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# TECHNICAL DOCUMENTATION OF I3E MODEL VERSION 2

KELLY DE BRUIN AND AYKUT MERT YAKUT



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## EXECUTIVE SUMMARY

This paper provides a technical description of the Ireland Environment, Energy and Economy (I3E) model. The I3E model is an intertemporal computable general equilibrium model with multiple firms, one representative household group, multiple commodities, government, enterprises, and rest of the world accounts. It describes the Irish economy in sectoral detail. This model includes a detailed description of energy inputs and concomitant greenhouse gas emissions and has been developed with the purpose of investigating the economic and environmental impacts of climate policies for Ireland.

# 1 Introduction

Climate change and the associated challenges are at the forefront of current policy debates. For example, all countries have committed to the Paris Climate agreement (with the exception of the US). In line with this, climate policy is becoming increasingly important domestically, as Ireland is obliged to decrease its emissions under the EU Commission's Climate and Energy Package. Ireland is required to deliver a 20% reduction in non-ETS greenhouse gas emissions by 2020 relative to 2005 levels, increasing to 30% in 2030. Designing appropriate energy policies is imperative to ensure a smooth and least-cost transition to a low-carbon economy. Furthermore, a better understanding of the implications of such a transition is necessary, especially on a sectoral level, to identify those sectors most hit by climate policies. In this sense data-based economic models can be very useful to advise policy, giving insights into the real-world effects of different policy measures. Such models can investigate the economic costs and sectoral implications of reaching the EU goals and which specific climate policies (e.g., carbon tax, energy tax or quotas) can be implemented to reach these goals, accounting for the economic behaviour of producers and consumers.

To analyse the impacts of energy-related policies on the Irish economy, a model with sectoral details and various energy inputs and technologies is needed. With this objective in mind, the Ireland Environment, Energy and Economy model (I3E model) is developed. This model focusses on the relationship between the economy, energy inputs and environmental impacts in the form of greenhouse gas (GHG) emissions. The I3E model is an intertemporal computable general equilibrium (CGE) model with multiple firms, one representative household group (RHG), multiple commodities, government, enterprises, and rest of the world accounts.

This paper serves as a reference document concerning the technical details of the I3E model. In what follows, the model structure and equations will be presented

This document is structured as follows. In the next section, the sets, which represent the dimensions of the model, will be defined and clarified. Section 3 presents the equations dictating the behaviour of each agent in turn. Section 4 discusses the description of commodities in the model. In Section 5, the quantities are discussed and in Section 6, the prices. The energy part of the model is discussed in Section 7. Section 8 describes how economic growth is incorporated in I3E. Section 9 explains the savings–investment behaviour, and Section 10 gives the equilibrium conditions. The lists of activities and commodities can be found in Appendix A, the list of endogenous variables is presented in Appendix B, and finally, Appendix C provides the list of exogenous variables and parameters. This document describes the second version of the I3E model. The main differences between the model versions can be found in Appendix E.

## 2 Sets

The variables and parameters used within the model are defined over various sets, which are collections of different variables or parameters with common characteristics. Each set has a unique definition and contains elements that meet the defined criteria. The set logic is used to define equations over appropriate sets and allows us to exclude variables in the solution process. Each set and potential subsets are described in what follows.

- **Activities**

All activities are assigned to the set  $a$ . This is the broadest set for domestic production activities. Along with several dimensions, activities are assigned to several subsets as follows

- **Firm behaviour**

An *Activity* determines its level of investment, i.e. investment by destination, by dividend maximisation as explained in Section 3.2.1. This procedure is based on Tobin's  $q$ , which is defined as the market value of a firm's capital stock divided by the replacement cost of it; in other words, the marginal value of capital. If a firm's total investment expenditure is high, the market value and the replacement cost of its capital stock are of a similar magnitude and Tobin's  $q$  approaches unity. If it is less than unity, the firm should not invest anymore. In the calibration process, some of the activities' Tobin's  $q$  values are calibrated as less than unity, and these activities are considered non-dividend maximisers. See Section 3.2.1 for details. Dividend maximising firms are assigned to the subset  $dm_a$  and non-dividend maximising firms to  $ndm_a$ .

- **Production across commodities**

Activities are allowed to produce multiple products, and the quantity of production is determined by revenue maximisation as explained in Section 3.2.4. However, some activities either produce a single commodity or their production of a single commodity exceeds 95% of their total production. For these activities, total output is a Leontief aggregate of the commodity (if the activity is the unique producer) or commodities (if the production of a commodity exceeds the cut-off point). Otherwise, the activity's output is a constant elasticity of transformation (CET) aggregate of commodities that are produced by the activity in the base-year.

Activities for which the output is defined by CET function are assigned to the subset  $acet_a$  and activities where the output is defined by a Leontief function are assigned to the subset  $aleo_a$ .

- **Energy demand**

The sets  $a1_a - a4_a$  are created to distinguish activities concerning the composition of their energy demands by setting different elasticity of substitution values. The nested structures of production are explained in Section 3.2.2. The list of activities is provided in Appendix A.

- **Commodities**

All commodities are assigned to the set  $c$  which includes the following subsets:

- **Origin**

The set  $cp_c$  contains commodities produced by domestic producers and sold in the domestic market. This set is defined over commodities for which the cells of activity rows and commodity columns on the Social Accounting Matrix (SAM) are positive. The set  $cpn_c$  covers *non-domestically produced commodities*

- **Exports**

The set  $ce_c$  is defined over export commodities, i.e. commodities for which the cells of commodity rows and the rest of the world columns on the SAM are positive. The set  $cen_c$  is the complementary set that covers *non-exported commodities*.

- **Imports**

The set  $cm_c$  contains import commodities, i.e. commodities for which the cells of the rest of the world rows and commodity columns on the SAM are positive. The set  $cmn_c$  covers non-imported commodities.

- **Homogeneity**

As explained in Section 3.2.4, activities are allowed to produce multiple products; however, there are a few commodities produced by a single activity, for instance, public administration. In this case, the commodity is homogeneous and assigned to the set  $ch_c$ . This set covers commodities for which the elasticity of substitution for the domestically produced commodity,  $\sigma_{c,t}^{qxcS}$ , is zero. The set  $chn_c$  is the complementary set that covers *non-homogeneous commodities*, i.e. commodities that are produced by at least two distinct activities and  $\sigma_{c,t}^{qxcS}$  is positive.

- **Composite goods**

The model includes detailed nested structures of private composite consumption and production. The primary objective of creating such detailed nested structures is to reflect the compositions of households' and activities' demands for energy commodities as accurately as possible. In this sense, the way the nested structure is defined determines the substitutability between inputs to production and between goods for consumption. If goods are nested together, this represents higher substitutability between these goods compared to others. A Leontief relationship assumes no substitutability, whereas a constant elasticity of substitution (CES) relationship assumes a substitution possibility. The values of the elasticity of substitution parameter,  $\sigma$ , for different elasticity relations are chosen in order to reflect the low and high substitution possibilities among the commodities based on expert judgement.

- **Margins**

The set  $m$  contains trade and transportation margins.



- **Factors of production**

The set  $f$  contains factors of production. In this version, it has two elements, Labour and Capital.

- **Capital**

One type of capital (physical capital) is defined in the model.

- **Labour**

The set  $l$  has three elements: low skilled labour (LSL), medium skilled labour (MSL), and high skilled labour (HSL). The sectoral composition of employment by skill is retrieved from the Labour Force Survey (LFS). For each type of households  $hh$ , the composition of wage income by skill is retrieved from the Survey on Income and Living Conditions (SILC).

- **Households**

The set  $hh$  is the broadest set for households. In this model version, there are ten RHGs (five groups in urban areas and five groups in rural areas). These RHGs have been generated based on disposable income by using the Household Budget Survey (HBS). The survey has also been used to derive the composition of consumption expenditures across commodities and the distribution of household disposable income across different income items.

- **Time**

The set  $t$  comprises the horizon of the model and takes values between 2014 and 2054. In conventional CGE modelling, parameters do not have a time index. However, several parameters of the I3E model have a time index to generate a flexible model which allows dynamic simulations to be run. For instance, if the government wants to implement a policy change in 2020, the value(s) of relevant policy parameter(s) can be changed by 2020. If the model parameters do not have a time index, then the experiment would assume that the policy change starts in 2014.

### 3 Agents

This section discusses the behaviour of each agent in the model in turn. The agents in the I3E model consist of households, activities, commodities, enterprises, and government,

#### 3.1 Households

In this version, it is assumed that there is one RHG that solves the following intertemporal utility maximisation problem where the utility function is in the form of Constant Relative Risk Aversion (CRRA):

$$\max_{CC_{hh,t}} \sum_{t=1}^{\infty} \left( \frac{1+grw_t}{1+\rho_{hh}} \right)^t \frac{(CC_{hh,t})^{1-\theta_{hh}}}{1-\theta_{hh}} \quad (1)$$

*s.t*

$$SAV_{hh,t} + PCC_{hh,t} CC_{hh,t} \leq WINC_{hh,t} + CINC_{hh,t} + TR_{hh,t} + PEN_{hh,t} + \overline{NFI}_{hh,t} ER_t + RCI_{hh,t} RCHH_t$$

where  $grw_t$  is the economic growth rate in the period  $t$ ,  $\rho_{hh}$  is time preference rate,  $\theta_{hh}$  is intertemporal elasticity of substitution,  $WINC_{hh,t}$ ,  $CINC_{hh,t}$ ,  $TR_{hh,t}$ ,  $PEN_{hh,t}$ , and  $\overline{NFI}_{hh,t}$  are net-of-tax wage income, net-of-tax capital income (distributed dividends of enterprises), transfers from the government, pension income from the government, and net factor incomes from abroad (fixed in real terms), respectively,  $ER_t$  is exchange rate,  $SAV_{hh,t}$  is savings,  $CC_{hh,t}$  is household-specific composite consumption and  $PCC_{hh,t}$  is its price. The term  $RCI_{hh,t}$  is the carbon tax recycling income from the government and  $RCHH_t$  is a parameter (which is equal to zero in the calibration process) that allows for running an experiment in which the government recycles the carbon tax collection to households. The RHS of the budget constraint represents household disposable income,  $INC_{hh,t}$ . The components of disposable income are as follows.<sup>1</sup>

$$WINC_{hh,t} = \sum_l \gamma_{l,hh,t}^{wage} \sum_a W_{l,t} \overline{WFDIST}_{a,l,t} LD_{a,l,t} (1 - wtax_{l,t}) \quad (2)$$

where  $\gamma_{l,hh,t}^{wage}$  is the share of household  $hh$  in the total wage income of labour type  $l$ , and  $wtax_{l,t}$  is the wage income tax rate of labour type  $l$ .

$$CINC_{hh,t} = \gamma_{hh,t}^{inc} DISDIV_t \quad (3)$$

where  $\gamma_{hh,t}^{inc}$  is the share of household  $hh$  in the total dividend (capital / asset) income. In this set-up, household savings are determined endogenously by

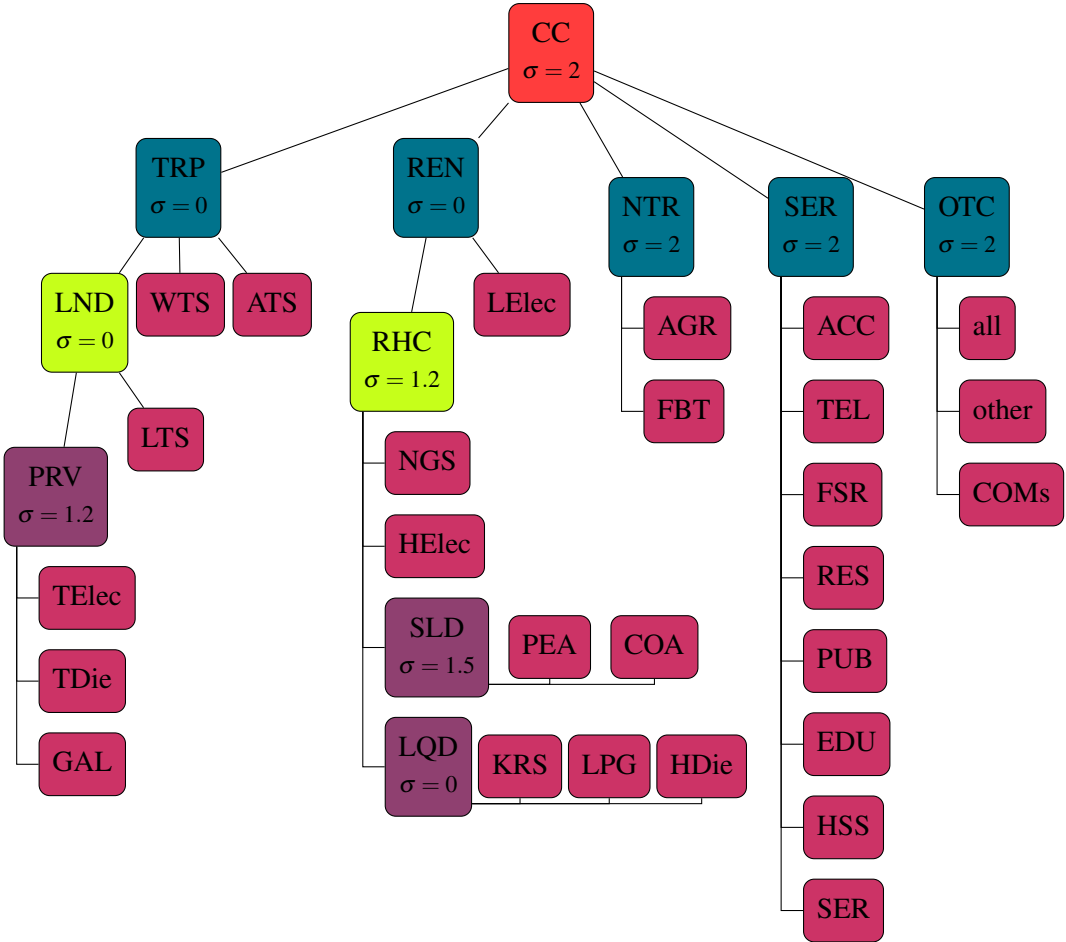
$$SAV_{hh,t} = INC_{hh,t} - PCC_{hh,t} CC_{hh,t} \quad (4)$$

<sup>1</sup> For the sake of space saving, explanations of variables and parameters will be given in the relevant subsections. You can see Section 3.2.1 and also Appendix B.

Household  $hh$  chooses the level of composite consumption to maximise the present discounted value of intertemporal utility. The first-order condition (FOC) of this problem, eq. (5), is the well-known consumption Euler equation and solves for the sequence of composite consumption where  $r_t$  is interest rate.

$$\frac{CC_{hh,t+1}}{CC_{hh,t}} = \left[ (1 + grw_t) \frac{1 + r_{t+1}}{1 + \rho_{hh}} \frac{PCC_{hh,t}}{PCC_{hh,t+1}} \right]^{\frac{1}{\theta_{hh}}} \tag{5}$$

In the second stage, household  $hh$  disaggregates her composite consumption across commodities by maximising her intratemporal utility. The nested structure of household composite consumption is depicted by Figure 1.



**Figure 1: Nested Structure of Consumption**

Several composite commodities are included in the consumption nest; these are either constant elasticity of substitution (CES) or Leontief aggregates of directly observed commodities and/or other composite commodities. For the CES aggregates of the composite commodities, the values of elasticity of substitution parameters,  $\sigma$  in Figure 1, are chosen in a range of 1.2 and 2 in order to reflect the different substitution possibilities across different commodities that constitute the composite commodity.

Household composite consumption,  $CC$ , is assumed to be a CES aggregate of composite commodities of Transportation ( $TRP$ ), Residential Energy ( $REN$ ), Nourishment ( $NTR$ ), Services ( $SER$ ), and other commodities ( $OTC$ ). As described above, this reflects that different services goods are easier to substitute with each other than, for example substituting services goods with nourishment goods. The logic here is that consumers are more likely to substitute food products with, e.g., agricultural products if prices of food products increase than to increase their consumption of services as food prices increase.

The composite commodity  $TRP$  is a Leontief aggregate of land, air, and water transportation commodities where land transportation ( $LND$ ) is also a Leontief aggregate of public and private transportation commodities. The choice of a Leontief relationship here is warranted by the low level of substitutability between transport types; a consumer will not substitute their daily car commute with air or water transport due to increases in petrol prices. It should be noted that the original land transportation commodity ( $LTS$  with NACE Code 49) covers the public transportation demand of households. In I3E, we assign a share of household demand energy commodities including gasoline, diesel, and electricity for private transportation purposes. We refer to private transportation energy use as the composite commodity  $LND$ , which is assumed to be a Leontief aggregate of that energy commodities.<sup>2</sup> The  $REN$  is disaggregated into lighting electricity and residential heating which is further disaggregated into natural gas supply, solid fuel, heating electricity, and liquid fuel. Moreover, solid (liquid) fuel is a CES (Leontief) aggregate of peat and coal (kerosene, liquid petroleum gas, and diesel for heating purposes). The total electricity consumption of households, the commodity  $ELC$ , is known from the SAM, and it is disaggregated into electricity demand by transportation, lighting, and heating purposes by using the data provided by [SEAI \(2013, Table 19\)](#). Similarly, total private consumption of diesel is disaggregated into diesel demand for transportation and heating by using the energy balances. The composite commodity  $NTR$  is a CES aggregate of the commodities agriculture and food, beverage, and tobacco while the composite commodity  $SER$  is a CES aggregate of several service commodities. The composite commodity  $OTC$  is a CES aggregate of all remaining commodities that are demanded by households.

The compositions of electricity and diesel demands across their sub-components are assumed to be identical across households as there are no data available to make a further distinction. Moreover, the values of elasticity of substitution shown in [Figure 1](#) are also assumed to be the same across households.

## 3.2 Activities

### 3.2.1 Investment by Destination

The term “investment by destination” refers to investment expenditures of firms / sectors. In the model we distinguish between dividend maximisers and non-dividend maximisers; we discuss each in turn. In a fully intertemporal setting, the investment decision can be endogenised via a dividend maximisation

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<sup>2</sup> According to the energy balances, the private consumption of liquid petroleum gas is devoted both to private transportation and to residential heating. Since the former is a quite tiny portion of the total demand, it is assumed to be zero and liquid petroleum gas is assumed to be a part of the residential heating demand.

problem where a firm maximises the present discounted value of its dividend stream,  $DIV_{dm,t}$  (i.e. the present value of firm,  $V_{dm,t}$ ) by choosing the level of physical investment,  $PSI_{dm,t}$ , and levels of factors of production (i.e. capital,  $K_{dm,t}$ , and composite labour,  $CLD_{dm,t}$ ).

$$\max_{PSI_{dm,t}, K_{dm,t}, CLD_{dm,t}} V_{dm,t} = \sum_{t=1}^{\infty} \left( \frac{1 + grw_t}{1 + r_t} \right)^t DIV_{dm,t} \quad s.t \quad (6)$$

$$K_{dm,t+1} (1 + grw_t) = (1 - \delta_{dm,t}) K_{dm,t} + PSI_{dm,t} \quad (7)$$

where  $\delta_{dm,t}$  is the depreciation rate. Activity-specific capital stock ( $K_{dm,t}$ ) evolves according to capital accumulation function, eq. (7). The Lagrange multiplier of this maximisation problem,  $q_{dm,t}$ , which is constrained by capital accumulation function is the well-known Tobin's  $q$ <sup>3</sup>, i.e. the marginal value of capital:

$$DIV_{dm,t} = (1 - corptax_t) WK_{dm,t} K_{dm,t} - INV_{dm,t} \quad (8)$$

$$INV_{dm,t} = PPSI_t PSI_{dm,t} + PVA_{dm,t} ADJ_{dm,t} \quad (9)$$

$$ADJ_{dm,t} = \phi_{dm,t} \frac{PSI_{dm,t}^2}{K_{dm,t}} \quad (10)$$

where  $WK_{dm,t}$  is the price of capital and  $PPSI_t$  is the price of the investment. The sectoral dividend is equal to net-of-corporate tax sectoral profit minus total investment expenditures,  $INV_{dm,t}$ , which includes the cost of new investment equipment and the adjustment cost. Adjustment cost is an increasing and convex function of investment; for a given level of sectoral capital stock, the cost of installing new capital equipment will be greater. Adjustment cost,  $ADJ_{dm,t}$ , is measured by the price of the value added,  $PVA_{dm,t}$ , because it is assumed that installation of new capital requires the resources of the firm, which leads to interruption of the production process and thus losses of output.

The FOCs of this dividend maximisation problem w.r.t. the levels of physical investment and capital stock, respectively, are as follows.

$$q_{dm,t} = PPSI_t + 2 PVA_{dm,t} \frac{ADJ_{dm,t}}{PSI_{dm,t}} \quad (11)$$

$$q_{dm,t} (1 + r_t) (1 + grw_t) = q_{dm,t+1} (1 - \delta_{dm,t}) + PVA_{dm,t+1} \frac{ADJ_{dm,t+1}}{K_{dm,t+1}} + (1 - corptax_t) WK_{dm,t+1} \quad (12)$$

<sup>3</sup> Tobin's  $q$  is the ratio of the market value of existing capital to its replacement cost. See Hayashi (1982) for further discussion.

For non-dividend maximisers, the investment (by destination) expenditure in period  $t$  of an activity is a fixed fraction of its total profits in period  $t$  as follows, where  $\gamma_{ndm,t}^{invdes}$  is a fixed parameter.

$$INV_{ndm,t} = \gamma_{ndm,t}^{invdes} WK_{ndm,t} K_{ndm,t} \quad (13)$$

The level of investment expenditures determines the level of physical investment demand of a firm ( $PSI_{ndm,t}$ ) which, in turn, determines the level of sectoral capital stock ( $K_{ndm,t}$ ).

$$INV_{ndm,t} = PPSI_t PSI_{ndm,t} \quad (14)$$

$$K_{ndm,t+1} (1 + grw_t) = (1 - \delta_{ndm,t}) K_{ndm,t} + PSI_{ndm,t} \quad (15)$$

The depreciation rate of these firms is arbitrarily set to be equal to 0.05.

For all firms, either in the subset of  $dm$  or  $ndm$ , the real value added,  $VA_{a,t}$ , is a CES aggregate of sectoral capital stock and sectoral composite labour

$$VA_{a,t} = \lambda_{a,t}^{va} [\gamma_{a,t}^{cva} K_{a,t}^{-\rho_{a,t}^{va}} + (1 - \gamma_{a,t}^{cva}) CLD_{a,t}^{-\rho_{a,t}^{va}}]^{-\frac{1}{\rho_{a,t}^{va}}} \quad (16)$$

where  $CLD_{a,t}$  is composite labour demand,  $\gamma_{a,t}^{cva}$  is the share parameter of capital in real value added,  $\lambda_{a,t}^{va}$  is the shift parameter, and  $\rho_{a,t}^{va}$  is the exponent parameter and obeys  $\sigma_{a,t}^{va} = 1/(1 + \rho_{a,t}^{va})$ .

The value added is equal to the sum of the payments to the factors of production

$$PVA_{a,t} VA_{a,t} = WK_{a,t} K_{a,t} + CW_{a,t} CLD_{a,t} \quad (17)$$

where  $CW_{a,t}$  is price of sectoral composite labour input.

As the sectoral capital stocks evolve by following the law of motion for capital (7 and 15), the FOC of the cost minimisation problem yields the optimal level of composite labour demand

$$CLD_{a,t} = \left[ \frac{PVA_{a,t} \gamma_{a,t}^{cva}}{CW_{a,t} (\lambda_{a,t}^{va})^{\rho_{a,t}^{va}}} \right]^{\sigma_{a,t}^{va}} VA_{a,t} \quad (18)$$

Since the model comprises there types of labours, it is assumed that an activity chooses a sector-specific composite labour input which is a CES aggregate of different types of labour. Each activity solves a cost minimisation problem to find the optimal combination of types of labours that minimises the cost of labour.

$$\min_{LD_{a,l,t}} CW_{a,t} CLD_{a,t} = \sum_l LD_{a,l,t} \tilde{W}_{l,t} \overline{WFDIST}_{a,l,t} \quad s.t \quad (19)$$

$$CLD_{a,t} = \left[ \sum_l \gamma_{a,l,t}^{cl} (BX_t LD_{a,l,t})^{-\rho_{a,t}^{cl}} \right]^{-\frac{1}{\rho_{a,t}^{cl}}} \quad (20)$$

where  $LD_{a,l,t}$  is labour demand of the activity,  $\tilde{W}_{l,t}$  is the wage per effective labour,  $\overline{WFDIST}_{a,l,t}$  is the ratio of factor payments to labour  $l$  from activity  $a$  to the wage rate of labour  $l$ ,  $BX_t$  is the level of labour productivity,  $\gamma_{a,l,t}^{cl}$  is the share of labour type  $l$  in composite labour input of activity  $a$ , and  $\rho_{a,t}^{cl}$  is an exponent parameter. We need the term  $\overline{WFDIST}_{a,l,t}$  since the wage rate is determined by equilibrium conditions in the labour market for each type of labour, i.e. the model solves for three wage rates. However, since productivity of each type of labour is different, payments to labour cannot be the same across sectors. The term captures the distance between average and sectoral payments to labour.

In the case of labour-augmented technological growth, activities choose the level of “efficient labour”, i.e. the multiplication of the labour demand and the level of productivity. Given the technological change, the wage per effective labour,  $\tilde{W}_{l,t}$ , is constant while the wage per labour,  $W_{l,t}$  – the wage received by labour – grows at the rate of technological change. For the given level of labour supply, labour is paid more in every period due to increases in its productivity. Hence, the following relation holds.

$$W_{l,t} = \tilde{W}_{l,t} BX_t$$

The optimal level of labour demand is determined by

$$BX_t LD_{a,l,t} = \left[ \frac{CW_{a,t} \gamma_{a,l,t}^{cl} BX_t}{W_{l,t} \overline{WFDIST}_{a,l,t}} \right]^{\sigma_{a,t}^{cl}} CLD_{a,t} \quad (21)$$

### 3.2.2 Production

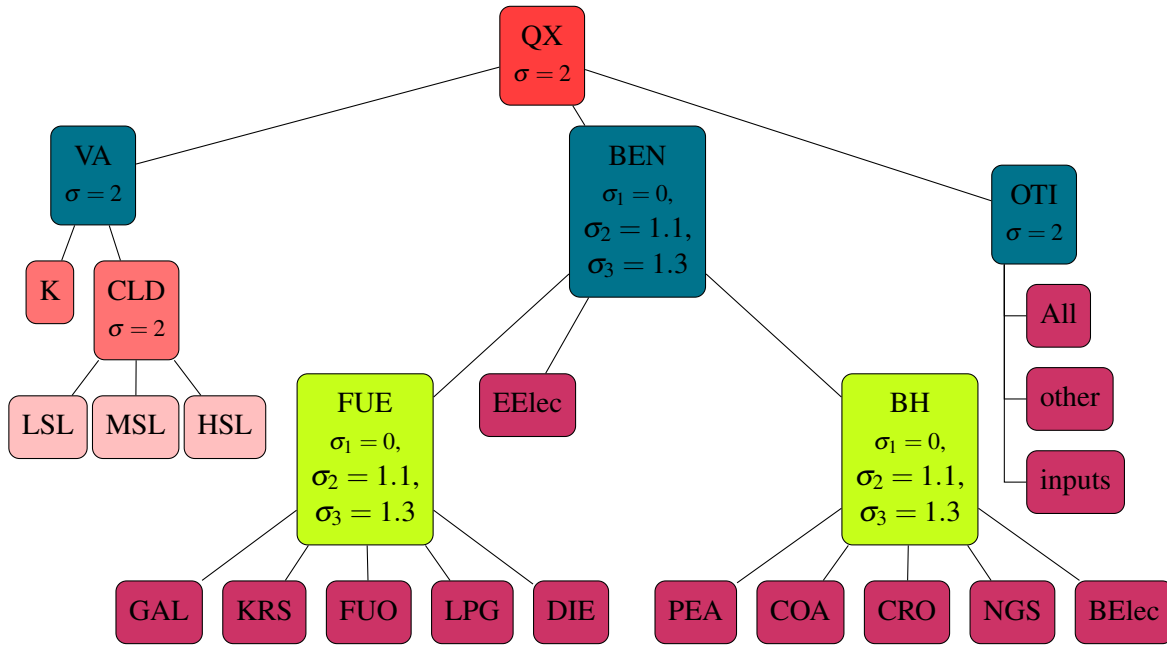
The total value of production, the LHS of eq. (22), is equal to payments to factors of production, i.e. value-added, production taxes paid to the government, the total cost of intermediate inputs, and the cost of the Emissions Trading System (ETS).

$$PX_{a,t} QX_{a,t} = PVA_{a,t} VA_{a,t} + PRODTAXS_{a,t} + PCIN_{a,t} CCIN_{a,t} + CETS_{a,t} \quad (22)$$

where  $QX_{a,t}$  is an activity’s total production and  $CCIN_a$  is composite intermediate input demand;  $PX_{a,t}$  and  $PCIN_{a,t}$  are their prices, respectively.

The production of activities in the model is represented by a nested structure as shown in Figure 2. To reflect the differences in energy demand compositions, activities are assigned to three distinct groups at two different layers of the nested production structure. The elasticity of substitution parameter,  $\sigma$ , takes the value of 0 for activities which have a quite strict composition of energy demand. On the other hand, its value is either 1.1 or 1.3 for activities whose composition of the energy demand is not dominated by a specific energy commodity.

The activities are assumed to produce a composite product  $QX$ , which is an aggregate of value added ( $VA$ ), business energy ( $BEN$ ), and other inputs ( $OTI$ ). The value added is a CES aggregate of factors of production, which are capital and composite labour, and the commodity  $OTI$  is an aggregate of all inter-



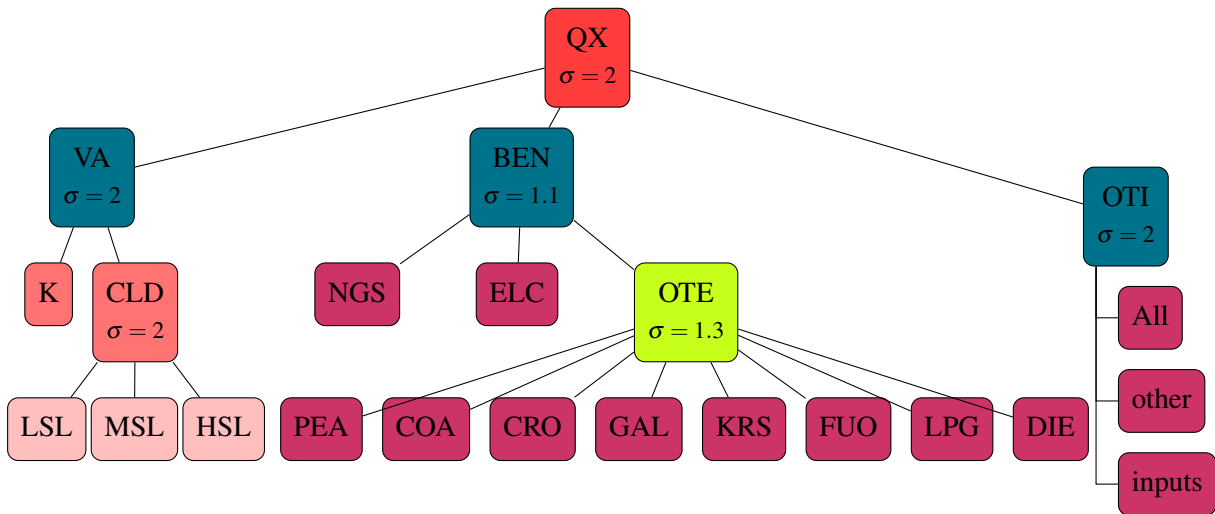
**Figure 2: Nested Structure of Production, except Electricity Production**

mediate inputs except the energy commodities. The composite labour is a CES aggregate of three types of labour. For all activities, except the electricity production, the commodity *BEN* is assumed to be an aggregate of energy electricity (*EElec*), fuel (*FUE*) and business heating (*BH*). The composite commodity *BH* is an aggregate of liquid and solid fuels including coal, peat, crude oil, natural gas supply, and business electricity for heating purposes. On the other hand, the composite commodity *FUE* is an aggregate of gasoline, kerosene, fuel oil, liquid petroleum gas, and diesel. The electricity demand of activities, except the electricity production, is disaggregated across demands for energy purposes and heating/combustion purposes.<sup>4</sup> The nested structure of production of all activities except electricity production is depicted by Figure 2.

The electricity production activity has a unique production technology and energy demand composition. The activity's business energy, *BEN*, is assumed to be a CES aggregate of electricity, natural gas, and other energy (*OTE*) which is a CES aggregate of all remaining energy commodities, as shown in Figure 3. The value of  $\sigma$  is 1.1 for the commodity of *BEN* while it is 1.3 for the *OTE* because the electricity production has more flexibility to substitute between liquid and solid fuels than natural gas and electricity's itself.

<sup>4</sup> At this stage, the disaggregation is done by arbitrarily assuming that 40% (60%) of the total sectoral electricity is used for heating/combustion (energy).





**Figure 3: Nested Structure of Electricity Production**

### 3.2.3 Emissions Trading System

The ETS is the European Union’s (EU) key tool to reduce industrial emissions. In the current phase of the system, Phase-III that covers the period of 2013–2020, each installation (i.e. a production unit) receives an amount of free allowances (EU-ETS, 2018). If the installation’s total emissions exceed its free allowances, the installation will need to purchase additional allowances<sup>5</sup> at the ETS price determined in the EU-wide ETS market. If the installation emits less than its free allowances, it can sell its unused allowances to other installations at the EU-ETS price. In other words, the system generates a cost (revenue) for those installations which emit more (less) than their allowances. An installation may generate more revenue from the ETS system by reducing its emissions as it invests more on cleaner production technologies. Therefore, the last term on the RHS of eq. (22) is a cost (revenue) item if it is positive (negative).

The coverage of sectoral emissions by the ETS is 100% for the energy production sectors; energy production installations do not receive any free allowances and need to purchase allowances to cover all their emissions. Petroleum refining, mineral, and aviation sectors’ emissions are also fully covered by the ETS, but installations operating in these industries receive some free allowances. On the contrary, land transportation (road or railway), agriculture, waste, and residential sectors are exempted from the ETS. For the other sectors, the ETS coverage varies based on the average size of production units regarding the combustion capacity, production capacity, etc. (EPA, 2018).

In the I3E model, economic activities directly internalise the cost of ETS in their cost minimisation problems. As an example, the optimal level of an intermediate input in the composite commodity of business heating (*BH*) is

<sup>5</sup> In the EU-ETS terminology, the term “allowance” is also used to define emission permits.

$$INT_{c,a,t} = \left[ \frac{PCIN_{BH,a,t} \gamma_{c,a,t}^{bh}}{PQD_{c,t} + ETSADJ_{c,a,t}} \right]^{\sigma_{a,t}^{bh}} CCIN_{BH,a,t} \quad (23)$$

where  $INT_{c,a,t}$  is intermediate input demand on commodity  $c$  by activity  $a$ ,  $CCIN_{BH,a,t}$  is intermediate demand on the composite commodity  $BH$  by activity  $a$  and  $PCIN_{BH,a,t}$  is its price, and  $\gamma_{c,a,t}^{bh}$  and  $\sigma_{a,t}^{bh}$  are share and exponent parameters of the CES function, respectively.<sup>6</sup> The denominator in the parentheses is the total unit cost of a commodity  $c$  for activity  $a$  where  $PQD_{c,t}$  is purchaser price of commodity  $c$  and  $ETSADJ_{c,a,t}$  is the ETS adjuster such that

$$ETSADJ_{c,a,t} = \overline{PETS}_t \overline{ETStoE}_{a,t} (1 - AtoT_{a,t}) carcon_c \quad (24)$$

where  $\overline{PETS}_t$  is the EU-ETS price of a per ton emission allowance,  $\overline{ETStoE}_{a,t}$  is the ratio of ETS emissions to total emissions of activity  $a$ ,  $AtoT_{a,t}$  is the ratio of ETS allowance to ETS emissions of activity  $a$ , and  $carcon_c$  is the carbon content of commodity  $c$  which is calibrated by dividing total emission of commodity  $c$  by its total consumption for the year of 2014.<sup>7</sup>

The EU-ETS price is an exogenous variable since it is determined in the EU-wide allowance market, which is not explicitly incorporated in the I3E model. The ratio of ETS emissions to total emissions of an activity is a parameter (i.e. its value for each activity is calibrated by using the realised values of total and ETS emissions in 2014), and its value is assumed to be constant through time. However, the ratio of ETS allowance to ETS emissions of an activity is an endogenous variable. Total sectoral fuel combustion emissions are calculated as:

$$emis_{act_{a,t}} = \sum_c carcon_c INT_{c,a,t} \quad (25)$$

The ETS emissions for the activity and the free allowances are given by:

$$\begin{aligned} emis_{ETS_{a,t}} &= emis_{act_{a,t}} \overline{ETStoE}_{a,t} \\ \overline{allowance}_{a,t} &= AtoT_{a,t} emis_{ETS_{a,t}} \end{aligned} \quad (26)$$

where  $\overline{allowance}_{a,t}$  is the free allowance,  $emis_{ETS_{a,t}}$  is the ETS emissions, and  $emis_{act_{a,t}}$  is the total emissions of activity  $a$ . In Phase III of the ETS, the freely allocated allowance of each installation and thus activity is known. In Phase IV that covers the period of 2021–2030, the free allowances of installations will be reduced by 1.74% (2.2%) annually until 2030 for all sectors (the aviation). Applying these reductions, the sequence of free allowances for each activity is determined until 2030. As the structure of Phase V of the ETS has not been declared by the European Commission, the level of allowances beyond 2031 are assumed to be constant at their 2030 levels. Along the business-as-usual path of the model economy, as the free allowances decline, the value of  $AtoT_{a,t}$  declines as well. Moreover, the positive trend in the EU-ETS price has also been applied in the BaU scenarios.

<sup>6</sup> A fully fledged proof of the equations on the ETS system is available in Appendix D.

<sup>7</sup> See Section 7 for details.

In the I3E model, ETS emissions are a fixed fraction of total emissions of an activity, hence for a given declining level of free ETS allowances, the allowances to ETS emissions ratio declines (eq. 26). For a given carbon content of the energy commodities and increasing level of the ETS price, the value of the ETS adjuster variable increases (eq. 24), which puts upward pressure on the unit cost of the intermediate input and thus reduces its demand.

The process emissions of the aviation and mineral sectors constitute the second biggest portion of the free ETS allowances in Ireland.<sup>8</sup> These emissions are calculated based on the level of sectoral output

$$emis\_OETS_{a,t} |_{\{a \in ATS, ONM\}} = emis\_oth_{a,t} QX_{a,t} \quad (27)$$

where  $emis\_OETS_{a,t}$  is the other ETS emissions (i.e process emissions) and  $emis\_oth_{a,t}$  is the share of the other ETS emissions in the sectoral output. For these emissions, the aforementioned two sectors also receive a free allowance,  $\overline{allowance\_oth}$ , the sequence of which is also given until 2030. The level of these allowances beyond 2031 is assumed to be constant at their 2030 levels. To allow the ATS and ONM sectors to internalise the free allowance for their process emissions, another variable, namely  $allow\_oth_{a,t}$ , which is the ratio of other ETS allowances to sectoral output, is generated where

$$\overline{allowance\_oth}_{a,t} = allow\_oth_{a,t} QX_{a,t} \quad (28)$$

The total cost of the ETS is given by

$$CETS_{a,t} = \overline{PETS}_t (emis\_ETS_{a,t} + emis\_OETS_{a,t} - \overline{allowance}_{a,t} - \overline{allowance\_oth}_{a,t}) \quad (29)$$

### 3.2.4 Multi-product Determination

Each activity can produce multiple products, and the levels of output for each product are determined by a revenue maximisation problem. Gross production is a constant elasticity of transformation (CET) aggregate of the products produced by the activity if the activity is not the producer of a single commodity and the level of production of a single commodity does not exceed 95% of the total production of the activity.

$$QX_{acet,t} = \lambda_{acet,t}^{qxac} \left[ \sum_c \gamma_{acet,c,t}^{qxac} QXAC_{acet,c,t}^{qxac} \right]^{\frac{1}{\rho_{acet,t}^{qxac}}} \quad (30)$$

while total value of output has to be equal to the sum of values of each commodity produced:

$$PX_{acet,t} QX_{acet,t} = \sum_c PXAC_{acet,c,t} QXAC_{acet,c,t} \quad (31)$$

<sup>8</sup> The aviation sector's process emissions come from the inconsistency between the verified ETS emissions of the sector and its fuel demand on the Energy Balance of Ireland. The inconsistency emerges from Ireland-based installations' fuel demand outside of Ireland. The energy demand appears in the corresponding country's energy balance sheet but the related emissions appear in the ETS account of the installation. The ATS sector's other ETS emissions eliminate this inconsistency.

where  $QXAC_{acet,c,t}$  is the volume of production of commodity  $c$  by activity  $acet$  and  $PXAC_{acet,c,t}$  is its price,  $\lambda_{acet,t}^{qxac}$  is the shift parameter and  $\gamma_{acet,c,t}^{qxac}$  is the share parameter of commodity  $c$ ,  $\sigma_{acet,t}^{qxac}$  is the elasticity of transformation and  $\rho_{acet,t}^{qxac} = 1/\sigma_{acet,t}^{qxac} + 1$  holds for  $\sigma_{acet,t}^{qxac} \in (0, \infty]$ . Each activity maximises (31) subject to (30) and the following FOC determines the level of production:

$$QXAC_{acet,c,t} \Big|_{\gamma_{acet,c,t}^{qxac}} = \left[ \frac{PXAC_{acet,c,t}}{PX_{acet,t} \gamma_{acet,c,t}^{qxac} (\lambda_{acet,t}^{qxac})^{\rho_{acet,t}^{qxac}}} \right]^{\sigma_{acet,t}^{qxac}} QX_{acet,t} \quad (32)$$

As the price of commodity  $c$  produced by activity  $acet$  increases, production increases. Note that eq. (32) is defined conditional on a positive production share of commodity  $c$  in the total output of activity  $acet$  on the SAM. This implies that an activity is not able to produce commodities which are not produced in the base case.

For activities which produce a single commodity or the level of production of a single commodity exceeds 95% of total production of the activity, the level of production of each commodity is a fixed share of total output of the activity.

$$QXAC_{aleo,c,t} \Big|_{\gamma_{aleo,c,t}^{qxac}} = \gamma_{aleo,c,t}^{qxac} QX_{acet,t} \quad (33)$$

where  $\gamma_{aleo,c,t}^{qxac}$  is the share parameter and obeys  $\sum_c \gamma_{aleo,c,t}^{qxac} = 1$ . The parameter is equal to 1 for a commodity  $c$  if the activity is the unique producer.

### 3.3 Enterprises

The model economy includes an “enterprises” account. The representative enterprise is assumed to be the owner of all firms. Such an assumption helps to simplify some details of the model and also avoids the need for detailed data which are not available. The enterprises account collects all gross sectoral profits and receives transfers from the government  $\overline{GTRENT}_t$  which is fixed in nominal terms, and pays corporate tax to the government,  $CORPTAXS_t$ . The remaining amount is either saved by the enterprise account,  $ESAV_t$ , or paid to households as dividend payments,  $DISDIV_t$ .

$$DISDIV_t = \sum_a WK_{a,t} K_{a,t} + \overline{GTRENT}_t - CORPTAXS_t - ESAV_t \quad (34)$$

It is assumed that enterprises savings is a fixed fraction,  $\gamma_t^{esav}$ , of net-of-tax profit receipts of the account:

$$ESAV_t = \gamma_t^{esav} (\sum_a WK_{a,t} K_{a,t} - CORPTAXS_t) \quad (35)$$

### 3.4 Government

In this model economy, the government collects direct taxes on labour incomes (eq. 36), and on sectoral profits (i.e. corporate tax) (eq. 37), indirect taxes on sales of commodities (eq. 38), carbon tax on energy

commodities (eq. 39), export tax on export of commodities (eq. 40), and production tax on production activities (eq. 41). The first half of the total cost of the ETS goes to the government whereas the second half goes to the EU Commission. The government allocates her total revenues to consumption and to transfers to households in terms of welfare transfers and pension. The real values of these transfers are fixed and their nominal values are indexed to the average wage rate. The total nominal values of these government transfers to households are distributed across households based on the fixed fractions.

Each type of labour pays a wage income tax at the rate  $wtax_{l,t}$  over their total income receipt from activities

$$WTAXS_{l,t} = wtax_{l,t} \sum_a W_{l,t} \overline{WFDIST}_{a,l,t} LD_{a,l,t} \quad (36)$$

Corporate tax is paid at a uniform rate across activities,  $corptax$ , by the enterprise account on total profits of activities

$$CORPTAXS_t = corptax_t \sum_a WK_{a,t} K_{a,t} + TCTAXS_t RCCT_t \quad (37)$$

where  $TCTAXS_t$  is total carbon tax collection of the government and  $RCCT_t$  is a parameter (which is equal to zero along the calibration process) that allows for running an experiment in which the government directs the carbon tax revenues to reduce the corporate tax rate.<sup>9</sup>

Sales tax on commodity  $c$  is imposed on total domestic supply of the commodity, which is the sum of import and domestic production:

$$SALTAX_{c,t} = stax_{c,t} (PM_{c,t} QM_{c,t} + PD_{c,t} QD_{c,t}) \quad (38)$$

where  $stax_{c,t}$  is ad-valorem sales tax rate on commodity  $c$ . Carbon tax is collected on domestic consumption of energy commodities at fixed price of per-tonne equivalent of carbon,  $\overline{PCAR}_{c,t}$ , which is exogenously determined by the government,

$$CTAXS_{c,t} = \alpha_{c,t} \overline{PCAR}_{c,t} carcon_c QS_{c,t} \quad (39)$$

The parameter  $\alpha_{c,t}$  equates the carbon tax collection of the government on commodity  $c$  to the given levels of variables and parameters.

Although the domestic sale of electricity is exempted from the carbon tax, its export is subject to a carbon tax. Therefore, the total carbon tax collection of the government includes the export tax on electricity. Since the inclusion of carbon tax affects the domestic purchaser prices of commodities, the carbon tax payments on the consumption of electricity are set to zero. This amount is transferred to the export tax account on the SAM.

$$EXPTAXS_{c,t} = exptax_{c,t} PWE_{c,t} QE_{c,t} ER_t \quad (40)$$

<sup>9</sup> For the other recycling schemes and their details, see Section 3.4.1.

where  $exptax_{c,t}$  is ad-valorem tax rate on the export of commodity  $c$ . The  $exptax_c$  is positive for only electricity and zero for the other commodities.

Activities pay production tax on the value of their total production

$$PRODTAXS_{a,t} = prodtax_{a,t} PX_{a,t} QX_{a,t} \quad (41)$$

where  $prodtax_{a,t}$  is ad-valorem tax rate on activity  $a$ .

Total revenue of the government is equal to summation of the six items above.

$$\begin{aligned} GOVREV_t = & \sum_a (PRODTAXS_{a,t} + CETS_{a,t} 0.5) + \sum_l WTAXS_{l,t} + CORPTAXS_t \\ & + \sum_c (SALTAX_{c,t} + CTAXS_{c,t} + EXPTAXS_{c,t}) \end{aligned} \quad (42)$$

The total government consumption of commodities has an autonomous part which is fixed in nominal terms,  $\overline{GOVCONA}_t$  and an induced part which is a positive function of the current period's nominal gross domestic product

$$GOVCON_t = \overline{GOVCONA}_t + mps GDP_t + TCTAXS_t RCGC_t \quad (43)$$

where the parameter  $mps$  stands for the government's marginal propensity to spend and  $RCGC_t$  is a parameter (which is equal to zero along the calibration process) that allows for running an experiment in which the government directs the carbon tax collection to the government consumption. A simple ordinary least square (OLS) analysis shows that the value of  $mps$  is 0.05 when the first difference of the government consumption is regressed on the first difference of GDP. If the level of government consumption is regressed on the level of GDP, the parameter  $mps$  becomes 0.106. To lower the sensitivity of the induced government expenditures, the first differences are used. The level of government consumption of each commodity is a simply fixed fraction,  $\gamma_{c,t}^{gc}$ , of its total consumption

$$PQD_{c,t} CG_{c,t} = \gamma_{c,t}^{gc} GOVCON_t \quad (44)$$

where  $CG_{c,t}$  is the government consumption demand for commodity  $c$ .

The difference between total revenues and expenditures of the government is public savings:

$$\begin{aligned} GSAV_t = & GOVREV_t - GOVCON_t - (\overline{GTRHH}_t + \overline{TOTPEN}_t) W\_mean_t - \\ & \overline{GTRENT}_t - GFINT_t ER_t - TCTAXS_t RCHH_t \end{aligned} \quad (45)$$

where  $\overline{GTRHH}_t$  and  $\overline{TOTPEN}_t$  are the real values of the total government welfare transfers and pension payments to households, respectively, and  $W\_mean_t$  is the average wage rate. The real values of these transfers are assumed to be fixed but the government updates them based on the change in the average

nominal wage. These transfers are distributed across households by fixed parameters,

$$\begin{aligned} TR_{hh,t} &= \gamma_{hh,t}^{grhh} \overline{GTRHH}_t W\_mean_t \\ PEN_{hh,t} &= \gamma_{hh,t}^{penhh} \overline{TOTPEN}_t W\_mean_t \end{aligned} \quad (46)$$

$GFINT_t$  is the interest payments of the government to the rest of the world over the outstanding foreign debt stock,  $GFDS_t$ , which grows by the level of government savings:

$$GFINT_t = r_t GFDS_t \quad (47)$$

$$GFDS_{t+1} = GFDS_t - GSAV_t / ER_t \quad (48)$$

### 3.4.1 Carbon Tax Revenue Recycling Schemes

In the context of environmental policies, achieving a double dividend (a decline in emissions and an economic efficiency gain through a reduction in distortionary taxes) and counteracting the regressive nature of environmental policies play an important role. In this respect, recycling of carbon / environmental tax collection to other agents in the economy rather than its usage by the government is considered the primary policy tool to achieve a double dividend and/or reduce inequality. In the design of recycling schemes, governments have several options such as transferring the funds to households directly (unconditionally) or reducing tax rates such as wage income tax rate, production tax rate of (selected) activities, corporate tax rate, and sales tax (VAT) rates. A government may choose not to recycle the tax collection and use it to increase government expenditures or to reduce debt stock. In order to simulate all those policy options, the I3E model includes a flexible recycling mechanism. This section is devoted to explaining how the system of equation is capable of evaluating all recycling options.

If the Irish government chooses not to recycle the carbon tax collection, the increased amount of tax collection is used to reduce the foreign debt stock. Since all policy switching parameters ( $RCCT_t$ ,  $RCGC_t$ ,  $RCHH_t$ ) are equal to zero, the entire carbon tax collection remains in the total government revenue which increases the public savings via eq. (45).

If the government directs the total carbon tax collection to households in the form of unilateral transfers, this would directly affect the government savings and household disposable income via positive recycling income

$$RCI_{hh,t} = \gamma_{hh,t}^{rch} TCTAXS_t RCHH_t \quad (49)$$

where  $\gamma_{hh,t}^{rch}$  is the share of household  $hh$  in the total carbon recycling income and is assumed to be equal to the households' share in government transfers  $\gamma_{hh,t}^{grhh}$ . This experiment is run by switching the value of  $RCHH_t$  from 0 to 1.

Similarly, if the government uses the total carbon tax collection to increase government consumption, the value of parameter  $RCGC_t$  in eq. (43) is switched from 0 to 1. In this case, the government expenditure increases by the total carbon tax collection and the government savings decrease.

In order to quantify the impacts of the other recycling schemes, the tax rates intended to be reduced must be endogenous variables rather than parameters. To hold the numbers of the equation and endogenous variables equal, a new set of equations must be defined for tax rates.

In the corporate tax rate reduction option, a new equation is not needed since the corporate tax rate has no activity or commodity dimension, i.e. it is a uniform rate across activities. Therefore, an experiment can be designed such that the value of parameter  $RCCT_t$  is switched from 0 to 1 in eq. (37) **and** the value of corporate tax collection is fixed to its baseline level. Since one variable is fixed, without having a new equation, the system of equations solves for the optimal level of the corporate tax rate,  $corptax_t$ . It is evident from eq. (37) that for the fixed  $CORPTAXS_t$ ,  $RCCT_t = 1$  increases the RHS of the equation and forces  $corptax_t$  to decline.

The following two equations govern the process of reducing the income tax rate by using the carbon tax collections

$$TWTAXS_t = \sum_l WTAXS_{l,t} + TCTAXS_t RCWT_t \quad (50)$$

$$wtax_{l,t} = wtax_{l,0} WTADJ_t - WTADJ_{P_t} WT01_{l,t} \quad (51)$$

where  $TWTAXS_t$  is the total wage tax collection,  $RCWT_t$  is a policy switching parameter (which is calibrated to 0),  $wtax_{l,0}$  is the calibrated wage income tax rate of labour type  $l$ ,  $WTADJ_t$  and  $WTADJ_{P_t}$  are full and partial tax rate adjuster variables which are set to 1 and 0, respectively, and  $WT01_{l,t}$  is a binary parameter for each type of labour. Without a policy change,  $RCWT_t$  and  $WTADJ_{P_t}$  are equal to 0,  $WTADJ_t$  is equal to 1,  $WT01_{l,t}$  is 0 for all labour types, and eq. (51) solves for the wage income tax rate that is exactly equal to its calibrated value.

In an experiment of wage income tax rate reduction, the value of  $TWTAXS_t$  is fixed to its baseline level **and** the value of parameter  $RCWT_t$  is switched from 0 to 1. In this case, for the fixed LHS of eq. (50), the RHS increases and forces  $WTAXS_{l,t}$  to decline via reduction in  $wtax_{l,t}$ .

As one variable is fixed,  $TWTAXS_t$ , another variable must freely adjust to obtain a consistent solution. In this policy setting, the government has two options; reducing all wage income tax rate equiproportionally or excluding some of the labour types in the wage income tax rate reduction. Alterations in the wage income tax rates are driven by  $WTADJ_t$  and  $WTADJ_{P_t}$ . In the first option, the variable  $WTADJ_t$  freely adjusts (while  $WTADJ_{P_t}$  is 0) and lowers all wage income tax rates by the same percentage change. If the government directs the carbon tax collections to reduce the wage income tax rates of low- and medium-skilled labours by excluding the high-skilled labour, the binary variable  $WT01_{l,t}$  is set to 1 for the former two labour types while its value for the latter labour type is still equal to 0. Then,  $WTADJ_{P_t}$  adjusts (while  $WTADJ_t$  is 1) to reach a solution by lowering the wage income tax rates of low- and medium-skilled labours by the same absolute value.

Another policy option of the government is using the carbon tax collection to reduce the production tax rates of activities,  $prodtax_{a,t}$ . A similar structure to that of the wage income tax reduction option is



built as follows

$$TPRODTAXS_t = \sum_a PRODTAXS_{a,t} + CTAXS_t RCPT_t \quad (52)$$

$$prodtax_{a,t} = prodtax_{a,0} PTADJ_t - PTADJ_{P_t} PT01_{a,t} \quad (53)$$

where  $TPRODTAXS_t$  is the total production tax collection,  $RCPT_t$  is a policy switching parameter (which is calibrated to 0),  $prodtax_{a,0}$  is the calibrated production tax rate of activity  $a$ ,  $PTADJ_t$  and  $PTADJ_{P_t}$  are full and partial tax rate adjuster variables which are set to 1 and 0, respectively, and  $PT01_{a,t}$  is a binary parameter for each activity. Without a policy change,  $RCPT_t$  and  $PTADJ_{P_t}$  are equal to 0,  $PTADJ_t$  is equal to 1,  $PT01_{a,t}$  is 0 for all activities, and eq. (53) solves for the production tax rate that is exactly equal to its calibrated value.

In an experiment of production tax rate reduction, the value of  $TPRODTAXS_t$  is fixed to its baseline level **and** the value of parameter  $RCPT_t$  is switched from 0 to 1. In this case, for the fixed LHS of eq. (52), the RHS increases and forces  $PRODTAXS_{a,t}$  to decline via reduction in  $prodtax_{a,t}$ .

In this policy option, the government may reduce all production tax rates equiproportionally (set  $PTADJ_{P_t}$  to 0, allow  $PTADJ_t$  to freely adjust, sectoral tax rates decline by the same percentage change) or may exempt some of the activities from the policy (set  $PTADJ_t$  to 1, allow  $PTADJ_{P_t}$  to freely adjust, the tax rates decline by the same absolute value). The latter option allows the government to use the carbon / environmental tax collection to reduce the cost of production of cleaner energy producers rather than treating all activities equally.

The last policy option is reducing the sales (value-added) tax rates. The following equations govern this process

$$TSALTAXS_t = \sum_c SALTAXS_{c,t} + CTAXS_t RCST_t \quad (54)$$

$$stax_{c,t} = stax_{c,0} STADJ_t - STADJ_{P_t} ST01_{c,t} \quad (55)$$

where  $TSALTAXS_t$  is the total sales tax collection,  $RCST_t$  is a policy switching parameter (which is calibrated to 0),  $stax_{c,0}$  is the calibrated sales tax rate of commodity  $c$ ,  $STADJ_t$  and  $STADJ_{P_t}$  are full and partial tax rate adjuster variables which are set to 1 and 0, respectively, and  $ST01_{c,t}$  is a binary parameter for each commodity. Without a policy change,  $RCST_t$  and  $STADJ_{P_t}$  are equal to 0,  $STADJ_t$  is equal to 1,  $ST01_{c,t}$  is 0 for all commodities, and eq. (55) solves for the sales tax rate that is exactly equal to its calibrated value.

In an experiment of sales tax rate reduction, the value of  $TSALTAXS_t$  is fixed to its baseline level **and** the value of parameter  $RCST_t$  is switched from 0 to 1. In this case, for the fixed LHS of eq. (54), the RHS increases and forces  $SALTAXS_{c,t}$  to decline via a reduction in  $stax_{c,t}$ .

In this policy option, the government may reduce all sales tax rates equiproportionally (set  $STADJ_{P_t}$  to 0, allow  $STADJ_t$  to freely adjust, the tax rates decline by the same percentage change) or may set some of the commodities exempted from the policy (set  $STADJ_t$  to 1, allow  $STADJ_{P_t}$  to freely adjust, the

commodity tax rates decline by the same absolute value). The latter option allows the government to use the carbon / environmental tax collection to reduce the sales tax rates of a selected set of commodities. Considering equiproportional declines in sales tax rates is not a meaningful policy option since it also covers the carbon commodities; the government would be increasing the carbon tax to lower the sales tax rates of the energy commodities. Hence excluding energy commodities from the tax reduction would make sense. Moreover, the government has room to extend the set of excluded commodities in the policy design, e.g. alcoholic beverages and tobacco products may also be excluded.

## 4 Commodities

Commodity supply consists of the production from domestic producers and imports. Commodity demand consists of the demand for intermediate usage, household (private) demand, government demand, investment demand, trade and transportation margins demand, and export demand.

Data concerning supply and demand of commodities are retrieved from the Supply and Use Tables (SUTs), where the commodity production per activity and the various demands are given. The SUTs, however, only provide the value of exports by commodities but not which activities' outputs are exported. For this reason, we define a commodity, namely  $QXC_{c,t}$ , that is a CES composite of domestically produced commodities. Its volume and value are as follows:

$$QXC_{c,t}|_{\{cp_c \text{ and } chn_c\}} = \lambda_{c,t}^{qxc} \left[ \sum_a \gamma_{a,c,t}^{qxc} QXAC_{a,c,t}^{-\rho_{c,t}^{qxc}} \right]^{\frac{-1}{\rho_{c,t}^{qxc}}} \quad (56)$$

$$PXC_{c,t} QXC_{c,t} = \sum_a PXAC_{a,c,t} QXAC_{a,c,t} \quad (57)$$

where  $PXC_{c,t}$  is price of composite commodity  $QXC_{c,t}$ . Equation (56) is defined over the set  $cp_c$  to control for positive values of  $QXAC_{a,c,t}$  and over the set  $chn_c$  to control for non-homogeneous commodities. For commodities that are not produced domestically,  $QXC_{c,t}|_{cp_c} = 0$  holds while for homogeneous commodities, the volume of the composite commodity  $QXC_{c,t}$  is equal to production of activity  $a$

$$QXC_{c,t}|_{\{ch_c\}} = \sum_a QXAC_{a,c,t} \quad (58)$$

The commodity  $QXC_{c,t}$  is either sold in the domestic market or exported. From the demand side, demand is assumed to be a CET aggregate of domestic sales and exports. The volumes of these components are determined by FOCs of a profit maximisation problem in which eq. (59) is maximised subject to eq. (60)

$$QXC_{c,t} PXC_{c,t}|_{\{cp_c\}} = PD_{c,t} QD_{c,t} + PE_{c,t} QE_{c,t} \quad (59)$$

$$QXC_{c,t}|\{ce_c\} = \lambda_{c,t}^{qxcd} \left[ \gamma_{c,t}^{qxcd} QD_{c,t}^{\rho_{c,t}^{qxcd}} + (1 - \gamma_{c,t}^{qxcd}) QE_{c,t}^{\rho_{c,t}^{qxcd}} \right]^{\frac{1}{\rho_{c,t}^{qxcd}}} \quad (60)$$

where  $QD_{c,t}$  is volume of sales in the domestic market and  $PD_{c,t}$  is its producer price,  $QE_{c,t}$  is volume of exports and  $PE_{c,t}$  is its price in domestic currency. The price of composite commodity ( $PXC_{c,t}$ ) is defined over the set of domestically produced commodities because supply of the composite commodity ( $QXC_{c,t}$ ) comes from the domestic producers. If it is not produced by domestic producers, its price is equal to zero,  $PXC_{c,t}|\{cpn_c\} = 0$ . The FOC of this problem yields

$$QE_{c,t}|\{ce_c\} = \left[ \frac{PE_{c,t} \gamma_{c,t}^{qxcd}}{PD_{c,t} (1 - \gamma_{c,t}^{qxcd})} \right]^{\sigma_{c,t}^{qxcd}} QD_c \quad (61)$$

The volume of export is a positive function of its price denominated in domestic currency. Equations (60) and (61) are defined over the set of export commodities only. This is because it is highly improbable that commodities are imported solely for the purpose of exporting them. For commodities that are not in the subset of export commodities, the volume of their domestic demands are equal to their supply

$$QD_{c,t}|\{cen_c\} = QXC_{c,t} \quad (62)$$

By following the convention in the literature, it is assumed that domestically produced commodities that are sold in the domestic market ( $QD_{c,t}$ ) and imports ( $QM_{c,t}$ ) are imperfect substitutes of each other. These two generate a composite commodity  $QS_{c,t}$  via a CES function.

$$QS_{c,t}|\{cp_c \text{ and } cm_c\} = \lambda_{c,t}^{arm} \left[ \gamma_{c,t}^{arm} QD_{c,t}^{-\rho_{c,t}^{arm}} + (1 - \gamma_{c,t}^{arm}) QE_{c,t}^{-\rho_{c,t}^{arm}} \right]^{\frac{-1}{\rho_{c,t}^{arm}}} \quad (63)$$

The FOC of the following cost minimisation problem yields the optimal volume of import demand,  $QM_{c,t}$ .

$$PQS_{c,t} QS_{c,t} = PD_{c,t} QD_{c,t} + PM_{c,t} QM_{c,t} \quad (64)$$

$$QM_{c,t}|\{cp_c \text{ and } cm_c\} = \left[ \frac{PD_{c,t} (1 - \gamma_{c,t}^{arm})}{PM_{c,t} \gamma_{c,t}^{arm}} \right]^{\sigma_{c,t}^{arm}} QD_{c,t} \quad (65)$$

where  $PQS_{c,t}$  is purchaser price of composite commodity  $c$ .  $PM_{c,t}$  is import price of commodity  $QM_{c,t}$  including tariffs and denominated in domestic currency.

Equation (63) is defined over both sets of domestically produced ( $cp_c$ ) and imported ( $cm_c$ ) commodities. For commodities either not domestically produced but imported ( $PD_{c,t}$  is undefined) or domestically produced but not imported ( $PM_{c,t}$  is undefined), total domestic demand is linear summation of imports and domestic sales

$$QS_{c,t}|\{cpn_c \text{ and } cm_c\} \text{ OR } \{cp_c \text{ and } cmn_c\} = QM_{c,t} + QD_{c,t} \quad (66)$$

## 4.1 Margins

Trade and transportation services are necessary to deliver commodities from factories and docks to markets. Producer prices,  $PD_{c,t}$ , do not comprise the cost of these margins since these are not part of the production process. These costs are paid by final users of commodities and are included in purchaser prices,  $PQS_{c,t}$ . Since a commodity is produced by several activities and the cost of trade and transportation margins is paid by consumers, margins are demanded by commodities. Each commodity demands margin  $m$  as a fixed fraction,  $marg-d_{m,c}$ , of its total composite supply, and the total volume of these demands is equal to the total supply of margins,  $QSTM_{m,t}$ :

$$QSTM_{m,t} = \sum_c marg-d_{m,c} QS_{c,t} \quad (67)$$

where the set  $m$  stands for margins and consists of a single element as we have information only on the total of trade and transportation margins in the national accounts. Then, margin demand in terms of commodities is a fixed fraction of this supply, i.e. via Leontief technology, and its price is simply equal to the weighted average of the commodities' purchaser prices

$$QDTM_{c,t} = \sum_m marg-s_{c,m} QSTM_{m,t} \quad (68)$$

$$PTM_{m,t} = \sum_c marg-s_{c,m} PQD_{c,t} \quad (69)$$

where  $QDTM_{c,t}$  is margin demand of commodity  $c$ ,  $marg-s_{c,m}$  is the share parameter of commodity  $c$  in total margin supply of  $m$  and  $PTM_{m,t}$  is margin price.

## 5 Quantities

The total intermediate demand of commodity  $c$  is simply equal to the summation of activities' intermediate demands, eq. (70), and total private, i.e. household, demand for commodity  $c$  is the summation of all households' demands, eq. (71).

$$QINT_{c,t} = \sum_a INT_{c,a,t} \quad (70)$$

$$TOTPRCON_{c,t} = \sum_{hh} CD_{c,hh,t} \quad (71)$$

$$GDP_t = \sum_a (PVA_{a,t} VA_{a,t} + PRODTAXS_{a,t}) + \sum_c (SALTAX_{c,t} + CTAXS_{c,t} + EXPTAXS_{c,t}) \quad (72)$$

The total value of the gross domestic product,  $GDP_t$ , by the value-added approach is equal to the summation of the value added in each sector and indirect taxes on production activities, sales of commodities and international trade.

## 6 Prices

Here we discuss the various prices used in the model. The export price of commodity  $c$  denominated in domestic currency received by the exporter,  $PE_{c,t}$ , is equal the product of world price of commodity  $c$ ,  $PWE_{c,t}$ , exchange rate,  $ER_t$  and export tax rate,  $exptax_c$ .

$$PE_{c,t}|_{\{ce_c\}} = PWE_{c,t} (1 - exptax_{c,t}) ER_t \quad (73)$$

If the value of  $exptax_{c,t}$  is positive (negative), the domestic price is lower (higher) than the international price, so it represents a tax (subsidy). World prices are conventionally defined as fixed parameters without a time subscript. However, due to the quite volatile pattern of international energy prices, the time subscript is added to allow us to give the time sequence of the prices.

The import price of commodity  $c$  denominated in domestic currency,  $PM_{c,t}$ , is equal the product of the world price of commodity  $c$ ,  $PWM_{c,t}$ , exchange rate,  $ER_t$ , and tariff rate,  $tariff_{c,t}$ .

$$PM_{c,t}|_{\{cm_c\}} = PWM_{c,t} (1 + tariff_{c,t}) ER_t \quad (74)$$

Since Ireland is a member of the Customs Union, the trade flows between the EU member states are exempted from import tariffs. On the other hand, the main trade partners of Ireland are the UK<sup>10</sup> (one-third and one-sixth of the total merchandise imports and exports, respectively) and the US (one-tenth and more than one-fifth of the total merchandise imports and exports, respectively) but tariff rates are quite low for the majority of commodities. In line with this structure of the foreign trade flows, data on tariff revenues of the government are not officially available. Therefore, tariff revenues and thus the tariff rates on imported commodities are assumed to be zero.

$$PQS_{c,t} QS_{c,t} = PM_{c,t} QM_{c,t} + PD_{c,t} QD_{c,t} \quad (75)$$

The value of total domestic demand by producer prices,  $PQS$ , is equal to the values of imports and domestic sales of commodities. However, sales tax, carbon tax, and trade and transportation margins are paid by consumers and they are included in the purchaser price,  $PQD_c$ , of commodity  $c$ .

$$PQD_{c,t} = (PQS_{c,t} + \overline{PCAR}_{c,t} carcon_c \alpha_c) (1 + stax_{c,t}) + \sum_m marg_{d_{m,c}} PTM_{m,t} \quad (76)$$

<sup>10</sup> The weight of the UK in total Irish international trade flows will be more important after full exit of the UK from the EU.

Equation (77) shows the material balance of commodity supply and demand. The LHS is equal to total supply of composite commodity  $c$  that is demanded for several purposes including trade and transportation margins.

$$QS_{c,t} = QINV_{c,t} + TOTPRCON_{c,t} + QINT_{c,t} + CG_{c,t} + QDTM_{c,t} \quad (77)$$

The solution of  $PXAC_{a,c,t}$ , the price of commodity  $c$  produced by activity  $a$ , comes from the solution of the problem defined by equations (56) and (57). Equation (78) is defined over the set of non-homogeneous commodities and of commodities produced in the base case, i.e.  $SAM_{a,c}$  is non-zero. The price of activity commodities for homogeneous commodities produced in the base case is equal to their composite domestic commodity price, eq. (79), and is equal to zero for the other cases.

$$PXAC_{a,c,t} |_{\{cp_c \text{ and } ch_c\}} = PXC_{c,t} QXC_{c,t} \gamma_{a,c,t}^{qxc_s} QXAC_{a,c,t}^{(-\rho_{c,t}^{qxc_s} - 1)} \sum_a \left[ \gamma_{a,c,t}^{qxc_s} QXAC_{a,c}^{-\rho_{c,t}^{qxc_s}} \right]^{-1} \quad (78)$$

$$PXAC_{a,c,t} |_{\{ch_c\}} = PXC_{c,t} \quad (79)$$

There are two more prices: the price of composite commodity  $c$  produced by domestic producers,  $PXC_{c,t}$ , and price of value added,  $PVA_{a,t}$ . These prices are solved by equations (59) and (22), respectively.

Since it is assumed that activities produce multiple products and eq. (57) is used in the multiple product determination problem, using it to solve for the price of output,  $PX_{a,t}$ , is not an option. Therefore, another variable namely  $\gamma_{a,c,t}^{qxacqx}$  is introduced as

$$QXAC_{a,c,t} = \gamma_{a,c,t}^{qxacqx} QX_{a,t} \quad (80)$$

Note that this equation does NOT imply that multiple product determination is fixed share of total output in which case  $\gamma_{a,c,t}^{qxacqx}$  must be a parameter rather than a variable. Then, the output price is equal to

$$PX_{a,t} = \sum_c \gamma_{a,c,t}^{qxacqx} PXAC_{a,c,t} \quad (81)$$

Price of (physical) investment is uniform across activities due to the fact that we can only observe the breakdown of total investment expenditures across commodities. To make this price activity-specific, we have to know the distribution of the activity's physical investment across commodities. In other words, we need to know the capital composition matrix of activity, which is not readily available in almost all countries.

$$PPSI_t = \prod_c PQD_{c,t}^{jworg} \quad (82)$$

Equations (83) and (84) define consumer price index (CPI) and producer price index (PPI), respectively. The commodity weights of the CPI are equal to shares of commodities in total private consumption

while the weights in the PPI are shares of commodities in total domestic production.

$$CPI_t = \sum_c wgt_{c,t}^{cpi} PQD_{c,t} \quad (83)$$

$$PPI_t = \sum_c wgt_{c,t}^{ppi} PD_{c,t} \quad (84)$$

$CPI_1$ , the value of consumer price index in the first period, is chosen as the *numéraire* of the system, i.e. all prices are solved for *relative to* the  $CPI_1$ .

## 7 Energy

The flow of emissions through the economy is captured in I3E by explicitly including energy/carbon commodities. The I3E model includes the following energy commodities: peat, coal, crude oil, gasoline, kerosene, fuel oil, liquid petroleum gas, diesel, electricity, natural gas, and other petroleum products. Crude oil and fuel oil are not subject to private consumption, i.e. households do not demand these energy commodities. The consumption of energy commodities regardless of the purpose causes CO<sub>2</sub> emissions, with the exception of crude oil (which is transformed into other energy commodities) and electricity (which is generated using energy commodities). The parameter  $carcon_c$  takes a positive value for these energy commodities, which cause emissions and reflects the amount of emissions released per unit of the carbon commodity in Mt of CO<sub>2</sub>. For the other commodities defined in the model, the value of  $carcon_c$  is equal to zero. Using this approach, the level of emissions needed to produce non-energy commodities can also be calculated. The level of emission for each energy commodity is calculated as

$$emis_{c,t}|_{\{carcon_c\}} = carcon_c QS_{c,t} \quad (85)$$

The economy-wide total emissions is the summation of emissions per commodity

$$emis_{tot_t} = \sum_c emis_{c,t} \quad (86)$$

The economy-wide total emissions consists of emissions caused by activities in the production process (based on the intermediate input demand, eq. 25), households (based on the consumption demand, eq. 87), and government (based on the public demand, eq. 88).

$$emis_{hh_{hh,t}} = \sum_c carcon_c CD_{c,hh,t} \quad (87)$$

$$emis_{gov_t} = \sum_c carcon_c CG_{c,t} \quad (88)$$

Equation (89) calculates the total residential emissions which includes private demand of energy commodities for residential heating purposes, i.e. the commodities under the nest of residential heating (*RHC*) on Figure 1. Households' total emissions can be distinguished as residential and non-residential emissions in order to measure the effects of policy shocks on their behaviours regarding private consumption for different purposes.

$$emis\_hhres_{hh,t} = emis\_hh_{hh,t} - \sum_{SPRV_c} carcon_c CD_{c,hh,t} \quad (89)$$

where  $SPRV_c$  is a subset of energy commodities demanded for private transportation purposes and comprises *TDie* and *GAL*, under the nest of *PRV* on Figure 1.

## 8 Economic Growth

Economic growth has three sources: the growth of employment due to population growth, the growth in capital stock driven by investment, and the growth in total factor productivity, which is known as the Solow residual. This is the component of the economic growth that is not explained by growth in the factors of production.

As stated by [Acemoglu \(2009\)](#), if the technological progress is not of the Harrod-neutral form with any constant returns to scale (CRS) production function, per capita or aggregate variables cannot grow at a constant rate. This is referred to as a balanced growth path (BGP). The type of technological progress (capital augmented – Solow-neutral, labour augmented – Hicks-neutral) matters if the CRS production function is in a CES form in which the elasticity of substitution between capital and labour is not unity. If it is equal to 1, which corresponds to a Cobb-Douglas type of production function, each technological progress produces identical results ([Acemoglu, 2009](#), 71–2). Since the value added function is assumed to be in the form of CES, the technical change has to be labour-augmented.

It is assumed that the total population grows at a constant rate,  $n_t$ , and the technology, i.e. the productivity of the labour force, grows at a constant rate,  $g_t$ .<sup>11</sup> In mathematical terms,

$$\begin{aligned} POP_{t+1} &= POP_t (1 + n_t) \\ BX_{t+1} &= BX_t (1 + g_t) \end{aligned}$$

where  $POP_t$  is the total population at period  $t$  and  $BX_t$  is the productivity level of the labour force at period  $t$ . In the case of labour-augmented technological growth, the growth of the productivity level of labour implies that the economy operates as if she has more labour ([Acemoglu, 2009](#), 69).

The balanced growth path in a discrete time model requires that a variable should evolve as follows:

$$x_{t+1} = x_t (1 + n_t) (1 + g_t) = x_t (1 + n_t + g_t + n_t g_t) = x_t (1 + grw_t)$$

<sup>11</sup> In the current version, all growth rates are uniform across the model horizon. The time subscript is added to allow us to incorporate different growth rates through time.



where  $grw_t$  is the economic growth rate. In the current version, the values of  $n$  and  $grw$  are retrieved from the medium-run estimates of the macroeconomic forecast model of the ESRI, namely COSMO (COre Structural MOdel for Ireland), [Bergin et al. \(2017\)](#). Accordingly,  $n_t$  and  $grw_t$  are calibrated as 0.8% and 3.3%. These population and economic growth rates imply 2.48% growth rate of the labour productivity.

## 9 Saving–Investment

Private, i.e. household, savings are determined endogenously as explained in the previous sections.

$$TOTPRSAV_t = \sum_{hh} SAV_{hh,t} \quad (90)$$

Government savings,  $GSAV_t$ , are endogenously determined via the government budget constraint and the enterprise savings,  $ESAV_t$ , are a fixed fraction of the total net-of-corporate tax profits, eq. (35).

In the absence of financial markets, a CGE model is able to solve for one of the following: foreign trade balance/savings, nominal exchange rate, and price index/real exchange rate ([Robinson, 1989](#), 921). Here, foreign savings,  $\overline{FSAV}_t$ , are assumed to be fixed, and the consumer price index's value in the first period is fixed, making it possible for the model to solve for the nominal exchange rate which equates to the foreign exchange movements. The value of total savings in the economy is

$$TOTSAV_t = TOTPRSAV_t + GSAV_t + ESAV_t + \overline{FSAV}_t ER_t \quad (91)$$

In the equilibrium, the sum of investment expenditures of activities, i.e. investment by destination, must be equal to the total demand of commodities for investment purposes, i.e. investment by origin.

$$TOTINV_t = \sum_a INV_{a,t} \quad (92)$$

$$PQD_{c,t} QINV_{c,t} = \gamma_{c,t}^{invorg} TOTINV_t \quad (93)$$

where  $\gamma_{c,t}^{invorg}$  is the share of commodity  $c$  in total volume of investment and  $QINV_c$  is the volume of investment demand for commodity  $c$ .

## 10 Equilibrium

For the economy to be in equilibrium, all markets must be in equilibrium. In the relevant sections, the price and quantity equilibrium conditions were discussed. Here we discuss the labour market and foreign market equilibrium conditions. The labour market equilibrium, eq. (94), solves for wage rates of each type of labour for the fixed supply of labour.

$$\overline{LSUP}_{l,t} = \sum_a LD_{a,l,t} \quad (94)$$

The foreign market equilibrium, eq. (95), ensures foreign exchange supply and demand and solves for the equilibrium exchange rate.

$$\sum_c PWE_{c,t} QE_{c,t} + \sum_{hh} \overline{NFI}_{hh,t} + \overline{FSAV}_t = \sum_c PWM_{c,t} QM_{c,t} + GFINT_t + \frac{\sum_a CETSa,t \cdot 0.5}{ER_t} \quad (95)$$

where the last item on the RHS of eq. (95) is the half of the cost of the ETS in foreign currency.

Up to this point, the number of endogenous variables and the number of variables is equal to each other, and there is no need to add an equation that shows investment-saving balance. If the system of equations is defined correctly, i.e. if there is no error in the model, investment and savings will be equal to each other by definition. However, rather than dropping this equation, it is conventionally associated with a slack variable, namely Walras. If there is no error in the model,  $WALRAS_t = 0$  should hold for every period  $t$  and experiment.

$$TOTSAV_t = TOTINV_t + WALRAS_t \quad (96)$$

## Appendix A Lists of Activities and Commodities

Table A.1: Activities

Abbreviation	Name	NACE Codes	$\sigma_{BEN}$	$\sigma_{FUE}$	$\sigma_{BH}$	$\sigma_{OTE}$
ACC	Accommodation and Hotel Services	55-56,79	1.5	0	0	
AGR	Agriculture	1-3	1.5	0	0	
ATS	Air Transportation	51	0	1.3	1.3	
BFM	Basic Metal Manufacturing	24-25	1.5	1.3	1.3	
BPP	Basic Pharmaceutical Products	21	1.5	0	0	
CHE	Chemical Products	20	1.5	1.5	1.5	
CON	Construction	41-43	1.5	0	1.5	
EDU	Education Sector	85	1.5	1.3	1.5	
FBT	Food, Beverage and Tobacco	10-12	1.5	1.5	1.5	
FSR	Financial Services	64-66,77	1.5	0	0	
HHS	Health Sector	86-88	1.5	1.3	1.3	
HTP	High-Technology Products	26-28	1.5	1.5	1.5	
LTS	Land Transportation	49	1.3	0	1.3	
NGS	Natural Gas Supply		0	1.3	0	
OIN	Other Industrial Products	17,18,33	1.5	1.5	1.3	
OMN	Other Mining Products		1.5	1.3	1.5	
ONM	Other Non-metallic Products	23	1.5	1.3	1.5	
OTM	Other Manufacturing	31-32	1.3	1.5	0	
PEA	Peat		1.5	1.3	1.5	
PET	Petroleum		0	0	0	
PUB	Public Sector	84	1.5	1.3	1.5	
RES	Real Estate Services	68	1.5	0	0	
RUP	Rubber and Plastic Products	22	1.5	1.5	1.3	
SER	Other Services	remaining*	1.5	0	0	
TEL	Telecommunication Services	61	1.5	0	0	
TEX	Textile	13-15	1.5	1.5	1.3	
TRD	Trade	45-47	1.5	1.5	0	
TRE	Transportation Equipment	29-30	1.5	1.5	0	
WAT	Water and Sewerage	36,37-39	1.5	1.5	1.3	
WTS	Water Transportation	50	0	0	0	
WWP	Wood and Wood Products	16	1.5	1.3	0	
ELC	Electricity		1.2			1.5

\*: It excludes NACE codes 5-9 (Mining, Quarrying and Extraction), 19 (Petroleum Products), and 35 (Electricity and Gas Supply).

*Note:* The activities without NACE codes are further disaggregated sectors.

**Table A.2: Commodities**

AGR	Agriculture	BFM	Basic metal manufacturing
PEA	Peat	HTP	High-technology products
COA	Coal	TRE	Transportation equipment
CRO*	Crude oil	ELC	Electricity
OMN*	Other mining products	NGS	Natural gas supply
FBT	Food, beverage and tobacco	WAT	Water and sewerage
TEX	Textile	CON	Construction
WWP	Wood and wood products	TRD	Trade
OIN	Other industrial products	LTS	Land transportation
GAL	Gasoline	WTS	Water transportation
KRS	Kerosene	ATS	Air transportation
FUO*	Fuel oil	ACC	Accommodation and hotel services
LPG	Liquid petroleum gas	TEL	Telecommunication services
DIE	Diesel	FSR	Financial services
OPP	Other petroleum products	RES	Real estate services
OTM	Other manufacturing	PUB	Public services
CHE	Chemicals and chemical products	EDU	Education sector
BPP	Basic pharmaceutical products	HHS	Health sector
RUP	Rubber and plastic products	SER	Other services
ONM	Other non-metallic products		

\*: Not subject to private consumption.

## Appendix B List of Endogenous Variables

Name	Description
$ADJ_{a,t}$	Adjustment cost
$allow_{oth_{a,t}}$	Other ETS allowance to output ratio
$AtoT_{a,t}$	Allowance to total ETS emissions ratio
$CD_{c,hh,t}$	Consumption by goods
$CETS_{a,t}$	Cost of ETS
$CG_{c,t}$	Consumption demand of government by sectors
$CINC_{hh,t}$	Capital (dividend) income
$CLD_{a,t}$	Composite labour demand
$corptax_t$	Corporate tax rate
$CORPTAXS_t$	Corporate tax payments
$CPI_t$	Consumer price index
$CTAXS_{c,t}$	Value of carbon tax revenues
$CW_{a,t}$	Composite wage
$DISDIV_t$	Total distributed dividends
$DIV_{a,t}$	Sectoral dividends
$emis_{act_{a,t}}$	Activity emission
$emis_{c,t}$	Commodity emission
$emis_{ETS_{a,t}}$	ETS emissions
$emis_{gov_t}$	Government emission
$emis_{hh_{hh,t}}$	Household emission
$emis_{OETS_{a,t}}$	Other ETS (process) emissions
$emis_{res_{hh,t}}$	Residential emissions of households
$emis_{tot_t}$	Economy-wide emissions
$ER_t$	Level of exchange rate
$ESAV_t$	Savings of enterprises
$ETSADJ_{c,a,t}$	ETS adjustment in unit cost for activities
$EXPTAXS_{c,t}$	Export tax
$GDP_t$	Gross domestic product
$GFDS_t$	Government foreign debt stock
$GFINT_t$	Interest payments of the government to RoW
$GOVCON_t$	Total government consumption expenditures
$GOVREV_t$	Government revenues
$GSAV_t$	Government savings
$INC_{hh,t}$	Disposable income

$INT_{c,a,t}$	Intermediate input demand of sector by good
$INV_{a,t}$	Investment by destination
$K_{a,t}$	Sectoral capital stock
$KSUP_t$	Total capital stock
$LD_{a,l,t}$	Labour demand
$PCIN_{c,a,t}$	Price of activity-specific composite commodity $c$
$PCQS_{c,hh,t}$	Price of household-specific composite commodity $c$
$PD_{c,t}$	Consumer price of domestic supply of commodity $c$
$PE_{c,t}$	Export price of commodity $c$ (including export tax) in domestic currency
$PEN_{hh,t}$	Pension income of households
$PM_{c,t}$	Price of import commodity $c$ including import tax) in domestic currency
$PPI_t$	Producer (domestic) price index
$PPSI_t$	Price of investment
$PQD_{c,t}$	Purchaser price of composite commodity $c$
$PQS_{c,t}$	Supply price of composite commodity $c$
$prodtax_{a,t}$	Production tax rate
$PRODTAXS_{a,t}$	Taxes on production payments by activities
$PSI_{a,t}$	Physical investment
$PTADJ_t$	Uniform production tax rate adjuster
$PTADJ_Pt$	Partial production tax rate adjuster
$PTM_{m,t}$	Price of margin
$PVA_{a,t}$	Price of sectoral value added
$PX_{a,t}$	Price of sectoral output
$PXAC_{a,c,t}$	Price of commodity $c$ produced by activity $a$
$PXC_{c,t}$	Price of composite domestic commodity $c$
$q_{a,t}$	Tobin $q$
$QD_{c,t}$	Total domestic supply of commodity $c$
$QDTM_{c,t}$	Total demand of margin
$QE_{c,t}$	Total export of commodity $c$
$QINT_{c,t}$	Total intermediate input demand of good
$QINV_{c,t}$	Investment demand of good
$QM_{c,t}$	Total import demand of commodity $c$
$QS_{c,t}$	Composite supply of commodity $c$
$QSTM_{m,t}$	Total supply of margin
$QX_{a,t}$	Sectoral output
$QXAC_{a,c,t}$	Production of commodity $c$ by activity $a$
$QXC_{c,t}$	Total production of commodity $c$ by domestic producers
$RCI_{hh,t}$	Revenue recycling income of hh

$SALTAX_{c,t}$	Value of sales tax revenues
$SAV_{hh,t}$	Savings
$shr\_qxaccqx_{a,c}$	Share of commodity $c$ in production of activity $a$
$STADJ_t$	Uniform sales tax rate adjuster
$STADJ_P_t$	Partial sales tax rate adjuster
$stax_{c,t}$	Sales tax rate
$TARIFS_{c,t}$	Sectoral tariff revenues
$TOTINV_t$	Total investment (expenditure)
$TOTPRCON_{c,t}$	Total private consumption by good
$TOTPRSAV_t$	Total private savings
$TOTSAV_t$	Total savings
$TPRODTAXS_t$	Total production tax collection
$TR_{hh,t}$	Transfer receipt
$TSALTAX_t$	Total sales tax collection
$TWTAXS_t$	Total wage tax collection
$VA_{a,t}$	Sectoral value added
$W_{l,t}$	Wage rate
$W\_mean_t$	Average wage
$WALRAS_t$	Slack variable for Walras law
$wgt_{c,t}^{cpi}$	Share of commodity $c$ in total private consumption = weights in CPI
$wgt_{c,t}^{ppi}$	Share of commodity $c$ in total domestic production = weights in PPI
$WINC_{hh,t}$	Wage income
$WK_{a,t}$	Price of capital
$WTADJ_t$	Uniform wage tax rate adjuster
$WTADJ_P_t$	Partial wage tax rate adjuster
$wtax_{l,t}$	Wage tax rate (including SSI premiums)
$WTAXS_{l,t}$	Wage income tax payments

## Appendix C List of Exogenous Variables and Parameters

Name	Description
<b>Exogenous Variables</b>	
$\overline{allowance}_{a,t}$	ETS allowance
$\overline{allowance\_oth}_{a,t}$	Other ETS (process) allowance
$\overline{BX}_t$	Level of technical efficiency
$\overline{FSAV}_t$	Foreign savings
$\overline{GOVCONA}_t$	Autonomous government consumption expenditures
$\overline{GTRENT}_t$	Transfers from government to enterprises
$\overline{GTRHH}_t$	Transfers from government to households
$\overline{LSUP}_{l,t}$	Labour supply
$\overline{NFI}_t$	Net factor income from abroad
$\overline{PCAR}_t$	Carbon price
$\overline{PETS}_t$	Price of ETS
$\overline{TOTPEN}_t$	Total pension transfer of government to households
$\overline{WFDIST}_{a,l,t}$	Sectoral proportion for factor prices
<b>Parameters</b>	
$\alpha_{c,t}$	Carbon tax adjustment parameter
$\beta_{hh,t}$	Discount rate
$carcon_c$	Carbon content of commodity
$\delta_{a,t}$	Depreciation rate
$emis\_oth_{a,t}$	Other ETS emissions per output for ATS and ONM
$ETStoE_{a,t}$	ETS emissions in total emissions
$exptax_{c,t}$	Export tax rate on ed commodity
$g_t$	Technology growth rate
$\gamma_{c,t}^{arm}$	Share parameter of Armington
$\gamma_{c,a,t}^{bh}$	Share parameter of CES of business heating (BH)
$\gamma_{hh,t}^{cinc}$	Share of household $hh$ in capital income
$\gamma_{a,l,t}^{cld}$	Share parameter of CES of CLD
$\gamma_{a,t}^{cva}$	Share parameter of CES of VA
$\gamma_t^{esav}$	Share of enterprises savings in net-of-tax profit
$\gamma_{c,t}^{gc}$	Share of commodity $c$ in total government demand
$\gamma_{hh,t}^{grhh}$	Share of household $hh$ in government transfers
$\gamma_{a,t}^{invdes}$	Share of investment demand (=inv. by destination) in gross profits (positive for NDM)
$\gamma_{c,t}^{invorg}$	Share of investment demand by commodity (=inv. by origin) in total investment
$\gamma_{hh,t}^{penhh}$	Share of household $hh$ in pensions



$\gamma_{a,c,t}^{qxac}$	Share parameter of CET of QX
$\gamma_{c,t}^{qxcd}$	Share parameter of CET (demand) of QXC
$\gamma_{a,c,t}^{qxcS}$	Share parameter of CES (supply) of QXC
$\gamma_{hh,t}^{rch}$	Share of household $hh$ in recycling income
$\gamma_{l,hh,t}^{wage}$	Share of household $hh$ in wage income
$grw_t$	Economic growth rate
$\lambda_{c,t}^{arm}$	Shift parameter of Armington
$\lambda_{a,t}^{qxac}$	Shift parameter of CET of QX
$\lambda_{c,t}^{qxcd}$	Shift parameter of CET (demand) of QXC
$\lambda_{c,t}^{qxcS}$	Shift parameter of CES (supply) of QXC
$\lambda_{a,t}^{va}$	Shift parameter of CES of VA
$marg_{d_{m,c}}$	Margin demand share in total composite commodity
$marg_{s_{c,m}}$	Margin supply share in total supply
$MPS_t$	Marginal propensity to spend over GDP
$n_t$	Population growth rate
$\phi_{a,t}$	Adjustment parameter of investment
$PT01_{a,t}$	Binary production tax rate adjuster
$PWE_{c,t}$	World price of exported goods
$PWM_{c,t}$	World price of imported goods
$\rho_{hh}$	Time-preference rate
$\rho_{c,t}^{arm}$	Exponent parameter of Armington
$\rho_{a,t}^{bh}$	Exponent parameter of CES of business heating (BH)
$\rho_{a,t}^{cld}$	Exponent parameter of CES of CLD
$\rho_{a,t}^{qxac}$	Exponent parameter of CET of QX
$\rho_{c,t}^{qxcd}$	Exponent parameter of CET (demand) of QXC
$\rho_{c,t}^{qxcS}$	Exponent parameter of CES (supply) of QXC
$\rho_{a,t}^{va}$	Exponent parameter of CES of VA
$RCCT_t$	Carbon tax revenue recycling switching parameter to corporate tax
$RCGC_t$	Carbon tax revenue recycling switching parameter to government consumption
$RCHH_t$	Carbon tax revenue recycling switching parameter for households
$RCPT_t$	Carbon tax revenue recycling switching parameter to production tax
$RCST_t$	Carbon tax revenue recycling switching parameter to sales tax
$RCWT_t$	Carbon tax revenue recycling switching parameter to wage tax
$r_f t$	Foreign interest rate
$\sigma_{c,t}^{arm}$	Elasticity of Armington
$\sigma_{a,t}^{bh}$	Elasticity of CES of business heating (BH)
$\sigma_{a,t}^{cld}$	Elasticity of CES of CLD
$\sigma_{a,t}^{qxac}$	Elasticity of CET of QX

$\sigma_{c,t}^{qxcd}$	Elasticity of CET (demand) of QXC
$\sigma_{c,t}^{qxcS}$	Elasticity of CES (supply) of QXC
$\sigma_{a,t}^{va}$	Elasticity of CES of VA
$ST01_{c,t}$	Binary sales tax rate adjuster
$tariff_{c,t}$	Tariff rate on imported commodity
$\theta_{hh}$	Intertemporal elasticity of substitution
$WT01_{l,t}$	Binary wage tax rate adjuster

## Appendix D How do activities internalise the cost of the ETS?

The value of production is given by eq. (22). By definition, the total cost of the intermediate input demand is equal to the total cost of the composite intermediate inputs (VA, BEN, OTI) which, in turn, is the sum of the cost of intermediate inputs (VA is equal to the cost of the factors of production, OTI is equal to the cost of the non-energy intermediate inputs, etc.). Plugging equations (41) and (29) into eq. (22) results in

$$PX_{a,t} QX_{a,t} = PVA_{a,t} VA_{a,t} + prodtax_{a,t} PX_{a,t} QX_{a,t} + \sum_c PQD_{c,t} INT_{c,a,t} + \overline{PETS}_t (emis\_ETS_{a,t} + emis\_OETS_{a,t} - \overline{allowance}_{a,t} - \overline{allowance\_oth}_{a,t}) \quad (D.1)$$

By further plugging equations (25–28)

$$(1 - prodtax_{a,t}) PX_{a,t} QX_{a,t} = PVA_{a,t} VA_{a,t} + \sum_c PQD_{c,t} INT_{c,a,t} + \overline{PETS}_t \sum_c [carcon_c INT_{c,a,t} ETStoE_{a,t} (1 - AtoT_{a,t})] + \overline{PETS}_t QX_{a,t} emis\_oth_{a,t} - allow\_oth_{a,t} QX_{a,t} \quad (D.2)$$

The second line of eq. (D.2) is the cost of ETS concerning the emissions of an activity based on its energy consumption, and the third line is the cost of ETS regarding the process emissions based on the level of output.

Rearranging yields

$$(1 - prodtax_{a,t}) PX_{a,t} QX_{a,t} = PVA_{a,t} VA_{a,t} + \sum_c (PQD_{c,t} + ETSADJ_{c,a,t}) INT_{c,a,t} + \overline{PETS}_t (emis\_oth_{a,t} - allow\_oth_{a,t}) QX_{a,t} \quad (D.3)$$

where  $ETSADJ_{c,a,t}$  is given by eq. (24). The existence of the ETS hence affects the unit cost of the intermediate input.

## **Appendix E Updates Between the Model Versions**

### **E.1 Major differences**

- In the previous model version along the baseline, households had received carbon recycle income from the government. Based on a discussion with the Department of Finance, there is no such transfer in the current policy scheme. Therefore, the baseline of the model has been updated accordingly.
- The purchaser price calculations have been updated. The carbon tax no longer enters linearly but as a unit tax. It is first added to the producer price and then the sales tax is calculated based on this price.
- The SAM has been updated mainly regarding the composition of energy demand by activities based on the CSO Business Energy Use Survey of 2014.
- In the previous version, government transfers to households had been introduced as a welfare transfer, and its fixed real value had been indexed to the CPI. In this version, firstly, the total government transfers have been disaggregated into government welfare transfers and pension transfers. Moreover, the fixed real values of both transfers, based on a discussion with the Department of Finance, have been indexed to average wage.
- The sectoral composition of employment by skill types has been updated by using the Labour Force Survey.

### **E.2 Extensions**

- The EU ETS has been introduced.
- The households sector has been disaggregated into ten RHGs based on disposable income with five groups in urban areas and five groups in rural.
- Labour has been disaggregated into three skill groups: low-, medium-, and high-skilled labour.
  1. Each household receives wage income from each type of labour depending on its share in total wage income per type of labour.
  2. Firms firstly choose the composite labour input, which is a CES aggregate of the types of labours. Then they decide on the composition of labour concerning skills.
- A carbon recycling policy mechanism has been developed where various carbon tax revenue recycling experiments can be run.

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