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TIMELY MEASUREMENT OF REAL EFFECTIVE EXCHANGE RATES

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We demonstrate that short-run real exchange effective rate changes are dominated by nominal effective exchange rate changes, while inflation rates are sticky and contribute little to short-run real exchange rate changes. These observations allow a rather accurate real-time approximation of the real effective exchange rate using actual nominal exchange rate data and forecast inflation data. We measure the approximation error and find it is minor for most countries and sizeable only for a few countries experiencing high and volatile inflation. For a set of countries, the revision in our estimates using real-time data is slightly lower than the revision in World Bank estimates and much lower than International Monetary Fund estimates. By considering two widely studied economic issues, unit root testing in real exchange rates and nominal exchange rate forecasting with the real exchange rate, we find that using a version of real exchange rates based on approximated monthly price level data instead of actual price level data hardly changes the conclusions on unit roots and forecasting. By combining alternative data sources for exchange rates and consumer prices, we calculate up-to-date monthly real effective exchange rates for 177 countries and the euro area. Our dataset, which is frequently updated, includes more than twice as many observations as the second most comprehensive dataset.

Keywords: effective exchange rates, price level forecasting, unit root testing, exchange rate forecasting
JEL code: F31, E37, F37

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1. Introduction

The real effective exchange rate (REER), which measures the development of the price (or cost) level adjusted value of a country's currency against a basket of the country's trading partners, is a frequently used indicator in theoretical and applied economic research and policy analysis. It is used for a wide variety of purposes, including assessing the equilibrium value of a currency, the change in price or cost competitiveness, the drivers of trade flows, or incentives for reallocation of production between the tradable and the non-tradable sectors.

Because of the importance of the REER in economic research and policy analysis, several multilateral institutions, including the Bank for International Settlements (BIS), Eurostat, the International Monetary Fund, the Organisation for Economic Cooperation and Development, and the World Bank, publish REER indicators for selected countries. In these datasets, data is available for all advanced and several emerging and developing countries, but coverage is insufficient, and some data is released with a delay.

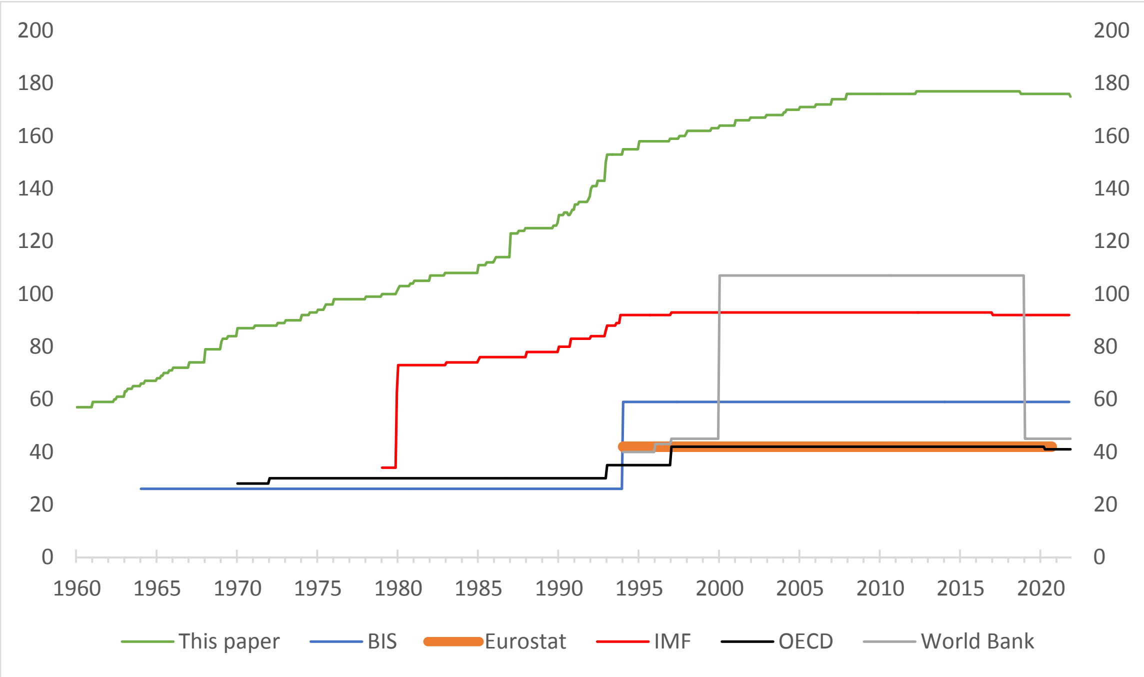
This paper contributes to the measurement of monthly consumer price index-based real effective exchange rates with two main novelties[1,2]. First, we develop a straightforward methodology to approximate the REER when data on nominal exchange rates is available, but data on consumer prices is not yet available due to publication delays. This allows the estimation of up-to-date REERs. Data on nominal exchange rates is readily available practically in real-time, and thus a monthly average exchange rate (which is used for our calculations) can be calculated already on the first day of the subsequent month. However, the publication of CPI data is typically delayed: for several countries, CPI is published in the middle of the subsequent month, and for others there are longer delays. We forecast the latest missing consumer price observations, enabling the approximation of the REER already on the first day of the subsequent month. We measure the uncertainty related to this forecast and find it minor for most countries. The approximation error is sizeable only for a few countries experiencing high and volatile levels of inflation.

Second, by combing various datasets for consumer price index and exchange rate data, we calculate monthly REERs for 177 countries (plus the euro area), many more than in any other dataset, and for generally longer periods. Our dataset includes more than twice as many observations for consumer-price based monthly REERs as the second most comprehensive dataset from the IMF (Figure 1). The

[1] This paper discusses REER estimation at the monthly frequency. Our dataset also includes annual data, see the online annex.
[2] Consumer prices are used the most often in the calculation of real effective exchange rate indices. Other indicators used include producer prices, GDP deflator and unit labour costs. See Chinn (2006) for a comprehensive overview of the theoretical underpinnings of various real effective exchange rate measures.

large number of countries included in our dataset allows us to calculate the REER relative to a broader set of trading partners than in other datasets, providing a more comprehensive picture of the development of the real value of currencies.

Figure 1: Availability of monthly consumer-price based REER indicators in the various datasets on 4 December 2021 (number of countries)



Source: Bruegel based on data collected from websites of the five institutions listed in the legend. Note: Only REERs for countries are included. Additionally, all datasets but the World Bank include the euro area, while the World Bank dataset includes 12 aggregate REER indices (the whole world plus 11 regions, like low-, middle- and high-income countries).

While our methodology can be applied to REERs based on any conceptual framework, the matrix we use to weigh the role of trading partners in foreign trade, which is based on Bayoumi *et al* (2006), was derived from gross trade value data. This implicitly assumes that only final goods cross the border (Bayoumi, 2018). The same approach is used by international organisations that regularly publish REER data. This approach is called the 'conventional approach'.

As already noted by Klau and Fung (2006), vertical specialisation implies that gross value trade differs from the value-added content made in the exporting country, and makes final goods exports and intermediate goods imports complements. Trade in intermediate products expanded rapidly with the development of global value chains in the past decades. Recent research developed new methods to take into account the influence of global value chains on the calculation of weighing matrices for REERs. For example, Bems and Johnson (2017) developed a value-added real effective exchange rate indicator, which aggregates bilateral value-added price changes. They estimated such REERs for 40 countries using annual data from 1995–2007. Patel *et al* (2019) extended the value-added real effective exchange rates by considering sectoral heterogeneity, and they made estimates for 40 countries for the 1995-2009 period. Unfortunately, neither a weighting matrix for value-added trade nor the underlying data to construct the matrix are available for the 177 countries we include in our dataset, while the available underlying data for a few dozen countries lags by many years. Thus, our current dataset improves the conventional REER measures. When value-added trade and price data becomes available for a larger set of countries and in a timely manner, our methodology can be extended readily to utilise that.

The rest of the paper is organised as follows. Section 2 presents our methodology after establishing stylised facts about the short-run drivers of real effective exchange rates. Section 3 briefly describes our data (full description, including weblinks, are included in the online annex). Section 4 presents our forecasting results for the consumer prices using alternative models and quantifies the resulting REER approximation errors in light of the volatility of REERs and the REER revisions of the IMF and the World Bank. Section 5 studies the role of approximated consumer price level data in the measurement of the REER in two economic applications: testing for a unit root in the REER and forecasting the nominal exchange rate with the REER. Section 6 compares REER estimates from alternative sources and section 7 concludes.

2. Methodology

2.1 Definition of the effective exchange rate

The real effective exchange rate is defined as:

$$(1) \qquad Q_{i,E,t} = S_{i,E,t} \cdot \frac{P_{i,t}}{P_{W,t}}$$

where $Q_{i,E,t}$ is the real effective exchange rate of country i against a basket of currencies ('E' in the subscript refers to 'effective', that is, by considering a basket of trading partners), $S_{i,E,t} = \prod_{j=1}^{N} S_{i,j,t}^{w(j)}$ is the nominal effective exchange rate of the country under study, which is in turn the geometrically weighted average of $S_{i,j,t}$, the nominal bilateral exchange rate between country i and j (measured as the foreign currency price of one unit of domestic currency and thus an increase indicates appreciation of the home currency), $P_{i,t}$ is the consumer price index of home country i, $P_{W,t} = \prod_{j=1}^{N} P_{j,t}^{w(j)}$ is the geometrically weighted average of price levels of trading partners ('W' in the subscript refers to 'world', that is, the aggregate of those countries that are included in the basket), $P_{j,t}$

is the price level of county j, $w^{(j)}$ is the weight of trading partner j, and N is the number of trading partners considered. The weights sum to one, ie $\sum_{j=1}^{N} w^{(j)} = 1$. We use geometrically weighted averages, the most frequently used method in the literature, because a geometrically weighted average treats increases and decreases in the exchange rate symmetrically and is not affected by the choice of the base year (Ellis, 2001).

2.2 Drivers of REER change variance

The logarithmic transformation of equation (1) is:

$$(2) \qquad q_{i,E,t} = s_{i,E,t} + p_{i,t} - p_{W,t}$$

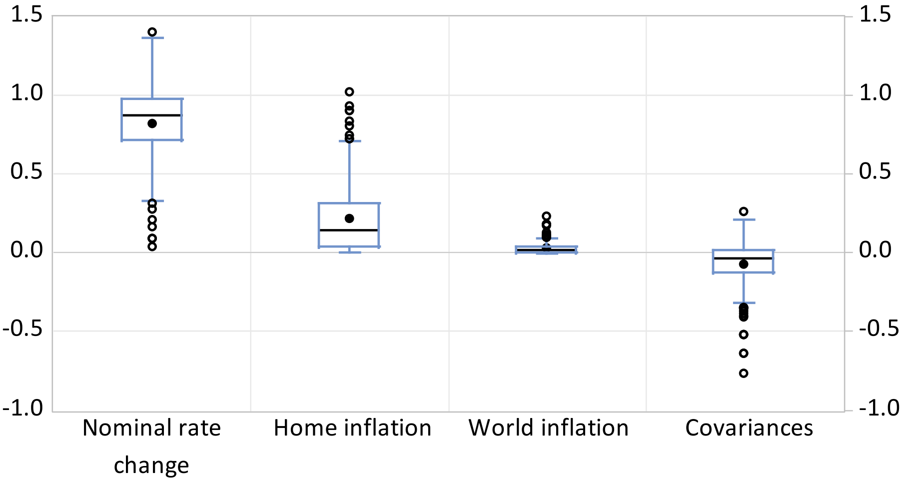
where $q_{i,E,t} = \ln(Q_{i,E,t})$, $s_{i,E,t} = \ln(S_{i,E,t})$, $p_{i,t} = \ln(P_{i,t})$ and $p_{W,t} = \ln(P_{W,t})$.

The variance of the change of the real effective exchange rate can be decomposed as:

$$(3) \qquad \sigma(\Delta q_{i,\mathrm{E},t}) = \sigma(\Delta s_{i,E,t}) + \sigma(\Delta p_{i,t}) + \sigma(p_{W,t}) + 2 \cdot \sigma(\Delta s_{i,E,t}, \Delta p_{i,t}) - 2 \cdot \sigma(\Delta s_{i,E,t}, p_{W,t}) - 2 \cdot \sigma(\Delta p_{i,t}, p_{W,t})$$

where $\sigma(x_t)$ denotes the variance of x_t and $\sigma(x_t, y_t)$ denotes the covariance between x_t and y_t. Figure 2 shows that the variance of the change in the real exchange rate is dominated by the variance of the nominal exchange rate change, as the ratio of the two variances are not far from one for most countries. Home inflation variance is on average about one-quarter of the real exchange rate change variance, while foreign inflation variance is a tiny fraction of the real exchange rate change variance. The sum of the three covariance terms has a small negative impact on the real exchange rate change variance in the case of most countries. The outliers to these observations mostly include countries with high inflation rates.

Figure 2: Contribution to the variance of the monthly real effective exchange rate change: distribution across 177 countries and the euro area



Source: Bruegel. Note: the four distributions plotted refer to: $\sigma(\Delta s_{i,E,t})/\sigma(\Delta q_{i,E,t})$, $\sigma(\Delta p_{i,t})/\sigma(\Delta q_{i,E,t})$, $\sigma(p_{W,t})/\sigma(\Delta q_{i,E,t})$, and $\left(2 \cdot \sigma(\Delta s_{i,E,t}, \Delta p_{i,t}) - 2 \cdot \sigma(\Delta s_{i,E,t}, p_{W,t}) - 2 \cdot \sigma(\Delta p_{i,t}, p_{W,t})\right)/\sigma(\Delta q_{i,E,t})$, respectively. The variances and covariances were calculated over the 2016-2021 period for 177 countries and the euro area. Our broad real effective exchange rate index (which is calculated relative to 120 trading partners) is used.

The box portion represents the first and third quartiles (middle 50 percent of the data, defining the interquartile range); the median is depicted using a line through the centre of the box; the mean is drawn using a filled circle within the box; the whiskers and the staples show the range of observations which are within the first quartile minus 1.5-times the interquartile range and the third quartile plus 1.5-times the interquartile range; outside this range, empty circles denote outliers.

We also find that inflation rates are rather persistent: the median of the estimated autoregressive parameter of 12-month inflation rates for 177 countries is 0.9. The relatively high persistence of inflation rates and the dominant role of nominal exchange rate changes in real exchange rate changes suggests that the real effective exchange rate could be approximated well when data on nominal exchange rates is available, even if recent price data has not yet been published but a forecast is used to approximate it. Exchange rate data is readily available and the monthly average for a particular month can be calculated on the first day of the subsequent month. We therefore use actual nominal exchange rate data and forecast price level data whenever data is not yet available.

2.3 Forecasting the price level

Consider the general case in which the number of missing recent observations is k with $k \geq 1$. Thus, at the last observation of the sample period, denoted as time T, the latest available actual data for the price level is $P_{i,T-k}$, while data is not yet available for $P_{i,T-k+1}, P_{i,T-k+2}, \dots, P_{i,T}$. We assess four alternative predictions for such missing data[3]. These include two naïve forecasts (unchanged 1-month or 12-month inflation rates), an autoregressive model and the forecasts made by the IMF. We do not set-up more comprehensive inflation forecasting models, partly because for the bulk of the countries only very short-horizon forecasts have to be made, for which the simple models we consider already provide rather accurate forecasts. Also, setting-up more complex models might face data limitations for several of the 177 countries we consider. The IMF forecasts we consider presumably include the influence of a large number of economic factors and, as we will see, for most countries, simple models outperform IMF forecasts in short-horizon forecasting.

<u>2.3.1 Unchanged monthly inflation rate from the latest available month</u>

(4)
$$P_{i,T-k+1|T} = P_{i,T-k} \cdot \left(\frac{P_{i,T-k}}{P_{i,T-k-1}}\right)$$

$$P_{i,T-k+2|T} = P_{i,T-k} \cdot \left(\frac{P_{i,T-k}}{P_{i,T-k-1}}\right)^2$$

ε

$$P_{i,T|T} = P_{i,T-k} \cdot \left(\frac{P_{i,T-k}}{P_{i,T-k-1}}\right)^k$$

where $P_{i,T-s|T}$ for $s = 0,1,\dots,k-1$ is the predicted vale of $P_{i,T-s}$ based on information available at time T (when the latest available data is for time $T-k$). Note that we use seasonally adjusted price levels and thus this prediction, as well as the other predictions considered in this paper, do not include a seasonal component.

<u>2.3.2 Unchanged 12-month inflation rate from the latest available month</u>

(5)
$$P_{i,T-k+1|T} = P_{i,T-k-11} \cdot \left(\frac{P_{i,T-k}}{P_{i,T-k-12}}\right)$$

$$P_{i,T-k+2|T} = P_{i,T-k-10} \cdot \left(\frac{P_{i,T-k}}{P_{i,T-k-12}}\right)$$

ε

$$P_{i,T|T} = P_{i,T-k} \cdot \left(\frac{P_{i,T-k}}{P_{i,T-k-12}}\right)$$

[3] The assumed frequency is monthly for the equations below, while data could be collected on various days of a month. We collect data on one of the first days of each month. Time *T* in the equations refers to the preceding full month, for example, to November 2021 when the data was collected on 4 December 2021.

2.3.3 Prediction based on an autoregressive model for the inflation rate

$$(6) \qquad p_{i,T-k+1|T} = p_{i,T-k} + \Delta p_{i,T-k+1|T}$$

$$p_{i,T-k+2|T} = p_{i,T-k} + \Delta p_{i,T-k+1|T} + \Delta p_{i,T-k+2|T}$$

ε

$$p_{i,T|T} = p_{i,T-k} + \Delta p_{i,T-k+1|T} + \Delta p_{i,T-k+2|T} + \cdots + \Delta p_{i,T|T}$$

where lowercase $p_{\ldots}$ indicates the natural logarithm $P_{\ldots}$, and $\Delta p_{i,T-s|T}$ is an autoregressive forecast of $\Delta p_{i,T-s}$ from a model estimated over the sample $t = 1, 2, \ldots, \ T - k$:

$$(7) \qquad \Delta p_{i,t} = \beta_0 + \beta_1 \cdot \Delta p_{i,t-1} + \beta_2 \cdot \Delta p_{i,t-2} + \cdots + \beta_l \cdot \Delta p_{i,t-l} + \varepsilon_t$$

where ε_t is the error term. We compare alternative lag structures for equation (7) and alterative estimation schemes.

2.3.4 IMF World Economic Outlook forecasts

The IMF presents its forecasts twice a year in its *World Economic Outlook* (WEO) database, typically in the first days of April and October, for five years ahead at the annual frequency[4]. The October 2021 WEO included inflation forecasts for 193 countries. To convert the IMF's annual forecasts to the monthly frequency that we use, we assume constant monthly inflation rates so that the resulting annual inflation rate is the same as the IMF's annual forecast. The IMF publishes two types of forecasts: annual average and end-of-period; that is, December of a year compared to the December of the preceding year. We use the latter measure to achieve the best consistency with our monthly data. In our out-of-sample forecasting exercise, we always use the most recently available IMF forecast: from April to September of a year we use the April IMF forecast of that year and from October to March we use the latest October IMF forecast.

Let us introduce a slightly altered notation due to the mixed-frequency data we use. For example, suppose that $k = 15$ recent monthly price level observations are missing in December of year τ. In this case, the latest available actual monthly price level data is for July of year $\tau - 1$ that we denote as $P_{i,\tau-1_{m07}}$. In December of year τ, the forecasts made by the IMF in October of year τ are available. We denote the IMF forecast made in October of year τ for December of the preceding year as $P^{(IMF)}_{i,\tau-1_{m12}|\tau_{m10}}$, and we denote the forecast made for December of year τ as $P^{(IMF)}_{i,\tau_{m12}|\tau_{m10}}$. In this

[4] The exception during our sample was April 2020, when forecasts only for 2020 and 2021 were published and not for five years ahead.

example, we calculate the forecasts for August-December of year $\tau - 1$ based on information available at time τ_{m12}:

(8)
$$P_{i,\tau-1_{m08}|\tau_{m12}} = P_{i,\tau-1_{m07}} \cdot \left(\left(\frac{P^{(IMF)}_{i,\tau-1_{m12}|\tau_{m10}}}{P_{i,\tau-2_{m12}}} \right) \Big/ \left(\frac{P_{i,\tau-1_{m07}}}{P_{i,\tau-2_{m12}}} \right) \right)^{\left(\frac{1}{12-7}\right)}$$

$$P_{i,\tau-1_{m09}|\tau_{m12}} = P_{i,\tau-1_{m08}|\tau_{m12}} \cdot \left(\left(\frac{P^{(IMF)}_{i,\tau-1_{m12}|\tau_{m10}}}{P_{i,\tau-2_{m12}}} \right) \Big/ \left(\frac{P_{i,\tau-1_{m07}}}{P_{i,\tau-2_{m12}}} \right) \right)^{\left(\frac{1}{12-7}\right)}$$

ε

$$P_{i,\tau-1_{m12}|\tau_{m12}} = P_{i,\tau-1_{m11}|\tau_{m12}} \cdot \left(\left(\frac{P^{(IMF)}_{i,\tau-1_{m12}|\tau_{m10}}}{P_{i,\tau-2_{m12}}} \right) \Big/ \left(\frac{P_{i,\tau-1_{m07}}}{P_{i,\tau-2_{m12}}} \right) \right)^{\left(\frac{1}{12-7}\right)}$$
$$= P^{(IMF)}_{i,\tau-1_{m12}|\tau_{m10}}$$

Forecasts for January-December of year τ:

(9)
$$P_{i,\tau_{m01}|\tau_{m12}} = P^{(IMF)}_{i,\tau-1_{m12}|\tau_{m10}} \cdot \left(\frac{P^{(IMF)}_{i,\tau_{m12}|\tau_{m10}}}{P^{(IMF)}_{i,\tau-1_{m12}|\tau_{m10}}} \right)^{\left(\frac{1}{12}\right)}$$

$$P_{i,\tau_{m02}|\tau_{m12}} = P_{i,\tau_{m01}|\tau_{m12}} \cdot \left(\frac{P^{(IMF)}_{i,\tau_{m12}|\tau_{m10}}}{P^{(IMF)}_{i,\tau-1_{m12}|\tau_{m10}}} \right)^{\left(\frac{1}{12}\right)}$$

ε

$$P_{i,\tau_{m12}|\tau_{m10}} = P_{i,\tau_{m11}|\tau_{m12}} \cdot \left(\frac{P^{(IMF)}_{i,\tau_{m12}|\tau_{m10}}}{P^{(IMF)}_{i,\tau-1_{m12}|\tau_{m10}}} \right)^{\left(\frac{1}{12}\right)} = P^{(IMF)}_{i,\tau_{m12}|\tau_{m10}}$$

2.4 Comparing price level forecasting results

In order to select a forecasting model for each country, we compare the forecasting ability of models in an out-of-sample forecasting exercise using the January 2015 – November 2021 period for evaluating out-of-sample forecasts. This is a reasonably long period including years with good global growth performance, the global economic contraction caused by the COVID-19 pandemic, and the initial period of economic recovery from that. The pre-pandemic years were generally characterised by low inflation, while inflation started to increase worldwide during the recovery from the pandemic recession. Since the main goal of our forecasting exercise is to forecast a few recent missing price level observations, the performance of the various models over much longer out-of-sample evaluation periods is not relevant for our study.

The equations in the previous section described the forecasting methods for the case when k recent observations were missing at the last observation of the sample period, time T. When assessing the forecasting ability of the various models in an out-of-sample forecasting exercise, we make forecasts for all forecasting horizons between 1 and 24 months from each month of the out-of-sample evaluation period[5]. Obviously, for converting the annual IMF forecasts to monthly forecasts, we use the latest version of the IMF forecast which was available ahead of the out-of-sample forecast evaluation period. For example, when forecasting for the January 2015 – December 2016 period, we use the October 2014 IMF forecasts.

The application of the naïve forecasts assuming unchanged inflation rates and the IMF forecasts do not require a model estimation. But the application of an autoregressive model as indicated in equations (6) and (7) necessitate multiple choices: the number of autoregressive lags included, the sampling method and the sample size to use for the estimation.

As regards the lag length, we compare models with lags from 1 to 6.

As regards the sample size and sampling method, observations several decades ago might not be informative about current inflation developments, but it is difficult to set a date for the start of the estimation sample size. We therefore compare three options:

- Rolling estimation over the past five years[6],
- Rolling estimation over the past ten years,
- Recursive estimation starting from January 2000[7].

We use two widely-employed loss functions to compare the forecasting ability of various models, evaluated at percent errors. One is the mean absolute percent forecast error (MAPFE):

[5] More precisely, the first date for evaluating out-of-sample forecasts is January 2015 and thus our first forecast iteration uses data up to December 2014 to make forecasts for the January 2015-December2016 period.

[6] That is, we first estimate the models on the January 2010 – December 2014 period, calculate forecasts for the January 2015 – December 2016 period and compare these forecasts with the actual price level. Next, we estimate the models on the February 2010 – January 2015 period, calculate forecasts for the February 2015 – January 2017 period and compare these forecasts with the actual price level, and so on.

[7] That is, we first estimate the models on the January 2000 – December 2014 period, calculate forecasts for the January 2015 – December 2016 period and compare these forecasts with the actual price level. Next, we estimate the models on the January 2000 – January 2015 period, calculate forecasts for the February 2015 – January 2017 period and compare these forecasts with the actual price level, and so on.

(10) $$\mathrm{MAPFE}_h = 100 \cdot \sum_{t=T-M+1}^{T} \frac{1}{M} \cdot \frac{\left|P_{i,t+h} - P_{i,t+h|t}\right|}{P_{i,t+h}}$$

where $P_{i,t+h|t}$ is the h-period ahead forecast of the price level of country i made at time t, M is the total number of forecasts made and $h = 1, 2, \ldots, 24$ is the forecast horizon. The other is the root mean squared percent forecast error (RMSPFE):

(11) $$\mathrm{RMSPFE}_h = 100 \cdot \sqrt{\sum_{t=T-M+1}^{T} \frac{1}{M} \cdot \left(\frac{P_{i,t+h} - P_{i,t+h|t}}{P_{i,t+h}}\right)^2}$$

We test whether the forecast is unbiased using the version of the traditional test that is regarded more satisfactory by Clements *et al* (2007), which is based on the regression:

(12) $$P_{i,t+h} - P_{i,t+h|t} = \alpha + v_t$$

where α is a parameter to estimate and v_t is the regression error which follows a MA(k-1) process. The null hypothesis of unbiasedness corresponds to the test of $\alpha = 0$. We estimate regression (12) with ordinary least squares using the Newey and West autocorrelation and heteroskedasticity consistent covariance matrix.

For each country, we select the model to use which results in the lowest RMSPFE indicator for the particular forecasting horizon corresponding to the number of missing observations as indicated in Table 1. There are various tests for comparing the forecast accuracy of alternative models, such as the ones developed by Diebold and Mariano (2002) and Clark and West (2007). Since our primary aim is predicting the few missing observations of the price level and not a ranking of the alternative forecasting models, it is not interesting for us, for example, whether the model with lowest RMSPFE is a statistically significantly better forecaster than the model with the second lowest RMSPFE. Thus, we do not report formal tests for comparing alterative forecasts.

2.5 Measuring the REER approximation error from our price level forecasts

Real exchange rate indicators are frequently revised, even if no forecast data is used, for two main reasons. First, consumer price data are sometimes revised. Second, we use seasonally adjusted consumer price data, because the seasonality of prices is less relevant for the underlying development of the real exchange rate. Even when there is no historical data revision, seasonal adjustment results in somewhat altered seasonally adjusted values when a new data point is added and the whole time series is seasonally adjusted again.

We measure the REER approximation error in two ways:

- Using the latest available data (collected on 4 December 2021), we do out-of-sample forecasting for the period 2015-2021 in order to measure the magnitude of the REER approximation error resulting only from our price level forecasts;
- We collected real-time data from October 2020-December 2021 and use that to measure REER revisions resulting from all possible sources, in comparison with the World Bank's REER estimate revisions over the same period.

3. Data

We collect consumer price index and US dollar exchange rate data from publicly available data sources for the longest available time periods and for the largest number of countries and the euro area. We collect exchange rate against the US dollar and use them to calculate the bilateral rates between all countries[8]. Our main data sources are the Global Economic Monitor[9] dataset of the World Bank, IMF International Financial Statistics, and the OECD's consumer price indices dataset. We also collect data from national statistical offices or central banks for 40 countries. Here we focus on monthly data, while the online annex details our data sources for both the monthly and annual frequencies.

For four countries, monthly CPI data is not available, but the highest data frequency in our full sample period is quarterly. We thus approximate monthly CPI based on quarterly CPI by linear interpolation so that these countries are included in our monthly REER dataset[10]. For an additional four countries, monthly CPI data is available now, but quarterly data starts earlier: for these countries, we use interpolated quarterly data up to the first observation of monthly data.

We combine the alternative datasets by checking the available time periods for each country and the similarities of the data in the overlapping sample periods. For consumer prices, we use seasonally adjusted values because within-year seasonality is not relevant for the underlying development of the real value of currencies. When the earliest and the most recent data points are from the same data source, we use that data source. When the earliest and the most recent data points are from different data sources, we chain to each other the data from alternative sources to obtain the maximum time period available.

[8] For euro-area members, since their entry to the euro area, we multiplied the euro/dollar exchange rate with the conversion rate to the euro in order to extend the exchange rate of their earlier national currencies.
[9] The World Bank's *Global Economic Monitor* has two versions: an irregularly updated online interface includes monthly data from January 1987 (in December 2021, the most recent observation was for June 2020), while a downloadable zip file is updated daily, but includes data only for the most recent 360 months. The country coverage also slightly differs across the two versions. We use both versions.
[10] Such an approximation has been done for other datasets too, because monthly CPI-based REER is published for some of these countries, like Australia and New Zealand, by the BIS, IMF, OECD and World Bank.

We also check for data recording errors. For example, on a few occasions, a price level time series with large values suddenly falls to close to zero and then continues to gradually increase from that level, which suggests there is break in the time series. We exclude data before such breaks.

With the combination of alternative data sources, we obtain monthly data for 177 countries plus the euro area. For 58 countries, the sample starts in 1960, while for the other countries the earliest available observation determines the starting date of our calculated REERs.

For several countries, CPI data is published with a lag, which is particularly relevant for the monthly frequency. Table 1 shows that on 4 December 2021, no data was missing for preceding full month (November 2021) for 18 countries, one data point (November 2021) was missing for 90 countries, two data points (October and November 2021) were missing for 10 countries, and so on. For seven countries, more than two years of recent data was missing.

Table 1: Distribution of the number of missing recent monthly CPI observations across 177 countries on 4 December 2021, and tests for forecast unbiasedness for out-of-sample forecasts made in 2015-2021

Distribution of the number of missing recent CPI observations			Price level forecast unbiasedness test			
Missing CPI data points (months)	Frequency (number of countries)	Cumulative (number of countries)	p>=10%	10%>p>=5%	5%>p>=1%	1%>p
0	18	18				
1	90	108	77	4	6	3
2	10	118	5	3	1	1
3	11	129	8	2		1
4	2	131	1			1
5	8	139	7			1
6	2	141	1	1		
7	2	143	1			1
8	6	149	5			1
9	0	149				
10	1	150	1			
11	5	155	5			
12	3	158	3			
13-24	12	170	7	1	2	2
More	7	177				

Source: Bruegel. Note: the number of missing observations refer to the month preceding the data collection date of 4 December 2021: zero is used when the CPI data for November 2021 is available; one is used when the most recent CPI data is for October 2021, and so on. There was no missing observation for the euro area. For the unbiasedness test, the table shows the p-value of testing the null hypothesis of $\alpha = 0$ in equation (12) using the January 2015 – November 2021 out-of-sample forecast evaluation period.

For both the monthly and annual frequencies, we calculate two versions of the REER: a broader one covering 120 trading partners at the monthly frequency (available from January 1993) and 170 trading partners at the annual frequency (available from 1992), and a narrower one covering 51 trading partners at the monthly frequency (available from January 1960) and 65 trading partners at the annual frequency (available from 1960). For the monthly frequency, we selected the countries to be included in the basket of trading partners by considering three criteria:

1) Price level and exchange rate data should be available from January 1960 (for our narrow index) or from January 1993 (for our broad index),
2) The mean absolute percent forecast error of the consumer price level over the 2015-2021 out-of-sample evaluation period should be less than 1 percent (see section 4.1)[11],
3) Countries that experienced hyperinflation sometime in the past are excluded.

We still calculate REERs for countries for which price level forecast errors were larger than 1 percent on average, but do not include them in the basket of trading partners to reduce end-point uncertainty of REER estimates for other countries. We also calculate REERs for countries that experienced hyperinflation sometime in the sample period, but do not include them in the basket due to the uncertainty of price and exchange rate measurement under hyperinflation.

On average across countries, the 51 trading partners included in the narrow basket account for 78 percent of trade, while the 120 countries included in the broad basket account for 95 percent of trade.

For the annual frequency, we do not create forecasts and hence data availability and lack of hyperinflation are the two criteria we use to include countries in the basket of trading partners.

Between October 2020 and December 2021, we also collected data from our data sources in order to perform an analysis using real-time data[12]. The data was collected between 3rd and 17th days of the various months with the average data collection day on the 6th day of the month.

[11] This criterion did not exclude any country from the narrow index, but excluded 23 countries from the broad index. The threshold of 1.5 percent would have excluded 13 countries from the broad index, the threshold of 2 percent would have excluded 10 countries, and the threshold of 3 percent would have excluded 6 countries.

[12] Among the three main datasets on consumer prices published by the institutions that we use, historical data vintages are available only for the IMF International Financial Statistics starting from August 2017.

4. Price level forecasts and REER estimates

4.1 Forecasting consumer price levels

We compare the price level forecast accuracy of the models listed in section 2.3 in the 2015-2021 out-of-sample evaluation period, for those 170 countries for which less than two years of data is missing. Instead of reporting detailed results for all 170 countries, we report the median across countries (Tables 2 and 3)[13]. The ranking of models depends little on whether we use the MAPFE or the RMSPFE statisics. The three main conclusions based on both loss functions:

- The autoregressive models provide somewhat more accurate forecasts than the naïve forecasts, which assume unchanged inflation rates,
- There are rather limited differences between the various autoregressive models, though at longer forecast horizons, the 5-year rolling estimation resulted in somewhat better forecasts than the 10-year rolling and recursive estimations,
- IMF forecasts have larger forecast errors than other methods at short forecast horizons, but improve relatively to the other methods over longer forecast horizons.

Among the two naïve forecasts, the assumption of unchanged monthly inflation rates results in somewhat better forecasts than the assumption of unchanged 12-month inflation rates only for one-month ahead forecast, while at longer forecast horizons, the assumption of unchanged one inflation rates results in by far the largest forecast errors among all models considered.

The naïve forecasts assuming unchanged 12-month inflation rates are inferior to all versions of the autoregressive model up to six-month forecasting horizons, yet for one-year ahead forecasts this naïve forecast is better than the best autoregressive model.

Among the autoregressive models, the differences are minor for short horizon forecasts. For example, the median values reported in Table 2 shows that the range of the 18 versions of the autoregressive model is 0.28 percent to 0.30 percent, a rather narrow range. The range widens somewhat with the increase of the forecast horizon: for one-year ahead forecasts, the range is 1.62 percent to 2.00 percent. Among the three alternative estimation samples considered, the five-year rolling estimation resulted in, on average across the 170 countries, the most accurate forecasts for longer forecasting horizons.

The IMF forecasts have on average the largest forecast errors for short horizon forecasting, possibly because IMF forecasts are updated only twice a year and hence, for most forecast rounds, the latest

[13] We report the median across countries, and not the average, because forecast errors are large for a few countries with very high levels of inflation and these outliers dominate a cross-country average calculation.

information is not taken into account. However, IMF forecasts relative to the other methods improve with the forecast horizon: at the one-month forecasting horizons, there is only one country for which the IMF forecasts proved to be the best among the methods we considered, there are 12 such countries at the 6-month forecasting horizons, 24 countries at the 1-year horizon and 26 countries at the 2-year horizon.

Tables 2 and 3 show the median across countries, though the ranking of the forecasting models can differ for individual countries. Considering the relevant forecasting horizons indicated in Table 1 (eg for 90 countries only one-month ahead forecast has to be made, for 10 countries two-month ahead forecast have to be made, and so on, up to the 24-month horizon), the most accurate forecasting model was a version of the recursively estimated autoregressive model for 63 countries, a version of the five-year rolling estimated autoregressive model for 52 countries, a version of the ten-year rolling estimated autoregressive model for 23 countries, the IMF forecast for eight countries, while the method assuming unchanged one-month inflation rate was most accurate for four countries and the method assuming unchanged twelve-month inflation rates was most accurate for one country.

By considering the most accurate method at the relevant forecast horizon for each country, the right block of Table 1 reports p values of the unbiasedness tests. For the bulk of countries, the null hypothesis of unbiased forecasts cannot be rejected at standard probability levels (ie the p-values is larger than 10 percent). There are only 11 countries for which forecasts are found to be biased at the 1 percent significance level.

Table 2: Out-of-sample price level forecasts in 2015-2021, mean absolute percent forecast error (MAPFE), median values across 170 countries

	Forecast horizon (in months)													
Model	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**18**	**24**
Unchanged monthly inflation	0.34	0.65	0.92	1.21	1.45	1.71	1.98	2.24	2.55	2.80	3.05	3.37	5.05	6.74
Unchanged 12-month inflation	0.41	0.65	0.83	0.95	1.05	1.18	1.26	1.35	1.45	1.49	1.53	1.58	2.24	2.78
AR(1) - 5-year rolling	0.28	0.45	0.58	0.71	0.82	0.95	1.10	1.20	1.29	1.42	1.53	1.64	2.12	2.51
AR(2) - 5-year rolling	0.28	0.44	0.60	0.71	0.83	0.96	1.07	1.20	1.31	1.42	1.51	1.62	2.11	2.48
AR(3) - 5-year rolling	0.28	0.47	0.61	0.73	0.86	0.95	1.08	1.18	1.32	1.43	1.54	1.65	2.10	2.51
AR(4) - 5-year rolling	0.28	0.46	0.61	0.74	0.86	0.98	1.07	1.17	1.32	1.43	1.56	1.63	2.09	2.49
AR(5) - 5-year rolling	0.28	0.47	0.62	0.75	0.85	0.96	1.11	1.18	1.31	1.42	1.54	1.63	2.09	2.55
AR(6) - 5-year rolling	0.29	0.47	0.62	0.74	0.86	0.97	1.08	1.19	1.32	1.42	1.53	1.62	2.10	2.55
AR(1) - 10-year rolling	0.29	0.50	0.64	0.77	0.94	1.09	1.20	1.32	1.45	1.57	1.68	1.81	2.52	3.28
AR(2) - 10-year rolling	0.28	0.49	0.64	0.78	0.94	1.05	1.19	1.31	1.43	1.54	1.67	1.80	2.51	3.16
AR(3) - 10-year rolling	0.29	0.48	0.63	0.75	0.91	1.03	1.17	1.30	1.42	1.54	1.65	1.77	2.47	3.15
AR(4) - 10-year rolling	0.29	0.47	0.62	0.74	0.88	1.02	1.15	1.26	1.39	1.50	1.62	1.74	2.46	3.10
AR(5) - 10-year rolling	0.29	0.47	0.62	0.74	0.86	1.01	1.15	1.25	1.37	1.50	1.62	1.73	2.42	3.15
AR(6) - 10-year rolling	0.29	0.47	0.61	0.74	0.87	0.97	1.09	1.24	1.36	1.49	1.61	1.73	2.34	3.02
AR(1) - Recursive	0.30	0.52	0.68	0.85	0.98	1.10	1.27	1.39	1.56	1.76	1.88	2.00	2.81	3.64
AR(2) - Recursive	0.30	0.51	0.67	0.84	1.00	1.09	1.21	1.38	1.52	1.67	1.82	1.97	2.78	3.64
AR(3) - Recursive	0.29	0.51	0.66	0.83	0.97	1.08	1.18	1.33	1.49	1.61	1.80	1.98	2.69	3.53
AR(4) - Recursive	0.29	0.50	0.65	0.81	0.98	1.12	1.18	1.32	1.46	1.58	1.73	1.90	2.73	3.39
AR(5) - Recursive	0.29	0.48	0.64	0.79	0.92	1.04	1.17	1.29	1.43	1.58	1.72	1.85	2.58	3.40
AR(6) - Recursive	0.29	0.47	0.63	0.79	0.92	1.03	1.15	1.29	1.42	1.54	1.66	1.84	2.49	3.39
IMF	0.46	0.72	0.92	1.08	1.21	1.33	1.42	1.52	1.63	1.73	1.81	1.92	2.39	2.95

Source: Bruegel. Note: price level forecasts are evaluated in the January 2015 – November 2021 out-of-sample forecasting period for countries with no missing observations, while for countries with missing observations, the last date of the evaluation period is determined by the availability of the latest data. Those 170 countries are considered for which no more than 24 recent observations are missing, see Table 1.

Table 3: Out-of-sample price level forecasts in 2015-2021, root mean squared percent forecast error (RMSPFE), median values across 170 countries

	Forecast horizon (in months)													
Model	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**18**	**24**
Unchanged monthly inflation	0.51	0.92	1.30	1.73	2.02	2.36	2.79	3.19	3.55	3.87	4.35	4.81	7.05	9.22
Unchanged 12-month inflation	0.59	0.88	1.07	1.23	1.38	1.52	1.62	1.71	1.83	1.93	2.01	2.07	2.78	3.51
AR(1) - 5-year rolling	0.39	0.60	0.77	0.92	1.07	1.19	1.36	1.48	1.61	1.76	1.90	1.98	2.53	3.09
AR(2) - 5-year rolling	0.40	0.59	0.77	0.92	1.07	1.19	1.34	1.46	1.60	1.75	1.87	2.00	2.53	3.05
AR(3) - 5-year rolling	0.40	0.61	0.80	0.93	1.08	1.18	1.34	1.46	1.60	1.74	1.84	1.95	2.51	2.99
AR(4) - 5-year rolling	0.40	0.62	0.79	0.93	1.06	1.19	1.34	1.47	1.60	1.74	1.84	1.94	2.52	2.95
AR(5) - 5-year rolling	0.40	0.62	0.78	0.93	1.09	1.20	1.35	1.49	1.65	1.77	1.88	1.99	2.52	3.10
AR(6) - 5-year rolling	0.40	0.62	0.79	0.94	1.08	1.20	1.37	1.50	1.63	1.77	1.87	1.99	2.55	3.10
AR(1) - 10-year rolling	0.40	0.66	0.83	1.02	1.20	1.36	1.50	1.65	1.80	1.93	2.12	2.26	3.05	3.87
AR(2) - 10-year rolling	0.40	0.63	0.85	1.01	1.16	1.34	1.49	1.61	1.76	1.92	2.04	2.18	3.01	3.67
AR(3) - 10-year rolling	0.40	0.63	0.82	0.99	1.14	1.28	1.47	1.60	1.76	1.90	2.04	2.17	2.84	3.62
AR(4) - 10-year rolling	0.39	0.62	0.81	0.99	1.14	1.25	1.42	1.57	1.73	1.86	2.00	2.15	2.83	3.60
AR(5) - 10-year rolling	0.40	0.61	0.81	0.99	1.12	1.25	1.42	1.57	1.71	1.83	1.98	2.14	2.81	3.68
AR(6) - 10-year rolling	0.40	0.62	0.81	0.97	1.12	1.23	1.37	1.51	1.66	1.83	1.96	2.10	2.77	3.60
AR(1) - Recursive	0.41	0.69	0.90	1.09	1.26	1.43	1.60	1.74	1.90	2.07	2.27	2.44	3.18	4.10
AR(2) - Recursive	0.40	0.68	0.88	1.07	1.25	1.38	1.57	1.74	1.88	2.03	2.17	2.34	3.12	4.06
AR(3) - Recursive	0.40	0.64	0.85	1.05	1.24	1.37	1.52	1.73	1.84	2.00	2.16	2.32	3.09	3.97
AR(4) - Recursive	0.40	0.63	0.85	1.03	1.21	1.37	1.54	1.71	1.84	1.97	2.15	2.27	3.10	3.98
AR(5) - Recursive	0.39	0.62	0.82	1.01	1.20	1.32	1.47	1.66	1.81	1.98	2.11	2.29	3.05	4.00
AR(6) - Recursive	0.39	0.62	0.79	0.99	1.15	1.33	1.46	1.65	1.80	1.96	2.09	2.24	2.96	3.87
IMF	0.66	0.97	1.20	1.36	1.53	1.67	1.80	1.90	2.01	2.12	2.22	2.31	2.75	3.34

Source: Bruegel. Note: price level forecasts are evaluated in the January 2015 – November 2021 out-of-sample forecasting period for countries with no missing observations, while for countries with missing observations, the last date of the evaluation period is determined by the availability of the latest data. Those 170 countries are considered for which no more than 24 recent observations are missing, see Table 1.

We compare the magnitude of the price level forecast errors to the average volatility of the nominal effective exchange rate, considering the relevant forecasting horizon of each country as indicated in Table 1. Thus, we compare the magnitude of the one-month ahead price level forecast errors to the one-month change in the nominal effective exchange rate for those 90 countries for which one-month ahead forecast has to be made. We compare the two-month ahead price level forecast errors to the two-month change in the nominal effective exchange rate for the 10 countries for which two-month ahead forecasts have to be made, and so on. For this comparison, we measure both the forecast error and the nominal effective exchange rate change as absolute value of percent changes.

Table 4 shows that for most forecasting horizons up to two years, the typical price level forecast errors are less than one-half of nominal exchange rate volatility, ratios that are not large. The exceptions are the forecasts made for eight and 21 months ahead.

Table 4: The ratio of mean absolute percent forecast error (MAPFE) from out-of-sample price level forecasts to the mean absolute percent change of the nominal effective exchange rate in 2015-2021, median values across countries

Missing CPI data points	Number of countries	CPI forecast error/NEER 51 change	CPI forecast error/NEER 120 change
1	90	0.31	0.29
2	10	0.46	0.46
3	11	0.40	0.40
4	2	0.30	0.31
5	8	0.38	0.38
6	2	0.49	0.39
7	2	0.45	0.38
8	6	0.75	0.72
9	0		
10	1	0.34	0.35
11	5	0.68	0.72
12	3	0.28	0.27
13	0		
14	1	0.42	0.40
15	1	0.35	0.36
16	1	0.25	0.22
17	0		
18	1	0.13	0.12
19	0		
20	3	0.46	0.46
21	3	1.35	1.38
22	0		
23	2	0.30	0.31
24	0		
All	**152**	**0.34**	**0.36**

Source: Bruegel. Note: the January 2015 – November 2021 period is used to evaluate out-of-sample price level forecasts and to calculate the change in the exchange rate. For each country, we calculate the ratio of mean absolute percent price level forecast error to the mean absolute percent change of the nominal effective exchange rate, both calculations were done over the horizon of missing price level data (i.e. one-month is considered for those 90 countries for which one -month CPI data was missing, see Table 1, and so on). This calculation considers 152 countries and not the full sample of 177 countries, because the 18 countries with no missing observations and the 7 countries with more than 24 missing observations are not included. NEER51 refers to the nominal effective exchange rate considering 51 trading partners, while NEER120 refers to the nominal effective exchange rate considering 120 trading partners.

To conclude, the findings of unbiased forecasts and the relatively small typical forecast errors compared to nominal exchange rate volatility suggests that an approximation of the REER with partially forecasted price levels could result in relatively small REER approximation errors.

4.2 Measuring the REER approximation error

We measure the REER approximation error resulting from our CPI forecast methodology using out-of-sample forecasts that we evaluate for the January 2015 – November 2021 period. For this exercise, we assume that in each month during this period, the number of missing recent CPI observations was the same as on 4 December 2021, the date of our most recent data collection. Thus, we assume that in each month from 2015 to 2021, for 18 countries no recent CPI observations were missing, for 60 countries only one recent CPI observation was missing, for 10 countries two observations were missing, and so, as indicated by Table 1. For example, for the first out-of-sample evaluation date, January 2015, we use the actual January 2015 CPI data for 18 countries, we make CPI forecasts using data up to December 2014 for 90 countries, we make CPI forecasts using data up to November 2014 for 10 countries, and so on, and use this CPI data (actual for 18 countries and forecast for all other countries) and actual nominal exchange rate data to calculate the REER. We compare these estimates to the estimates based on actual data from the most recent data collection.

For the 18 countries with no recent missing CPI observations, REER forecast errors result only from the forecasts of the missing CPI observations of trading partners.

Table 5 shows that the absolute value of the typical REER forecast error for the 18 countries with no missing recent CPI observations is 0.05 percent. That is, if value of an initial estimate of the REER was 100, this estimate is likely revised to either 100.05 or 99.95, which is a very minor revision. The typical REER forecast error for the 90 countries with one missing CPI observations is 0.23 percent, still a very low number. REER forecast errors tend to increase with the number of missing CPI observations and with the rate of inflation (last column of Table 5).

Overall, for the bulk of the countries, the REER forecast errors arising from CPI level forecasts is relatively small, while the error is larger only for a few countries characterised by higher inflation rates. Nevertheless, even for most of these countries, the REER forecasts are unbiased.

An important question is whether we should also report REER estimates for those countries for which the forecast error is relatively large, or set a forecast error threshold above which we do not report REER estimates. Since there is no straightforward way to set a threshold, we report REER estimates for all countries and at the same time provide sufficient information for the users of the dataset to decide whether to use the full sample period available in the dataset, or only that period for which the REER was calculated using actual price level data. Thus, we provide the following information for all countries (see the online annex and the downloadable dataset):

- The number of recent months with missing price level data and thus the time periods for which price level forecasts were made,
- The price level forecast method,
- The MAPFE and RMSPFE statistics for price level forecasting from our January 2015 – November 2021 out-of-sample forecasting exercise,
- The MAPFE and RMSPFE statistics for REER forecasting from our January 2015 – November 2021 out-of-sample forecasting exercise.

This can help users to assess the uncertainty of our REER estimates in the periods for which forecasts were made.

Another question is whether we should report estimates for the seven countries for which more than 24 recent monthly price level observations are missing. We did not include these countries in our out-of-sample forecasting exercise, because the available sample period is significantly shortened by the missing observations. However, the IMF WEO includes annual price level data for these countries either up to 2020 (Sao Tome and Principe, Libya, Tonga, Venezuela, Yemen), or up to 2019 (Comoros and Liberia). Hence, for five of these countries, only the 2021 values are forecasts, while for two such countries the 2020-2021 values are forecasts. In section 5, we find that results of certain analyses are practically unchanged when we use smoothed monthly price level data by assuming that actual annual 12-month inflation is distributed evenly across the months of a year. For this reason, we also include those seven countries in our dataset for which more than 24 recent price level observations are missing, and use evenly distributed actual annual inflation over the months of the year, plus IMF forecasts (for 2021 for five countries and for 2020-2021 for two countries).

Table 5: The mean absolute percent forecast error (MAPFE) and root mean squared percent forecast error (RMSPFE) from out-of-sample REER forecasts in 2015-2021, median values across countries

Missing CPI observations	Number of countries	MAPFE	RMSPFE	Average 12-month inflation
0	18	0.05	0.07	2.4
1	90	0.23	0.32	2.7
2	10	0.77	1.00	2.2
3	11	0.62	0.77	1.8
4	2	1.16	1.56	7.4
5	8	0.70	0.88	0.8
6	2	0.76	1.05	3.1
7	2	0.65	0.80	1.8
8	6	2.26	3.05	3.9
9	0			
10	1	1.52	1.90	4.0
11	5	2.13	2.66	1.6
12	3	1.46	1.71	4.0
13	0			
14	1	1.59	1.98	4.9
15	1	4.13	4.95	13.1
16	1	0.86	1.17	1.1
17	0			
18	1	0.66	1.02	0.4
19	0			
20	3	3.31	4.58	3.0
21	3	3.10	4.26	1.5
22	0			
23	2	3.21	3.93	1.1
24	0			
all	**170**	**0.30**	**0.44**	**2.2**

Source: Bruegel. Note: Out-of-sample forecast errors were calculated over the sample of January 2015 – November 2021. For each month within this period and for each country, a forecast for the particular month was made for the national price level by assuming that the number of recent missing CPI observation is the same as at the 4 December 2021 data collection. The REER was calculated using the forecast price levels, or the actual price level for those 18 countries for which no missing observations are assumed. These REER forecasts are compared to the latest REER estimation using data downloaded on 4 December 2021. Those 170 countries are considered for which no more than 24 recent observations are missing, see Table 1. For each country, the model proved to be the best CPI forecaster for the time horizon of missing data was used. Results refer to the broad REER index relative to 120 trading partners; results for the narrow REER index are rather similar.

4.3 Comparing REER estimate revisions from alternative sources

We compare our REER estimate revision with the World Bank and IMF REER estimate revisions, using the real-time data we collected between early October 2020 and early December 2021. Thus, this out-of-sample evaluation period is much shorter than the 2015-2021 evaluation period considered so far.

The World Bank publishes REER indicators in a timely manner: for all our real-time data collection dates, on average on the sixth day of each month, the REER estimate for the preceding month was already available. IMF REER publication is delayed by one month. Thus, when comparing our real-time estimates with the World Bank estimates, we compare the estimates for the latest month (eg for the data collected on 4 October 2020, we look at the revision in the estimates made for September 2020), while for the comparison with the IMF, we assess estimates for the preceding month (eg for the data collected on 4 October 2020, we look at the revision in the estimates made for August 2020).

The World Bank publishes REER data for 107 countries, but the most recent data is January 2018 for 61 counties and updated estimates only are published for the remaining 45 countries. These 45 countries include Venezuela, a country currently experiencing hyperinflation. The latest monthly inflation data we were able to collect for Venezuela is for April 2019 and thus we did not include Venezuela in our REER revision calculations. We therefore compare the revision in our and the World Bank REER estimates for the remaining 44 countries in the September 2020 – November 2021 out-of-sample evaluation period. We find that our estimates have slightly lower forecast errors that the World Bank estimates: the median across 44 countries of the MAPFE statistic for the latest month is 0.33 for our REER considering 120 trading partners, 0.34 for our REER considering 51 trading partners, and 0.36 for the World Bank REER. Considering the RMSPFE statistics, the values are 0.42, 0.43 and 0.46, respectively. Among the 44 countries, the revisions in our estimates are smaller than World Bank revisions for 31 countries, while for 13 countries World Bank revisions are smaller.

The IMF publishes data for 94 countries but the latest estimate for Venezuela is for December 2016. For the other 93 countries, the median of the MAPFE statistic for the month preceding the latest month is 0.10 for our REER considering 120 trading partners, 0.11 for our REER considering 51 trading partners, and 0.41 for the IMF REER. Considering the RMSPFE statistics, the values are 0.16, 0.15 and 0.54, respectively. Thus, on average, IMF REER estimate revisions are considerably larger than the revisions in our estimates. Among the 93 countries, the revisions in our estimates are smaller than IMF revisions for 81 countries, while for 12 countries IMF revisions are smaller.

5. How much does monthly price level variation matter for the REER in certain economic analyses?

The use of the forecast price level implicitly assumes smooth monthly inflation rates. An important question is to what extent the neglect of the actual dynamics of the price level distorts the conclusions from economic analyses using the REER. It seems reasonable to presume that when only one month of price level data is forecasted and actual price level data is used for the rest of the sample, there would be hardly any difference in results compared to the case when even the last price level observation was an actual data. Yet, when more recent observations are missing and hence a longer horizon forecast has to be made for the price level, results of economic analyses might differ compared to the case when only actual data is used for calculating the REER.

To assess the relevance of the neglect of monthly price level dynamics for REER calculation, we take two widely analysed economic topics, testing for a unit root in real exchange rates and forecasting the nominal exchange rate with the REER. We compare the results based on alternative versions of the REER, which are all based on the actual nominal exchange rates (as we do for the calculation of our REER indicators), but differ in terms of the price level data used:

- Actual monthly consumer prices for the full period;
- Actual monthly consumer prices for the full period except for last year, for which constant monthly inflation rate (corresponding to the actual annual inflation rate) is assumed;
- Actual monthly consumer prices for the full period except for the last 5 years, for which a constant monthly inflation rate (corresponding to the actual annual inflation rate) is assumed within each year;
- Approximated monthly consumer prices for the full period, for which a constant monthly inflation rate (corresponding to the actual annual inflation rate) is assumed within each year.

Whenever we use approximated data, we use the actual December values for each year and interpolate the values for the 11 months between two Decembers by assuming a constant monthly inflation rate.

5.1 Testing for a unit root in real exchange rates

There are numerous tests for unit roots. We employ the popular method developed by Phillips and Perron (1988), which uses a nonparametric method of controlling for serial correlation in the test equation. We include a constant, but no linear time trend in the test regression, and use the Bartlett Kernel to estimate the residual spectrum at frequency zero with the Newey-West bandwidth selection method. We test for unit root in the REER after logarithmic transformation.

Table 6 summarises the cases when the null hypothesis of a unit root is rejected. The conclusion about this null hypothesis hardly changes when approximated price level data is used instead of actual price level data for calculating the REER. When approximating price level only in the latest year, the conclusion differs slightly only for one country when considering the standard 1 percent, 5 percent and 10 percent significance levels: for the Bahamas, the p-value is 4.98 percent when using full sample actual price levels and 5.06 percent when price levels of the latest year are approximated. No country changes position when approximated data is used for the last five years instead of only the last year, and very few countries change position when we use approximated price level data for the full sample.

Table 6: Testing for a unit root in real effective exchange rates (number of countries)

	REER relative to 51 trading partners				REER relative to 120 trading partners			
Price level data used:	Full sample actual	Last year approximated	Last five years approximated	Full sample approximated	Full sample actual	Last year approximated	Last five years approximated	Full sample approximated
The null hypothesis of unit root is rejected:								
* at 1%	33	33	33	34	36	36	36	32
* between 1 and 5%	16	15	15	13	15	14	14	16
* between 5 and 10%	8	9	9	11	9	10	10	5
not rejected at 10%	120	120	120	119	117	117	117	124
total	177	177	177	177	177	177	177	177

Source: Bruegel. Note: the numbers in the table indicate the number of countries for which the null hypothesis of a unit root in the real effective exchange rate is rejected at the significance level indicated in the first column, depending on the type of price level used for calculating real effective exchange rate as indicated in the second row. The test of Phillips and Perron (1988) is used.

Table 6 also indicates that for about 120 of the 177 countries considered, the null hypothesis of a unit root in the real effective exchange rate is not rejected at the 10 percent significance level, according to the test of Phillips and Perron (1988).

5.2 Forecasting the nominal exchange rate with the REER

Recent research found that the real exchange rates of major currencies are stationary, which implies a co-integrating relationship between the non-stationary nominal exchange rate and home and foreign price levels. Some studies found that it is the nominal exchange rate that does most of the adjustment when the real exchange rate deviates from its long-run level. Among the eight models studied by Cheung *et al* (2019), the real exchange rate model led to the most promising exchange rate forecasts. Ca'Zorzi and Rubaszek (2020) used the real exchange rate to predict the nominal exchange rate and found that long-horizon (two- to five-year) forecasts are better than that of the random walk, while forecasts for shorter horizons are not.

Since long-horizon regressions[14] suffer from econometric problems (Berkowitz and Giorgianni, 2001; Rossi, 2007; Darvas, 2008), our longer-horizon forecasts are based on the iteration of one-period ahead forecasts using the simple two-equation model:

$$(13) \qquad \begin{aligned} s_{i,E,t+1} - s_{i,E,t} &= \theta_{i,1} + \theta_{i,2} \cdot q_{i,E,t} + \varepsilon_{i,t+1}^{(1)} \\ q_{i,E,t+1} &= \theta_{i,3} + \theta_{i,4} \cdot q_{i,E,t} + \varepsilon_{i,t+1}^{(2)} \end{aligned}.$$

That is, we use information up to time *t* to estimate the four parameters for each country i that we denote $\hat{\theta}_{i,j|t}$, *j*=1, 2, 3, 4. We then first calculate one-period ahead forecasts: $s_{i,E,t+1|t} = s_{i,E,t} + \hat{\theta}_{i,1|t} + \hat{\theta}_{i,2|t} \cdot q_{i,E,t}$ and $q_{i,Et+1|t} = \hat{\theta}_{i,3|t} + \hat{\theta}_{i,4|t} \cdot q_{i,E,t}$, where $s_{i,E,t+1|t}$ and $q_{i,E,t+1|t}$ denote the one-period ahead forecasts based on time *t* information. Note that $s_{i,E,t}$ and $q_{i,E,t}$ are observed at time *t*. The two-period ahead forecasts are obtained as: $s_{i,E,t+2|t} = s_{i,E,t+1|t} + \hat{\theta}_{i,1|t} + \hat{\theta}_{i,2|t} \cdot q_{i,E,t+1|t}$ and $q_{i,E,t+2|t} = \hat{\theta}_{i,3|t} + \hat{\theta}_{i,4|t} \cdot q_{i,E,t+1|t}$. Thus, the two-period ahead forecasts use information available only up to time *t*. And so on; we iterate the two equations forward based on information available only up to time *t*. The structure of model (13) is the same as analysed by Pincheira and West (2016).

The main benchmark in exchange rate forecasting is the driftless random walk, which, however, is nested in model (13). When comparing nested models, standard asymptotic tests do not apply when testing the null hypothesis of equal forecast accuracy. Clark and West (2007) suggested an adjustment of mean squared prediction error statistics, which leads to an approximately normal test. This test was based on the assumption that a long-horizon regression is used for forecasting and the out-of-sample forecasts are evaluated using a rolling-window estimation technique. However, Pincheira and West (2016) found that the Clark and West (2007) statistics also worked reasonably well when the iterated method is used to obtain multi-step forecasts and the recursive estimation scheme is used, which is our baseline setup. For the iterated method they considered a simple first-order autoregression for the predictor, in the same way as in our forecasting model (13). We therefore use the Clark and West (2007) statistics for testing the null hypothesis equal forecast accuracy of model (13) and the driftless random walk.

A co-integrating relationship can be better estimated over long estimation periods and thus we use our narrow REER index which is available since 1960 for several countries. Another important consideration for selecting the forecasting sample is the exchange rate regime: only under a floating

[14] A long-horizon regression includes the multi-period change of a variable on the left side of the regression. In our case, the long-horizon regression would be: $s_{i,E,t+\mathrm{h}} - s_{i,E,t} = \theta_{\mathrm{i},1} + \theta_{\mathrm{i},2} \cdot q_{i,E,t} + \varepsilon_{i,t+h}$.

exchange rate regime can the nominal exchange rate freely adjust when the real exchange rate deviates from its long-run level. We therefore exclude from our sample period the period of the Breton-Woods exchange rate system, when most countries adopted fixed exchange rates, and also exclude countries that have adopted a fixed exchange rate to the US dollar since then. We also exclude most of the 1970s from our sample period when nominal exchange rates were adjusting to the shocks caused by the collapse of the Bretton Woods system and the rise in oil prices. Based on these considerations, January 1979 seems to be a reasonable starting date for our estimation. To allow an initial estimation of model parameters, we evaluate the out-of-sample forecasts in the January 1990 – November 2021 period. There are 100 countries (plus the euro area) for which our narrow REER indicator is available from January 1979, of which 20 countries still employ a fixed rate to the US dollar. We, therefore, analyse the remaining 80 countries and the euro area.

To save space, we report detailed results for a few countries and the median for all analysed countries. We only compare the cases when either actual price level data or approximated price level data are used over the full period to calculate the REER. Results are rather robust for the use of alternative price level data (Table 7). For example, the ratio of the one-period-ahead mean squared forecast error of the nominal effective exchange rate based on model (13), to the mean squared forecast error of the random walk (and multiplied by 100), is 99.8 for the euro area when the actual price level data is used to calculate the REER and 99.7 when approximated price level data is used. The p-values of testing the null hypothesis of equal forecast accuracy are also rather similar.

Table 7 suggests mixed results for the forecasting ability of the real exchange rate model. For some countries, such as the United Kingdom, the real exchange rate model outperforms the random walk at least in long-horizon forecasts (consistent with the results of Ca'Zorzi and Rubaszek, 2020). But for others, like the United States, this is not the case. It should be noted, however, that Ca'Zorzi and Rubaszek (2020) studied bilateral exchange rates relative to the US dollar, while we analysed the nominal effective exchange rate relative to 51 trading partners, which could influence the findings.

Table 7: Mean squared forecast error of the real exchange rate model (random walk = 100)

	Using actual price level data for the REER					Using approximated price level data for the REER				
	forecast horizon in months									
	1	6	12	36	60	1	6	12	36	60
Australia	100.7	101.3	102.6	106.6	105.0	100.6	101.2	102.5	106.3	104.4
	(0.559)	(0.396)	(0.35)	(0.209)	(0.096)	(0.534)	(0.371)	(0.322)	(0.181)	(0.077)
Canada	100.6	102.7	105.1	110.4	116.9	100.5	102.4	104.6	108.9	114.1
	(0.746)	(0.801)	(0.813)	(0.835)	(0.922)	(0.694)	(0.746)	(0.77)	(0.738)	(0.817)
Euro area	99.8	98.6	96.0	89.7	79.0	99.7	98.3	95.5	88.2	76.9
	(0.214)	(0.127)	(0.044)	(0.006)	(0)	(0.193)	(0.106)	(0.034)	(0.005)	(0)
India	95.8	83.0	72.6	117.4	180.4	95.9	82.9	72.7	119.2	182.9
	(0.018)	(0.009)	(0.004)	(0.001)	(0.001)	(0.018)	(0.009)	(0.005)	(0.001)	(0.001)
Japan	101.5	105.2	110.1	123.0	164.1	101.5	105.2	110.0	121.7	161.1
	(0.373)	(0.322)	(0.238)	(0.072)	(0.018)	(0.375)	(0.318)	(0.237)	(0.065)	(0.017)
Kenya	93.8	85.8	78.7	51.1	57.9	93.9	86.3	78.8	52.8	58.8
	(0.028)	(0.025)	(0.021)	(0.006)	(0.002)	(0.029)	(0.026)	(0.023)	(0.005)	(0.002)
Korea South	100.0	96.7	90.8	71.1	66.1	99.7	95.7	89.3	69.7	63.7
	(0.132)	(0.027)	(0.005)	(0)	(0)	(0.051)	(0.013)	(0.002)	(0)	(0)
Switzerland	100.7	100.3	98.2	72.1	51.7	100.8	100.7	98.7	72.7	52.7
	(0.598)	(0.255)	(0.059)	(0)	(0)	(0.623)	(0.289)	(0.079)	(0)	(0)
Thailand	101.2	105.1	109.0	116.0	116.3	101.2	104.9	108.6	115.3	114.5
	(0.671)	(0.854)	(0.865)	(0.638)	(0.404)	(0.689)	(0.854)	(0.865)	(0.66)	(0.396)
United Kingdom	100.6	98.2	95.4	86.4	83.0	100.6	98.1	95.2	85.4	80.8
	(0.265)	(0.034)	(0.007)	(0.001)	(0)	(0.232)	(0.024)	(0.005)	(0)	(0)
United States	104.7	120.8	142.4	229.0	313.2	104.5	120.0	140.7	224.9	307.9
	(0.147)	(0.169)	(0.102)	(0.032)	(0.035)	(0.143)	(0.159)	(0.094)	(0.028)	(0.029)
Median	100.3	99.6	100.7	107.8	116.3	100.3	99.1	100.0	106.3	114.0
	(0.853)	(0.168)	(0.514)	(0)	(0)	(0.146)	(0.037)	(0.043)	(0.181)	(0)

Source: Bruegel. Notes: The real exchange rate forecasting model is defined in equation (13). Values in the first line for each country show the ratio of the mean squared forecast error (MSFE) of the real exchange rate model divided by the MSFE of the random walk and multiplied by 100. Thus, a value below 100 indicates that the model forecast errors are smaller than random walk forecast errors on average in 1990-2021. p values are reported in parentheses of testing the null hypothesis that the model MSFE is the same as that of the random walk against the one-sided alternative hypothesis that the model is better, based on the test of Clark and West (2007). 'Median' in the last line of the table refers to the median across 80 countries. The sample period includes monthly data from January 1979 to November 2021. Using recursive estimation windows, an out-of-sample evaluation of forecasts was performed in the 1990-2021 period.

To summarise, our results show minor differences between the results based on the actual and approximated versions of the real exchange rate. The most likely explanation for this finding is that the short-run movements of the real exchange rate are dominated by nominal exchange rates, for which we always use actual data.

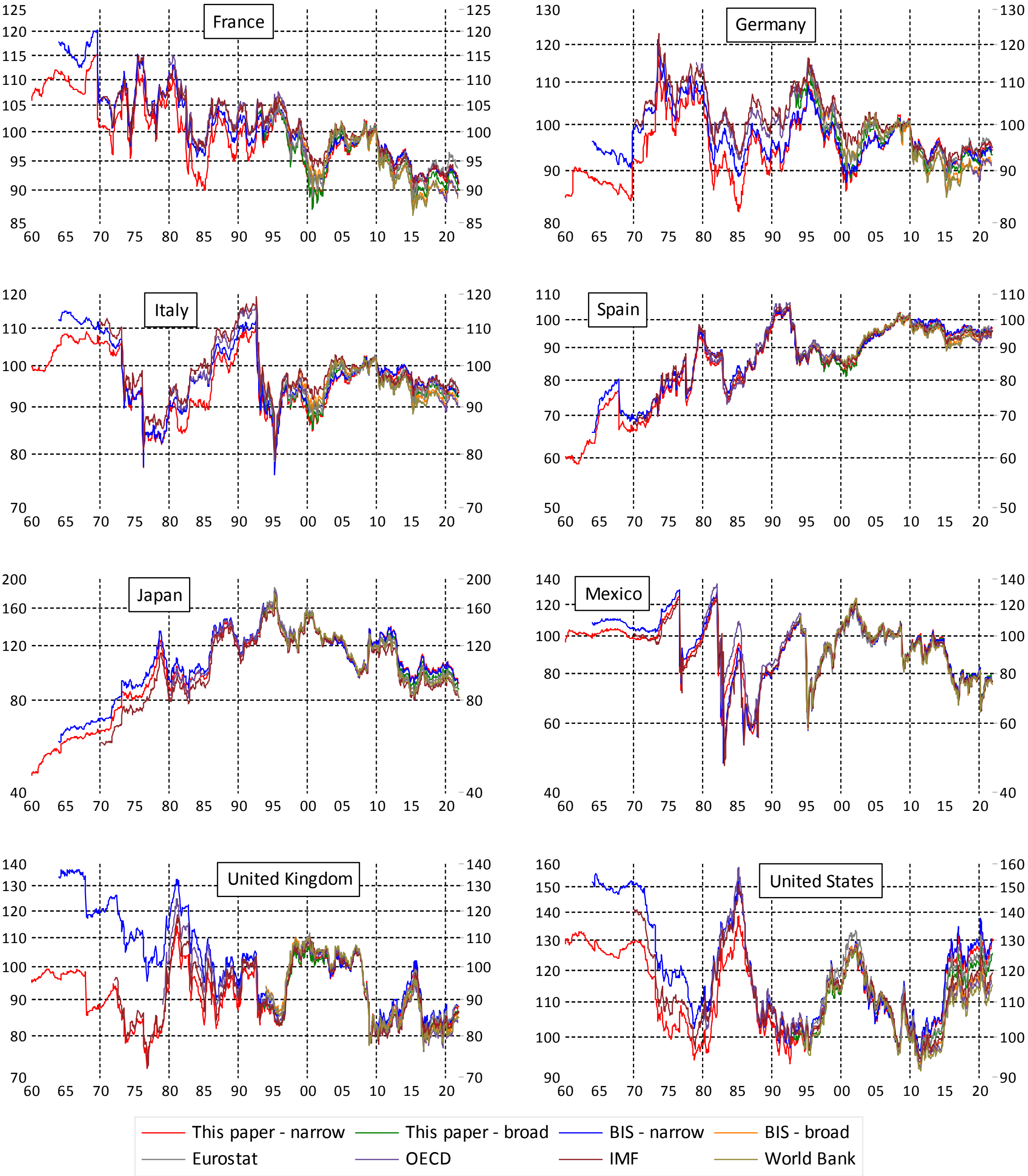
The implication of these results is that whenever annual price level data is available for a longer time period than monthly price level data, then for the period when monthly price level data is not available, the monthly real effective exchange rate can be well approximated with the use of actual monthly nominal exchange rate data and an approximated monthly price level data that assumes a constant within-year monthly inflation rate corresponding to actual annual inflation rate.

6. Comparing the latest REER estimate levels from alternative sources

Figure 3 compares REER estimates for eight selected countries that are included in the datasets of the BIS, Eurostat, IMF, OECD and World Bank. Differences in the estimates arise from two main sources: the different set of countries considered for the basket of trading partners and differences in the derivation of the weight matrix. For example, our narrow index considers 51 trading partners of which 25 are advanced countries and 26 are emerging countries, while the narrow index of the BIS considers 26 countries, of which 25 are advanced countries (a slightly different set to our 25 advanced countries) and one emerging country.

The alternative REER estimates move together and large jumps are visible for all. The levels of the REER estimates are rather similar even over more than half a century for Italy, Japan, Mexico and Spain. On the contrary, there are notable differences for the United Kingdom and United States. Our estimates for the United Kingdom are almost the same as the IMF estimates in the 1970s, but BIS estimates suggest an approximately 30 percent higher REER for the UK in the 1970s. For the United States, BIS estimates suggest the US dollar was about 15 percent stronger in 1970 than our estimate, while the estimate of the IMF is about half-way between our and the BIS estimates.

Figure 3: Comparison of alternative REER estimates for selected countries (December 2007=100)



Source: Bruegel. Note: REER data from the BIS, Eurostat, IMF, OECD and the World Bank were collected on 4 December 2021 and our REER calculations use data collected on this date.

7. Conclusions

The real effective exchange rate is an important indicator for researchers and policymakers. Current datasets from the BIS, Eurostat, the IMF, OECD and the World Bank include all advanced and several emerging and developing countries, but data for many countries is not available and some data is released with a delay. This paper develops a methodology to estimate monthly consumer-price based real effective exchange rates for 177 countries plus the euro area without any delay (eg the indicators can be estimated on the first day of each month for the preceding month). Our dataset includes more than twice as many observations as the second most comprehensive dataset from the IMF.

Our methodology is based on the observations that short-run real exchange effective rate changes are dominated by nominal effective exchange rate changes, while inflation rates are sticky and contribute little to short-run real exchange rate changes. Thus, we use actual nominal exchange rate data and forecast price level data whenever actual price level data has not yet been published. Our out-of-sample forecasting exercise over the 2015-2021 period demonstrates that for most countries, price level forecasts and the corresponding real effective exchange rate forecasts are rather accurate. Using real-time data from October 2020-December 2021, we find that the revisions in our real effective exchange rate estimates are marginally smaller than the revisions in World Bank estimates on average for 44 countries, and our revisions are considerably smaller than the revisions in IMF estimates on average for 93 countries.

We also found that in two frequently analysed research topics, testing for a unit root in real exchange rates and forecasting the nominal exchange rate with the real exchange rate, neglect of the actual monthly dynamics of the price level for the calculation of the real effective exchange rate hardly changes the results of the analysis. This finding suggests that whenever annual price level data is available for a longer period than monthly price level data, then for the period when monthly price level data is not available, the monthly real effective exchange rate can be well approximated using actual monthly nominal exchange rate data and approximated monthly price level data that assumes constant within-year monthly inflation rate corresponding to actual annual inflation rate.

The nominal and real effective exchange rates calculated in this paper are freely downloadable at:

https://www.bruegel.org/publications/datasets/real-effective-exchange-rates-for-178-countries-a-new-database/

The dataset will be regularly updated.

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