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THE ECONOMIC AND SOCIAL RESEARCH INSTITUTE

AN INTERDISCIPLINARY APPROACH TO THE MEASUREMENT OF UTILITY OR WELFARE

Fifth Geary Lecture, 1972

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*An Interdisciplinary Approach to the Measurement of Utility or Welfare**

1. *Nature of this Essay*

(*a*) The prevailing opinion of economists today on the utility function is one of *agnosticism*. Usually it is held that utility cannot be measured. This is comparable to the attitude of physicists *vis-à-vis* heat before the concepts of temperature and quantity of heat were introduced. The idea that utility cannot be measured is unsatisfactory. It is unsatisfactory because utility is the basic concept of economic theory and represents something that should enter into the set of aims of socio-economic policy. In particular, comparison of the utility experienced by various groups in a population is desirable once we admit that we must aim at some type of social optimality or of social justice. We must consider present-day practice concerning these matters as pre-scientific and try to deal with them in a scientific rather than an intuitive way. Today's practice is, at best, an attempt by politicians to compare intuitively the utilities experienced by various groups and attain as satisfactory a situation as possible, especially with regard to income distribution [5]. At worst that practice is only seen, by many critics of today's societies, as the outcome of a struggle for power between various classes or groups in society.

(*b*) In this essay I will understand by an individual's *utility* or welfare his happiness in so far as this is dependent on social elements,

*I want to express my sincere thanks to my long-time friend, Dr J. B. D. Derksen who, in personal discussions, contributed to what I have to say in this text. All errors are mine, of course.

leaving out such very personal elements of a religious, cultural or psychological character as art or love. Expressed positively, such elements, also psychological, as satisfaction from consumption (personal and collective), from work and from effort, both absolutely and in comparison to others, will be considered elements making for social happiness or welfare.

(*c*) Economists normally consider that the content to be given to such concepts as social welfare and justice is not part of their science. Often they say that statements on such matters imply *value judgements* or are the subject of ethics or morals. The way I see for tackling the problem of the definition and measurement contains not only such *ethical* aspects but also elements of a *methodological* nature. The latter may also be considered as a subject of the philosophy of science. I see my attempt as essentially scientific and in a sense, therefore, objective; it is at least a first step along this road. But, for the reasons just set out, it cannot be a chapter of economics only. Essentially it is *interdisciplinary*, in that it contains some elements of an ethical, and others of a methodological, nature. *Objectivity*, in this context, is reached if different scholars agree on the content to be given to the underlying concepts and if repeating the measurements involved produces essentially the same result. Objectivity is close to accuracy, if measurement is at stake; but no series of repeated measurements yields exactly the same results and we can only hope for agreement among a large *majority* of scholars. This majority is larger in, say, physics than in socio-metrics, because of the nature of the subjects; but it never amounts to unanimity.

(*d*) In the good econometric tradition set out in so masterly a way by R. C. Geary [1], I am presenting not only a theory in the form of a refutable hypothesis, but also a few *applications*. The latter will constitute by necessity some very simple first steps along the road indicated. From the preceding remarks, the reader will also understand that in this essay I assume the attitude, common to scientific workers, of mentioning explicitly all assumptions made, including, as does Gunnar Myrdal [3], the *explicit* formulation of the value judgements used, even though I present them as contributions to the realm of ethics. On many points the ideas presented here are close to those of Kolm [2].

2. *Elements Entering into the Utility (or Welfare) Concept*

Utility will be defined as a measurable function based on the contributions mentioned in Section 1. This function contains three types of elements to be called variables, parameters and coefficients.

(*a*) *Variables* represent phenomena that can assume different numerical values, sometimes by the individual's own choice, sometimes beyond his personal choice as when they reflect natural or social conditions. Examples are the figure or figures representing the nature of the job chosen and the consumable income going with it, as the consequence of salary and wage scales and tax rates. In a more detailed treatment of the subject, the quantities consumed of each of a number of items may also belong to this class of elements. Such details will not, however, be considered here.

(*b*) *Parameters* are figures characterising an individual's *nature;* among them appear his productive *abilities* as well as his *needs.* Examples of the former are his IQ, and other innate data such as the ability to guide other persons; examples of the latter are family size (a datum in the short run at least), and the individual's health or tastes. In the long run only *innate* characteristics should be among the parameters. Capabilities which can be attained by a learning process are among the variables; but the easiness by which this adaptation takes place again constitutes a parameter.

(*c*) *Coefficients* are numbers, dependent on the units in which utility is expressed as well as on those in which the variables or parameters are measured, indicating the effect of changes in variables or parameters on utility or welfare. Examples are the influence of a unit more of consumption on the individual's welfare, or the influence exerted on welfare by a given change in job. In some theories also the impact of an hour of additional effort will be indicated by a coefficient. Coefficients are constants; if a given variable exerts a curvilinear influence on an individual's utility, more than one constant or another mathematical form may be needed; for instance, a logarithmic form of the utility function may be sufficient.

3. *Methodological and Ethical Principles Proposed*

(*a*) In order to arrive at an objective theory of the measurement of utility the methodological and ethical principles to be introduced must appeal to a large majority of scientists and citizens. The methodological principles can best be submitted to scientists (including here not only natural, but also social and human scientists); the ethical principles must appeal to a large majority of citizens. Both requirements imply that the principles to be used must be of a rather general nature, so as to unite many; or to put it somewhat differently, they should express what unites people rather than what separates them. The implication of the approach is the assumption that there exist such almost universal principles. I purposely speak only of large majorities and not of principles accepted by all. Even in *physics*, large parts of which are considered to be objective, examples occur where unanimity does not exist. The accepted measurement of temperature by the degree of expansion of a number of substances constitutes an example. While almost all substances expand regularly upon the successive application of equal quantities of heat, in many temperature ranges exceptions to this behaviour occur. They occur wherever some substances change from the solid into the liquid phase (melting point) or from the liquid into the gas phase (point of evaporation) and they also occur on other occasions; thus, between 0^0 and 4^0 centigrade water contracts instead of expanding. In human matters we intuitively expect larger differences in behaviour and hence should not hesitate to call a method or statement objective even if the majority is less than in physics.

(*b*) The methodological principle I propose to apply is the one common to all natural sciences, including biology, namely that we select, from the theories conceived, the *simplest that does not contradict observation.* As soon as new observations are made which do not fit that theory, we choose, from the remaining theories, the simplest fitting the extended body of evidence. The process of testing a theory against observations requires that theories be formulated as "refutable hypotheses", an expression current in statistical testing [1]. Seemingly qualitative statements or propositions can often be transformed into quantitative form and so obtain the form of refutable hypotheses. Theories that do not permit such a

transformation will not, as a rule, be of much use to the aim of defining an optimal social order or social justice.

(*c*) The ethical principle I propose to introduce will be called the *principle of the fundamental equality of man.* The principle will be interpreted as meaning that we assume human beings to be equal except for those aspects where measurement has shown them to be unequal. This interpretation is eminently scientific in that it accepts the results of any generally accepted (or not disqualified) observations. It is scientific also in the sense that it introduces something which distinguishes man from other living beings. It is in the best of ethical traditions, where time and again, during the cultural history of man, some aspect of equality has been assumed to exist. In Christian theology the phrase "equality before God" is among the fundamental concepts concerning man. Among human rights, we find the legal maxim of "equality before the law", implying equal punishment for equally illegal acts; and also the rejection of many forms of discrimination. Equal voting rights are given to all citizens in a long list of countries. According to some critics in some social systems some people are more equal than others; and in some Swiss cantons women are only on their way to franchise. But, as I warned above, we cannot expect complete unanimity in human matters. The precise content to be given to the principle of equality of man unless inequality can be proven is that the *coefficients of the utility functions of all individuals are identical.* Every new observation of inequality gives rise to the introduction of a new parameter into the utility function.

(*d*) The two main principles proposed open up a road leading to the continual refinement of measured utility functions, once a start has been made. One possible start will be presented in this essay, fitting the main facts of observed income distribution in the Netherlands reasonably well. Programmes of further research, theoretical as well as statistical, will be proposed. Concrete, though very provisional, content will be given to the concepts of an optimal social order and of social justice on the basis of this start in utility measurement.

4. *Initial Hypotheses Made on the Utility Function*

(*a*) The results shown here may be seen as an elaboration of

previous work dealing with the theories of income distribution, positive and normative [4], and of the optimal social order [5] on the one hand and empirical work on income distribution and its determinants [6] on the other. The statistical material available for the Netherlands, not very different from that for some other countries, severely limits the degree of refinement of this first exercise, but still provides a first example of the feed-back from observation to theory. The only example of a parameter used in the first verification of the theory offered above is the *level of schooling* attained. While strictly speaking this level does not represent a parameter, it was taken as such as a first approximation. For a considerable number of professional, industrial and social classes figures are available on the frequency distribution of schooling level attained and of average incomes (male working population over 25 years old). The level attained was measured in units of three years' successful school attendance, ranging from 2 (primary school completed) to 6 (university master degree obtained), called v, and was supposed to constitute the parameter for each group, whereas the upper quartile s was supposed to represent simultaneously the *level required* and the *variable* characterising the *job chosen.* For some ten larger social groups comparable American figures are available and the actual American level of schooling v' (median) used as a test of the required level in the Netherlands. There was a reasonable correlation between s and v'. Since in some previous work quoted already the level of schooling was found to be able to explain the larger part of income differences, an attempt only to use this one parameter v together with the corresponding variable s was not considered hopeless beforehand.

(*b*) On the basis of the methodological principle of simplicity the first assumption made on the mathematical shape of the utility function for a person i of the active population was*

$$\omega_i = ln\{x_i - c_0 s_i - c_1 v_i - \tfrac{1}{2}c_2(s_i - v_i)^2\} \qquad (4.1)$$

where x_i represents consumable income (income after tax) in hfl 1000 per annum;

s_i required level of schooling (in units of 3 years);
v_i actual attained level of schooling, in the same units;

*Unlike the other work quoted, I neglected family size as a possible parameter in this essay.

and c_0, c_1, and c_2 represent coefficients. The expression { } may be referred to as consumable income corrected for some inconveniences, to be discussed below. The choice of the mathematical form of the (natural) logarithm expresses a well-known psychological assumption, namely that equal percentage increases in (corrected) income produce equal absolute increases in welfare. The choice also appears to simplify the problem of the optimal social order (cf. Section 7). The form of the impact of increases in s_i and ν_i was assumed to be quadratic, as a first approximation. The linear terms $c_0 s_i$ and $c_1 \nu_i$ are the simplest way of representing a monotonic impact. A positive value of c_0 indicates increasing sacrifices needed for jobs which require increased schooling; a negative sign is also possible, however, and indicates increasing pleasure in work with jobs requiring more schooling. It is uncertain whether a coefficient c_1 should be introduced at all, since nobody can experience a change in ν_i, representing here an innate property. We will return to this question later (Section 6). Considerable psychological realism underlies the introduction of the quadratic terms in the special form of the square of the difference $s_i - \nu_i$, to be indicated by the *tension* between the ability required from the individual and the ability possessed by him. The essence of the quadratic terms is that an individual feels happiest if required and possessed capabilities coincide, other things assumed equal, and that both a positive and a negative tension make the individual less happy—about equally so for equal positive and negative tensions. Since large deviations between s_i and ν_i are hardly possible, the form of the square seems to be a sufficient approximation.

(*c*) Whether all coefficients introduced can be measured depends further on the *structure of the model* used to describe the operation of the economy (or society) considered. The well-known phenomenon of identification has to be used in order to find out which coefficients of a model can and which cannot be estimated.

Moreover, any measurement, in order to be acceptable, requires a set of sufficiently high correlation coefficients obtained in the process of testing. A theory must be rejected if some or one of the correlation coefficients are below some assumed standard. The choice of such a standard is subjective. My personal preference is for multiple correlation coefficients around 0·85, implying that more than two-thirds of the observed variance in the variable to be

explained can be accounted for by the relation considered. The standard to be chosen also depends, however, on the form of the relation tested and may be set lower for relations explaining differences or small balances than for relations containing absolute values of the variables involved.

5. *The Need for Further Data*

(*a*) As already observed, the examples to be given in this essay (Sections 6 and 7) can be seen only as an initial attempt to illustrate the approach. A continuous series of improvements in both the model and the figures used is desirable and constitutes a natural follow-up which I hope will develop. Part of this series is being prepared already. For the time being our efforts are limited by the availability of data. Three stages in the use of better data can be suggested. Improvements can be undertaken by more extensive use of *existing data*. Existing data include data for other countries, for incomes and schooling as well as for a factor to be discussed later, namely the degree of independence of a job and the most suitable individuals to fill such a job. Another example of data already available relates years of practical experience, which can be deduced from age and years of schooling of the same person. Finally, it is probable that I did not use the material available on the factors included in the correct way.

(*b*) A second, more elaborate, follow-up can be obtained by the use of data which have not yet been put into an appropriate statistical form, but *can be collected* from such sources as job evaluation and career planning. Job evaluation may supply us with data on job requirements, whereas data on career planning, available in large enterprises and government services, may supply the corresponding figures on capabilities offered. Furthermore, studies examining which components of capability are innate, and which can be acquired by learning, will be helpful in refining the choice of parameters and variables in the utility function. Finally, studies on the correspondence between capabilities required, as described in job evaluation, and capabilities offered by the population, as available in career planning, will also be of great importance to our subject.

(*c*) Apart from the data which can be made available from existing sources, one can imagine the development of *future sources* as a consequence of scientific development in general. Not only can job evaluation be improved, but also the correspondence between job evaluation aspects and aspects of human capabilities. Moreover, new and better measures may be found to ascertain differences in needs or tastes. More direct and more objective ways of measurement may be developed, thus helping to improve the material described under (*a*) and (*b*).

6. *First Results of Testing*

(*a*) In the models used for the initial series of tests mentioned in Sections 3 and 4, the supply of labour of different quality was derived from utility functions as described in Section 4. For each individual the possible incomes to be obtained from alternative choices of jobs were considered to be given, i.e. free competition in the labour market was assumed. These (spendable) incomes are the difference between gross income and direct taxes, with the latter assumed to be given. Gross incomes from alternative jobs were conceived as marginal productivities derived from a *unique production function* for the economy as a whole, of a *generalised Cobb-Douglas character*. Information about the demand side of the labour market appears to be irrelevant, in this simplified model, for the estimation of the utility or welfare function. It becomes important, however, when the gross income scale, reflecting the relative scarcity of each type of labour considered, has to be explained or when the possibilities of changing it, in order to change income distribution, have to be considered. For the testing of our assumptions on the utility function the demand side is irrelevant, as stated; the essential characteristic of the model is its subdivision of the labour market into 21 compartments, later reduced to 19, each compartment being characterised by its s and ν as discussed in Section 4. The statistical material used is shown in Table 1 and the way it has been estimated described in the Appendix. Since ν is supposed to reflect the (only) parameter in the utility function and free competition is assumed to exist, a free choice of all individuals with a given ν_i between those compartments with differing values of s where that value of ν is admissible enables us to test the relation, derived from (4.1):

$$x_i - c_0 s_i - c_1 v_i - \tfrac{1}{2} c_2 (s_i - v_i)^2 = x_j - c_0 s_j - c_1 v_i - \tfrac{1}{2} c_2 (s_j - v_i)^2 \quad (6.1)$$

where some terms in v_i cancel. Because of our assumption of the fundamental equality of man, we can use as observation pairs any two pairs with equal v_i for all values of i.

The correlations found are unsatisfactory, however: cf. Table 2, Cases A–E. As a consequence, the theory in its initial form *had to be rejected.* The correlation coefficient between $x_i - x_j$ and $z_y = (s_i - v_i)^2 - (s_j - v_i)^2$ indicated by x and z in Table 2, is low (0·14) and the regression coefficients, even after the introduction of other explanatory variables, are insignificant. A new variable which seemed to have high explanatory power will now be discussed.

(*b*) Consideration of the differences in social groups suggested the relevance of an additional capability, namely the capacity to take independent decisions. The groups considered were divided into three groups, ranked according to the increasing importance of such decisions, represented by a parameter W. Persons having to take decisions of minor importance only and usually described as socially dependent were given the value $W = 0$; for persons heading small and medium-sized enterprises $W = 1$; and for heads of limited companies or persons in liberal professions $W = 2$. As can be seen from Cases A–E in Table 3, satisfactory correlations could only be obtained if W were included. This state of affairs constitutes a textbook example of the necessity of introducing an additional element into a theory in order to fit the facts in a more satisfactory way. The basic idea may also be expressed as the capacity to give leadership to other people, which is given a central place in the analysis of income distribution by Tuck [7].

(*c*) The *interpretation* of the role of W, however, poses some questions. Must we consider W as a parameter, as suggested, which implies that it indicates an innate personal capability, or must we consider it as a job variable? I opted for the first interpretation, which gives to W a role comparable to the one of v in the simplest model. This implies that we have to consider as separate groups not

TABLE 1: *Estimated Income Distribution of the Netherlands, ca 1965, According to Schooling (v), Required Schooling (s) and Degree of Independence (W) or (W')*

s ↓	v=	2	3	4	5	6	W W'	Description of Main Groups Included
6	f			0·1	0·3	0·9		Professions
	l			42·3	42·3	42·3	2	Directors of limited companies
	l'			19·0	19·0	19·0	6	
	x			14·0	14·0	14·0		
5	f		1·5	1·1	0·5	0·7		Independent industrialists, teaching staff of secondary and third level
	l		19·5	19·5	19·5	19·5	1	
	l'		14·2	14·2	14·2	14·2	5	
	x		11·3	11·3	11·3	11·3		
4	f	0·7	5·3	4·4	1·3	0·1		Civil Servants in general service; private employees
	l	10·2	10·2	10·2	10·2	10·2	0	
	l'	9·7	9·7	9·7	9·7	9·7	2	
	x	8·3	8·3	8·3	8·3	8·3		
3	f	0·1	4·0	1·0	0·1			Independents in trade and services
	l	15·0	16·2	17·4	18·6		1	
	l'	12·0	12·5	13·0	13·4		3	
	x	9·9	10·2	10·5	10·8			
3	f	10·7	6·6	0·1				Primary school teachers; Part of administrative personnel; Police; Mining workers; Farmers
	l	8·9	10·8	12·7			0	
	l'	8·5	10·2	11·3			2	
	x	7·4	9·0	9·4				
2	f	46·6	13·8					Workers, including retail trade employees, part of administrative personnel
	l	5·3	8·4				0	
	l'	4·9	8·1				2	
	x	4·4	7·1					

f: frequency in per cent of number of tax payers
l: income in thousands of guilders
l': labour income contained in *l*
x: labour income after tax

TABLE 2: *Results of Testing the Two Theories by Equating Utility between Cells with Same v (Theory I, Cases A through E) or between Cells with Same v and W (Cases F, G, H)*

Case No.	N	Regression Coefficients for x on:							Simple Corr. between x and				Const. Term e	Mult. Corr. Coeff.
		ζ	w	q	q'	q''	m	ν	ζ	w	q	ν		
A	16	0·09 (0·15)	1·64 (0·66)	−0·18 (0·22)	.	.	.	.	0·14	0·89	−0·83	.	+0·55 (0·29)	0·90
B	16	0·10 (0·16)	.	−0·05 (0·39)	.	.	0·47 (0·28)	.	0·14	0·89	−0·83	.	+0·46 (0·33)	0·87
C	16	0·09 (0·15)	1·60 (1·01)	−0·17 (0·37)	.	.	0·02 (0·39)	.	0·14	0·89	−0·83	.	+0·55 (0·33)	0·90
D	12	0·12 (0·17)	1·73 (0·56)	.	−0·06 (0·16)	0·07 (0·07)	.	.	0·14	0·89	−0·83	.	+0·42 (0·35)	0·90
E	16	0·08 (0·14)	2·11 (0·30)	.	.	.	.	.	0·14	0·89	−0·83	.	+0·54 (0·28)	0·89
F	6	0·44 (0·12)	.	.	.	.	.	1·10 (0·40)	0·63	.	.	−0·20	−3·6 (1·42)	0·91
G	6	0·28 (0·28)	.	.	.	.	.	−0·17 (0·44)	0·79	.	.	−0·70	+1·47 (3·34)	0·79
H	6	0·32 (0·06)	.	.	.	.	.	.	0·79	.	.	.	+0·9 (0·5)	0·79

N: number of observations (differences between two cells with same utility).
x: differences in labour income after tax between two cells or $x_1 - x_2$.
ζ: $s_1^2 - s_2^2 - 2\nu(s_1 - s_2)$; s schooling required, ν schooling available.
q: $W_1^2 - W_2^2 - \nu(W_1 - W_2)$; W degree of independence (as in Table 1).
q': $W'^2_1 - W'^2_2 - \nu(W'_1 - W'_2)$; W' degree of independence (same, but 3 instead of 2).
q'': $W'^2_1 - W'^2_2$; $w = W_1 - W_2$; $m = w\nu$.
In Case F lowest row of observations in Table 1 has been excluded.
In Cases G and H lowest row of observations in Table 1 have been combined.

only those having a different s and ν, but those having a different set of ν and W; Table 3 indicates the arrangement of our statistical material corresponding to that interpretation. This arrangement reduces the number of observations to six pairs (each with another set ν, W).

It appears that significant results have now been obtained (Table 2, Cases F, G and H). They differ because in Case F the lowest line of Table 2 has not been used, since the category for $\nu = 2$ contains a group of holiday workers which somewhat depress the income figure. Since, however, the figure covers also a large portion of ordinary workers, it was decided to include them, by combining them with the second group with $s = 3$. This yielded Cases G and H. Further, since in Case G the influence of ν became insignificant, Case H was added, using the original theory with the pure tension $s - \nu$ instead of the generalised tension $s - 0{\cdot}45\ \nu$ resulting from Case F. This provided us with some further insight about the sensitivity of e, representing the influence of s on x and hence on ω. While in Case F (valid for the half of the population with high incomes) the influence of s was found to be negative, in Case H (valid for the population as a whole) this influence was found to be positive. These findings are interesting since their meaning is that for higher incomes the attractiveness of a higher s job surpasses its disutility, while for lower incomes the opposite applies. Notwithstanding its lower multiple correlation coefficient we take Case H as the most satisfactory result, because (i) its coefficients take an intermediate place between those of Cases F and G, and (ii) are theoretically more attractive, while (iii) it will keep us, because of the sign of the constant term, on the safe side in our estimation of the optimal and the just income distribution. Written in the form of (6.1) the relationship runs:

$$x_1 - 0{\cdot}45\ s_1 - \tfrac{1}{2}.\,0{\cdot}64\,(s_1 - \nu_j)^2 = x_2 - 0{\cdot}45\ s_2 - \tfrac{1}{2}.\,0{\cdot}64\,(s_2 - \nu_j)^2 \quad (6.1')$$

where $s_1 = 2, 2, 3, 2, 3, 3$; $s_2 = s_1 + 2$; and $\nu_j = 2, 3, 3, 4, 4, 5$. This corresponds with

$$\omega = ln\{x - 0{\cdot}45\ s - 0{\cdot}32\ (s - \nu)^2\} \quad (6.1'')$$

as the resulting utility or welfare function.

TABLE 3: *Distribution of Labour Income After Tax Over Values of s, v and W (degree of independence)*

s ↓	v =	2		3		4			5			6		
	W =	0	1	0	1	0	1	2	0	1	2	0	1	2
6								14·0			14·0			14·0
5					11·3		11·3			11·3			11·3	
4		8·3		8·3		8·3			8·3			8·3		
3			9·9		10·2		10·5			10·8				
3		7·4		9·0		9·4								
2		4·4		7·1										

(*d*) Whether W is a parameter or a variable, in both cases we must face the problem of what could be the corresponding variable or parameter in order to define the "tension" concept introduced in Section 4 (*b*). For lack of observations the only alternative I investigated assumes that in the case where W is considered a job variable rather than a personal parameter, the corresponding parameter is again ν, requiring the introduction of an expression $(W-\beta\nu)^2$ instead of W as an explanatory variable. The result of this attempt will be found in Table 2, Cases A through E, from which it becomes clear that:

(i) correlations are substantially improved ($R \gg r_{xz}$) by the introduction of $W_1 - W_2$ or w, showing itself already an $r_{xw} = 0{\cdot}89$, and

(ii) attempts to introduce $(W-\beta\nu)^2$ were unsuccessful: they did not improve the multiple correlation and the expression $(W-\beta\nu)^2$ obtained insignificant regression coefficients. From this I conclude that W is a personal parameter rather than a job variable to be acquired by schooling.

(*e*) As long as, in fact, parameters ν or W are innate, income will depend on them as a consequence of a relative scarcity of these human properties. Comparisons of the type of (6.1) using different values of ν (or W) instead of s and the same values of s on both sides will not reflect the psychological impact on ω of ν (or W), but rather their relative scarcity, that is, the impact of the difference between demand for, and supply of, the capability considered. Since the frontier between scarcity and power is difficult to draw, one may even interpret the additional scarcity income received by those endowed with a high value of ν or W as an income derived from "power" and view it as an element of "exploitation".*

In order to estimate the scarcity element in the income scale, the demand side of the labour market, discussed in Section 6 (*a*), has to be used. Sticking to the generalised Cobb-Douglas function, introduced in that sub-section, we may write this part of the model in the following way, where for simplicity sake we assume the existence of three levels only of ν' and of s' respectively (where

*Interestingly enough, a case of "exploitation" by *human* capital.

$\nu' = \frac{1}{2}\nu$ and $s' = \frac{1}{2}s$), and where we indicate by $\phi_{s\nu}$ the number of individuals in the active population. In a situation of scarcity of qualified labour the $\phi_{s\nu}$ for $\nu' > s'$ will be zero and the active population F will be distributed over the various combinations of s' and ν' as follows:

TABLE 4: *Numbers of Active Population in each Cell* (s'. ν')

s' ↓ \ $\nu' =$	1	2	3
1	ϕ_{11}	0	0
2	ϕ_{21}	ϕ_{22}	0
3	0	ϕ_{32}	ϕ_{33}
TOTAL	F_1	F_2	F_3

Here F_i equals the total number of people with endowment $\nu' = i$, whereas we have assumed $\phi_{31} = 0$ in accordance with Table 1, where no people appear with education $\nu = 2$ and jobs $s = 5$ or 6.

The production function is taken to be:

$$y = p\phi_{11}{}^{\rho_1}(\phi_{22} + \pi_{21}\phi_{21})^{\rho_2}(\phi_{33} + \pi_{32}\phi_{32})^{\rho_3} \qquad (6.2)$$

where y stands for total product, p for a constant depending on units used multiplied by the factor representing the share of production to be attributed to capital, π_{21} for the productivity of people ($s' = 2$, $\nu' = 1$) relative to people ($s' = \nu' = 2$) and π_{32} for the relative productivity of (3, 2) to (3, 3). It is plausible to assume that

$$\rho_1 + \rho_2 + \rho_3 < 1$$

Gross incomes of the various categories (s', ν') will be written as $l_{s\nu}$ and under the assumption of free competition between entrepreneurs they will be equal to their marginal product. In the well-known way we will find:

$$l_{11}=\frac{\rho_1 y}{\phi_{11}}$$

$$l_{21}=\frac{\pi_{21}\rho_2 y}{\phi_{22}+\pi_{21}\phi_{21}}$$

$$l_{22}=\frac{\rho_2 y}{\phi_{22}+\pi_{21}\phi_{21}} \qquad (6.3)$$

$$l_{32}=\frac{\pi_{32}\rho_3 y}{\phi_{33}+\pi_{32}\phi_{32}}$$

$$l_{33}=\frac{\rho_3 y}{\phi_{33}+\pi_{33}\phi_{32}}$$

Since the numerators of these expressions show ratios which are constant and given, and the numbers of people appear in the denominators only, the influence of scarcity on gross income in this model is clear and simple. Under practically all existing tax systems, income after tax, x, is a monotonic function of l and the scale of x will reflect, therefore, the relative scarcity of the various endowment combinations*.

*As an illustration the figures of Table 1 have been used to test the figures for the ρ_i ($i=1, 2, 3$) and ϕ_{ij} in formulae (6.3). From the relative labour income shares of people with $v=2$, 4, 6 (corresponding roughly with primary, secondary and tertiary level schooling) we find:

$$\rho_1 : \rho_2 : \rho_3 = 795 : 115 : 90 \qquad (6.31)$$

The relative frequencies of the ϕ_{ij} are:

$$\phi_{11} : \phi_{21} : \phi_{22} : \phi_{32} : \phi_{33} = 775 : 111 : 31 : 25 : 29 \qquad (6.32)$$

Adding ϕ's with identical level of schooling v' we find:

$$\phi_{\cdot 1} : \phi_{\cdot 2} : \phi_{\cdot 3} = 866 : 56 : 29 \qquad (6.33)$$

The proportions of the corresponding l follow:

$$l_1 : l_2 : l_3 = 92 : 205 : 310 \qquad (6.34)$$

For labour incomes after tax we find from Table 1, taking income modes:

$$x_{\cdot 1} : x_{\cdot 2} : x_{\cdot 3} = 4{\cdot}4 : 8{\cdot}3 : 14 \qquad (6.35)$$

Putting the middle figure at 205, these proportions become:

$$x_{\cdot 1} : x_{\cdot 2} : x_{\cdot 3} = 108 : 205 : 345 \qquad (6.36)$$

The similarity is clear. In a simple formula the scarcity relationship (6.34) is:

$$l = 110\, v' - 20 = 55\, v - 20 \qquad (6.37)$$

if we take l to stand for the (arbitrary) absolute figures in (6.34).

From (6.3) we can even deduce the conditions under which complete equalisation of income can be attained. This situation can be attained if:

(1) the Cobb-Douglas function is valid;

(2) free competition among entrepreneurs exists;

(3) the supply of individuals in each compartment is equal to its demand for them, interpreted by the following formulae:

$$\phi_{21}=\phi_{32}=0 \qquad (6.4)$$

(4) there is proportionality between the numbers needed and the elasticities of production *vis-à-vis* the number of each category:

$$\frac{\rho_1}{\phi_{11}}=\frac{\rho_2}{\phi_{22}}=\frac{\rho_3}{\phi_{33}} \qquad (6.5)$$

Using thousands of guilders as the units for l, we have to replace the figure 205 in (6.34) by 10·2 and we find instead of (6.37):

$$l = 2{\cdot}75\ \nu - 1 \qquad (6.38)$$

In our new approach, where both ν and W are parameters, we must express the scarcity income scale in terms of ν and W, or s and W. For the latter we find, with the aid of our original 21 observations of Table 1,

$$x = \underset{(0{\cdot}22)}{0{\cdot}43}\ s + \underset{(0{\cdot}42)}{2{\cdot}44}\ W + \underset{(0{\cdot}7)}{6{\cdot}6} \qquad (R = 0{\cdot}93) \qquad (6.39)$$

Written in terms of s it is the (labour) income (after tax) with which the individuals are confronted when choosing their job. This choice consists of maximising ω subject to condition (6.39). With our formula for ω (6.1'') we find as the maximum condition:

$$s = \nu - 0{\cdot}03$$

which fits the facts of Table 1 for two-thirds of the active population (those on the main diagonal).

If, therefore, we want to know the relation between x and the parameters ν and W we can simply replace s in formula (6.39) by ν; if moreover, we want to disregard W to make (6.39) comparable to (6.38), we may, as a crude approximation, replace W by $\frac{1}{2}(\nu - 2)$ and we find:

$$x = 1{\cdot}65\ \nu + 4{\cdot}2.$$

From Table 1 we may also derive a crude approximation of the relation between l and x: $l = 1{\cdot}5\ x - 2$ which yields

$$l = 2{\cdot}5\ \nu + 4{\cdot}3$$

meaning that the simpler model used to analyse the demand side is not too far from the more accurate model used for the supply side.

The obstacles that may prevent this equality from occurring relate to the innate qualities. The quantity of some kind of labour needed simply may not be present.

In addition, equality of x may not necessarily require equality of l: some tax systems may exist which attain equality of all x, without equality of all l.

(f) Parameters v and W are not strictly innate. Through a learning process they may be subject to change. If so, such a process should be translated into equations and the individual's choice may determine to what extent the process will be used. Innate parameters of a partial character will still remain, however, to which our preceding analyses apply. Thus, instead of the number of units (three-year periods) of schooling, the innate component of the individual's IQ may be the true parameter.

(g) For the innate parameters the coefficients such as c_1 in (6.1) cannot be measured by the procedures discussed so far. This poses two questions: (1) do we have *other procedures to measure* them? and (2) is it necessary to assume the existence of such coefficients? I see two answers to question (1): (i) by some accident, or as a result of surgery, the same individual may live in two situations in succession and thus be able to compare the welfare experienced. Even without accident or surgery, some individuals have in the past deliberately undertaken to diminish their physical condition, if by so doing they could improve their income from social benefits. The other answer to (1) is (ii) that if v is not strictly a parameter, the sacrifices needed to attain one unit more of v by a learning process can be interpreted to constitute c_1. For the Netherlands such sacrifices have been calculated. In a country where education is free and the income foregone paid to the student $c_1=0$. For the "marginal" student this is true also in Holland.

Question (2) raises a *fundamental* philosophical or ethical question. If for the overwhelming majority of people the transition from their value of v (or, more generally, any parameter) to another value is impossible, does it make sense to introduce a coefficient such as c_1? My answer is that at least it does not make sense to give a positive value to such a coefficient, meaning that a higher value of v constitutes a burden or a sacrifice. In the next section, dealing

with the application of our findings to the concepts of an optimal and a just income distribution, this consideration underlies my propositions, which are based on $c_1=0$.

7. *Consequences for the Numerical Aspects of Optimality and Justice of Income Distribution*

(*a*) In the preceding sections I proposed a multidisciplinary approach to the measurement of utility or welfare and elaborated an initial numerical test of the method proposed. During the process I indicated ways and means for successive improvements of both statistics and theories (or models) and showed one step towards improvement of the theory. In this section the results obtained will be used in order to give concrete content to the concepts of an optimal income distribution and a just income distribution. For the former concept we need one more assumption, namely the specification of the social welfare function Ω of a group of individuals, such as the national or the world economy. Applying again the methodological principle of the simplest theory I opt for the definition of the sum of individual utilities or

$$\Omega=\sum_i \omega_i \tag{7.1}$$

This implies that I do not enter into the discussion of the dynamic aspects; otherwise some weighted summation over time would have to be superimposed. The time aspect of the social welfare function and some of its implications have been dealt with by others. The static aspects implied in (7.1) are themselves interesting enough to give them some special attention. No doubt, however, a combination of the two aspects should be on our agenda. The main defence of (7.1) is the negative one that it does not discriminate in favour of some or against other individuals.

The optimal socio-economic situation will then be defined as the one maximising (7.1) under some restrictions such as the availability of given quantities of resources and given production functions. In our present exercise I consider as given the quantities of people with their parameters ν and W, the quantity of capital and the production function. In the simple model sketched out so far, and for the reasons given in Section 6 (*a*), the question of income

distribution can be singled out from the total problem of determining the optimum position of the economy. In more complicated models this would not always be so. It appears that the optimum is characterised by the equality of all individual utilities, or

$$x_{ij} - c_0 s_i - \tfrac{1}{2} c_2 (s_i - \nu_j)^2 = k, \text{ for all } i, j \qquad (7.2)$$

where k is a constant depending on the data of the problem.

This also means that in our case the same formula applies to the definition of a just income distribution, although this is not generally true.

Our omission of any terms containing ν or W is based on what was said in Section 6 (g). With values of c_0, and c_2 given, we can calculate an optimal or a just income distribution in the restricted sense of an income scale deserving the title of optimal or just; the complete picture of the income distribution also requires the calculation of the numbers ϕ_{ij}, possible in principle, but not to be undertaken in this essay. The result is given in Table 5, where (7.2) has been given the numerical specification

$$x = +0{\cdot}45\, s + 0{\cdot}32\, (s-\nu)^2 + 5{\cdot}7 \qquad (7.3)$$

the constant having been chosen so as to obtain the average labour income after tax for the two corresponding combinations ($s=2$, $\nu=3$) and ($s=\nu=2$).

The reader should be reminded that our assumptions are that ν and W constitute parameters in the strict sense, and hence cannot be changed by some learning process. To the extent that ν or W, or both, can be obtained by efforts constituting a sacrifice, our formula (7.3) should contain terms in ν or W, or both, expressing the corresponding sacrifices. These terms cannot outweigh the additional scarcity incomes at present enjoyed by those endowed with higher values of ν or W and reflected in formula (6.39). If ν and W are parameters in the strict sense, then the optimal income distribution can be reached by measures counteracting the scarcity incomes, without killing the stimuli for the better endowed individuals. It is not certain that such measures are yet feasible; but they may become feasible through the improvement of the tax raising machinery. If the so-called lump-sum taxes on personal endowment were to become feasible, through for instance the use

TABLE 5: *Actual and Optimal Income Distribution under the Assumption Set Out in Text (Optimal in brackets) (Thousands of Guilders, ca 1965)*

s ↓ / ν =	2	3	4	5	6
6	.	.	14·0	14.0	14·0
			(9·7)	(8·7)	(8·4)
5	.	11·3	11·3	11·3	11·3
		(9·2)	(8·3)	(8·0)	(8·3)
4	8·3	8·3	8·3	8·3	8·3
	(8·8)	(7·8)	(7·5)	(7·8)	(8·8)
3	9·9	10·2	10·5	10·8	.
	(7·4)	(7·1)	(7·4)	(8·3)	
2*	4·9	7·7	.	.	.
	(6·6)	(6·9)			

*Weighted average of two groups for $\nu = 2$ mentioned in Table 1.

of better psychotechnical tests, then the optimal distribution could be attained without the application of forms of compulsion. As long as the lump-sum tax on personal endowment is not practicable, we may have to use less attractive measures such as minimum wage legislation, restrictions on income from dividends and royalties, as well as on professional earnings, and stricter rules on the use of business expense accounts for personal use. I regard these measures as less attractive because of their strongly bureaucratic character. Even so, some of them, especially the imposition of stricter rules on the abuse of expense accounts and the like, are valuable *per se*. Alongside such measures, better access to the appropriate types of education will also help to further reduce income inequality. Elsewhere [6] I have concluded from econometric analysis that a realisable increase in access to education could reduce inequality in the distribution of primary income by one-third. The reduction required in order to attain the optimal income distribution estimated in Table 5 would involve a lowering of the range of labour income after tax to about one-third of its present level, namely, from a range of 9:1 between top and bottom to a range of 3:1. These figures suggest that a substantial contribution to reduced inequality in welfare can be made through better access to appropriate education.

8. *Summary and Conclusions*

In this essay an attempt was made to give objective content to the concept of utility or welfare by the introduction of a number of assumptions believed to be acceptable to a considerable majority of scientists (as far as methodology is concerned) and citizens (as far as ethics are involved). If this belief is indeed shared by such majorities, the way is clear for a series of refinements to the start made in the numerical example presented here.

The assumptions made are repeated briefly here; for their precise motivation the reader is referred to the complete text.

1. In order to define and measure utility or welfare we use the simplest theory that is not rejected by numerical observation.

2. The ethical principle needed as a basis for the concept of measurable welfare is that which asserts the fundamental equality of all human beings—meaning by this that human beings are equal in all relevant characteristics other than those where inequality has been established through measurement.

3. In our introductory example, the main characteristic of a human being for socio-economic problems is his level of schooling while the main characteristic of a job is the required level of schooling.

4. Utility is defined as the logarithm of consumable income, corrected for inconveniences due to (i) job chosen and (ii) tension between schooling required for that job and actual schooling.

5. Welfare depends positively on higher qualification or, at least, is not dependent on innate characteristics.

6. A relevant innate characteristic is the capacity for independent decision-making.

7. Total social welfare of a group is equal to the unweighted sum of welfare functions for the individuals.

8. As a start, meant as a challenge for more refined work on the subject, Table 5 gives figures on the optimal distribution of consumable income for 18 groups, into which the active population

of the Netherlands was divided, compared with the actual distribution around 1965.

9. A number of refinements which can be undertaken at short notice is mentioned in Section 5.

Appendix on Estimation of Figures of Table 1

These figures have been estimated mainly with the aid of (i) the "Netherlands Census of Population" for 1960 which gives information on the school achievements of a number of two-digit social groups; (ii) tax statistics for 1964 giving incomes before and after tax for a smaller number (21) of socio-economic groups and (iii) income distribution statistics for 1966 where labour income and income from other sources are shown. The exact names of the three sources are: (i) 13*e Algemene Volkstelling, Deel 8* (13th General Census, Part 8, p. 62, 72ff); (ii) *Nota over de inkomensverdeling, bijlage* 15, Miljoenennota 1970 (Memorandum on Income Distribution, Appendix 15, Budget Proposals 1970), p. 15; (iii) *Inkomensverdeling 1966 en vermogensverdeling 1967* (Income Distribution 1966 and Distribution of Wealth 1967), pp. 80-83.

Sources (i) and (iii) are publications of the Netherlands Central Bureau of Statistics; source (ii) is a parliamentary document. All can be obtained from the "Staatsuitgeverij" (Government Stationery Office), The Hague.

Since Source (ii) refers to a smaller number of social groups than Source (i), the groups in the latter have been combined so as to cover the larger groups of (ii); even these have not been kept separate, however, but have been combined if they showed almost equal incomes and almost equal social status (especially comparable degree of independence). As already observed in the main text, v has been taken to be the number of schooling periods of 3 years completed and s the corresponding upper quartile. For about ten one-digit social groups s has been compared with the median of school years (in units of 1 year) completed by the male American active population between 18 and 64 years (US Department of Commerce, Current Population Reports, Series P 20, No. 207, p. 27, and US Census of Population 1960, Educational Attainment, Final Report PC (2)—5B, p. 136). There was no

systematic deviation, apart from a few years of average difference. The comparison is crude, anyway, because the Dutch figures are expressed in multiples of three years only; the number of years of schooling is not given in the publications, but only the frequency distribution over five main school types. The reader should again be reminded of the illustrative character of the entire econometric material offered in this essay. The figures for the large social groups are given below.

TABLE A: *Comparison of USA median and Dutch upper quartile schooling*

US Groups		*NL Groups*		*US median (18–64 years old)*	*NL upper quartile*
No.	*Brief Description*	*No.*	*Brief Description*		
A	Professional, technical, etc.	0	Professional, specialists	16·3	15
B/I	Farmers and farm workers	4	Agrarian occupations, fishing	8·3	9
C	Managerial, official, proprietors	1	Managers, high officials	12·5	12
D	Clerical and kindred workers	2	Administrative occupations	12·4	9
E	Sales workers	3	Commercial occupations	12·5	9
F/G	Craftsmen, foremen, operators	7/8	Handicraft and industrial	10·2	9
H	Service workers	6/9	Services	10·0	9
J	Labourers excluding farmers, miners	5	Mining, peat digging	8·7	6

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