therefore could not be computed at the aggregate level. The latter income variable was defined as GDP in real constant prices.

In terms of the opportunity cost of holding money some researchers have tried different variables such as expected inflation or the rate of return on equity holdings in order to better specify the opportunity costs involved in holding money. Incorporating an expected inflation term did not prove fruitful in the present research. In addition, given the multicollinearity problem, it was necessary to choose a single interest rate variable to represent the opportunity costs of holding money. Experiments with both long and short term rates came down in favour of the latter.

In the end, therefore, a simple specification was adopted. Some testing of alternative, more complicated, specifications was carried out but none proved as robust as the preferred specification which has the demand for real money balances as being positively determined by income and negatively related to its opportunity cost, the short-term interest rate.

**SECTION 3: ESTABLISHING THE TIME SERIES PROPERTIES OF THE DATA: UNIT ROOT TESTS**

The empirical analysis stage of the research starts by testing for unit roots in the predetermined set of variables, real broad money (M2/M3), real income (GDP) and short-run interest rates. As mentioned above, all series except interest rates are expressed in logarithms.

The first thing to establish is the order of integration of the 3 series. It is crucial to determine these properties of the data using formal unit root testing and graphical analysis i.e. are the variables stationary in their levels or do they have to be differenced a number of times before they become stationary. If, for example, first differencing eliminates the trending behaviour in all the variables, then we can say that the variables are I(1), i.e. integrated of order one.

Testing for the presence of a unit root can be carried out in several different ways with the approach adopted here being one of graphical analysis combined with the augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests. In testing for the presence of a unit root, it is necessary to allow both an intercept and a time trend to enter the regression model. The null hypothesis to be tested is that a time series is non-stationary (i.e. it contains a unit root) against the alternative hypothesis of stationarity.

\[^3\] Interest rates are not expressed in logarithms since this would impose an unrealistic constant elasticity.
It must be stressed that unit root testing is neither a simple nor definitive process but use of both the ADF test, which allows for both constant and trend terms and has an adequate lag structure, combined with the sequential testing strategy which underpins the PP tests is widely accepted as being an appropriate testing strategy.

ADF and PP statistics for the level variables are shown in Tables 1 and 2 and when compared with the MacKinnon critical values clearly suggest that for both the Euro Zone countries and for EC15, all 3 series, real money, real GDP and the short term interest rate are non-stationary. The next step is to examine the first differences of the 3 series for both the country zones. On the basis of the ADF statistics, all of the differenced series reject the null hypothesis of non-stationarity at the 5% level and some reject it at the 1% level. This conclusion is broadly corroborated when one looks at the graphs of the first differences of the variables (Graphs 1 and 2). Given that the sample autocorrelations are decaying rapidly to zero and that there are no obvious or marked trends in the time paths of the variables one can reasonably conclude that stationarity is suggested. Overall therefore the results suggest that all variables correspond to I(1).

**SECTION 4: COINTEGRATION ANALYSIS: THE MULTIVARIATE APPROACH**

Given that our three basic series, real money, real GDP and interest rates appear to be I(1), then we must test the possibility that there is a long-run cointegrating relationship between these variables in their levels. If a long-run equilibrium relationship links two or more non-stationary time series, these series, despite containing stochastic trends, will remain closely linked over time since the “difference” between them will be stationary (i.e. stable).

The number of cointegrating equations linking the time series, i.e. the cointegrating rank, can be determined using the Johansen procedure. Johansen’s maximum likelihood approach is the recommended estimation method in the multivariate case since it determines the number of cointegrating vectors combined with providing estimates both of the latter vectors as well as of the adjustment parameters. Testing to see how many cointegrating vectors are present in the model involves an analysis of two likelihood ratio tests i.e. Johansen’s trace and maximal-eigenvalue statistics.

If, on the basis of the Johansen procedure, cointegration amongst the variables cannot be rejected, then at least one long-run linear relationship must exist linking the latter variables. If, on the other hand, the Johansen procedure shows an absence of cointegration between the variables then any posited relationship between the latter based on regression analysis is spurious i.e. there is an absence of a long-run equilibrium to which the system of variables converges over time.

Applying the Johansen approach to the annual data for the period 1970-1996 resulted in at most one cointegrating vector being accepted between the three I(1) variables, real money, real GDP and short-term interest rates, after testing for reduced rank.
TABLE 1  EURO ZONE
STATISTICAL STATIONARITY TESTS

A. Augmented Dickey-Fuller and Phillips-Perron Unit Root Tests in Levels*

A.1. Intercept included in test equation (1 lagged difference)

<table>
<thead>
<tr>
<th></th>
<th>EURO ZONE</th>
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<tbody>
<tr>
<td></td>
<td>ADF TEST STATISTIC</td>
</tr>
<tr>
<td>Real money</td>
<td>- 0.56</td>
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<tr>
<td>Real GDP</td>
<td>- 1.47</td>
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<tr>
<td>Interest rate</td>
<td>- 2.57</td>
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</table>

A.2. Trend and intercept included in test equation (1 lagged difference)

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td></td>
<td>ADF TEST STATISTIC</td>
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<tr>
<td>Real money</td>
<td>- 3.79</td>
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<tr>
<td>Real GDP</td>
<td>- 3.05</td>
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<td>Interest rate</td>
<td>- 2.58</td>
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* MacKinnon critical values for rejection of hypothesis of a unit root

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<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>Intercept + Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>1% critical value =</td>
<td>- 3.71</td>
<td>- 4.36</td>
</tr>
<tr>
<td>5% critical value  =</td>
<td>- 2.98</td>
<td>- 3.59</td>
</tr>
<tr>
<td>10% critical value =</td>
<td>- 2.63</td>
<td>- 3.23</td>
</tr>
</tbody>
</table>

GRAPH 1 : EURO ZONE : FIRST DIFFERENCES OF VARIABLES +
CORRELOGRAMS

![Graphs of Real Money, Real GDP, and Interest Rates with Correlograms](image-url)
A. Augmented Dickey-Fuller and Phillips-Perron Unit Root Tests in Levels*

A.1. Intercept included in test equation (1 lagged difference)

<table>
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<tr>
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<th>EUR 15</th>
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<tbody>
<tr>
<td></td>
<td>ADF TEST STATISTIC</td>
<td>PP TEST STATISTIC</td>
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<tr>
<td>Real money</td>
<td>- 0.32</td>
<td>- 0.84</td>
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<tr>
<td>Real GDP</td>
<td>- 1.29</td>
<td>- 1.68</td>
</tr>
<tr>
<td>Interest rate</td>
<td>- 2.66</td>
<td>- 1.97</td>
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A.2. Trend and intercept included in test equation (1 lagged difference)

<table>
<thead>
<tr>
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<th>EUR 15</th>
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<tr>
<td></td>
<td>ADF TEST STATISTIC</td>
<td>PP TEST STATISTIC</td>
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<tr>
<td>Real money</td>
<td>- 3.13</td>
<td>- 1.96</td>
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<tr>
<td>Real GDP</td>
<td>- 3.26</td>
<td>- 2.50</td>
</tr>
<tr>
<td>Interest rate</td>
<td>- 2.57</td>
<td>- 1.82</td>
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* MacKinnon critical values for rejection of hypothesis of a unit root

\[
\begin{align*}
1\% \text{ critical value} &= -3.71 \\
5\% \text{ critical value} &= -2.98 \\
10\% \text{ critical value} &= -2.63
\end{align*}
\]

Intercept \quad \text{Intercept + Trend}

\[
\begin{align*}
&-4.36 \\
&-3.59 \\
&-3.23
\end{align*}
\]

GRAPH 2 : EC15 : FIRST DIFFERENCES OF VARIABLES + CORRELOGRAMS
Table 3 shows that we can easily reject the null hypothesis that there are no cointegrating vectors for both the Euro Zone and EC15 groups of countries, while the hypothesis of a rank of less than or equal to 1 cointegrating vectors cannot be rejected by the likelihood ratio test.

One can therefore tentatively conclude that there is at least one statistically significant cointegrating vector relating the variables. This view is corroborated by the existence of one large eigenvalue for both the Euro-Zone and EC15 tests. It is reasonably evident that eigenvalues of between 0.6 and 0.8 for both country zones are significant values strongly indicating the presence of at least one cointegrating vector. The size of the eigenvalue suggests that there is at least one equilibrium condition that keeps the 3 time series in proportion to each other over the long-run. Graph 3 shows plots of this money demand cointegrating vector and of the actual and fitted values as well as recursive estimates of the eigenvalues.

This conclusion of one unique vector is also strengthened by looking at the normalised cointegrating coefficients which, on the assumption of 1 cointegrating equation, show values which are close to what economic theory would suggest. Checking that the normalised cointegrating equations are economically meaningful in terms of the theoretical predictions concerning the long-run relations amongst the variables is a crucial aspect of the analysis of the Johansen results. The respective unique cointegrating vectors for the Euro Zone and EC15 groups are reported in Table 3 as the unrestricted Beta eigenvectors where the cointegrating parameters have been normalised on real money. These equations are recognisably money demand functions with the coefficient on the opportunity cost variable (i.e. the interest rate) and on income both being of the correct order of magnitude and being correctly signed.

Since there would appear to be at most only one cointegrating vector, use of a multivariate (i.e. systems), as opposed to a single equation approach to the analysis, would not appear necessary. However, it is not valid to move down to the single equation approach unless we can show that all the variables in the cointegrating vector are weakly exogenous. If this can be shown then it is valid to abandon the multivariate model and move to a single equation approach.

**TESTING FOR WEAK EXOGENEITY:** To test for weak exogeneity one must place weak exogeneity restrictions on the alpha (i.e. $\alpha$) vector of the II-matrix and test, using a likelihood ratio test, whether the restrictions are valid. The II-matrix = $\alpha\beta$ with $\alpha$ representing the speed of adjustment to disequilibrium and $\beta$ is a matrix of long-run coefficients. Testing for weak exogeneity in the Euro Zone and EC15 money demand models is a test that real income and the short term interest rate are weakly exogenous.

The test results showed that the $\alpha$ restrictions imposed were valid and therefore that it was justifiable to use a single equation estimation of the cointegration vector. This conclusion is further confirmed, as we shall see later in the study, by the mutually similar results for the money demand model using the Johansen multivariate estimation method compared with the single equation approach. These results strongly suggest that the single or systems approaches to the estimation of an EC money demand equation are equivalent. In conclusion, therefore, a single equation analysis is justifiable despite the system context.
### EURO ZONE: Cointegration analysis 1971 to 1996

<table>
<thead>
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<th>EIGENVALUE</th>
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<tr>
<td>200.021</td>
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<tr>
<td>0.747184</td>
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<td>218.649</td>
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<td>0.000884848</td>
<td>218.660</td>
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</table>

**Ho:** $\text{rank} = p$

- $-T\log(1-\mu)$ **using** $T-NM$
- **95%** $-T\Sigma\log(.)$

| $p = 0$ | 34.38** | 30.25** | 21.0 | 37.28** |
| $p <= 1$ | 2.879 | 2.534 | 14.1 | 2.901 |
| $p <= 2$ | 0.02213 | 0.01948 | 3.8 | 0.02213 |

**STANDARDIZED \beta' EIGENVECTORS**

<table>
<thead>
<tr>
<th>L11_rm</th>
<th>L11_gdp</th>
<th>ec11_is</th>
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<tr>
<td>1.0000</td>
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<td>-52.070</td>
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### EC15: Cointegration analysis 1971 to 1996

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<th>EIGENVALUE</th>
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</table>

**Ho:** $\text{rank} = p$

- $-T\log(1-\mu)$ **using** $T-NM$
- **95%** $-T\Sigma\log(.)$

| $p = 0$ | 28.65** | 25.34* | 21.0 | 30.44* |
| $p <= 1$ | 1.768 | 1.564 | 14.1 | 1.785 |
| $p <= 2$ | 0.01767 | 0.01563 | 3.8 | 0.01767 |

**STANDARDIZED \beta' EIGENVECTORS**

<table>
<thead>
<tr>
<th>Leu_r2m3</th>
<th>Leu_gdpq</th>
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<tr>
<td>1.0000</td>
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<tr>
<td>-0.74504</td>
<td>1.0000</td>
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<td>-53.492</td>
<td>35.347</td>
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EURO ZONE

MONEY DEMAND COINTEGRATING VECTOR

ACTUAL AND FITTED VALUES

RECURSIVE EIGENVALUES

EC15

MONEY DEMAND COINTEGRATING VECTOR

ACTUAL AND FITTED VALUES

RECURSIVE EIGENVALUES
SECTION 5: SINGLE EQUATION COINTEGRATION ANALYSIS: 
THE ENGLE-GRANGER PROCEDURE FOR ESTIMATING THE 
ERROR CORRECTION MODEL

It needs to be underlined at this stage that since the use of a single equation approach is only valid when there is only one cointegrating vector and when all the right-hand side variables are shown to be weakly exogenous, any analysis should consequently always start by using the multivariate framework provided by the Johansen procedure since the systems approach considers all possible relationships between the variables. This latter point is, in fact, essential if one wishes to adopt the Engle-Granger single equation method for estimating long-run cointegrating relationships and error correction mechanisms (ECM’s) between variables. Given that we have more than 2 variables in the present model, checking that there are no more than one cointegrating vectors between them is obligatory because the Engle-Granger procedure is not applicable in cases where I(1) variables are linked by more than one long-run relationship (i.e. for “n” variables in a system there are at most (n-1) possible cointegrating vectors).

ESTABLISHING COINTEGRATION USING THE ENGLE-GRANGER TWO-STAGE PROCEDURE

SHORT OVERVIEW: The most widely used single equation approach to cointegration is the Engle-Granger two-stage procedure. In Stage One the static real money demand equation is estimated by OLS and the residuals from this equation are tested for the presence of unit roots. If the residuals are stationary, then in Stage Two they enter (lagged one period) into the short-run ECM with the change in real money as the dependent variable. This short-run ECM is then estimated using the residuals as estimates of the true disequilibrium errors, with the objective of obtaining information on the speed of adjustment to equilibrium.

STAGE 1: ESTIMATION OF THE LONG-RUN PARAMETERS BY SIMPLY ESTIMATING THE STATIC REGRESSION AND CHECKING THE RESIDUALS OF THE COINTEGRATING EQUATION TO SEE WHETHER THEY ARE I(0).

In order to test for cointegration using the Engle-Granger method, we must firstly establish that the time series, which are assumed to form the long-term equilibrium relationship, are all non-stationary i.e. I(1) series – this has already been done earlier in the paper for the real money, real GDP and short term interest rate series. Secondly we must check whether the disequilibrium errors (i.e. the residuals) associated with the static regression are stationary i.e. I(0). If the latter residuals are stationary then this implies that the variables are indeed cointegrated. These two latter processes are necessary to prove cointegration exists since if a long-run equilibrium relationship exists between a set of non-stationary variables then any linear combination of the variables, such as that given by the series formed by the disequilibrium errors associated with this cointegrating relationship, should form a stationary series and have a mean of zero.

Therefore, in order to test for cointegration we need to examine the residuals from the static equation for stationarity. When testing for stationarity in the residuals, it is normal to suppress both the intercept and time trend from the Dickey-Fuller and Phillips-Perron regressions since the OLS residuals are assumed to have a zero mean and the latter residuals are unlikely to have a deterministic trend. As one can see in Table 4 from a comparison of the ADF and PP statistics with the MacKinnon critical values, we can reject the hypothesis of non-stationary residuals and therefore it would appear that the
real money supply, real GDP and short-term interest rate series are cointegrated. This is further corroborated by the fact that if you exclude the interest rate variable from the static equation then the residuals are not I(0), therefore tentatively suggesting that no cointegration relationship exists between just real money and GDP. It appears that interest rates, as strongly suggested by theory, play a long-run role.

In conclusion, therefore, since the residuals from the static cointegrating regression are stationary i.e. they are integrated of order zero, we can tentatively conclude that the 3 variables cointegrate i.e. a long-run money demand function involving the 3 variables appears to exist.

<table>
<thead>
<tr>
<th>Table 4  STATIONARY TESTS ON RESIDUALS OF LONG RUN EQUATIONS*</th>
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<tbody>
<tr>
<td><strong>Augmented Dickey-Fuller Unit Root Test in Levels</strong></td>
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<tr>
<td><strong>EURO ZONE</strong></td>
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<tr>
<td>ADF TEST STATISTIC</td>
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<td><strong>Phillips-Perron Unit Root Test in Levels</strong></td>
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<td><strong>EURO ZONE</strong></td>
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<td>PP TEST STATISTIC</td>
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</tbody>
</table>

**Mac Kinnon critical values for rejection of hypothesis of a unit root**

$1\%$ critical value $= -2.66$, $5\%$ critical value $= -1.96$, $10\%$ critical value $= -1.62$

**STAGE 2: ESTIMATING THE ERROR CORRECTION MODEL (ECM):**

**ESTIMATING THE ECM USING LOG-DIFFERENCED VARIABLES FOR REAL MONEY AND REAL GDP AND THE DIFFERENCED INTEREST RATE VARIABLE GIVES ESTIMATES OF THE SHORT AND LONG-RUN RELATIONSHIPS BETWEEN REAL MONEY, REAL GDP AND SHORT-TERM INTEREST RATES.**

With the 3 time series apparently cointegrated, the short-term disequilibrium relationship between them can be expressed in error correction form, with any disequilibrium from the long-run relationship involving the 3 variables being progressively corrected. The residuals from the cointegrating regression in Stage 1 are utilised in Stage 2 as estimates of the true disequilibrium errors. As mentioned earlier, converting the dynamic model into an error-correction formulation (ECM) carries many advantages. An ECM contains information on both the short-run and long-run properties of the data, with disequilibrium being seen as a short-run process of adjustment to the long-run equilibrium state. It is important to stress that the existence of a dynamic error correction model is implied by cointegration. From a policy viewpoint, information regarding the short-run structure of the model is at least as interesting as estimates of the long-run because of the picture it conveys of the short-run adjustment behaviour of economic variables (i.e. the dynamics of adjustment).
As shown below in the box the second stage regressions include the stationary differenced series from stage one and an error-correction term which imposes the long-run properties of the model on the dynamic structure:

\[
\Delta \text{real money}_t = \text{lagged } (\Delta \text{real money}, \Delta \text{real GDP}, \Delta \text{Interest Rates}) - \lambda u_{t-1} + \epsilon_t
\]

where \( \lambda \) is a short-run adjustment parameter and \( u_{t-1} \), the disequilibrium error, provides an indication of the extent of the departure from the long-run relationship.

The specific dynamics of the short-run model were arrived at following a process of reduction from a more general model which included several lags of the explanatory variables. One then tested down to the simplified model eliminating in the intervening stages any differenced terms which were not significant in statistical terms i.e. which had insignificant t-ratios. This process of reduction resulted in the acceptance of the parsimonious money demand function indicated in Tables 5 and 6. Initially, in fact, lagged differences of up to the second order for the variables were included but Tables 5 (Euro Zone) and 6 (EC15) only report the final preferred money demand equation obtained by the Engle-Granger procedure.

**SECTION 6: EVALUATION OF THE ROBUSTNESS OF THE ERROR CORRECTION MODELS: DIAGNOSTIC AND STABILITY TESTS**

Requirements for a “stable” demand for money function include parameter values which remain constant over time (i.e. when estimated over different time periods) and a well behaved disturbance term, as reflected, in for example, a small equation standard error, which would indicate that variables other than income or the rate of interest could not play a significant role in determining the demand for money. If both these conditions hold, as the following analysis will suggest, then accurate predictions for the demand for money can be made, with the forecasting performance of such equations easily testable by out of sample forecasts.

In selecting the above, final, dynamic error correction models for the Euro Zone and EC15 countries, two main checks on the adequacy of the models were undertaken:

(a) diagnostic checks on the residuals;

(b) checks to ensure that parameter constancy holds;

In carrying out these tests attention was focussed on the need to ensure that the following criteria for model selection were respected:

- \( A \): the models had to be well specified in that they adequately explained the existing data not only in terms of having a high R² and a low residual sum of squares but also in that the residuals were completely random i.e. they were non-autocorrelated and homoscedastic;
### TABLE 5: ERROR CORRECTION MODEL: EURO ZONE

#### SHORT AND LONG-RUN PARAMETER ESTIMATES

**DYNAMIC SHORT RUN MODEL**

\[
DL_{11}\text{ rm} = +0.5067 \ DL_{11}\text{ rm}_1 + 0.5886 \ DL_{11}\text{ gdp} \\
[\text{HCSE}] [0.1032] [0.1245] \\
-0.003111 \ Dec_{11}\text{ is} -0.2726 \ Residual_1 \\
[0.0009323] [0.08227]
\]

\[R^2 = 0.944\] \[\sigma = 0.0086\] \[DW = 1.90\] \[RSS = 0.0014\]

**SOLVED STATIC LONG RUN EQUATION**

\[
DL_{11}\text{ rm} = +1.193\ DL_{11}\text{ gdp} -0.006306 \ Dec_{11}\text{ is} \\
(SE) (0.1374) (0.002186) \\
-0.5525 \ Residual \\
(0.2107)
\]

\[ECM = DL_{11}\text{ rm} - 1.19317*DL_{11}\text{ gdp} + 0.00630609*Dec_{11}\text{ is} + 0.552512*Residual;\]

---

**GRAPH 4: ERROR CORRECTION MODEL: EURO ZONE**

**ACTUAL AND FITTED VALUES + SCALED RESIDUALS**

![Graph showing fitted and residual values](image-url)
### DYNAMIC SHORT RUN MODEL

\[
\text{Deu}_r2m3 = +0.4539 \text{ Deu}_r2m3_1 + 0.5748 \text{ Deu}_gdpq \\
\text{[HCSE]} \quad [0.08338] \quad [0.1021] \\
-0.002623 \text{ Deu}_i + -0.2315 \text{ Residual}_1 \\
\text{[0.001675] \quad [0.06764]}
\]

\[R^2 = 0.89 \quad \sigma = 0.011 \quad DW = 1.78 \quad RSS = 0.00242\]

### SOLVED STATIC LONG RUN EQUATION

\[
\text{Deu}_r2m3 = +1.053 \text{ Deu}_gdpq + -0.4239 \text{ Residual}_1 \\
\text{[SE]} \quad (0.1799) \quad (0.1779) \\
-0.004803 \text{ Deu}_i \\
(0.002701)
\]

\[\text{ECM} = \text{Deu}_r2m3 - 1.05259*\text{Deu}_gdpq + 0.42388*\text{Residual}_1 + 0.00480263*\text{Deu}_i;\]

---

### GRAPH 5: ERROR CORRECTION MODEL: EC15

**ACTUAL AND FITTED VALUES + SCALED RESIDUALS**
• **B**: the model parameters had to be constant as indicated by the Chow tests for parameter stability and predictive failure as well as by the use of recursive coefficient estimation techniques;

• **C**: the correctness of the functional form used for the models had to be verified to ensure that no problems had arisen in this regard;

• **D**: finally, the models had to have sensible long-run interpretations i.e. the results had to be consistent with economic theory.

**ANALYSIS OF THE RESULTS OF STAGE 2 OF THE ENGLE-GRANGER PROCEDURE**: The main results from estimating the ECM’s for the Euro Zone and EC15 countries using Engle-Granger are included, as mentioned above, in Tables 5 and 6 and Graphs 4 and 5. In overall terms, one can say that the diagnostic statistics all appear satisfactory. In parentheses under each coefficient estimate are the heteroscedastic-consistent standard errors. The model fits the data well i.e. it tracks real money growth in a satisfactory fashion. The estimated equation standard error is very low and this is reflected in the graph of the actual and fitted values for the dynamic model which shows a tight fit.

While the interest elasticity may appear low (see Table 7 on elasticities) it should be noted that estimation using short-term interest rates and a broad definition of money always yields lower elasticities than with long rates and a narrow money definition. With a real income elasticity of greater than 1 for both the Euro Zone and EC15, these equations clearly do not support the microeconomic theories pioneered by Baumol & Tobin of economies of scale in cash holdings. In addition it would appear that income elasticities in excess of unity are not incompatible with the maintenance of nominal stability.

| Table 7  LONG AND SHORT-RUN ELASTICITIES |
|------------------|------------------|------------------|------------------|
| **SHORT-RUN**    | **LONG-RUN**     | **SHORT-RUN**    | **LONG-RUN**     |
| **EURO ZONE**    | **EC15**         | **EURO ZONE**    | **EC15**         |
| Income           | 0.59             | 0.57             | 1.19             | 1.05             |
| Rate of Interest*| -0.003           | -0.003           | -0.006           | -0.005           |

* Semi-elasticity: percentage change in money demand given a 1 percentage point increase in the short-term rate of interest. For example a 1 percentage point increase in the short-term rate induces a decline in real money demand in the short-run of the order of 0.3% in both the Euro Zone and EC15 groups of countries and a decline of between 0.5-0.6% over the long-run.

As regards the money demand disequilibrium term $u_{t-1}$ (i.e. the error correction term which is a proxy for the cointegration relationship) it has the correct sign and has a significant t-ratio, although its size is perhaps on the low side given that annual data is being used for estimation purposes. The coefficient on this disequilibrium or feedback term shows the amount of the disequilibrium which is present in period t-1, which is corrected for in the current period i.e. it is a speed of adjustment coefficient in that it

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4 It is important to stress that while the coefficient estimates are in excess of one they are not statistically different from one which means that the theoretically sound null hypothesis of homogeneity cannot be rejected.

5 **Note**: The error-correction coefficient is equal to the coefficient on the lagged residual term.
gives an indication of the speed of adjustment of the real money stock to deviations from its desired long-run position. As regards the latter it would appear that the Euro Zone and EC15 money supplies adjust relatively sluggishly to shifts in the long-run equilibrium relationship, with the parameter estimates on the lagged error term showing that only 20-30 percent of the disequilibrium was removed each period.

**Diagnostic Checks on the Residuals:** As indicated in Tables 8 and 9, the diagnostic checks on the residuals of the final models were satisfactory in all respects. In terms of the test statistics, failure at the 1% and 5% significance levels would be indicated by asterisks. Since no asterisks are shown, all the residual test results are satisfactory and therefore there is nothing to suggest that the model is misspecified.

**Checks that Parameter Constancy Holds: this checking exercise involved graphing the recursive properties of the model including Chow Tests and recursive estimation of the coefficients.**

Diagnostic testing for structural breaks in the model has been carried out in order to verify that the parameter estimates are constant. Checking in this way for parameter constancy is, in fact, the crucial stage in policy terms with the stability/instability of the money demand equation in the Euro-Zone area having enormous implications for the efficacy of the monetary policy course to be adopted by the European Central Bank.

Plots of the 1-step residuals and the breakpoint and forecast Chow Tests, obtained by applying recursive least squares to the model over successive time periods are given in Graphs 6 and 7 for the estimated short-run ECM. The final equations chosen above successfully pass all these normal Chow tests for predictive failure etc. Of course, if the Chow Tests for parameter stability are failed, this is an indication that the money demand function is unstable.

In addition to checking for the adequacy of the model as a whole, the stability of the individual parameters can be effectively tested and displayed by making use of the recursive least squares estimation procedure with the time paths of the recursive, OLS, estimates of the coefficients also being shown in Graphs 6 and 7. These recursive estimates for all the parameters were computed, as with the Chow tests above, in order to assess the stability of the dynamic error-correction model over time. The graphs show broad stability over the period in question for all of the coefficients, with no real evidence to suggest parameter instability resulting, for example, from the failure to allow for structural breaks.6

**Final Comment:** In conclusion, therefore, the extensive series of diagnostic tests and statistics indicated in the various Tables and Graphs suggests that the final models exhibit no problems in terms of specification. The diagnostic statistics are satisfactory. Short-run fluctuations in the demand for broad money in the Euro-Zone and EC15 areas would therefore appear to be a stable function of changes in real income, interest rates and lagged real money, as well as the ECM term which captures deviations of the real money stock from its desired equilibrium position.

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6 It should be pointed out that this lack of evidence of structural breaks is more a reflection of both the aggregation of country data and of the fact that annual as opposed to quarterly data is being used for estimation.
### Table 8: Diagnostic Tests for Assessing the Robustness of the Euro Zone Money Demand Equation

<table>
<thead>
<tr>
<th></th>
<th>Euro Zone</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>INSTABILITY TESTS</strong></td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td>0.20</td>
</tr>
<tr>
<td>Joint</td>
<td>1.14</td>
</tr>
<tr>
<td><strong>RESIDUAL TESTS</strong></td>
<td></td>
</tr>
<tr>
<td>Residual autocorrelation (AR)</td>
<td>F (2,18) 1.35 (0.28)</td>
</tr>
<tr>
<td>Conditional heteroscedasticity (ARCH)</td>
<td>F (1,18) 0.31 (0.58)</td>
</tr>
<tr>
<td>Normality Chi^2 (2)</td>
<td>1.73 (0.42)</td>
</tr>
<tr>
<td>Heteroscedasticity (Xi^2)</td>
<td>F (8,11) 2.86 (0.05)</td>
</tr>
<tr>
<td><strong>FUNCTIONAL FORM TESTS</strong></td>
<td></td>
</tr>
<tr>
<td>- Xi *Xj</td>
<td>F (14,5) 1.79 (0.27)</td>
</tr>
<tr>
<td>- Reset</td>
<td>F (1,19) 0.08 (0.78)</td>
</tr>
</tbody>
</table>

### Table 9: Diagnostic Tests for Assessing the Robustness of the EC15 Money Demand Equation

<table>
<thead>
<tr>
<th></th>
<th>EC15</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>INSTABILITY TESTS</strong></td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td>0.07</td>
</tr>
<tr>
<td>Joint</td>
<td>1.09</td>
</tr>
<tr>
<td><strong>RESIDUAL TESTS</strong></td>
<td></td>
</tr>
<tr>
<td>Residual autocorrelation (AR)</td>
<td>F (2,18) 0.12 (0.89)</td>
</tr>
<tr>
<td>Conditional heteroscedasticity (ARCH)</td>
<td>F (1,18) 0.29 (0.60)</td>
</tr>
<tr>
<td>Normality Chi^2 (2)</td>
<td>1.27 (0.53)</td>
</tr>
<tr>
<td>Heteroscedasticity (Xi^2)</td>
<td>F (8,11) 2.41 (0.10)</td>
</tr>
<tr>
<td><strong>FUNCTIONAL FORM TESTS</strong></td>
<td></td>
</tr>
<tr>
<td>- Xi *Xj</td>
<td>F (14,5) 1.23 (0.46)</td>
</tr>
<tr>
<td>- Reset</td>
<td>F (1,19) 0.31 (0.59)</td>
</tr>
</tbody>
</table>
Graph 6: STABILITY TESTING OF THE SHORT-RUN ERROR CORRECTION MODEL (EURO ZONE)

A: RECURSIVE ESTIMATES OF (LAGGED) MONEY, INCOME, INTEREST RATE AND ERROR CORRECTION COEFFICIENTS

B: 1-STEP RESIDUALS AND CHOW TESTS
Graph 7: Stability Testing of the Short-Run Error Correction Model (EC15)

A: Recursive Estimates of (Lagged) Money, Income, Interest Rate and Error Correction Coefficients

B: 1-Step Residuals and Chow Tests
OVERVIEW AND CONCLUDING REMARKS

Economic theory suggests that the real demand for money depends on an income or “scale” variable, such as real GDP, and on the opportunity cost of money as reflected by the short/long-term interest rate. Using an unrestricted dynamic modelling approach involving stationarity tests, cointegration analysis and the formulation of error correction models this paper provides estimates of a money demand function for the Euro-Zone and EC15 group of countries over the period 1970-1996.

The evidence presented in the paper strongly suggests that a stable, cointegrating, money demand function exists for both country groupings. Consequently, since the foundation for any reliable relationship between money and nominal income is a stable demand for money function, it could be tentatively concluded that with such a constant, well-specified, equation operating at the Euro-zone level that money targeting will at least be an option for the guidance of the ECB’s monetary policy course.

RESEARCH PROCESS: Following stationarity tests on the selected variables of real money, real GDP and interest rates, the paper proceeds to test for cointegration amongst the 3 variables concerned by estimating a VAR for these variables and applying the Johansen procedure. Two likelihood ratio tests (i.e. the maximum eigenvalue and the trace tests) determined the number of cointegrating vectors. The Johansen procedure results were interpreted as confirming the appropriateness of using a single equation modelling approach since the multivariate procedure indicated the existence of at most only one cointegrating vector and the real income and interest rate variables passed weak exogeneity tests. This was confirmed further by the fact that the estimated, normalised, cointegrating coefficients produced using the VAR, multivariate, approach were not only sensible in economic terms but were also roughly equivalent in size to those estimated by the preferred, Engle-Granger, single equation method.

The Engle-Granger approach was therefore adopted in the paper with the dynamic model being estimated in error-correction form. Underlying the long-run / short-run distinction in the error correction model is the notion of equilibrium, with the long-run referring to an equilibrium state where economic forces are in balance (i.e. there is no intrinsic tendency to shift) and with the short-run referring to the disequilibrium state. The Engle-Granger approach is divided into two stages, with the second, and most important stage, estimating the error correction model and using the residuals from the static equation in stage one as estimates of the disequilibrium errors.

ASSESSMENT OF FINAL MODEL: In practical operational terms, the key tests concerning the acceptability or “adequacy” of a model boils down to it having satisfactory diagnostic statistics, in particular residuals which are “white noise” and parameters which are constant over time, and that the model is consistent with a clearly enunciated economic theory. In this regard the final Euro Zone and EC15 models are adequate in all respects with the diagnostic statistics being satisfactory, the disequilibrium term \( u_{t-1} \) (i.e. the error correction term) having a significant t-ratio and being correctly signed and with the short and long-run parameter estimates appearing reasonable from an economic perspective. The ECM’s for the Euro Zone and EC15 countries were also shown to be stable with the graphs of the actual and fitted dynamic models showing close fits, indicating low equation standard errors.
**Policy Conclusions:** It should be underlined at this stage that the evidence presented in the paper suggesting well-behaved and stable Euro-zone and EC15 money demand equations contrasts sharply with the instability which appears to characterise most of such functions at the individual country level.

The key question for policy makers, therefore, is to evaluate whether the stability evident at the aggregate Euro-Zone and Community levels is simply a function of aggregation bias and therefore is potentially misleading from the point of view of policy formulation or whether it results from currency substitution being a key feature of the financial behaviour of the individual countries involved and consequently is positive in policy terms. At this stage such an evaluation is difficult to make. Consequently while the econometric evidence pointing to the existence of a stable and well behaved money demand function at the Euro-Zone level may suggest a role for a broad monetary aggregate in the conduct of a Euro-Zone wide monetary policy, there can be no guarantees that such a stable relationship will persist if the monetary authorities were to actually target such an aggregate, especially given the regime change heralded by the establishment of the Euro area. Prudence, therefore, would suggest that money growth rates should be just one of a set of indicators which policy makers should regularly monitor at the Euro Zone level.
REFERENCES


