MEASUREMENT ISSUES FOR ADEQUACY COMPARISONS AMONG PENSION SYSTEMS

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Measurement Issues for Adequacy Comparisons among Pension Systems

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Abstract

In this paper indexes are proposed in order to capture the degree to which a pension scheme (i) prevents from poverty among the elderly, (ii) enables living-standard smoothing after retirement, (iii) induces both intra- and inter-generational solidarity.

1 Introduction

In the Laeken summit (2001), members of the EU-15 fixed eleven objectives for pension systems with particular emphasis on three general targets: adequacy, financial sustainability and modernization. Three of the eleven objectives were specifically aimed at the adequacy of pension systems: poverty among the elderly, living-standard smoothing after retirement and (intra- and inter-generational) solidarity (EC 2003).

Hereafter, it seems reasonable to believe that policy proposals in this field will be supported in the name of the three targets above, by which a deeper analysis is definitely opportune. Also, it might be of interest to understand if and how recent sustainability-oriented pension reforms may have affected the adequacy of pension systems.

In this study we focus on the definition of indexes for each of the three adequacy objectives. The basic methodology is mostly drawn from the existing literature on poverty and inequality comparisons, by which indexes are expected to satisfy, at the very least, main statistical properties and \textit{fair} norms.

In this context, peculiarities of the pension system have been opportunely taken into account.

The paper is organized as follows. The basic concepts behind the three objectives for an adequate pension system are discussed in section 2. In section
3, indexes are proposed for each of the adequacy objectives. \(^1\) Section 4 concludes.

## 2 Conceptual framework

Adequacy concerns the ability of the pension system to achieve its objectives, which, indeed, are not uniquely defined \textit{a priori}. In the Laeken summit, three objectives have been agreed in order to identify adequate pension systems: (1) “to ensure that older people are not placed at risk of poverty and can enjoy a decent standard of living”, (2) “to enable people to maintain, to a reasonable degree, their living standards after retirement”, and (3) “to promote solidarity within and between generations” (EC 2003).

In order to avoid misunderstandings, it is worth observing that adequacy orderings do not immediately imply social desirability ones. The latter involve both adequacy and financial sustainability issues. In this sense, social desirability judgements might be formulated in terms of adequacy within the sole set of equally financially sustainable pension schemes.\(^2\)

An important observation about the nature of the three objectives is required at the outset. Living-standard smoothing differs with respect to the others because of (i) the basic perspective and (ii) the logical foundation. First, living-standard smoothing relies on the individual/longitudinal perspective, while the other two objectives refer to an aggregate/cross section perspective. Second, it traces back to the well-known \textit{life-cycle hypothesis}, while poverty and solidarity issues find their origins in the \textit{theory of distributive justice}. In some way, the opportunity for smoothing policies is mostly supported by the need for efficiency improvements within second-best economies, instead, redistribution (within and between) generations aims at equity-improvements within economies characterized by unfair/brute allocations of endowments/outcomes.

\textit{Poverty risk}

This is probably the most straightforward objective for an adequate pension system. The social security concept emerged in difficult historical circumstances, when fight against extreme poverty was a key preoccupation of the social decision-maker. Nowadays, as then, because of the fall in productivity

\(^1\) Since most of the analytical results in this paper are common knowledge in the literature on poverty, inequality and social welfare comparisons, indexes are not presented in the standard axiom-theorem framework. In this sense, the main concern is about implications of pension systems’ peculiarities for standard poverty and inequality indexes.

\(^2\) The lack of financial sustainability aspects automatically implies that adequacy comparisons might be not very informative when comparing tax- and contribution-sustained pension schemes.
with aging, the main rationale behind mandatory pension schemes is still represented by the prevention of poverty among the elderly.

**Living-standard smoothing after retirement**
The opportunity for living-standard smoothing finds its origins in the well-known life-cycle hypothesis, by which it is highlighted that individuals usually prefer stable to unstable consumption paths: they save during high-productivity stages in order to dissave during low-productivity ones. Unfortunately, individual smoothing targets might be hijacked because of incomplete markets as well as incomplete information, and, as a result, efficiency improvements in second-best economies might be well attained by enabling more reasonable degrees of smoothing. Apart of the origins, the Laeken summit refers to living standards, i.e. the quantity/quality of goods/services an individual may have access to,° not consumption. Living-standards after retirement are inevitably affected by the intertemporal allocation of consumption, but the two concepts are not equivalent (Morduch 1995): perfect consumption smoothing does not necessarily imply perfect living-standard smoothing, and vice versa.

**Solidarity within and between generations**
By virtue of objective (3), an adequate pension system should promote solidarity within and between generations. However, it is worth drawing a separating line between intra-generational (within generations) and inter-generational solidarity (between generations). Equity issues are involved in both cases, but with different implications. Inter-generational solidarity is aimed at the prevention of large income-gaps among the active and the retired part of the same population, by which, in the primordial state of the world, it allows for partial/full insurance against demographic and financial risks (rates of economic, wage and population growth, inflation and financial returns). Intra-generational solidarity, instead, focuses on the risk of large and unfair pension income disparities among members of the same cohort, and it might be appropriately referred to as progressivity.°

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° This value is inevitably affected by subjective (needs) as well as social living conditions.

° On the one hand, it is usually observed that the main drawback of progressivity consists of the efficiency loss due to disincentives in the labor market. On the other hand, within a primordial risk-averse scenario, progressivity may well be welfare improving, since it acts as an insurance against random/systematic/brute income disparities (Mirrlees 1974, Varian 1980, Fleurbaey 2001), “that is not available in the market” (Diamond 2004).
3 Adequacy comparisons in a multi-objective scenario

3.1 Basic premises

In this paper, poverty, smoothing, progressivity and solidarity are investigated through the use of selected indexes within a complete ordering approach. However, indexes are not the only feasible alternative. For the sake of robustness, stochastic dominance conditions, as defined under the unanimity requirement, might be preferred. For instance, we may claim that society A is poorer than society B if and only if any poverty-averse individual would label A as poorer than B (Foster and Shorrocks 1988). On the one hand, indexes might be preferred since they allow for complete orderings, and, as a result, facilitate policy decisions. On the other hand, indexes automatically entail a loss of robustness since orderings are required to satisfy consistency, not unanimity, where an index is said to be consistent if its orderings never contradict the ones obtained under the corresponding dominance condition.

For our purposes, a separating line has to be drawn between structural and effective indexes. The former is strictly concerned with the structure of tax or pension schemes and it ignores the pre-tax income distribution. On the contrary, effective indexes measure the impact of the tax or pension scheme on the pre-tax population. Here, the scheme matters since there are individuals affected by it, which seems to suit better the analysis of poverty as well as inequality issues.

Most of the aspects considered for adequacy purposes are well-known to be sensitive to the timing approach. By virtue of the ex-ante approach, current pension systems are assessed today with respect to the expected evolution of micro and macro variables like working income, longevity, financial and economic growth rates. Within the ex-post framework, instead, pension systems are assessed once all relevant variables are revealed. These two approaches are not necessarily equivalent. In this study we refer to micro-simulated data, so that, given the relevant probability distributions, fertility and mortality rates, financial and growth variables, individual profiles (e.g. income, household size, longevity) are identified under certainty (ex-post) conditions.

Since individuals are not expected to be homogenous in needs as well as in the quality and quantity of goods/services they have access to, any aggregation of

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5 For instance, it is usually observed that, ex-ante, not ex-post, the redistribution induced by the positive correlation between working income and longevity would be totally neglected (Creedy et al. 1993). However, as will be clearer later on, ex-ante, it is still possible to capture the expected perverse/fair redistribution whenever heterogenous longevity expectations are opportunely considered in the construction of the actuarially equivalent (solidarity neutral) distribution of pension incomes. We'll return this topic in section 4.
incomes among heterogenous income units would be a non sense: it might well be the case that equally endowed individuals enjoy different living standards. In order to tackle the heterogeneity in needs characterizing income units, we refer to disposable equivalent incomes (living standards) as obtained through standard multiplicative equilalizing transformations (Ebert and Moyes 2003). Also, we assume that members of the same household equally share the same living standard.

In order to capture the redistribution induced by pension systems we refer to both actual and virtual pension incomes, where the latter is obtained under the hypothesis of no redistribution at all. With this purpose in mind, two different approaches might be considered: (i) actuarial fairness (no-pension system hypothesis), or, (ii) actuarial equivalence (null solidarity). By the former, it is hypothesized that individuals accumulate their savings in a risk-free asset at the market return (Creedy et al. 1993, Disney 2004). By actuarial equivalence, instead, it is hypothesized that individuals get (a) the market return on savings accumulated in funded pension schemes, and (b) the sum of labor force and productivity growth for savings accumulated in PAYG systems (Samuelson 1958, Aaron 1966). The null solidarity hypothesis is more convenient for our purposes, since it excludes a priori any sort of adequacy-relevant redistribution (poverty, intra- and inter-generational).  

We indicate by $y_{t-k,h} := \{y_{1,t-k,h}, \ldots, y_{n_{t-k,h},t-k,h}\} \in \mathbb{R}^{n_{t-k,h}}$ the vector of net disposable equivalent incomes in the working life (at the net value of voluntary pension-oriented savings) for active individuals at time $t-k$ belonging to cohort $h$, by $p_{t,h} := \{p_{1,t,h}, \ldots, p_{r_{t,h},t,h}\} \in \mathbb{R}^{r_{t,h}}$ the vector of disposable equivalent pension incomes for retired individuals at time $t$ of cohort $h$, and, by $p^*_{t,h} := \{p^*_{1,t,h}, \ldots, p^*_{r_{t,h},t,h}\} \in \mathbb{R}^{r_{t,h}}$ the vector of disposable equivalent virtual pension incomes - as calculated under the hypothesis of actuarial equivalence - for retired individuals at time $t$ of cohort $h$. Obviously, the size of these vectors is allowed to vary over both time and cohort. From a longitudinal point of view, we indicate by $\ell_{i,h}, \bar{y}_{i,h}$ and $\bar{p}_{i,h}$ respectively lifetime income, average working and average pension income received by the $i$th income unit of cohort $h$. From a cross-section point of view, instead, we indicate by $\mu(y_t)$ and $\mu(p_t)$ respectively the average working and pension income among living individuals at time $t$. Henceforth, we refer to working and pension income respectively in

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6 If each individual is allowed to participate in both funded ($\lambda_i$) and unfunded schemes ($1-\lambda_i$) at the corresponding rates of return ($r,g$), then, unless of particular circumstances ($\lambda_i = \lambda \forall i$) the comparison between actual and virtual income distributions as obtained under actuarial fairness would naturally entail some fictitious intra-generational redistribution, whose origins are indeed of the inter-generational kind (Disney 2004).

7 The distribution of virtual pension incomes is constructed assuming no individual behavioral response under the null solidarity hypothesis. In this sense, individual needs and the quality/quantity of services individuals have access to are assumed to be independent of the degree of solidarity (inter- and intra-generational).
place of equivalent disposable income in the working and elder life.

3.2 Poverty

Poverty has been widely investigated in the existing literature, but some basic notations are required for our purposes. First, we are not interested in the level of poverty itself, but in poverty-relevant implications of pension systems. In order to capture these aspects, we refer to a sort of Reynolds and Smolensky (1977) index which is defined as the difference between virtual and actual poverty, i.e. how much poverty would have been under the hypothesis of an actuarially equivalent pension scheme and how much poverty really is. Second, a generally accepted approach to the definition of the poverty line does not exist yet, and hopefully won’t. In this study, we neglect poverty line debates; we assume that, for each period, there exists a generally agreed absolute poverty line, $z_t$, which is invariant with respect to income distributions. Third, individual pension incomes as well as poverty lines are not necessarily constant over time. Indeed, living standards after retirement might be affected by both indexing factors as well as changes of individual needs over time. As a matter of fact, it might well be the case that some retired income units fall below the poverty line in some, not all periods.

In order to simplify the basic notations, let’s suppose that members of the same cohort receive the first pension income at time $t+1$ (results are independent of this simplification). Then the $i$th pension income profile is $p_{i,h} := \{p_{i,t+1,h}, p_{i,t+2,h}, \ldots, p_{i,t+\nu_i,h}\} \in \mathbb{R}^{\nu_i}$ with $p_{t+\nu_i}$ indicating $i$th pension income in last living period. Also, let’s define by $z := \{z_{t+1}, z_{t+2}, \ldots, z_{t+\nu_i}\}$ the vector of poverty lines associated to each period, which applies to all members of the same population. Under the assumption of additive separability, the multidimensional nature of the problem can be tackled by virtue of a sequential aggregation, by which the poverty index among the elderly of cohort $h$, $I_h$, is defined as the additively separable aggregation of individual poverty rates, $I_i$, which, in turn, is a function of $i$th pension income profile (Bourguignon and
Chakravarty 2003), \( \gamma \) ie.,

\[
I_h^\gamma = \frac{1}{r_h} \sum_{i=1}^{r_h} \left[ I_{i,h}(p_{i,t+1,h}, \ldots, p_{i,t+\nu_i,h}) \right]^\gamma \\
= \frac{1}{r_h} \sum_{i=1}^{r_h} \left[ \alpha_1 \max \left( 1 - \frac{p_{i,t+1,h}}{z_{t+1}}, 0 \right)^{\theta_i} + \ldots + \alpha_{\nu_i} \max \left( 1 - \frac{p_{i,t+\nu_i,h}}{z_{t+\nu_i}}, 0 \right)^{\theta_i} \right]^{\gamma} \\
\]

(1)

where \( r_h, \alpha_i, \theta \) and \( \gamma \) indicate respectively the number of individuals of cohort \( h \) achieving retirement, the positive weights associated to each dimension (period), the parametrization of the elasticity of substitution and the coefficient of poverty aversion. In particular, since poverty attributes consist of poverty gaps in terms of living standards over different periods, \( \alpha_1 = \ldots = \alpha_{\nu_i} = \frac{1}{\nu_i} \forall i \) can be assumed without loss of generality.  

Main properties of (1) are given in the appendix (A).

As observed in Bourguignon and Chakravarty (2003), the poverty index in (1) is a multidimensional extension of the well-known Foster et al. (1984)'s poverty index and it captures both the intensity and recurrence of poverty within a multi-dimensional framework. In particular, it is worth observing that if \( \gamma = 0 \), then (1) is a multi-dimensional headcount ratio, \( H \) while, if \( \gamma = 1 \), then (1) is a poverty gap ratio as defined under the CES aggregation of univariate poverty gaps. Formally speaking, decision-maker’s preferences are defined such that (i) the poverty index must be increasing with individual poverty rates (\( \gamma > 0 \)) and (ii) the poverty index is decreasing for non re-ranking rich-to-poor transfers among individual poverty rates (\( \gamma > 1 \)).

For \( \theta_i = 1 \forall i \) a linear aggregation of relative poverty gaps is imposed. As special case, it is worth observing that, given \( \alpha_1 = \ldots = \alpha_{\nu_i} = \frac{1}{\nu_i} \forall i \) and

\( \gamma \) The main drawback of the sequential aggregation consists of the lack of consistency with its immediate counterpart. The poverty index might be constructed aggregating first individual incomes in each period, and then poverty indexes over time. These two procedures are not expected to be ordinally equivalent each other. However, within the field of additively separable (sub-group decomposable) poverty measures, Bourguignon-Chakravarty’s procedure might be easily supported in order to avoid very restrictive separability assumptions on the intertemporal aggregation function, eg. the counterpart discussed above would automatically imply

\[
\frac{\partial I}{\partial p_{i,t} \partial p_{i,t+1}} = 0.
\]

\( \theta \) Because of homogeneity of degree 0, actuarial transformations can be neglected. In addition, because of the decision-maker’s ex-post-based perspective, the discount factor has been assumed to be 1.

\( H \) In order to get this result, the summation should be considered with respect to the sole first \( q \) individuals, ie. the income units whose attributes are below the poverty line at least in one dimension (period).

\( H \) In addition, for \( \gamma > 2 \) it must be the case that the same transfer among (non-null) individual poverty rates is more affecting the lower is the poverty rate of the poorest income unit, ie. the transfer sensitivity axiom (Foster et al. 1984).
θ_i = 1 \forall i, under some circumstances, (1) orders income distributions consistently with poverty dominance conditions as defined on the univariate distribution of average and/or primary pension incomes (appendix B).

For θ_i > 1 \forall i, individual poverty rates are a concave aggregation of poverty gaps over time. In this context, a convenient assumption might be the following, θ_i = \max \left\{ \frac{z_t}{p_i,t+1,h}, \ldots, \frac{z_t+\nu}{p_i,t+\nu,h} \right\}, by which individuals are assumed to be heterogeneously averse to inter-temporal fluctuations of poverty gaps as well as pension incomes. In this case, if an individual is extremely and maximally poor at time t+d (dth dimension), then any reduction of the poverty gap at some time t+j (jth dimension) is almost useless in order to alleviate from poverty.\textsuperscript{12}

Once the poverty index has been defined, the reduction of poverty among the elderly induced by the redistributive nature of pension systems can be measured \textit{sic et simpliciter} as

\[ P_h = I_h^\gamma(p^\ast, z) - I_h^\gamma(p, z) \] (2)

where \( p^\ast, p \in \mathbb{R}_+ \) indicate respectively the vectors of virtual and actual pension incomes. In particular, it can be shown that \( P_h^\gamma \) satisfies the following properties: (a) monotonicity, meaning that, if the pension system causes some pension income below the poverty line to increase, then \( P_h^\gamma \) increases, (b) strong focus, ie. the index is neutral with respect to income changes above the poverty line, (c) scale invariance (appendix), (d) replication invariance, (e) \( P_h^\gamma \in [-1, 1] \) and (f) sub-group decomposability.

3.3 Living-standard smoothing after retirement

By virtue of the Laeken summit, pension systems should “enable people to maintain, to a reasonable degree, their living standards after retirement”. This statement allows for several observations. First, the decision-maker is not directly concerned with the intertemporal allocation of consumption, but with the more general allocation of living standards. As a result, the decision-maker is not just concerned with the ability of individuals to smooth consumption, but with the ability of pension schemes to allow for living-standard smoothing. Second, the adequacy of pension system is a non-increasing function of the gap between pre- and post-retirement living standards. Third, there is no concern for the degree of smoothing during the working life. Fourth, the pension system is expected to enable, not to impose, smoothing, by which the focus shouldn’t be restricted to the sole benefit formula and mandatory.

\textsuperscript{12} As observed in Bourguignon and Chakravarty (2003), the poverty index is increasing with respect to \textit{correlation-increasing switches} if \( \gamma > \theta \), and vice versa (Atkinson and Bourguignon 1982, Duclos 2006).
In order to measure the living-standard gap at retirement, “living standards” are defined in terms of average disposable equivalent working incomes at the net value of voluntary pension savings ($\bar{y}_i$), where different intervals of the working life might be considered depending on basic assumptions. More complicated is the identification of some reference living standard after retirement. As for poverty, living standards associated to pension incomes are not expected to be constant over time, meaning that, dependence on either average or primary insurance benefits might be cause a relevant loss of information. More generally, it might be said that the $i$th degree of smoothing concerns with the vector of living-standard gaps between average working income and pension incomes at different periods, by which each individual degree of smoothing is some function $\phi(||\bar{y}_{1,h} - p_{1,t,h}||, ||\bar{y}_{1,h} - p_{1,t+1,h}||, ...)$ with $\phi'(\cdot) < 0$. Notice that this specification automatically implies that both under- and over-savings are non-desirable attributes of a pension system. This is not irrelevant for measurement purposes. In the existing literature, living-standard smoothing is usually measured through replacement ratios, ie. $RR_i = \frac{p_i}{\bar{y}_i}$, where the numerator is often replaced by the primary insurance benefit. Unfortunately, this index would be seriously misleading when looking at the average degree of consumption smoothing in the society: the average replacement ratio, $\bar{RR} = \frac{1}{n} \sum_i RR_i$, would automatically entail compensation among $< 1$ and $> 1$ individual replacement ratios. As a matter of fact, it might be observed that the average replacement ratio does not necessarily satisfy the basic monotonicity requirement, by which, at the very least, the degree of living-standard smoothing in a society should be increasing with respect to any individual-based (horizontal) gap-reducing income redistribution. However, it might be convenient to retain the intuitive idea behind the replacement ratio. With this purpose in mind, let’s restrict the domain of possible living-standard vectors such that $p_{i,t+k,h} \leq 2\bar{y}_{i,h} \forall i,k$. Then, a generalized version of the $i$th replacement ratio at time $t+k$ might be defined as $RR_{i,t+k,h} = 1 - \frac{||\bar{y}_{i,h} - p_{i,t+k,h}||}{\bar{y}_{i,h}}$. Following the same aggregation as for relative poverty gaps, a complete ordering in terms of living-standard smoothing can be defined as the un-weighted aggregation of individual generalized replace-

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13 For instance, one should be taking into account that information and education can reasonably affect the degree of living-standard smoothing through an appropriate demand for voluntary savings.

14 In other words, living standards after retirement never double living standards in the working life. This restriction is not very strong since living standards (not incomes) matter. Also, it is worth observing that truncation is a very common practice in empirical income studies. Alternatively, in line with the Gini’s reformulation in Sen (1973), relative living-standard gaps might be defined as $(\bar{y}_{i,h} - p_{i,t+k,h})/(\bar{y}_{i,h} + p_{i,t+k,h})$, by which the domain restriction would be not required any further.
ment ratios, ie.,

\[ C^\eta_h = \frac{1}{r_h} \sum_{i=1}^{r_h} \left[ \sum_{k=1}^{\nu_i} \alpha_k \left( 1 - \frac{|\bar{y}_{i,k} - p_{i,t+k,h}|}{\bar{y}_{i,h}} \right) \right]^{\frac{\eta}{\beta_i}} \]

(3)

where \( i = 1, ..., r_h \) refers to income units of cohort \( h \) at retirement, \( k = 1, ..., \nu_i \) indicates the periods in which the \( i \)th income unit receives pension incomes, \( \beta_i \) is the degree of substitutability among individual degrees of smoothing over time and \( \eta > 0 \) is the degree of aversion to living-standard gaps at retirement (or, degree of care for smoothing). As for the poverty index, \( \alpha_1 = ... = \alpha_{\nu_i} = \frac{1}{\nu_i} \forall i \) can be assumed without loss of generality. Notice that, in contrast with (1), \( \beta_i \) is just the parametrization of the degree of substitutability among degrees of smoothing, not attributes (pension incomes). For \( \beta > 1 \), iso-smoothing contours are convex to the origin in the space of degrees of smoothing and individuals are averse to intertemporal fluctuations of the degrees of smoothing. However, for \( \beta > 1 \) iso-smoothing contours in the space of pension incomes are convex if \( \bar{y}_{i,h} > p_{i,t+k,h} \forall k \), but concave if \( \bar{y}_{i,h} < p_{i,t+k,h} \forall k \). In particular, for \( \beta = 0 \) degrees of smoothing as well as pension incomes over time are perfect substitutes and the iso-smoothing contour is a straight line. If \( \eta = 0 \), then the degree of living-standard smoothing is always maximum since null aversion to gaps is assumed. For \( 0 < \eta < 1 \), \( C^\eta_h \) is a concave aggregation of individual degrees of smoothing, meaning that, the policy-maker attaches a larger weight at the margin to individuals with a low degree of smoothing.\(^{15}\) For \( \eta = 1 \), the consumption smoothing index is defined \textit{sic et simpliciter} as the average of individual degrees of smoothing.

The smoothing index in (3) satisfies the following properties: (a) monotonicity with respect to gap-reducing income redistribution at the individual level, (b) scale invariance, (c) \( C^\eta_h \in [0, 1] \), with \( C_h^\eta = 1 \) if and only if perfect smoothing occurs at retirement for all income units, (d) replication invariance, and (e) sub-group decomposability, ie. the degree of living-standard smoothing in a society is equivalent to the weighted (sub-group share) aggregation of living-standard smoothing in each sub-group. Finally, under some circumstances, \( C_h^\eta \) is equivalent to the average replacement ratio by construction.\(^{16}\)

In order to measure the ability of pension schemes to enable living-standard smoothing, we may want to consider the gap between the actual degree of smoothing in (3) and the virtual degree of smoothing which would have been occurred in absence of mandatory saving plans. Unfortunately, the latter is

\(^{15}\)Low degree of smoothing does not imply either low or high living standards. In this sense, value judgements may be not very ethically founded in this context.\(^{16}\) From (3), it must be the case that, if \( \beta_i = 1 \forall i \), then

\[ C_h^1 = \begin{cases} \frac{1}{r_h} \sum_{i=1}^{r_h} \frac{\bar{y}_{i,h}}{\bar{y}_{i,h}} & \text{if } \bar{y}_{i,h} \geq p_{i,t+k,h} \forall k, i, \\ 2 - \frac{1}{r_h} \sum_{i=1}^{r_h} \frac{\bar{y}_{i,h}}{\bar{y}_{i,h}} & \text{if } \bar{y}_{i,h} \leq p_{i,t+k,h} \forall k, i. \end{cases} \]
not straightforward at all. In this paper, we stick to (3), which is equivalent to assuming that, in absence of mandatory saving plans, populations would achieve the same degree of (virtual) smoothing.

3.4 Intra- and inter-generational solidarity

3.4.1 Progressivity

Progressivity concerns with the equality improvement/loss induced by the pension system. This should be not confused with actuarial unfairness since the former is independent of identities, not the latter. In particular, in this section we refer exclusively to effective progressivity measures, which have been widely investigated in the exiting literature on fiscal progressivity. Especially the Reynolds and Smolensky (1977) (RS) and the Kakwani (1977) (K) index have been widely used in progressivity comparisons.\(^{17}\) RS is defined \textit{sic et simpliciter} as the difference between pre- and post-tax Gini indexes. K, instead, consists of the difference between the concentration index, as calculated on the distribution of tax liabilities, and the Gini index, as calculated on the pre-tax income distribution. The latter measures the disproportionality of taxes with respect to the allocation of pre-tax income.\(^{18}\) For our purposes, the pre-tax income distribution consists of the virtual distribution of pension incomes as obtained under actuarial equivalence, while the post-tax distribution is the actual distribution of pension incomes in the society.\(^{19}\)

\(^{17}\) In the field of pension systems, the existing literature has been mostly focusing on the \textit{effective progression index} (Nelissen 1995, Coronado et al. 2000, Oshio 2002), that is informationally (not ordinally) equivalent to the RS.

\(^{18}\) For the implications of different pre-tax income distributions in progressivity comparisons see Lambert and Pfahler (1992).

\(^{19}\) In this paper we refer to pension incomes not lifetime ones, even if the existing literature is mostly oriented to the latter. Actually, lifetime-based progressivity is not independent of the distribution of working incomes and, since progressivity is regarded as adequacy improving, it automatically implies that pension systems aim at working income disparities compensation, ie. the -1 correlation between working and pension incomes would be generally preferred. Differences between these two views are intuitive whenever a perfectly egalitarian distribution of virtual pension incomes is assumed. In this scenario, progressivity improvements would be feasible within the lifetime, not pension view. In general, the two views are partially conciliable each other whenever pension schemes are ordering-preserving with respect to the three relevant income vectors \((y, p^*, p)\): if, for each couple, the richest income unit in the working life is the one saving more (in the working life) as well as benefitting more (after retirement), then any progressivity improving policy in the pension view must be lifetime progressivity improving, not vice versa. Summing up, in this paper we opt for the pension view in order to avoid any interference of the working income distribution, whose relevance in terms of adequacy has been
Let \( p_h^t, p_h \in \mathbb{R}^n_+ \) be the virtual and actual pension income vectors of cohort \( h \). The pension system is \textit{revenue neutral} (financially sustainable) whenever \( \sum_i (p_h^t - p_i,h) = 0 \). As observed in Creedy et al. (1993), \textit{revenue neutrality} is crucial in order to avoid any interference between financial sustainability of the pension system and intra-generational redistribution.\(^\text{20}\) Technically speaking, given the distribution of virtual pension income, we may want the progressivity index \( (\mathcal{P}) \) to be independent of total tax liabilities (or subsidies), ie., given \( p_h^t, p_h \in \mathbb{R}^n_+ \), if \( p_h^t = (1 - \lambda)p_h^t + \lambda p_i,h \forall i \), then \( \mathcal{P}(p_h^t, p_h^t) = \mathcal{P}(p_h^t, p_h^t) \) \( \forall \lambda > 0 \). In fiscal progressivity, \( K \) (not \( RS \)) is well-known to satisfy this property. However, when dealing with pension systems, it should be taken into account that (i) taxes might be negative for some individuals and (ii) the average tax rate might be negative. The former is not sufficient to jeopardize either the opportunity for Gini-based inequality orderings or the relationship between \( RS \) and \( K \). However, since the concentration curve is expected to be negative for the poorest income units, the concentration index is not necessarily bounded on \([0,1]\) any longer (Chen et al. 1982, Berrebi and Silber 1985). On the contrary, some observations are required for the latter. If the average tax rate is negative \((g < 0)\), then the pension system is progressive whenever the concentration curve of (negative) tax liabilities lies no where below the Lorenz curve of virtual (pre-tax) pension incomes, ie. if subsidies (negative tax liabilities) are more equally distributed than virtual pension incomes.

Formally, given \( \sum_i (p_{i,h}^t - p_{i,h}) < 0 \), the pension system is progressive if and only if \( \mu_j(p_h^t) - \mu_j(p_h)/[\mu(p_h^t) - \mu(p_h)] \geq \mu(p_h)/\mu_j(p_h) \forall j \): 1, ..., \( n \). Even more, if the largest shares of subsidies are associated to poor income units, ie. \( \mu_j(p_h^t) - \mu_j(p_h)/[\mu(p_h^t) - \mu(p_h)] \geq 1 \forall j \), then the concentration curve lies above the bisectrix.

\textbf{Claim 3.1}

\textit{Given (i) \( (p_{i,h}^t - p_{i,h}) \in \mathbb{R}^n_+ \) with \( \sum_{i=1}^n (p_{i,h}^t - p_{i,h}) \neq 0 \) and (ii) \( p_h \) ordering-preserving with respect to \( p_h^t \), the Kakwani index is defined as follows}

\[ K = \begin{cases} C(p_h^t - p_h) - G(p_h^t) & \text{if } \sum_i (p_{i,h}^t - p_{i,h}) > 0 \implies RS = \frac{a}{1-a} K, \\ -C(p_h^t - p_h) + G(p_h^t) & \text{if } \sum_i (p_{i,h}^t - p_{i,h}) < 0 \implies RS = \frac{a}{1-a} K \end{cases} \tag{4} \]

where \( a = [\sum_i (p_{i,h}^t - p_{i,h})]/\sum_i p_{i,h} \) is the average subsidy rate as calculated with respect to the total amount of actual pension incomes. (Proof in appendix C).

Then, (4) is independent of total tax liabilities or subsidies. However, for our purposes, the same property should be satisfied within a multi-period framework. Here, it might be convenient to assume that the progressivity index is already discussed in the previous section.

\(^{20}\) In Creedy et al. (1993), the \textit{revenue neutrality condition} implies equivalence between the actuarial value of total contributions and the benefits paid by/to a generation.
independent of the total amount of tax liabilities or subsidies received over the whole pension income profile independently of the intertemporal allocation.

Claim 3.2

Given \( \mathcal{P} = K \{ C[\phi(p_{t+1,h} - p_{t,t+h}, \ldots, p_{t+\nu_i,h} - p_{t,t+\nu_i,h})], G[\phi(p_{t+1,h}, \ldots, p_{t+\nu_i,h})] \} \) with \( \phi(\cdot) \) additively separable,\(^{21}\) if (i) \( \sum_{i=1}^{\nu_i} \sum_{k=1}^{\nu_i} (p_{t+k,h} - p_{t+k,h}) \neq 0 \) and (ii) \( \frac{1}{\nu_i} \sum_{i=1}^{\nu_i} p_{t,t+k,h} \) is ordering-preserving with respect to \( \frac{1}{\nu_i} \sum_{i=1}^{\nu_i} p_{t,t+k,h} \), then

\[
\mathcal{P}_h = K \left\{ C \left[ \sum_{k=1}^{\nu_i} \alpha_k (p_{t+k,h} - p_{t+k,h}) \right], G \left[ \sum_{k=1}^{\nu_i} \alpha_k p_{t+k,h} \right] \right\} \tag{5}
\]

where \( t + \nu_i \) is the last period with \( i \) receiving pension benefits and \( K \) is defined as in (4), is independent of total tax liability or subsidies.\(^{22}\) (Appendix D).

As for poverty and smoothing, \( \alpha_1, \ldots, \alpha_{\nu_i} = 1/\nu_i \forall i \) may be assumed without loss of generality.\(^{23}\) In particular, \( \mathcal{P} \) satisfies the following properties: (a) monotonicity, ie. given an ordering preserving pension system at each period, if society A is obtained from society B by virtue of some additional subsidies or taxes redistributed in favor of poor income units in some period, then it must be the case that \( \mathcal{P}^A \geq \mathcal{P}^B \), (b) scale invariance with respect to actual and virtual pension incomes (\( [\lambda p^*, \lambda p] \)), (c) multi-period independence of total tax liabilities (appendix D), (d) symmetry, (e) replication invariance and (f) sub-group decomposability in Dagum (1997)'s sense.\(^{24}\)

3.4.2 Inter-generational solidarity

Pension incomes are inevitably affected by demographic and economic growth factors. In this sense different generations face a gamble, by which, ex-post, there may be “losers” or “winners” without any immediate individual responsibility. By virtue of the Laeken summit, it is observed that adequate pension systems should prevent large gaps among the active and retired part of the

\(^{21}\)Once again, a sequential aggregation has been preferred, by which the progressivity index is defined aggregating first individual incomes and taxes over time, next individual positions across the population. Given \( \phi(\cdot) \) additively separable, the converse would automatically imply \( \frac{\partial \mathcal{P}}{\partial p_{t,t+k,h} \partial p_{t+1,t,k,h}} = 0 \forall i \), that is a very strong assumption.

\(^{22}\)In order to simplify notations, individuals are assumed retire in the same period.

\(^{23}\)Alternatively, the market rate of return might be used in order to account for actuarial values. In this case, it has to be observed that the two ‘if’ conditions in (4) apply in actuarial terms.

\(^{24}\)Dagum’s k-group decomposition involves three components: (i) the net contribution of the Gini inequality between sub-populations, (ii) the contribution of the Gini inequality within sub-populations and (iii) the contributions of the intensity of transvariation between sub-populations.
population at each period, that is, gap-reducing inter-generational redistribution is adequacy improving.
The peculiarity of inter-generational solidarity with respect to former objectives consists of the two basic agents to be considered for measuring purposes, respectively the active and the retired part of the population within an overlapping generations perspective. We indicate by \( \mu(y_t) \) and \( \mu(p_{t,h}) \), respectively the average income of the working population at time \( t \) and the average income of retirees of cohort \( h \) at time \( t \). As for the previous section, the measurement of solidarity aspects opportune-ly relies on the comparison between what has really occurred and what would have occurred in the absence of inter-generational redistribution. In line with our observations above, we refer to the virtual income distribution as obtained under the hypothesis of actuarial equivalence, which implies null inter-generational transfers whatever the pension system.

In the existing literature on the measurement of solidarity, two different approaches have been considered. On the one hand, (generational) replacement ratios (GRR) - the ratio between the average pension income of the retirees and the average working income of the active generation at time \( t \) - are usually exploited in order to capture the degree of inter-generational solidarity. Unfortunately, this measure captures the sole gap among active and retired income units, not the inter-generational redistribution occurring between them. On the other hand, governments intertemporal budget constraints have been exploited in order to measure how much future generations must pay in net taxes (generational accounts) in order to tackle the current spending commitment of the government (Auerbach et al. 1991). This approach focuses on the amount of transfer received or paid by each generation, but it wouldn’t capture the gap variation among active and retired income units of the same population.

Then, in order to capture the amount of inter-generational solidarity, the two approaches above might be merged in such a way to focus on how pension schemes affect the income-gap among active and retired generations, i.e. \(|\mu(y_t) - \mu(p_{t,h}^\ast)| - |\mu(y_t) - \mu(p_{t,h})|\). This difference is positive in case of (fair) gap reducing pension schemes, and vice versa. In particular, it can be shown that this indicator is orderly equivalent to the absolute re-formulation of the Gini’s index as calculated on the distribution of average income among the active and retired part of the population.

Without loss of generality, we can restrict the domain of possible income distributions such that \(|\mu(y_t) - \mu(p_{t,h}^\ast)| - |\mu(y_t) - \mu(p_{t,h})|\) \(\leq \mu(y_t)\). Under this assumption, inter-generational solidarity of cohort \( h \) can be defined as the

\[ G_t^\ast = \frac{|\mu(y_t) - \mu(p_{t,h}^\ast)|}{\mu(y_t) + \mu(p_{t,h}^\ast)} \quad \text{and} \quad G_t = \frac{|\mu(y_t) - \mu(p_{t,h})|}{\mu(y_t) + \mu(p_{t,h})}, \]

which indicate respectively virtual and actual income disparities among the active and retired part of the population.

\[ 25 \] Reference to average incomes allows to avoid any interference with intra-generational aspects as well as any dependency of the size of the two sub-populations (even if the latter enters the analysis through the PAYG’s rate of return).

\[ 26 \] From Sen (1973)’s re-formulation of the Gini, it must be the case that \( G_t^\ast = \frac{|\mu(y_t) - \mu(p_{t,h}^\ast)|}{\mu(y_t) + \mu(p_{t,h}^\ast)} \) and \( G_t = \frac{|\mu(y_t) - \mu(p_{t,h})|}{\mu(y_t) + \mu(p_{t,h})} \), which indicate respectively virtual and actual income disparities among the active and retired part of the population.
un-weighted linear aggregation of the degrees of solidarity at each period, \(^{27}\) ie.

\[
S_h = \frac{1}{\nu} \sum_{k=1}^{\nu} \frac{|\mu(y_{t+k}) - \mu(p^*_t+h,k)|}{\mu(y_{t+k})} - \frac{1}{\nu} \sum_{k=1}^{\nu} \frac{|\mu(y_{t+k}) - \mu(p_t+h,k)|}{\mu(y_{t+k})}
\]  

(6)

where \(t + \nu\) is the last period with at least one survived retiree of cohort \(h\). \(S_h\) is positive in the presence of gap-reducing redistribution on average, and vice versa. For \(\mu(y_{t+k}) \geq \mu(p^*_t+h,k)\), \(\mu(p_t+h,k) \forall k\), (6) is just the difference between average generational replacement ratios, ie. \(S_h = \overline{GRR}_h - \overline{GRR}^{*}_h\).

Formally, solidarity orderings obtained through (6) must be consistent with the more robust partial ordering criterion, by which, if the area between the virtual and actual Lorenz curves of society A in each period is larger than B’s one for any p% share of poorest income units, then A has at least as much solidarity as B. In addition, it can be shown that (6) is (a) scale invariant, (b) symmetric, (c) replication invariant and that (d) \(S_h \in [-1, +1]\).

4 Concluding remarks

In the recent years political debates on pension systems have allowed for the definition of the basic objectives of pension system. Among all, pension systems are required to be adequate, that is, to prevent from poverty among the elderly, to allow for consumption smoothing at (and after) retirement, and to redistribute within and across cohorts in order to account for unfair income disparities due to non-responsible choices. As a matter of fact, measurement issues becomes crucial in order to assess current pension systems and political proposals.

In this paper we have proposed indexes for the measurement of the adequacy of pension systems for each of the three perspectives agreed in the Laeken Summit. We show that the the prevention of poverty due to the pension system can be measured as the gap of FGT poverty indexes obtained, respectively, from the virtual and the actual income distribution (the former is obtained under the hypothesis of no pension system). In order to measure the degree to which pension systems allow for smoothing at (and after) retirement, we propose a modified version of the replacement rate which is obtained from the aggregation, for each income unit, of income gaps at each period after retirement. In order to measure the solidarity of pension systems two different perspectives have been considered. First, the Kakwani index, opportunely

\(^{27}\) The un-weighted linear aggregation has been preferred since the main focus in the case of inter-generational solidarity is on the transfer of resources, not its implications. However, if the effective progression index (Musgrave and Thin 1948) - defined in \([0, +\infty]\) - is preferred with respect to the Reynolds-Smolensky’s - negative in case of \textit{unfair} (gap-increasing) inter-generational redistribution - the more general CES aggregation might be considered as well.
modified in order to account for the possibility of negative tax-rates, is shown to match the measurement of intra-generational solidarity induced by pension systems. Second, a Gini-based index of inter-generational redistribution due to pension systems has been proposed. The latter accounts for both relevant aspects of inter-generational redistribution: (a) the gap among the active and retired part of the population, that is usually measured through generational replacement rates and (b) the amount of transfers received or paid by each generation, that is usually measured through generational accounts.
A

Axiom A.1 Strong Focus
Let’s consider two populations A and B of r income units each in m periods, whose pension incomes at time $t+k$ for the $i$th income unit are indicated respectively by $p^A_{i,t+k}$ and $p^B_{i,t+k}$. If (i) $p^A_{i,t+k} \geq z_{t+k}$, (ii) $p^B_{i,t+k} = p^A_{i,t+k} + \Delta$ with $\Delta > 0$ and (iii) $p^A_j + s = p^A_{i,t+s}$ $\forall$ $j \neq i$, $p^B_{i,t+s} = p^A_{i,t+s}$ $\forall$ $s \neq k$ and $\forall i$, then it must be the case that $P(A) = P(B)$.

Axiom A.2 Scale invariance
Let’s consider two populations A and B of r income units each over m periods, whose pension incomes at time $t+k$ for the $i$th income unit are indicated respectively by $p^A_{i,t+k}$ and $p^B_{i,t+k}$. If $z^A_{t+k} = \lambda z^A_{t+k}$ and $p^B_{i,t+k} = \lambda p^A_{i,t+k}$ $\forall$ $i, k$ with $\lambda > 0$, then $P(A) = P(B)$.

Axiom A.3 Monotonicity
Let’s consider two populations A and B of r income units each in m periods, whose pension incomes at time $t+k$ for the $i$th income unit are indicated respectively by $p^A_{i,t+k}$ and $p^B_{i,t+k}$. If (i) $p^B_{i,t+k} = p^A_{i,t+k} + \Delta$ with $\Delta > 0$, $p^A_{i,t+k} < z_{t+k}$, and (ii) $p^A_j + s = p^A_{i,t+s}$ $\forall$ $j \neq i$, $p^B_{i,t+s} = p^A_{i,t+s}$ $\forall$ $s \neq k$ and $\forall i$, then it must be the case that $P(A) \geq P(B)$.

Axiom A.4 Multi-dimensional principle of transfer
Let’s A be the $r \times m$ matrix $A$ indicating the allocation of pension incomes among r individuals in m periods. If $B = H \cdot A$ where $H$ is a bi-stochastic matrix whose elements $h_{ij} \neq 1$ $\forall$ $i, j$, then $P(A) \geq P(B)$.

Axiom A.5 Replication invariance
Given an allocation A of pension incomes among a population of r income units over m periods, if population B is the k-fold replication of population A, then $P(A) = P(B)$.

Axiom A.6 Sub-group decomposability
Let A,...,Z be the $n^i \times m$ matrices obtained as a partition into non-empty subgroups from the $n \times m$ matrix X indicating the allocation of pension incomes among n income units over m periods such that $\sum n^i = n$, then $P(X) = \sum n^i P(A) + ... + \sum n^Z P(Z)$. 

17
Claim B.1
Given two societies A and B such that \( z_{t+k} = z \ \forall \ k \ (z^A = z^B) \) and \( p_{i,t+k,h} < z \ \forall \ k \) or \( p_{i,t+k,h} > z \ \forall \ k \) for each individual, if (i) multi-dimensional poverty is defined as some additively separable aggregation of the linear and un-weighted aggregation of individual poverty gaps over time and (ii) any non re-ranking rich-to-poor transfer is poverty reducing, then the two following statements are equivalent:

- A is at least as poor as B for all poverty averse and transfer sensitive individuals;
- the cumulated difference between A’s and B’s distribution function of average pension incomes is non negative for all \( \bar{p}_h < z \).

Proof B.1
Given

\[
P = \psi \left\{ \varphi \left[ \left( 1 - \frac{p_{1,t+1,h}}{z} \right) \right], \ldots, \left( 1 - \frac{p_{1,t+v_i,h}}{z} \right) \right\}, \ldots, \varphi \left[ \left( 1 - \frac{p_{r_h,t+1,h}}{z} \right) \right], \ldots, \left( 1 - \frac{p_{r_h,t+v_i,h}}{z} \right) \right\}
\]

such that \( \psi(\cdot) : \mathbb{R}^{h, h+} \rightarrow \mathbb{R}_+ \) is additively separable and \( \varphi(\cdot) : \mathbb{R}^h \rightarrow \mathbb{R}_+ \) linear and un-weighted, then

\[
P = \int_0^z \phi \left( 1 - \frac{\bar{p}_h}{z} \right) f(\bar{p}_h) d\bar{p}_h
\]

where \( f(\bar{p}_h) \) is the density function of individual average incomes. Given \( \phi(0) = 0 \) and \( \phi'(0) = 0 \). In addition, from Foster et al. (1984), if the social decision-maker is averse to poverty and any rich-to-poor transfer is poverty enhancing, then \( \phi'(\cdot) > 0 \) and \( \phi''(\cdot) \geq 0 \). Given,

\[
P(A) - P(B) = \int_0^z \phi \left( 1 - \frac{\bar{p}_h}{z} \right) [f^A(\bar{p}_h) - f^B(\bar{p}_h)]d\bar{p}_h \quad (B.1)
\]

and, integrating by parts two times,

\[
P(A) - P(B) = \frac{1}{z^2} \int_0^z \phi'' \left( 1 - \frac{\bar{p}_h}{z} \right) S^z(\bar{p}_h) d\bar{p}_h \quad (B.2)
\]

where \( F(\cdot) = \int f(\cdot)d(\cdot) \) and \( S^z(\bar{p}_h) = \int_0^{\bar{p}_h} [F^A(x) - F^B(x)] dx \). By contradiction, it can be shown that \( P(A) - P(B) \geq 0 \) if and only if \( S^z(\bar{p}_h) \geq 0 \ \forall \ x < z \).

Claim B.2
From (1), if (i) all individuals of cohort h retire at the same period (ii) \( \theta_i =

\[28\] This result might be particularly interesting when measuring poverty from micro-simulated data.
$\forall i, (iii) \alpha_k = \frac{1}{\nu_i} \forall k$ and $(iv) z_{t+k} = \xi_k z_{t+k-1} \forall k, p_{i,t+k,h} = \xi_k p_{i,t+k-1,h} \forall i$ and $\forall k$ with $\xi_k > 0 \forall k$, then

$$I^\gamma_h = \frac{1}{r_h} \sum_{i=1}^{r_h} \max \left(1 - \frac{p_{i,t+1,h}}{z_{t+1}}, 0\right) \gamma$$

**Proof B.2** Straightforward from (1).

**C**

**Proof C.1** (Claim 3.1) $K$ is defined as twice the area ($A$) between the Lorenz curves of tax liabilities and virtual pension incomes. Since the first line in (4) is well-known, let’s focus on the second line only. Indicating the Lorenz curve and tax liabilities by $L(\cdot)$ and $t$ respectively, it must be the case that

$$2A = 2 \int_0^1 L(t_h)dt_h - 2 \int_0^1 L(p^*_h)dp^*_h = 2 - 2 \int_0^1 L(-t_h)d(-t_h) - 2 \int_0^1 L(p^*_h)dp^*_h = G(-t_h) + G(p^*_h) = -G(t_h) + G(p^*_h) = -C(t_h) + G(p^*_h),$$

where the last step holds since $p^*_h$ is ordering-preserving with respect to $p^*_h$.

**D**

**Axiom D.1** (Multi-period independence of total tax liabilities)

Given $p^*_{t+k,h}, p_{t+k,h} \in \mathbb{R}^n$ with $k := 1, \ldots, \nu_i$, if $p'_{t+k,h}$ is constructed such that

$$\sum_{k=1}^{\nu_i} p^*_{i,t+k,h} + \lambda \sum_{k=1}^{\nu_i} p_{i,t+k,h} \forall i,$$

then $P(p^*_h, p_h) = P(p^*_h, p'_h)$.

**Proof D.1** (Claim 3.2) Since $C$ and $G$ are h.d.0, $P$ satisfies axiom 3.2 if $\phi(\cdot)$ is homogenous. Additive separability over time implies $\phi(p^*_{t+1,h} - p_{t+1,h}, \ldots, p^*_{t+\nu_i,h} - p_{t+\nu_i,h}) = \psi(p^*_{t+1,h} - p_{t+1,h}) + \ldots + \psi(p^*_{t+\nu_i,h} - p_{t+\nu_i,h})$, while $\sum_{k=1}^{\nu_i} p^*_{i,t+k,h} = (1 - \lambda) \sum_{k=1}^{\nu_i} p^*_{i,t+k,h} + \lambda \sum_{k=1}^{\nu_i} p_{i,t+k,h} \forall i$ implies both (i) independence of total generational tax liabilities and (ii) independence of intertemporal allocation of tax liabilities over time. Then, $\psi(\cdot)$ must be linear and multiplicative.


The AIM project aims at providing a strengthened conceptual and scientific basis for assessing the capacity of European pension systems to deliver adequate old age income maintenance in a context of low fertility and steadily increasing life expectancy. The main focus is on the capacity of social security systems to contribute to preventing poverty among the old and elderly and more generally to enable persons to take all appropriate measures to ensure stable or “desired” distribution of income over the full life cycle. In addition it will explore and examine the capacity of pension systems to attain broad social objectives with respect to inter- and intra generational solidarity.

Furthermore it will examine the capacity of pension systems to allow workers to change job or to move temporarily out of the labour market and to adapt career patterns without losing vesting of pensions rights. The project will also address the specific challenges with respect to providing appropriate old age income for women.

A general objective of the research project is to clearly identify and analyse the potential trade-offs between certain social policy objectives and overall stability of public debt.

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**Participating institutes**

- Centre for European Policy Studies, CEPS, Belgium, coordinator
- Federal Planning Bureau, FPB, Belgium
- Deutsches Institut für Wirtschaftsforschung (German Institute for Economic Research), DIW, Germany
- Elinkeinoelämän tutkimuslaitos, (Research Institute of the Finnish Economy), ETLA, Finland
- Fundación de Estudios de Economía Aplicada , FEDEA, Spain
- Social and Cultural Planning Office, SCP, Netherlands
- Instituto di Studi e Analisi Economica (Institute for Studies and Economic Analysis), ISAE, Italy
- National Institute for Economic and Social Research, NIESR, United Kingdom
- Centrum Analiz Spololeczno-Ekonomicznych (Center for Social and Economic Research), CASE, Poland
- Tarsadalomkutatasi Informatikai Egyesules (TARKI Social Research Informatics Centre), TARKI, Hungary
- Centre for Research on Pensions and Welfare Policies, CeRP, Italy
- Institute for Economic Research, IER, Slovak Republic
- Inštitut za ekonomska raziskovanja (Institute for economic research), IER, Slovenia