THE INSURANCE PROPERTIES OF COMMON DEBT ISSUANCE

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The insurance properties of common debt issuance

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Abstract

In a federation of sovereign states, common debt can provide insurance against idiosyncratic shocks even without any intended, ex ante transfers. This insurance property arises automatically when the common debt service is financed by a levy on members that is proportional to national income. This is the case in the EU. It implies that if the economy of a member state is hit by a negative shock, i.e., if it grows less than the Union average, its contribution to the service of the common debt is correspondingly reduced. By contrast, the service of national debt, which is typically fixed in nominal terms, becomes more difficult in the case of a negative idiosyncratic shock. Ceteris paribus, common debt issuance is thus akin to linking debt service to GDP growth. Uncertainty about growth increases with the time horizon. The insurance property of common debt thus increases with its maturity.

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1. Introduction

The Covid-19 crisis has led to a common European fiscal response in the form of the Next Generation EU (NGEU) package. One important novelty of this package is that it will involve for the first time the issuance of substantial common European debt. Part of this debt will finance loans that have to be repaid by member states. But a substantial part, around €360 billion, finances grants and will thus have to be serviced by EU sources of funding.

This contribution focuses on the economics of this grant component, and the implications of issuing common debt in general.

Common European debt does not represent free resources; in the end it has to be serviced by all member states in one way or another. There has been some discussion about whether a country would gain if the grants received were equal to its share in EU GDP and thus its share in the EU budget (e.g., Merler, 2020). In this case the grants received today should be equal to the share of the country in EU debt service, implying that there would be no net transfer (on an expected value basis).

This paper argues that common debt issuance can provide an important insurance function even if there is no (expected) transfer of resources once future debt service has been taken into account. A number of real-world complications illustrate this point, such as differences in interest rates (EU bonds carry a lower interest rate than the bonds of the highly indebted member states), the nature of the expenditure, the explicit redistribution in the EU budget towards poorer regions and countries, etc.

A key problem for national debt, usually fixed in nominal amounts, is uncertainty about growth (nominal and real). Debt service problems usually arise when growth is lower than expected, making it more costly to finance debt service.

Section 2 provides a simple model to illustrate how common debt, which is served out of a common pool, provides an important insurance benefit. Section 3 documents the uncertainty about growth performance in the EU. Section 4 illustrates different ways in which debt service costs result in convex costs. Section 5 concludes.

2. An illustrative model

This two-period model describes a federation consisting of an arbitrary number of identical member states, which all have the same (ex ante) growth prospects, but whose economies are subject to idiosyncratic growth shocks. In the first period a certain amount of debt, indicated by d, is issued. The amount is fixed exogenously. The only decision variable is the distribution between national debt and common debt. The debt has to be repaid (with interest) in the second period. To illustrate the basic point, interest payments are thus incorporated into the debt service due during the second period.

An essential element of the model is a conventional loss function, which expresses the idea that increasing tax revenues lead to increasing distortions (Mankiw, 1987). This implies that, at the margin, the higher the tax rate the more and more costly it becomes to obtain higher tax revenues. The social loss from obtaining tax revenues, measured as a percentage of GDP, is thus assumed to be given by:

$$(1) \qquad L = \beta q_{t+1}^2 \qquad \beta>0$$

Where q indicates the ratio of tax revenues to GDP, or the overall effective tax rate. The parameter β represents the efficiency of the tax system. A higher value of β implies a lower efficiency of the tax system.

As explained above, debt, denoted by d, is issued in the first period and repaid during the second period. Tax revenues are needed in the second period only to service debt. This implies that $q_{t+1} = d$.

To concentrate on the implications of different forms of debt, it is assumed that there are two forms of debt: national or common. The total amount of debt, and on what the proceeds are spent, is taken as given here.

The tax rate q_{t+1} needed at the national level to service debt in the second period depends on the amount of national debt and the share of the country concerned in common debt. The former is fixed in (nominal) units, but the national contribution on the service of the latter depends on the evolution of the economy of the home country.

Output in the second period is subject to uncertainty. To keep the analytics as simple as possible, it is assumed here that output in the second period will be equal to 1+θ with probability 0.5 and with equal probability 1- θ (of course 0<θ<1). The parameter θ thus describes the uncertainty about future growth, an increase in θ represents an increase in the mean preserving spread. The parameter θ could also be considered as 'GDP at risk' (Adrian et al., 2019).[1]

Output is here understood relative to the Union average. Ex ante, the home country has thus the same growth prospects as the average. All debt service takes place in the future. The subscript 't+1' will not be indicated in the remainder as it applies to all variables.

The key element of the model is that the home country contributes its GDP pro rata to the service of the common debt. This is the case in Europe where the main element of the EU budget has become what is called the 'GNI own resource', which provides about 70% of the EU budget (European Commission; for a survey, see Iara, (2016)). No decision has been taken yet on how to service the debt arising through the grant element in the NGEU programme. Given

[1] In a 'plucking' model (Dupraz et al., 2019) the distribution would be asymmetric, with a low probability of a large negative value and a higher probability of smaller positive values.

the status-quo bias in the EU (unanimity is needed to introduce new revenue sources) it is likely that that the GNI resource will continue to remain the main source for revenues when the NGEU debt has to be repaid.

This immediately implies that for any one unit of common debt, the debt service will amount to (1+θ) in the good state of nature and (1-θ) in the bad state of nature. But the GDP out of which this debt service has to be generated co-varies perfectly with the debt service. For one unit of common debt the debt service relative to GDP will be equal to (1+θ)/ (1+θ) = 1. The burden of national debt service, by contrast, is very much impacted by shocks. For one unit of debt the tax burden falls to 1/ (1+θ) in the case of a positive shock and rises to 1/ (1-θ) in the case of a negative shock.

This insurance property of common debt is absent in most of the variants of 'eurobonds' that have been proposed over the years (Esteves and Tuncer, 2016; and Leandro and Zettelmeyer, 2019) because they keep the responsibility for debt service at the national level.

It can now be shown that the insurance element survives when both types of debt, national and common, coexist.

The share of total debt incurred at the EU level is denoted by γ; the share that remains national is thus equal to (1-γ). This implies that in the good state of nature, when the country achieves an income of (1+θ), debt service cost is given by d{ (1-γ) + γ(1+ θ) }. Average expected debt service cost (taking into account the equal probability of good and bad states) is given by:

Expected debt service = d{0.5*[(1-γ) + γ(1+ θ)] + 0.5*[(1-γ) + γ(1-θ)]}=d

This implies that, ex ante, the issuance of common debt does not imply a transfer. The home country can expect to contribute as much to the service of the common debt as its share of the economy is in the first period. In terms of expected debt service, the home country would thus be indifferent between national and common debt.

However, expected debt service is not the key issue here; it is the expected value of the burden debt service brings to bear on the economy, and this depends on the state of the economy. Any given amount of debt service leads to higher social costs when income is low because in this case higher tax rates are needed.

For example, in the good state of nature, the tax rate (debt service cost divided by output) needed to service the debt would be equal to:

$$(1)\quad q = \mathrm{d}\left[\frac{(1-\gamma)}{(1+\theta)} + \gamma\right] = \mathrm{d}\left[\frac{(1-\gamma)+(1+\theta)\gamma}{(1+\theta)}\right] = \mathrm{d}\left[\frac{(1+\gamma\theta)}{(1+\theta)}\right]$$

which is lower the higher θ (the larger positive shock) and the higher the share of national debt.

When the home country experiences a boom, national debt looks more attractive. A similar equation determines the tax rate in the bad state of nature. This implies that the expected loss arising from debt service (when a fraction γ is financed by common European debt) is given by:

$$(2)\ E(L) = d^2\beta\frac{1}{2}\left\{\left[\frac{(1-\gamma)}{(1+\theta)}+\gamma\right]^2+\left[\frac{(1-\gamma)}{(1-\theta)}+\gamma\right]^2\right\}$$

As expected, given the loss function, the expected loss is proportional to the square of the debt (relative to GDP).

The result for the two limiting cases can be seen immediately:

1. If all debt is common (γ=1), the impact of income uncertainty is completely neutralised. The expected loss is independent of θ, equal to βd.[2] In this case debt service is determined only by the total amount of debt (relative to GDP) and the efficiency of the domestic tax system, as represented by the parameter β.

$$(1)\ E(L)_{\gamma=1} = \beta d^2$$

2. If all debt is national (γ=0), income uncertainly results in a higher expected loss because of the convexity of the loss function. In this case the two remaining expressions can be combined to yield:

$$(2)\ E(L)_{\gamma=0} = \beta d^2\left\{\frac{(1+\theta^2)}{(1-\theta^2)^2}\right\} > \beta d^2$$

This expression is larger than the loss under 100% common debt (equal to βd^2) since national debt exposes the country to the danger of having to increase tax rates during bad times, thus leading to a higher social loss. The expected social loss with only national debt increases with the parameter representing uncertainty: θ.

The intuition for the two extremes is thus relatively straightforward.

It can also be shown that the overall expected loss is always decreasing in γ. This implies that in this set-up the issuance of more common debt instruments, even without any redistributive element, should be welfare-increasing.

A key element in all the results is that the welfare loss is proportional to the square of total debt. The benefits from common debt thus become particularly important when debt levels are high.

The basic insight that convex debt service costs make common debt attractive has a wider application than the specific model used here. Proposals to emit GDP-linked bonds, i.e. bonds whose interest, and maybe even principal, would be linked to the state of the economy (Kopf, 2018; and Blanchard et al., 2016) have their roots in the same insight. Given the way the EU budget is financed, common European debt is equivalent to issuing GDP linkers.

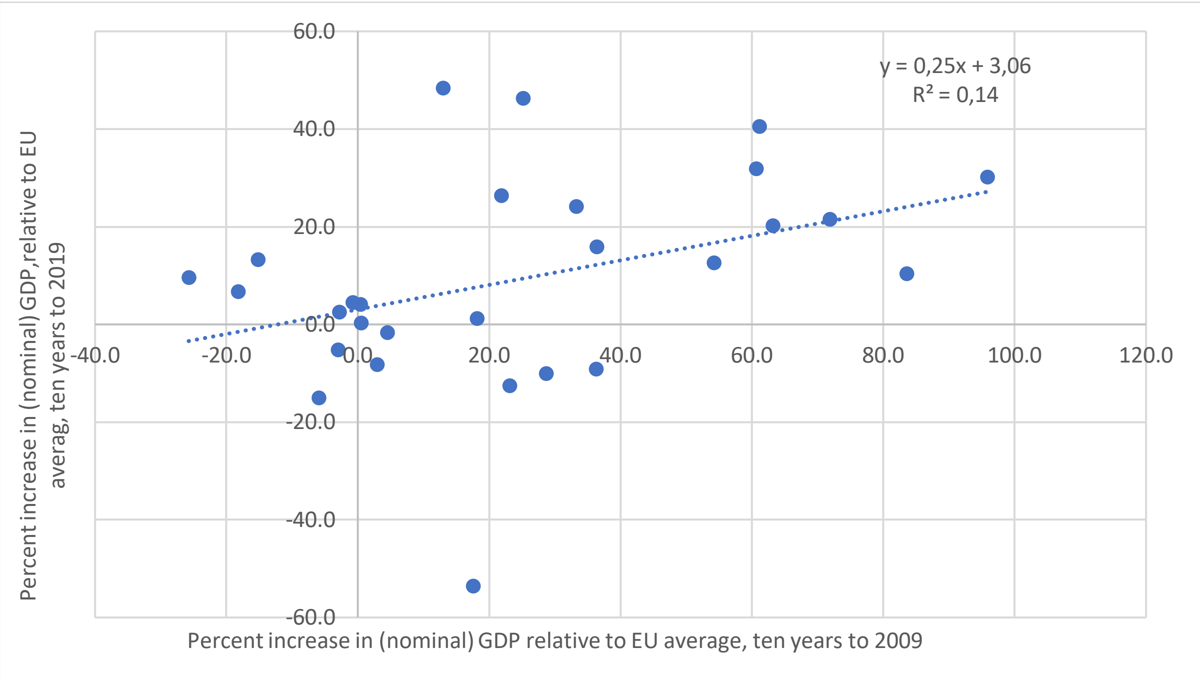
There are other mechanisms that lead to a convex cost of debt service that are discussed separately below. For example, risk premia typically increase with the debt-to-GDP ratio. As shown in Alcidi and Gros (2018), this implies that the overall interest burden becomes a quadratic function of the debt-to-GDP ratio. This would lead to a convex cost of public debt even if the cost of collecting revenues for debt service are constant.

Higher debt ratios might, in the extreme, lead to default, which also implies convexity. This means that financing common expenditure with common debt instruments is especially important for countries for which debt sustainability is borderline. The gain would arise from reducing the probability of a tail outcome – even without any transfer in terms of expected debt service costs.

3. How important is uncertainty about growth in the EU?

Figure 1 below shows the increase in nominal GDP over 10 years (relative to the EU average), comparing the pre- and post-financial crisis period. There is very little persistence in growth performance over time. It is too early to tell whether the current crisis will lead to a similar change in growth prospects. (Nominal GDP was chosen because contributions to the EU budget are determined as 1% of GDP (with small variations across member states).

Figure 1. Relative growth performance changes over time



Source: own calculations based on AMECO data.

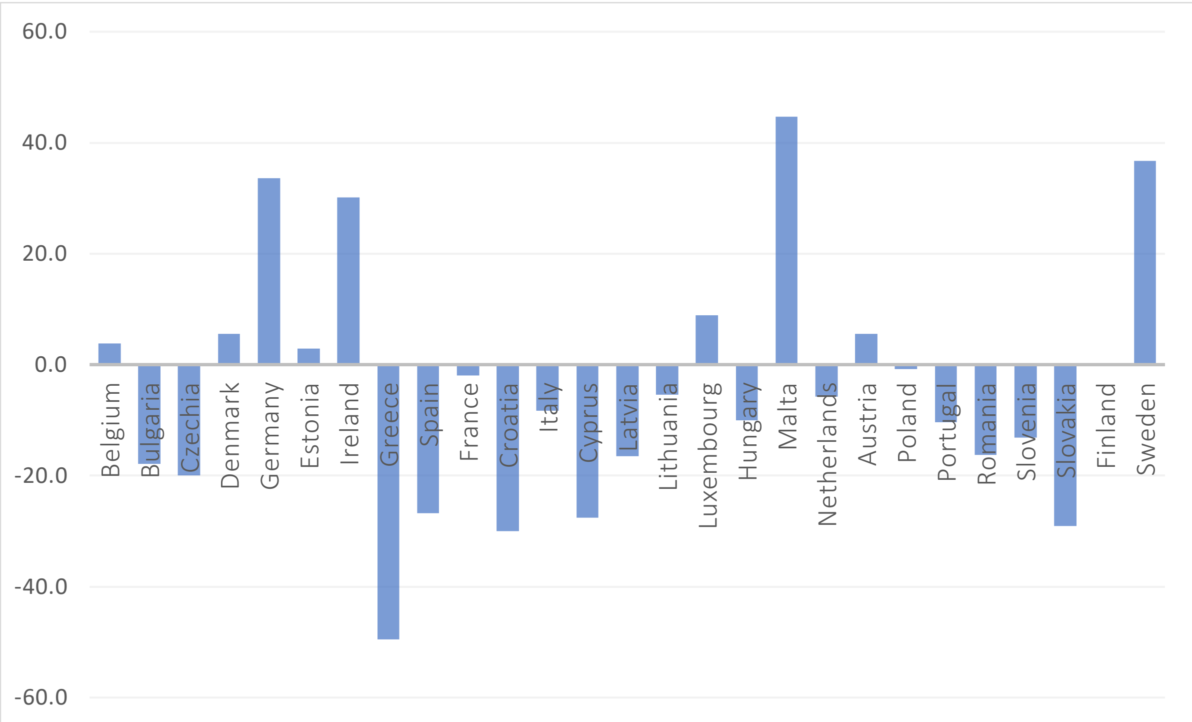
Another way to illustrate the importance of this is to compare the actual shares in EU GDP (and thus in contributions to the EU budget) with the prediction one would have made in 2010 based on the growth performance of the first decade of the euro (2000–10). There are rather large differences between the actual shares and those one would have predicted in 2010.

For example, Germany's share had been declining in the first 10 years of the euro (1999 to 2009, a period which includes the first two years of the Great Financial Crisis), whereas it increased over the following decade. The combined impact of these two developments was that the share of Germany in EU GDP is, as of 2019, about 30% larger than one would have expected 10 years ago. This implies that Germany's contribution to the EU budget is now (2019 data) about 30% larger than one would have expected only ten years ago. The current crisis is likely to accentuate this discrepancy between expected and actual.

The opposite is the case for Greece, whose share in EU GDP (and thus its contribution to the budget) is almost 40% lower than one might have expected 10 years ago on the basis of its performance until then.

Figure 2 shows the difference between actual shares in EU GDP and those predicted (based on naïve trend extrapolation) in 2010.

Figure 2. Percentage difference in shares in EU GDP as predicted for 2019 – actual vs 2010 prediction



Source: own calculations based on the AMECO dataset of the Commission.

The standard deviation of the forecast error in the shares in EU GDP across all 27 EU member states is 19%, implying that over 10 years the error in predicting the share each country would have in financing the EU budget and thus also its debt service, might on average be close to 20%.

For the 20-30 years of maturities foreseen for the financing of the Recovery Fund, the uncertainty would be even greater. If growth surprises are not correlated, as implied by the data presented above, the standard deviation of the differences in shares in EU GDP over 30 years should be equal to $19*3^{0.5}$ = about 3%.

4. Convex costs of public debt

The model above used increasing collection costs as the most straightforward way to motivate a convex cost for public debt.

Another way would be to acknowledge that high debt ratios lead to substantial risk premia. Experience and numerous studies show that, above a certain threshold of debt, the (market) interest rate on public debt increases with the debt level (as a percentage of GDP). This has been confirmed by many empirical studies (e.g., Laubach, 2009; Ardagna et al., 2007; and Engen and Hubbard, 2004), and plays a key role in debt sustainability analysis (DSA) exercises (Alcidi and Gros, 2018).[2]

This leads to a convex cost of debt: if a government incurs more debt, it has to pay interest on that additional debt. As Blanchard (2019) notes, a risk premium is justified for a country with a high debt level because debt might become unsustainable. However, by incurring more debt the risk premium will increase, thus increasing the cost of refinancing the entire stock of existing debt. This secondary effect produces a non-linear relationship.

This reasoning can be illustrated more formally. The link between the interest rate on government debt and public debt if often expressed by the following equation:

$$Average\ cost\ of\ debt\ \equiv i_t = r_t + \alpha(b_t - 60) \text{ (for b>60)}$$

where i_t is the interest rate on public debt, r_t represents the risk-less rate (e.g. the rate on German government bonds, for a euro government debt). The variable b_t represents public

[2] For example, the IMF's (2017) and the European Commission's projections of debt in Greece used a simple rule of thumb that the risk premium, defined as the difference between the riskless rate and the interest rate on public debt of any particular country, increases by 3 to 4 basis points for every 1 percentage point increase in the debt to GDP ratio above 60%. Alcidi and Gros (2018) and (2019). Blanchard et al., (2020) uses the same approach.

debt as a percentage of GDP. The second term on the right-hand side (RHS) of this equation represents the risk premium, which depends on the parameter α and the difference between the excess of the debt to GDP over 60%. The key parameter is α, which represents the strength of the link between debt and the risk premium.

The conjecture that the interest rate on public debt increases with the debt ratio could be motivated in several ways. The simplest view would be that a high debt level indicates a chronic inability of the political system of a country to levy enough taxes to pay for government expenditures. As debt increases, so does the temptation for the government to devalue the real value of its debt. In a country with its own currency, this could take the form of (unexpected) inflation, e.g. when the government forces the central bank to finance the deficit by creating additional money. Empirical studies for the US and other countries have thus focused on the real interest rate on government debt. For countries in the euro area, the risk for holders of government debt might be more one of outright default, as in the case of Greece.[3] The risk premium is thus calculated as the difference between the interest rate a specific euro area country pays on its debt, relative to that of a country where the risk of default is perceived as negligible (i.e. Germany). For risk-neutral investors, the risk premium in either case would be the expected loss of the real value of debt, which in turn is usually given as the probability of default (or inflation) times the expected loss of value in case of default. Another interpretation of alpha would be that it represents the 'risk aversion' of investors. When there is even a low probability of a loss of real value of a long-term government bond there is uncertainty. The price for this uncertainty is a measure of the degree of risk aversion of investors. The higher the degree of risk aversion, the higher the risk premium. This has an important implication: even for a given expected loss, and thus a given degree of uncertainty about default or inflation, the risk premium might vary over time as the risk aversion of investors changes (e.g. Delatte et al., 2017).

Alcidi and Gros (2018 and 2019) report that this feedback from debt to interest cost plays a key role in the sustainability analysis of the IMF and the European Commission. The precise value of the parameter α is not central to the analysis here. (It has been estimated by the IMF to be 0.04 and by the European Commission to be 0.03, see Alcidi and Gros, (2018); see also Blanchard et al., (2020)). The key point here is that as long as α is positive, i.e. a higher debt level leads to a higher risk premium, the relationship between debt service costs and the debt ratio becomes non-linear.

[3] Gros (2018) shows that in some cases (e.g., Italy in 2018) the risk premium observed in the market might contain two elements: the potential for a default while remaining in the euro and the potential risk that the country leaves the euro area (and devalues).

The total debt service cost can be represented by the simple product of debt times the interest rate:

$$Interest\ expense \equiv\ b_t \cdot i_t = b_t\,[r_t + \alpha(b_t - 60)] = b_t\,r_t + \alpha(b_t^2 - 60b_t\,)$$

Interest rate expenditure (as a % of GDP) thus increases with the square of the debt to GDP ratio, b^2 (as long as α >0). If one accepts that the risk premium increases with the debt ratio one does not need any recourse to increasing collection costs to motivate a convex cost of public debt.

Another reason why high debt ratios can be costly is that they increase the danger of default. Models of default assume in general that the act of defaulting itself has a considerable economic cost in terms of lost output. A large literature exists on sovereign default, both theoretical and empirical (see Das et al., 2012, who confirm that a default or restructuring has an important negative impact on output). However, almost all of the episodes of default considered in the empirical literature concern emerging markets, with the majority being small ones. It is thus questionable whether this experience is relevant in the European context.

The experience in Greece would also confirm the finding of Benjamin and Wright (2009), that a country often finds itself with higher debt/GDP ratio after default then before. The idea that the act of defaulting itself depresses output, and thus reduces debt service capacity, opens the way to multiple equilibria and many types of strategic interactions between the government and its creditors (Pitchford and Wright, 2013). There are of course many ways in which one can model the game between the country and its creditors. But the overall result is clear. A highly indebted country is much more likely to end up in costly default than a country with little debt, thus confirming the idea that the cost of debt is convex in the debt ratio.

5. Conclusion

The basic message can be restated in a very simple way. The cost of public debt increases more than proportionally with the debt/GDP ratio. This convexity has one immediate implication in the presence of uncertainty about growth: the average social cost of public debt is higher than the contractual debt service cost embedded in the interest rate.

Common European debt, which is financed by a pro-rata levy on the output of member states, provides an insurance function because countries which grow less have to contribute less (and vice versa). This insurance function becomes more important the longer the horizon and thus the uncertainty about (relative) economic performance.

The famous debt assumption by Secretary Hamilton in 1790, under which the federal US government assumed all the debt of the 13 states, can be understood in the context of this model. The most important aspect of this measure might not have been to create a safe asset.

US Treasury debt usually traded below par and was thus not considered particularly safe during the first decade of the US. The main advantage of this debt mutualisation was that it distributed the risk across the individual states (and their inhabitants). This distribution of risk proved important because after the turn of the 19th century the states on the southern seaboard entered into relative decline and thus contributed much less to the repayment of the federal debt when it finally occurred during the 1820s and 30s (Lindert, 2013).

The financing of the NGEU package through common EU bonds thus represents an important step forward, by providing an incipient shock-absorption mechanism.

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Annex 1. Details of calculations

To arrive at a tractable expression equation (2) can be rewritten as

$$(1)\ E(L) = d^2\beta\frac{1}{2}\left\{\left[\frac{(1-\gamma)(1-\theta)}{(1+\theta)(1-\theta)} + \gamma\right]^2 + \left[\frac{(1-\gamma)(1+\theta)}{(1-\theta)(1+\theta)} + \gamma\right]^2\right\}$$

Simplifying the denominators:

$$(2)\ E(L) = d^2\beta\frac{1}{2}\left\{\left[\frac{(1-\gamma)(1-\theta)}{(1-\theta^2)} + \gamma\right]^2 + \left[\frac{(1-\gamma)(1+\theta)}{(1-\theta^2)} + \gamma\right]^2\right\}$$

Multiplying out the quadratic expressions yields:

$$(3)\ E(L) = d^2\beta\frac{1}{2}\left\{\frac{(1-\gamma)^2(1-\theta)}{(1-\theta^2)}^2 + 2\gamma\frac{(1-\gamma)(1-\theta)}{(1-\theta^2)} + \gamma^2 + \frac{(1-\gamma)^2(1+\theta)}{(1-\theta^2)}^2 + 2\gamma\frac{(1-\gamma)(1+\theta)}{(1-\theta^2)} + \gamma^2\right\}$$

Collecting terms:

$$(4)\ E(L) = d^2\beta\frac{1}{2}\left\{\frac{(1-\gamma)^2(1-\theta)^2}{(1-\theta^2)^2} + \frac{(1-\gamma)^2(1+\theta)^2}{(1-\theta^2)^2} + 2\gamma\frac{2(1-\gamma)}{(1-\theta^2)} + 2\gamma^2\right\}$$

$$(5)\ E(L) = d^2\beta\left\{\frac{(1-\gamma)^2(1+\theta^2)}{(1-\theta^2)^2} + 2\gamma\frac{(1-\gamma)}{(1-\theta^2)} + \gamma^2\right\}$$

The result for the limiting cases can be seen immediately:

If all debt is European (γ=1), the impact of income uncertainty is completely neutralised. The expected loss is independent of θ, equal to dβ. In this case debt service is determined only by the total amount of debt (relative to GDP) and the efficiency of the domestic tax system, as represented by the parameter β.

If all debt is national (γ=0), income uncertainty results in a higher expected loss because of the convexity of the loss function. In this case the expected loss is equal to:

$$(3)\ E(L) = d^2\beta\left\{\frac{(1+\theta^2)}{(1-\theta^2)^2}\right\} > d\beta$$

This expression is larger than dβ as one would expect; national debt exposes the country to the danger of having to increase tax rates during bad times, thus leading to a higher social loss. The expected social loss with only national debt increases with the parameter representing uncertainty, θ.

To show that increasing the proportion of common debt (γ) always reduces expected losses, it is convenient to rewrite expression (5) above as:

$$(1)\ E(L) = d^2\beta\left\{\left[\frac{(1-\gamma)}{(1-\theta^2)} + \gamma\right]^2 + \frac{\theta^2(1-\gamma)^2}{(1-\theta^2)^2}\right\}$$

$$(2)\ E(L) = d^2\beta\left\{\left[\frac{1}{(1-\theta^2)} + \gamma\left(1 - \frac{1}{(1-\theta^2)}\right)\right]^2 + \frac{\theta^2(1-\gamma)^2}{(1-\theta^2)^2}\right\}$$

With 0<θ<1, the second expression in the square brackets is negative. This implies that the overall expected loss is decreasing in γ.

Annex 2. Extensions/robustness: analytical results with more general distribution of shocks

The analytical results obtained above can be generalised to any symmetric distribution of shocks with a mean of zero. We concentrate here on this type of distribution because it implies that the common bonds do not imply a transfer on an expected value basis.

The probability distribution of a shock of size θ is denoted by f(θ). By symmetry, f(θ)= f(-θ). Moreover, the integral for f(θ) between -∞ and 0 must equal ½.

This implies that the expected loss (burden from debt) is given by:

$$(1)\ E(L) = \beta d^2 \int_{-1}^{0} \left\{ \left[\frac{(1-\gamma)(1-\theta)}{(1+\theta)(1-\theta)} + \gamma \right]^2 + \left[\frac{(1-\gamma)(1+\theta)}{(1-\theta)(1+\theta)} + \gamma \right]^2 \right\} f(\theta) d\theta$$

Where the integration starts at -1 since this is the lower bound for the shock. This can be somewhat simplified to:

$$(6)\ E(L) = \beta d^2 \int_{-1}^{0} \left\{ \frac{(1-\gamma)^2(1+\theta^2)}{(1-\theta^2)^2} + 2\gamma \frac{(1-\gamma)}{(1-\theta^2)} + \gamma^2 \right\} f(\theta) d\theta$$

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